Quiz problem 1: Construct a thermodynamic potential with natural variables temperature T pressure p, and particle number N : Gibbs free energy $G(T, p, N)$

1. Derive the thermodynamic identity for G.

   Solution:

Recall that,

$$dU = TdS - pdV + \mu dN \tag{1}$$

where,

$$\left.\frac{\partial U}{\partial S}\right|_{V,N} = T \qquad \left.\frac{\partial U}{\partial V}\right|_{S,N} = -p \qquad \left.\frac{\partial U}{\partial N}\right|_{S,V} = \mu \tag{2}$$

We want to transform $U(S, V, N)$ to $G(T, p, N)$, so let's define

$$G \equiv U - S\left.\frac{\partial U}{\partial S}\right|_{V} + p\left.\frac{\partial U}{\partial V}\right|_{S,N} \tag{3}$$

$$\equiv U - ST + pV \tag{4}$$

So,

$$\boxed{G = U - ST + pV} \tag{5}$$

2. Calculate all Maxwell relations that can be derived from G.

   Solution:

The total differential of G is,

$$dG = \left.\frac{\partial G}{\partial T}\right|_{p,N} dT + \left.\frac{\partial G}{\partial p}\right|_{T,N} dp + \left.\frac{\partial G}{\partial N}\right|_{T,p} dN \tag{6}$$

$$= \left.\frac{\partial U}{\partial T}\right|_{p,N} dT + \left.\frac{\partial U}{\partial p}\right|_{T,N} dp + \left.\frac{\partial U}{\partial N}\right|_{T,p} dN + \left.\frac{\partial\left[ST + pV\right]}{\partial T}\right|_{p,N} dT + \left.\frac{\partial\left[ST + pV\right]}{\partial p}\right|_{T,N} dp + \left.\frac{\partial\left[ST + pV\right]}{\partial N}\right|_{T,p} dN \tag{7}$$

$$= -SdT + Vdp + \mu dN \tag{8}$$

Submitted by: Paige Lorson

3. Show that for extensive systems the chemical potential is given by:

$$\mu = \frac{G}{N} \tag{9}$$

Solution:

Recall the Euler equation, for an extensive system

$$U = TS - pV + \mu N \tag{10}$$

We can use this in our definition of G,

$$G = \left[\cancel{TS} - \cancel{pV} + \mu N\right] - \cancel{ST} + \cancel{pV} \tag{11}$$

So for a extensive system,

$$\boxed{\mu = \frac{G}{N}} \tag{12}$$