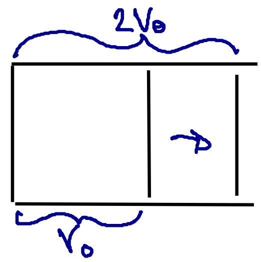
April 10, 2020. Quiz 1, due 5pm, Saturday 4/11. :

Quiz problem 1: Given the equation of state below, what is the amount of work done when a system isothermally expands from initial volume V_0 to final volume $2V_0$?

$$p = \frac{Nk_BT}{V - Nb}$$



Notes: $N = \text{Const}$

$$dU = \delta Q_{in} - \delta W_{out}$$

$$\int_1^2 \delta W_{out} = \int_1^2 p\,dV = \int_1^2 \frac{Nk_BT}{V - Nb}\,dV$$

$$W = \Big[Nk_BT \ln|V - Nb|\Big]_{V_0}^{2V_0}$$

$$\boxed{W = Nk_BT \ln\left|\frac{2V_0 - Nb}{V_0 - Nb}\right|}$$

Quiz problem 2: An ideal gas is contained in a cylinder with a tightly fitting piston. Several small masses are on the piston. (Neglect friction between the piston and cylinder walls.) The cylinder is placed in an insulating jacket, and a large number of masses are then added to the piston.

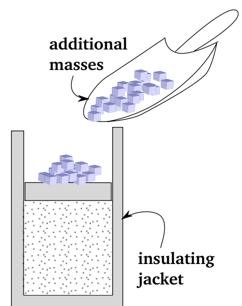
You may use the following properties of an ideal gas:

- The internal energy is a function of N and T only.
- $pV = Nk_BT$

Tell whether the pressure, temperature, and volume of the gas will increase, decrease, or remain the same. Explain.

For an ideal gas,

$$p\forall = Nk_bT \quad \& \quad U = f(T,N)$$



Adding mass to the piston will impart boundary work into the system

$$\delta W_{in} = p\,d\forall = \left(\frac{Nk_BT}{\forall}\right)d\forall$$

The added mass will move the piston down, meaning, $d\forall < 0$.

An Ideal gas can be represented Polytropically where

$$P_1\forall_1^{\,n} = P_2\forall_2^{\,n} \quad , \text{ with } n = \text{const}$$

Since Volume decreases from $1 \to 2$, Pressure must increase from $1 \to 2$, $dP > 0$.

Finally, since there is no Q in or out, but there is work into the system so by the 1st Law U must increase from $1 \to 2$.

$$dU = \cancel{\delta Q_{in}}^{\,0} - \delta W_{out}$$

However, Recall $U = f(T)$ so if U increases, so does Temperature, $dT > 0$

In Summary:

Temp ↑

Volume ↓

Pressure ↑

Quiz problem 3: Given the definitions below, evaluate the requested partial derivative.

$$U = x^2 + y^2 + z^2$$
$$z = \ln(y - x) + x^2$$

Find $\left(\frac{\partial U}{\partial z}\right)_y$

$$dU = \frac{\partial U}{\partial x}\bigg|_{yz} dx + \frac{\partial U}{\partial y}\bigg|_{xz} dy + \frac{\partial U}{\partial z}\bigg|_{xy} dz$$

$$dU = 2x\,dx + 2y\,dy + 2z\,dz$$

$$dz = \left[\frac{-1}{y-x} + 2x\right] dx + \frac{1}{y-x}\,dy$$

$$\hookrightarrow \frac{dx}{dz}\bigg|_y = \frac{-1}{\frac{1}{y-x} + 2x}$$

$$\boxed{\frac{\partial U}{\partial x}\bigg|_y = 2z - \frac{2x}{\frac{1}{y-x} + 2x}}$$