April 10, 2020. Quiz 1, due 5pm, Saturday 4/11. :

Quiz problem 1: Given the equation of state below, what is the amount of work done when a system isothermally expands from initial volume V_0 to final volume $2V_0$?

$$p = \frac{Nk_BT}{V - Nb}$$

Quiz problem 2: An ideal gas is contained in a cylinder with a tightly fitting piston. Several small masses are on the piston. (Neglect friction between the piston and cylinder walls.) The cylinder is placed in an insulating jacket, and a large number of masses are then added to the piston.

You may use the following properties of an ideal gas:

• The internal energy is a function of N and T only.

• $pV = Nk_BT$

Tell whether the pressure, temperature, and volume of the gas will increase, decrease, or remain the same. Explain.

additional masses

insulating jacket
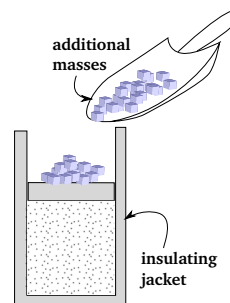
Quiz problem 3: Given the definitions below, evaluate the requested partial derivative.

$$U = x^2 + y^2 + z^2$$
$$z = \ln(y - x) + x^2$$
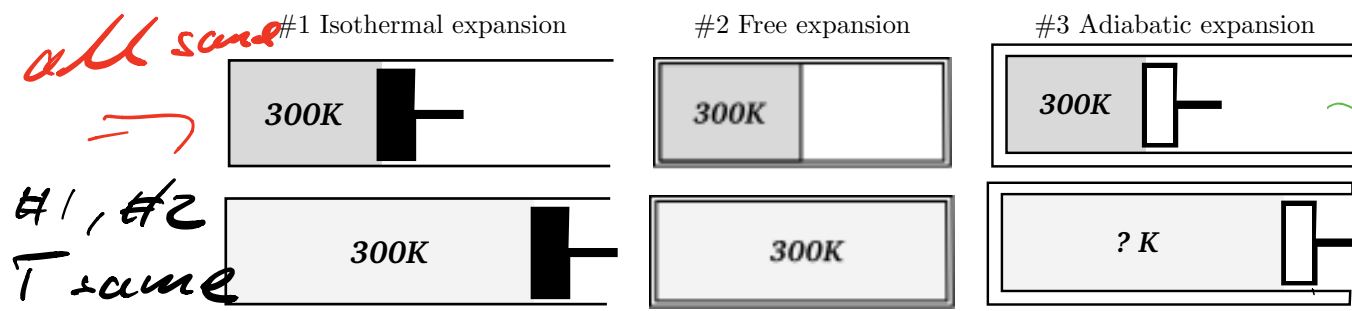
Find $\left(\dfrac{\partial U}{\partial z}\right)_y$

Activity: How many ways can you expand a gas in a piston?

Consider the three processes described below.

Process #1 Five moles of an ideal gas are initially confined in a one-liter cylinder with a movable piston, at a temperature of 300 K. Slowly the gas expands against the movable piston, while the cylinder is in contact with a thermal reservoir at 300 K. The temperature of the gas remains constant at 300 K while the volume increases to two liters.

Process #2 A thin plastic sheet divides an insulated two-liter container in half. Five moles of the same ideal gas are confined to one half of the container, at a temperature of 300 K. The other half of the container is a vacuum. The plastic divider is suddenly removed and the gas expands to fill the container. Because it is a free expansion of an ideal gas (no work is done on or by the gas), the final temperature of the gas is also 300 K.

Process #3 The same cylinder as in process #1 is thermally insulated and then allowed to slowly expand, starting at 300 K, to twice its original size (two liters).

#1 Isothermal expansion   #2 Free expansion   #3 Adiabatic expansion

(handwritten annotations:)
all same

⟹

#1, #2
T same
V same

⟶ use EOS: $pV = NkT$ ⟹ p same

(right margin handwritten:)
$Q = 0$
?
$\Delta U = Q + W$
$W < 0$
$\Delta U < 0$
$\Delta U \propto T$
$T_f < T_i$

1. Are $\Delta S_{\text{isothermal}}$, ΔS_{free} and $\Delta S_{\text{adiabatic}}$, the change in entropy of the gas for each process, positive, negative, or zero? Please explain your reasoning.

2. Is $\Delta S_{\text{isothermal}}$ greater than, less than, or equal to ΔS_{free}? How do each of these compare with $\Delta S_{\text{adiabatic}}$? Please explain.

3. Are $\Delta S_{\text{surr-isothermal}}$, $\Delta S_{\text{surr-free}}$ and $\Delta S_{\text{surr-adiab}}$, the change in entropy of the surroundings for each process, positive, negative, or zero? Please explain.

- isothermal: quasistatic.
? free : not q.s.
- adiabatic : q.s.

? Find q.s process with same
initial and final states.

→ isothermal expansion
has identical initial and
final states.

Look at isothermal exp.:
p-V diagram.

EOS: $pV = NkT$

$p = NkT \frac{1}{V}$

$\propto \frac{1}{V}$

free expansion
(not in $p-V$)

$T_i > T_f$

isotherm

T_f

adiabat

• adiabatic
final state
V same
p? V?
think about this
$pV = NkT$
$\frac{?}{?} = \quad ?$

$\propto \frac{1}{V^Q}$

V_1 $V_2 = 2V_1$ V

Entropy changes in gas
(system)

#1) T=const

Does heating take place?

$\rightarrow \Delta U = Q + W$ $\leftarrow$ useful if 1 var. is $= 0$

$T = const$ $\quad U \propto T$

$\rightarrow \Delta U = 0 \implies Q = -W$

$W < 0 \qquad Q > 0 \rightarrow \Delta S_{isoT}^{sys} = \frac{Q}{T}$

not the case $\qquad\qquad$ exact because $T = const$

$dS = \frac{dQ}{T} = -\frac{dW}{T} \rightarrow$ integrate

$\Delta S_{isoT}^{sys} = \frac{Q}{T} > 0$
$\qquad > 0$
$\qquad\qquad - > 0$

#2: same as #1 $\quad \Delta S_{free} = \Delta S_{isoT}$
$\qquad\qquad\qquad\qquad\qquad > 0$

#3: $\quad \Delta U = \overset{=0}{\cancel{Q}} + W$

$\Delta S_{ad}^{sys} = \int_i^f dS = \int_i^f \frac{dQ}{T}^{=0} = 0$

2) $\Delta S_{isoT} = \Delta S_{free} > \Delta S_{ad}$

3) ΔS in environment for the 3 processes

free, ad. are not connected

to environment $\Delta S_{ad}^{env} = \Delta S_{free}^{env}$

$\Delta S_{isoT}^{env} = \dfrac{?}{?} > 0 \qquad \dfrac{Q^{env}}{T^{env}} = -\dfrac{Q^{sys}}{T^{sys}} \qquad T^{env} = T^{sys} = T$

$= -\dfrac{Q}{T} > 0 \quad < 0$

$\Delta S_{isoT}^{env} = -\Delta S_{isoT}^{sys}$

$isoT: \quad \Delta S_{isoT} = \Delta S_{isoT}^{env} + \Delta S_{isoT}^{sys} = 0$

$free: \quad \Delta S_{free} = 0 \quad + \Delta S_{isoT}^{sys} > 0$

$adi: \quad \Delta S_{adi} = 0 \quad + 0 \quad = 0$

Reversible: 2 meanings

1) system is reversible

2) system + environment is reversible

reversible requires quasistatic

$\Delta S^{adi} = \Delta S^{sys} + \Delta S^{env} = 0 + 0 = 0$

$\Delta S^{isoT} = \Delta S^{sys} + \Delta S^{env} = +\dfrac{Q}{T} + \dfrac{-Q}{T} = 0$

$\delta U = 0 \Leftarrow T = const \quad U \not\propto T$

#1 $\Delta S_{sys}^{isoT} = \dfrac{Q}{T} = \dfrac{-W}{T} = -\dfrac{1}{T} \displaystyle\int_{U_i}^{U_f} -p \, dV$

$= \dfrac{1}{T} \displaystyle\int_{V_i}^{V_f} N k T \dfrac{1}{V} \, dV = N k \ln \dfrac{V_f}{V_i}$

$$\#3 \qquad \Delta S_{sys}^{adi} = \int dS = \int \frac{dQ}{T} = \int \frac{dU - dW}{T}$$

$$dU = ? \qquad U = \frac{3}{2} NkT \rightarrow dU = \frac{3}{2} Nk \, dT$$

$$\underbrace{}_{EOS} \qquad \underset{\text{linearize, total diff.}}{\nearrow}$$

$$dW = -p\, dV$$

$$\Delta S_{sys}^{adi} = \int_{T_i}^{T_A} \frac{3}{2} NkT \, dT - \int_{U_i}^{U_A} -p\, dV$$

$$dT = dT(d_p, dU)$$

Hint: $\qquad EOS : \qquad pV = NkT$

$\longrightarrow$ linearize

$$V\, dp + p\, dV = Nk\, dT$$

Note, we already know that the entropy change for the adiabatic process is 0, what we calculate here is
the functional form of adiabates in a p-V diagram.

$$U(\lambda S) = \lambda\, U(S)$$

$$\boxed{T_H} \xrightarrow{\quad Q \quad} \boxed{T_c}$$