$$\Omega(E)\,\delta E = \int_{E<H<E+\delta E} d\mathbb{P}\, d\mathbb{Q}$$

$$= \underbrace{\int d\mathbb{Q}}_{V^N} \cdot \int_{E<H<E+\delta E} d\mathbb{P}$$

$$E = \sum_{j=1}^{3N} \frac{1}{2} m_j v_j^2 = \sum_{j=1}^{3N} \frac{p_j^2}{2m_j} \underset{m_j = m}{=} \frac{\mathbb{P}^2}{2m}$$

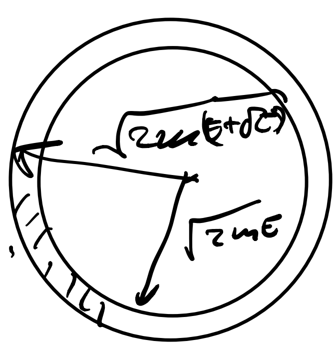
$E = \text{const.} \rightarrow$ on sphere in 3N DIMENSIONAL SPACE WITH RADIUS $R = \sqrt{2mE}$. $\longrightarrow$ 3N−1 SPHERE: S_R^{3N-1}

VOLUME: $\mu\left(S_R^{\ell-1}\right) = \pi^{\ell/2} R^{\ell} \dfrac{1}{(\ell/2)!}$

CIRCLE IN 2D : $\ell=2$ $\quad \frac{2}{2}! = 1$ ✓, πR^2

SPHERE IN 3D : $\ell = 3$ $\quad \frac{3}{2}! = \frac{3}{4}\sqrt{\pi}$

$V_2 = \frac{4}{3}\pi R^3$



$3N-1$

$$\frac{\text{SHELL}}{\delta E} = \frac{\mu\left(S^{3N-1}_{\sqrt{2m(E+\delta E)}}\right) - \mu\left(S^{3N-1}_{\sqrt{2mE}}\right)}{\delta E}$$

$$= \frac{d\mu\left(S^{3N-1}_{\sqrt{2mE}}\right)}{dE}$$

$$\frac{d}{dE}\left(\frac{\pi^{\frac{3N}{2}}(2mE)^{\frac{3N}{2}}}{\left(\frac{3N}{2}\right)!}\right)$$

$$= \pi^{\frac{3N}{2}}\, 3Nm\, (2mE)^{\frac{3N}{2}-1} \Big/ \left(\frac{3N}{2}\right)!$$

$$R = \sqrt{2mE}$$

$$= 3Nm\left(\pi^{\frac{3N}{2}}\right) R^{3N-2} \Big/ \left(\frac{3N}{2}\right)!$$

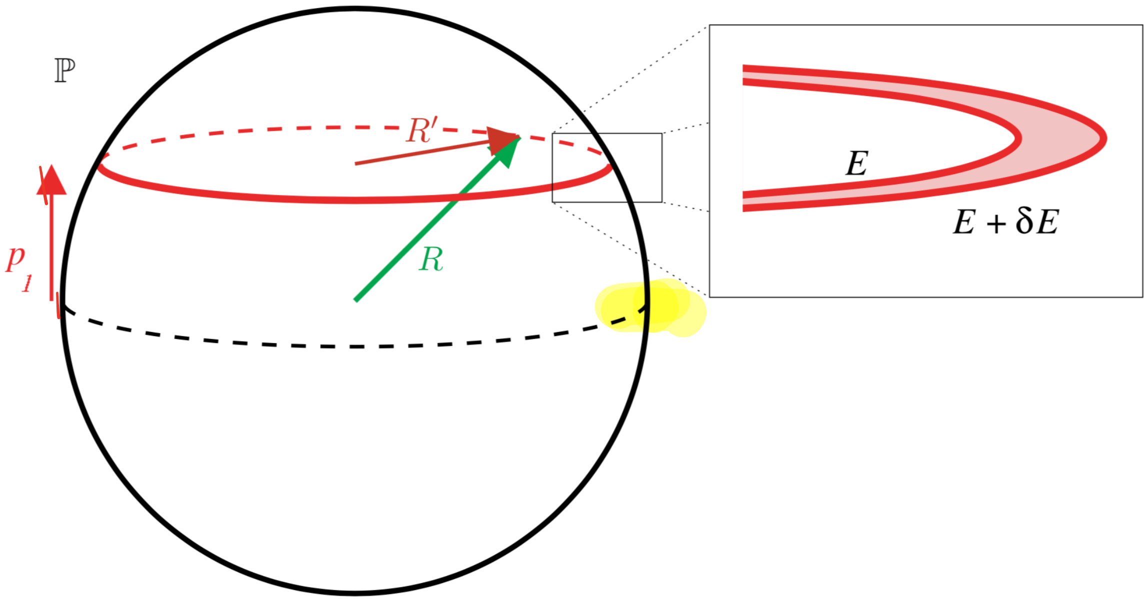
$\rho(p_1) =:$ probability distribution of $p_1 = p_x$ of particle 1.

→ Example of an UNCONDITIONAL PDF.

Here, USE GEOMETRY

$$R = \sqrt{2mE}$$

$$\text{fix } p_1 \to R' = \sqrt{2mE - p_1^2}$$



2 "CIRCLES" FROM CUTTING "SPHERES" BY PLANE $p_1 = \text{const.}$

S^{3N-2}

$$\frac{\text{ANNULAR AREA}}{\delta E} = \frac{d\mu\left(\sqrt{2mE - p_1^2}^{\,3N-2}\right)}{dE}$$

$$= \frac{\pi^{\frac{3N-1}{2}}(3N-1)m\left(2mE-p_1^2\right)^{\frac{3N-3}{2}}}{\left(\frac{3N-1}{2}\right)!}$$

R_1 $\tilde{R} = R'$

$$= \frac{(3N-1)m\,\pi^{\frac{3N-1}{2}}\tilde{R}^{3N-3}}{\left(\frac{3N-1}{2}\right)!}$$

$$\rho(p_1) = \frac{\text{ANNULAR AREA}}{\text{SHELL VOLUME}}$$

$$= \frac{(3N-1)m\pi^{\frac{3N-1}{2}}\tilde{R}^{3N-3}\left(\frac{3N}{2}\right)!}{3Nm\pi^{\frac{3N}{2}}R^{3N-2}\left(\frac{3N-1}{2}\right)!}$$

$$\propto \frac{R^2}{\tilde{R}^3}\left(\frac{\tilde{R}}{R}\right)^{3N}$$

$$\left(\frac{\tilde{R}}{R}\right)^{3N} = \left(\frac{2mE - p_1^2}{2mE}\right)^{\frac{3N}{2}} = \left(1 - \frac{p^2}{2mE}\right)^{\frac{3N}{2}}$$

$$N \to \infty \quad x^N = \begin{cases} \sim 0 & x < 1 \\ \sim 1 & x \equiv 1 \end{cases}$$

$$\to \frac{p^2}{2mE} \ll 1$$

$$\frac{R^2}{\tilde{R}^3} = \frac{2mE}{\left(2mE\left(1 - \frac{p_1^2}{2mE}\right)\right)^{3/2}}$$

$$= \frac{1}{\sqrt{2mE}} \frac{1}{\left(1 - \frac{p_1^2}{2mE}\right)^{3/2}} \approx \frac{1}{\sqrt{2mE}} \left(1 + \frac{3}{2}\frac{p_1^2}{2mE} \pm \dots \right)$$

$$= \frac{1}{\sqrt{2mE}} = R$$

$$1 - \frac{p_1^2}{2mE} = 1 - x = e^{-x} = e^{-\frac{p_1^2}{2mE}}$$

$x \ll 1$

$$\rho(p_1) = \frac{1}{\sqrt{2\pi}\sqrt{m\frac{2E}{3N}}} e^{-\frac{p_1^2}{2m}\left(\frac{3N}{2E}\right)}$$

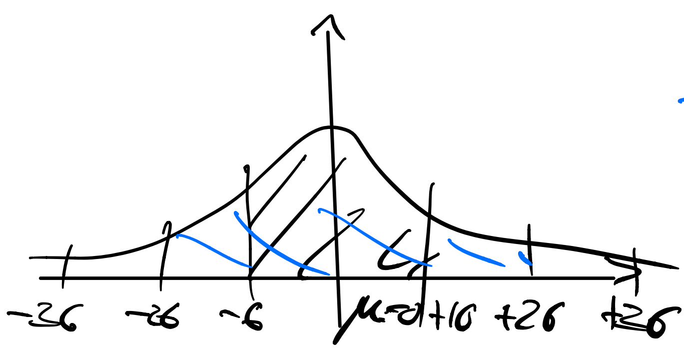
$\uparrow E_{kin}$ $\quad \uparrow 1/k_B T$

STANDARD DEV. $\sigma = \sqrt{\frac{2mE}{3N}}$

NOTES: RESULTS TRUE FOR ANY p_j

$p_1 \rightarrow S^{3N-2}$ SPHERE HAS ALL AREA AT THE EQUATOR

$\delta p \pm n\sigma$



$n\sigma$	$P = \int_{-n\sigma}^{+n\sigma} \rho(p_i)$
1	0.683
2	0.95
3	0.997
4	0.9999
5	0.99999

KEY RESULTS

1) TEMPERATURE

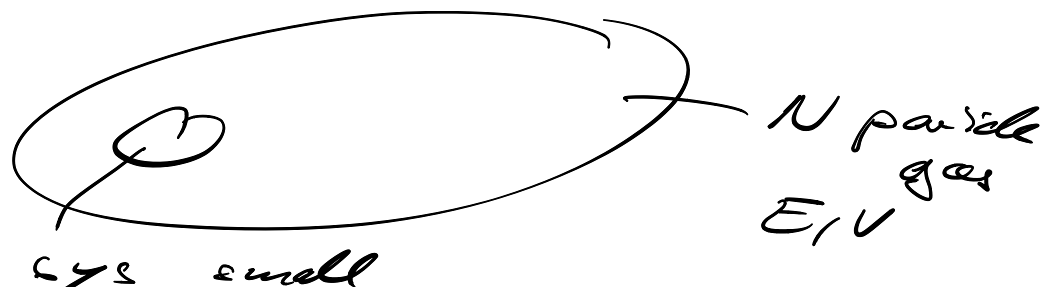
$$k_B T = \frac{2E}{3N} \qquad \rho(p_i) = \frac{1}{\sqrt{2\pi m kT}}\, e^{-\frac{p_i^2}{2mkT}}$$

2) BOLTZMANN FACTOR $\quad E_{kin} = \frac{p_i^2}{2m}$

$$\propto e^{-\frac{E_{kin}}{kT}}$$

$$\propto e^{-\frac{E_{sys}}{kT}}$$

system is one momentum



3) EQUIPARTITION

$$\underbrace{\frac{p_i^2}{2m}}_{\frac{E}{3N}} = \frac{kT}{2}$$

EVERY HARMONIC DEGREE OF FREEDOM RECEIVES $\frac{1}{2}kT$ ENERGY.

4) GENERAL CLASSICAL MOMENTUM DISTRIBUTION $\leftarrow$ MULTIPLE GASES AND INTERACTIONS