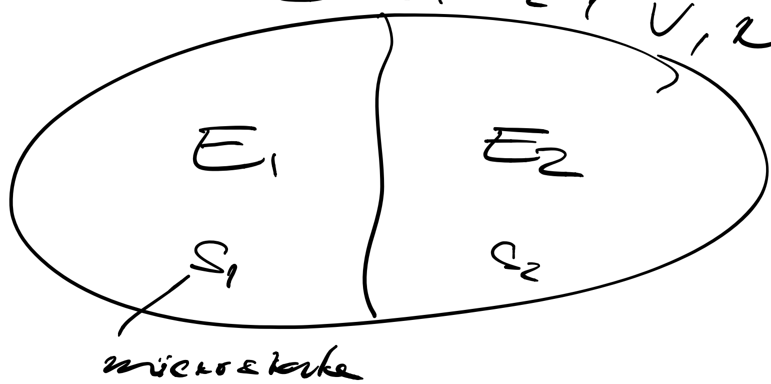
$$\Omega(E,V,N) = \frac{V^N 3N m \pi^{\frac{3N}{2}} (2mE)^{\frac{3N}{2}}}{\left(\frac{3N}{2}\right)!}$$

$E = E_1 + E_2$, V, N



$S = \{s_1, s_2\}$

$$E = E_1 + E_2 + \underbrace{E_{int}(1,2)}$$

interaction energy across BOUNDARY

ASSUME INTERACTIONS ARE SHORT RANGE, CAN BE RESTRICTED TO SHELL OF FINITE WIDTH ΔL

$$E_1, E_2 \propto V_1, V_2$$

$$E_{int} \propto \underset{\uparrow \atop L^2}{A} \cdot \Delta L = V^{2/3} \Delta L$$

$$\frac{E_{int}}{E} \propto \frac{V^{2/3}}{V} = \frac{1}{V^{1/3}} \xrightarrow[V \to \infty]{} 0$$

WANT: $\mathcal{S}(E_1)$ OF SUBSYSTEM 1 HAVING ENERGY E_1

$$\mathcal{S}(s_1) \propto \Omega_2(E - E_1)$$

$$\Omega_1(E_1)\delta E = \int_{E_1 < E < E_1 + \delta E} dE_1$$

$$\rho(E_1) \propto \Omega_1(E_1)\,\Omega_2(E - E_1)$$

$$\Omega(E) = \int dE_1\, \Omega_1(E_1)\,\Omega_2(E - E_1)$$

$$\Rightarrow \rho(E_1) = \frac{\Omega_1(E_1)\,\Omega_2(E - E_1)}{\Omega(E)} \qquad (1)$$

MOST LIKELY STATE OF SUBSYSTEM 1 HAS MAXIMUM $\rho(E_1)$ at $E_1 = E_1^*$.

MAXIMIZE $\rho(E_1)$ (1)

$$0 \overset{!}{=} \frac{\partial \rho(E)}{\partial E_1} = \left(\frac{\partial \Omega_1}{\partial E_1}\bigg|_{E_1^*} \Omega_2 + \frac{\partial \Omega_2}{\partial E_2}\bigg|_{E-E_1^*} \cdot \otimes \right.$$

$$\left. \otimes \underbrace{\frac{\partial E_2}{\partial E_1}}_{\substack{=-1 \\ E_2 = E - E_1}} \Omega_1 \right) \frac{1}{\Omega(E)}$$

$$\Leftrightarrow \frac{1}{\Omega_1} \frac{\partial \Omega_1}{\partial E_1}\bigg|_{E_1^*} = \frac{1}{\Omega_2} \frac{\partial \Omega_2}{\partial E_2}\bigg|_{E-E_1^*} \qquad (2)$$

$$\frac{\partial \ln \Omega_1}{\partial E_1}\bigg|_{E_1^*} = \frac{\partial \ln \Omega_2}{\partial E_2}\bigg|_{E-E_1^*} \qquad (3)$$

DEFINE (FOR CONVENIENCE)
EXTENSIVITY

$$\ln \Omega = \frac{S}{k_B}$$ $\Leftarrow$ UNIT CONVERSION FACTOR

$$\boxed{S = k_B \ln \Omega} \qquad (4)$$

$$\frac{\partial S_1}{\partial E_1}\bigg|_{E_1^*} = \frac{\partial S_2}{\partial E_2}\bigg|_{E-E_1^*} \qquad (5)$$

DEFINE $$\frac{\partial S}{\partial E}\bigg|_{V,N} = \frac{1}{T} \qquad (6)$$

$$S(E,E_1) = S_1(E_1) + S_2(E-E_1)$$

$$= k_B\left(\ln \Omega_1(E_1) + \ln \Omega_2(E-E_1)\right)$$

$$= k_B \ln\big(\underbrace{\Omega_1(E_1)\cdot\Omega_2(E-E_1)}_{\propto\, \mathcal{S}(E_1)}\big)$$

$$\propto \ln \mathcal{S}(E_1)$$

EQUILIBRIUM (MOST LIKELY "STATE")

$$\equiv \max \mathcal{S}(E_1)$$

$\Leftrightarrow$ EQUILIBRIUM $\equiv$ MAX $\left(S_1(E_1) + S_2(E-E_1)\right)$

S is not just extremum $\rightarrow$

BUT S IS MAXIMUM !

VERIFY THAT $\rho(E_1)$ IS STRONGLY PEAKED AT $E_1 = E_1^*$

$\rho(E_1) \propto$

$$\Omega_1(E_1)\,\Omega_2(E-E_1) = \exp\left(\frac{1}{k_B}\Big(S_1(E_1) + S_2(E-E_1)\Big)\right)$$

TAYLOR EXPAND BOTH S_1 AND S_2

$$\approx \exp\left(\frac{1}{k_B}\cdot\left[S_1(E_1^*) + \cancel{(E_1-E_1^*)\frac{\partial S_1}{\partial E_1}} + \frac{1}{2}(E_1-E_1^*)^2\frac{\partial^2 S_1}{\partial E_1^2} + S_2(E-E_1^*) + \underbrace{\cancel{(E_1-E_1^*)\frac{\partial S_2}{\partial E_2}}}_{(5)}\cdot\underbrace{\frac{\partial E_2}{\partial E_1}}_{-1} + \frac{1}{2}(E_1-E_1^*)^2\frac{\partial^2 S_2}{\partial E_2^2}\cdot\underbrace{\frac{\partial E_2^2}{\partial E_1^2}}_{(-1)^2}\right]\right)$$

$$= \Omega_1(E_1^*)\,\Omega_2(E_2^* = E - E_1^*) \times \exp\left(\frac{1}{2k_B}(E_1-E_1^*)^2\left(\frac{\partial^2 S_1}{\partial E_1^2} + \frac{\partial^2 S_2}{\partial E_2^2}\right)\right) \quad (7)$$

$$\rho(E_1) \propto \frac{1}{\sqrt{2\pi}\,\sigma_E}\, e^{-\frac{(E_1-E_1^*)^2}{2\sigma_E^2}} \quad (8)$$

$\to$ NORMALIZATION FIXES PREFACTOR

$$\frac{1}{\sigma_E^2} = -\frac{1}{k_B}\left(\frac{\partial^2 S_1}{\partial E_1^2} + \frac{\partial^2 S_2}{\partial E_2^2}\right)$$

MINUS SIGN !!

$$\frac{\partial S}{\partial E^2} = \frac{\partial(\frac{1}{T})}{\partial E} < 0 \qquad E\uparrow \to \frac{1}{T}\downarrow$$

TYPO IN BOOK FOOTNOTE 27: "INVERSE" TEMP

$\frac{\partial^2 S}{\partial E^2} < 0 \to S$ IS CONVEX DOWNWARD FACING FUNCTION.

$S_1(E_1) + S_2(E-E_1)$

$$\frac{1}{\sigma_E^2} \propto \frac{S}{E^2} \propto \frac{N}{N^2} = \frac{1}{N}$$

$\frac{\sigma_E}{N}$ =: ENERGY FLUCTUATIONS PER PARTICLE $\propto \frac{1}{\sqrt{N}}$

VERIFY ENTROPY IS EXTENSIVE

$$S_{tot} = k \ln \Omega(E)$$

$$= k \ln \int dE_1\, \Omega_1(E_1)\, \Omega_2(E-E_1)$$

$$\overset{(7)(8)}{=} k \ln \Omega_1(E_1^*)\, \Omega_2(E-E_1^*) \int dE_1\, e^{-\frac{(E_1-E_1^*)^2}{2\sigma_E^2}}$$

$$= S_1(E_1^*) + S_2(E-E_1^*) + k_B \ln \sqrt{2\pi\sigma_E^2}$$

FOOTN. 35 $\frac{1}{\sigma_\varepsilon^2}$

WORM OF GAUSSIAN

$\propto \frac{1}{2} \ln N$

WHAT IS ENTROPY?

IN STATISTICAL MECHANICS, THE SECOND LAW ARISES FROM PROBILITY ARGUMENTS ABOUT THE MOST LIKELY STATE IN A SUBSYSTEM OF A CLOSED SYSTEM.