Buoyancy Effects $\Rightarrow l_b$ or $L_O = \left(\epsilon / N^3\right)^{1/2}$

we reviewed B as a source of tke

examples of convectively driven mixing

in daytime atmospheric boundary layer

nighttime ocean boundary layer.

at least the upper boundary layer

Notes on Evolution of Turbulence in the Diurnal Warm layer Moulin et al 2018

neglecting transport terms:

$$\frac{\partial\left(\frac{1}{2}\overline{u_i u_i}\right)}{\partial t} = \underbrace{-\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}}_{P} - \underbrace{\frac{g}{\bar{\rho}}\overline{u_3 \rho'}}_{B} - \epsilon$$

Scale $\frac{1}{2}\overline{u_i u_i} \sim \frac{3}{2} u^2$

Re stress $\overline{u_i u_j} \sim C_{uv} u^2$ ← lab empirical result for shear flow $C_{uv} \sim 0.3$

$\frac{\partial U_i}{\partial x_j} \sim S$ (for shear)

B is a tke sink

Important aside: how to quantify B?

Consider steady state $\mathcal{P} = B + \epsilon$ ← B is a sink

define flux Richardson # $R_f = \frac{B}{\mathcal{P}}$

$= \text{ratio of } \frac{\text{PE gained by mixing}}{\text{mechanical prod. of tke}}$

tells us how much of tke
goes to ↑ system PE

where $B > 0$, $R_f > 0$ since $\mathcal{P} > 0$

In a lab or DNS we can compute changes
in KE & PE due to mixing events

- this is impossible in natural flows

→ $u_i(z)$, $\rho_i(z)$ initial u, ρ profiles

→ $u_f(z)$, $\rho_f(z)$ final state.

$$\delta KE = \int_0^h \tfrac{1}{2}\rho_f(z)\, u_f^2(z)\, dz - \int_0^h \rho_i(z)\, u_i^2(z)\, dz$$

$$\sim \rho_0 \int (u_f^2 - u_i^2)\, dz$$

$$\delta PE = \int_0^h \rho_f(z)\, g z\, dz - \int_0^h \rho_i(z)\, g z\, dz$$

lab studies suggest that turbulence cannot be
sustained if $R_f \gtrsim 0.2$

that is turbulence decays if $\gtrsim 20\%$ of the
is lost to B.

let's rewrite $B = -g/\rho \, \overline{w'\rho'}$ also $N^2 = -\frac{g}{\rho}\frac{\partial \rho}{\partial z}$

define eddy diffusivity $\overline{w'\rho'} = K_\rho \frac{\partial \rho}{\partial z}$

$$g/\rho \, \overline{w'\rho'} = g/\rho \, K_\rho \frac{\partial \rho}{\partial z} = -K_\rho N^2$$

$$B = K_\rho N^2$$

$$P = \epsilon + B$$

$$P/B = \epsilon/B + 1$$

$$1/R_f = \epsilon/K_\rho N^2 + 1$$

$$\boxed{K_\rho = \frac{R_f}{(1-R_f)} \, \epsilon/N^2}$$

let $\Gamma = \frac{R_f}{1-R_f}$, R_f define by lab measurements

$$K_\rho = \Gamma \, \epsilon/N^2$$

$$\Gamma \simeq 0.2$$

$$B = \Gamma \epsilon$$

$$\frac{d}{dt}(3/2\, u^2) = C_{uv}\, u^2 S - \epsilon(\Gamma + 1)$$

inviscid estimate of ϵ $\quad \epsilon = C_\epsilon u^3/l$ $\quad C_\epsilon \sim 0.5$ empirical

replace l with l_b

$$\frac{d\epsilon}{dt} = \frac{2}{3}\epsilon\left[C_{uv}S - C_\epsilon^{2/3}N(1+\Gamma)\right]$$

assume that $N(t)$ changes slowly relative to ϵ
constant

general solution $\epsilon(t) = C_0 e^{t/\gamma}$
const. of integration

$$\gamma = \left[\tfrac{2}{3}\left(C_{uv}S - \alpha N\right)\right]^{-1} \qquad \alpha = C_\epsilon^{2/3}(1+\Gamma) \simeq .75$$

when $\gamma < 0$, or $C_{uv}S < \alpha N$, $\epsilon(t)$ decays.

when $\gamma > 0$, or $C_{uv}S > \alpha N$, $\epsilon(t)$ grows.

↳ let's go back to paper Fig 7.

Dynamics of Temperature (Scalar) Fluctuations

temperature variance eq'n. ← related to tke

$$\underbrace{\frac{\partial}{\partial t}\left(\tfrac{1}{2}\overline{\theta^2}\right)}_{\text{time}} + \underbrace{U_j\frac{\partial}{\partial x_j}\left(\tfrac{1}{2}\overline{\theta^2}\right)}_{\text{adv}} = \underbrace{-\frac{\partial}{\partial x_j}\left[\tfrac{1}{2}\overline{u_j\theta^2} - \gamma\frac{\partial}{\partial x_j}\left(\tfrac{1}{2}\overline{\theta^2}\right)\right]}_{\text{divergence}}$$

$$\underbrace{-\overline{u_j\theta}\frac{\partial\Theta}{\partial x_j}}_{P_\theta} - \underbrace{\gamma\overline{\frac{\partial\theta}{\partial x_j}\frac{\partial\theta}{\partial x_j}}}_{\text{diffusive}}$$

molecular diffusion of $\overline{\theta^2}$

consider steady-state ($\partial_t = 0$), homogeneous.

stratified $\frac{\partial \Theta}{\partial z} \neq 0$, neglect transport

$$-\overline{w\theta}\,\frac{\partial \Theta}{\partial z} = \gamma\,\overline{\frac{\partial \theta}{\partial x_j}\frac{\partial \theta}{\partial x_j}}$$

gradient ρ_θ = molecular diffusion of $\overline{\theta^2}$

$$\chi = 2\gamma\,\overline{\frac{\partial \theta}{\partial x_j}\frac{\partial \theta}{\partial x_j}}$$

← something that can be measured using fast T sensors.

$$\overline{w\theta}\,\frac{\partial \Theta}{\partial z} = \frac{\chi}{2}$$

$$\overline{w\theta} = -K_T\frac{\partial \Theta}{\partial z}$$

$$K_T = \frac{\chi_\theta}{2\Theta_z^2} \leftarrow K_\rho = \frac{\Gamma \epsilon_{ij}}{N^2}$$

$$K_T = K_\rho \Rightarrow \Gamma = \frac{\chi N^2}{2\epsilon\,\Theta_z^2}$$

extensive comparisons $\Rightarrow$ $\Gamma \simeq 0.2$