Ph641 - Statistical Thermophysics - Spring 2020

Homework 1 - due Wednesday, Apr 15

1. Internal energy: I.

The function U is given by $U(S, V, N) = (S^2 + S - V^2)/N$. Give three reasons why this is not a correct function describing the internal energy of a material.

2. Internal energy: II.

The internal energy of system is given by $U(S, V, N) = S^\alpha V^\beta N^\gamma$.

a) Using the fact that U is extensive, find a relation between α, β, and γ.
b) Calculate the pressure p, temperature T, and chemical potential μ as a function of S, V, and N.
c) Calculate the heat capacity at constant volume V (and constant N).
d) Calculate the adiabatic compressibility (at constant N).
e) Based on the sign of these response functions (assume $C_V \geq 0$, $\kappa_S \geq 0$, we'll prove this later), find inequalities for α and β.

3. Total differentials and partial derivatives

Given the equations below (N, k_B, a, and b are constants):

$$p = \frac{Nk_BT}{V - bN} - \frac{aN^2}{V^2} \qquad U = \frac{3}{2}Nk_BT - \frac{aN^2}{V}$$

Evaluate the requested partial derivatives:

$$\text{a)} \quad \left(\frac{\partial p}{\partial V}\right)_T \text{ and } \quad \text{b)} \quad \left(\frac{\partial U}{\partial V}\right)_p$$

4. Triple product rule

Problem 3.10 in textbook (EOPC-book by Sethna).

5. Simple cycle in p-V diagram

Problem 5.5 in textbook (EOPC-book by Sethna).

6. Two coupled systems

A thermodynamic system consists of two identical subsystems, $\alpha = \{1, 2\}$, described by thermodynamic variables T_α, $U_\alpha = C(T_\alpha - T_0) + U_0$, and $S_\alpha(T_\alpha)$.

a) Determine $S_\alpha(T_\alpha)$ for the individual subsystems.

b) Determine $U(T_1, T_2)$ and $S(T_1, T_2)$ for the total system.

Now, consider the total system, where the two subsystems can exchange energy with each other but not with any external environment. The initial energies and initial temperatures of the two subsystems are $U_{i,1}$ and $U_{i,2}$ and $T_{i,1}$ and $T_{i,2}$ respectively.

c) Determine and plot the entropy of the total system and discuss it as a function of U_1. (For the graph of $S(U_1)$ use $U_{i,1} = 3CT_0$ and $U_{i,2} = CT_0$.)

d) Using the maximum principle for the entropy, determine the equilibrium temperatures and energies for the two subsystems.

e) Determine the entropy increase for the total system after reaching equilibrium as a function of $T_{i,1}$ and $T_{i,2}$.

f) Now consider the case where $T_{i,1}$ and $T_{i,2}$ differ only slightly from the equilibrium temperature T_f of the total system: $T_{i,1} = T_f + \Delta T/2$ and $U_{i,2} = T_f - \Delta T/2$. Determine the entropy gain $\Delta S = S_f - S_i$ up to second order in ΔT and up to first order in ΔT for the individual subsystems. Interpret the result.

g) Express the entropy $S(U_f)$ and energy $U(T_f)$ of the total system after reaching equilibrium in terms of the total energy U_f and equilibrium temperature T_f. Discuss your result.