IDEAL GAS $\quad pV = Nk_BT$

$\xrightarrow{\text{IMPLIES}}$ $\quad U = U(T) \neq U(T,V)$

(assume $N = \text{const}$)

SHOW: $\left.\frac{\partial U}{\partial V}\right|_T \overset{!}{=} 0 \qquad (*)$

need $U(T,V)$

$U = U(\underline{\underline{S}},V)$

$dU = T\,\underline{dS} - p\,dV \qquad (1)$

$S(T,V)$

$\underline{dS} = \left.\frac{\partial S}{\partial T}\right|_V dT + \left.\frac{\partial S}{\partial V}\right|_T dV \qquad (2)$

(2) in (1)

$$dU = T\left.\frac{\partial S}{\partial T}\right|_V dT + \underbrace{\left(T\left.\frac{\partial S}{\partial V}\right|_T - p\right)}_{\left.\frac{\partial U}{\partial V}\right|_T \overset{!\text{show}}{=} 0}dV$$

$(*)$

ELIMINATE S VIA MAXWELL-RELATION

$$\left.\frac{\partial S}{\partial V}\right|_T \rightarrow V, T, N \Rightarrow F(T, V, N)$$

$$dF = -S\,dT - p\,dV + \mu\,dN$$

$$\left.\frac{\partial F}{\partial T}\right|_{V,N} = -S \qquad \left(\frac{\partial F}{\partial V}\right)_{T,N} = -p$$

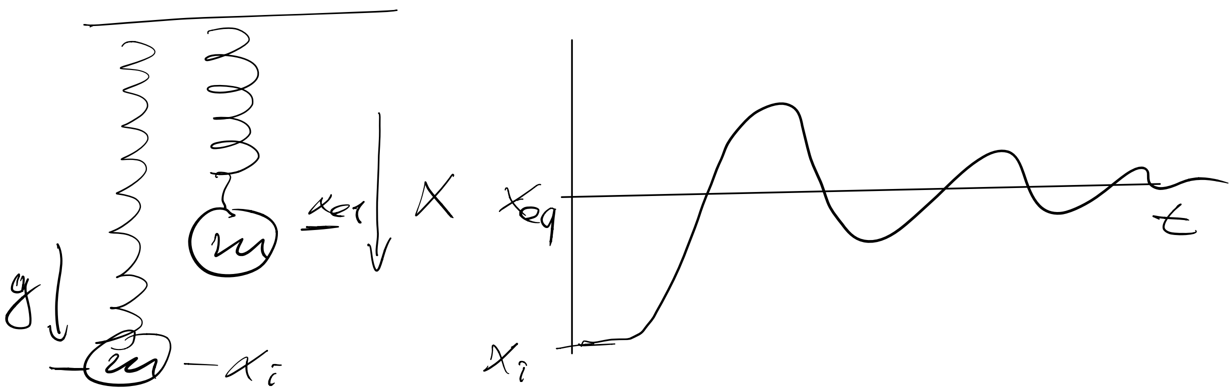
$$\left.\frac{\partial S}{\partial V}\right|_T = \left.\frac{\partial p}{\partial T}\right|_V$$

$$\left.\frac{\partial U}{\partial V}\right|_T = T \left.\frac{\partial p}{\partial T}\right|_V - p \underset{pV = NkT}{=} \underbrace{T \cdot \frac{Nk}{V}}_{p} - p = ?$$

$$? = p - p = 0$$

$$U \propto kT$$

MECHANICAL (STATIC) EQUILIBRIUM



Find x_{eq}?

a) Use Newton: FORCE EQUILIBRIUM

b) MINIMIZE POTENTIAL ENERGY

SPRING: $U_k = \frac{1}{2}k(x-x_0)^2$

$$U_m = -mgx$$

$$\min U = \min(U_k + U_m)$$

$$U = \frac{1}{2}k(x-x_0)^2 - mgx$$

EXTREMUM: $\frac{\partial U}{\partial x} = kx - mg \overset{!}{=} 0$

$$x_{eq} = \frac{mg}{k}$$

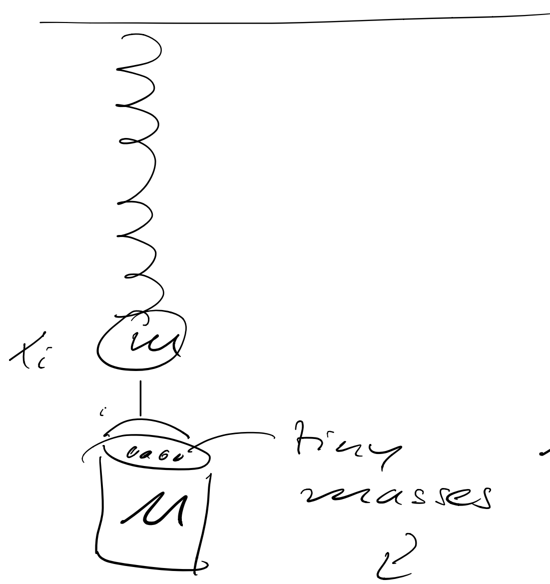
a) $\sum \vec{F} = 0 \qquad F_k + F_g = 0$

$$F_k = -\frac{\partial U_k}{\partial x} = -kx \quad F_g = mg$$

$$\rightarrow \; -kx + mg = 0$$

$$\rightarrow \; x_{eq} = \frac{mg}{k}$$

EXTRACT WORK FROM THE MINIMIZATION OF U:



tiny masses

remove one by one

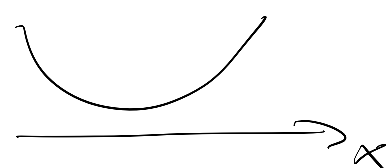
M such that

$$x_i = \frac{(m+M)g}{k}$$

$\Rightarrow$ QUASISTATIC MECHANICAL PROCESS TO EXTRACT MAX WORK.

$U(x)$ is MINIMUM:

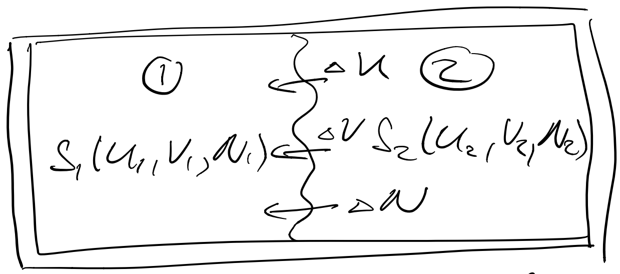
$$\frac{\partial^2 U}{\partial x^2} > 0$$



$$\frac{\partial^2 U}{\partial x^2} = k > 0$$

"DERIVED THAT $k>0$"

CLOSED SYSTEM



use 2nd LAW of TD (max S)
to DETERMINE EQUILIBRIUM

$$\Delta S = \Delta S_1 + \Delta S_2$$

if $\Delta S_1, \Delta S_2$ ARE SMALL

$$\Delta S = \left.\frac{\partial S_1}{\partial U_1}\right|_{V,N} \Delta U_1 + \left.\frac{\partial S_1}{\partial V_1}\right|_{U,N} \Delta V_1 + \left.\frac{\partial S_1}{\partial N_1}\right|_{U,V} dN_1$$

δS

$$+ \left.\frac{\partial S_2}{\partial U_2}\right|_{V,N} \Delta U_2 + \ldots$$

$$\Delta U = \Delta U_1 = -\Delta U_2$$
$$\Delta V = \Delta V_1 = -\Delta V_2$$
$$\Delta N = \Delta N_1 = -\Delta N_2$$

$$\Delta S = \left(\left.\frac{\partial S_1}{\partial U_1}\right|_{V,N} - \left.\frac{\partial S_2}{\partial U_2}\right|_{V,N} \right) \Delta U$$

$$+ (\ldots) \Delta V$$

$$+ \left(\qquad \right) \Delta N$$

Use $dS = \frac{1}{T}\, dU + \frac{p}{T}\, dV - \frac{\mu}{T}\, dN$

$$\Rightarrow \Delta S = \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \Delta U + \left(\frac{p_1}{T_1} - \frac{p_2}{T_2} \right) \Delta V$$

$$+ \left(-\frac{\mu_1}{T_1} - \left(-\frac{\mu_2}{T_2} \right) \right) \Delta N \overset{!}{=} 0$$

EXTREMUM

$$\Rightarrow T_1 = T_2, \quad p_1 = p_2, \quad \mu_1 = \mu_2$$

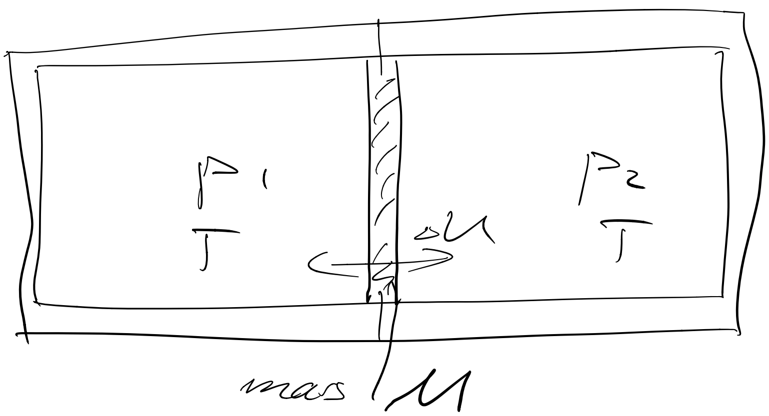
NOTATION

dS, ΔS, δS

dS: infinitesimal quasistatic

ΔS: finite non-equilibrium

δS: infinitesimal



$p_1 > p_2$

$F_1 = p_1 \cdot A \qquad F_2 = p_2 A$

$$E = U_1 + U_2 + \frac{1}{2} M v^2$$