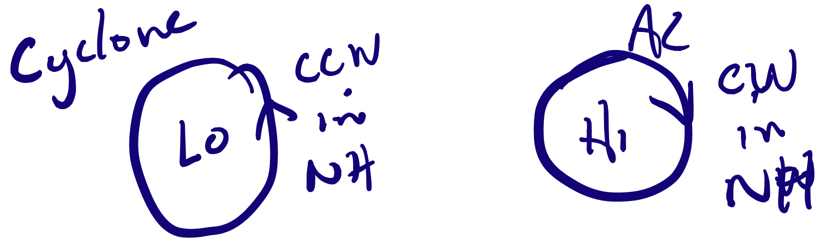
Lecture 7. – <u>Buoyancy Effects</u>

① rotation – in a rotating reference frame,

$\frac{\partial \hat{u}}{\partial t} \propto f \hat{u}$ f - rotation rate of earth.

$$Ro = \frac{\text{non-linear terms}}{\text{Coriolis force}} \sim \frac{U^2/L}{fU} = \frac{U}{fL}$$



② Electromagnetic forces – Magnetohydrodynamics

applies to electrically-conducting fluids
– plasmas, liquid metals, salt water

" flow of conducting fluid thru a magnetic field induces electrical current"

" current applied to a conducting fluid in a magnetic field can induce a fluid motion"

body force $\tilde{J} \times \tilde{B}$ $\tilde{J}$ – current density
$\hat{B}$ – magnetic field.

Maxwell's Eq'n.

$\hat{u}$

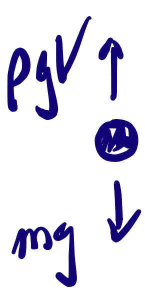
③ Buoyancy.

practical examples where varying density causes a gravitational force.

- circulation of coolant thru a nuclear reactor
- cooling power transistors on PC boards
- critical to geophysical flows where ultimate source of energy is solar radiation.
- salinity variations.

momentum eq'n with gravitational force:

$$\rho \frac{D\hat{u}}{Dt} = -\nabla p + \rho g + \rho \nu \nabla^2 \hat{u}$$



↑ force per unit volume $\int dV$ of a displaced object to get the buoyant force object $\rho g V$

consider a static fluid (not moving)

density ρ_0 = constant

pressure $p_0 = p_0(z)$

then $\nabla p_0 = \rho_0 g$

$$\frac{\partial p_0}{\partial z} = \rho_0 g$$

$$p_0 = \int \rho_0 g \, dz$$

Suppose $p = p_0 + p'$; $\rho = \rho_0 + \rho'$ not turbulence
small fluctuations
incl. waves.

$$\rho \frac{D\tilde{u}}{Dt} = \cancel{-\nabla p_0} - \nabla p' + \cancel{\rho_0 g} + \rho' g + \mu \nabla^2 \tilde{u}$$

$\div \rho_0$

- $$\left(1 + \frac{\rho'}{\rho_0}\right) \frac{D\tilde{u}}{Dt} = -\frac{1}{\rho_0} \nabla p' + \frac{\rho'}{\rho_0} g + \nu \nabla^2 \tilde{u}$$

if $\rho'/\rho_0 << 1$, ρ' only affects momentum in the way that it relates to g

$$\frac{Du}{Dt} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial x} + \nu \nabla^2 u$$

$$\frac{Dv}{Dt} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial y} + \nu \nabla^2 v$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial z} + \nu \nabla^2 w + \frac{\rho'}{\rho_0} g \leftarrow$$

typically primes are dropped

$\frac{\rho'}{\rho_0} g$ is the same OM as $\frac{\partial w}{\partial t}$, $\nu \nabla^2 w$

TKE eq'n $\quad q^2 = \frac{1}{2}\overline{u_i u_i}$

$$\frac{\partial}{\partial t} q^2 + U_j \frac{\partial q^2}{\partial x_j} = -\frac{1}{\partial x_j}\left(\frac{1}{\rho}\overline{u_j p} + \overline{u_j q^2} - 2\nu \overline{u_i A_{ij}}\right) - \overline{u_i u_j} S_{ij} - 2\nu \overline{A_{ij} A_{ij}} - \underbrace{\frac{g}{\rho}\overline{u_i \rho'}\delta_{i3}}_{J_b}$$

P is usually >0 $\qquad \epsilon > 0$

buoyancy flux $J_b = -\frac{g}{\rho}\overline{u_i \rho'}\delta_{i3} = -\frac{g}{\rho}\overline{u_3 \rho'} = -\frac{g}{\rho}\overline{w\rho'}$

$$\frac{\partial q^2}{\partial t} + U_j \frac{\partial q^2}{\partial x_j} = -\frac{\partial}{\partial x_j}(\quad) + P + J_b - \epsilon$$

time rate of change — advection by mean flow — transport terms (divergence terms) — source — ? — sink

if all sources removed + consider effect on CV (neglect adv)

$$\frac{\partial}{\partial t} q^2 = -\epsilon$$ ← turbulence is a decaying process

with sources

$$\boxed{\frac{\partial q^2}{\partial t} = P + J_b - \epsilon}$$

?

$J_b = -g\,\overline{\rho' w'}$ = buoyant production / consumption

→ in a stratified fluid J_b is generally a sink term

→ in unstable conditions (dense fluid over light fluid) then J_b is a source of tke [convection]

Consider 2 cases:

① light fluid rises, $\rho'<0$, $w'>0$
or
dense fluid sinks, $\rho'>0$, $w'<0$

$$\overline{\rho' w'}<0 \quad \therefore \frac{\partial q^2}{\partial t}>0$$

↑ TKE by reducing system PE

② light fluid down $\rho'<0$, $w'<0$
or
dense up $\rho'>0$, $w'>0$

$$\overline{\rho' w'}>0 \quad \therefore \frac{\partial q^2}{\partial t}<0$$

↓ TKE acts to increase system PE.