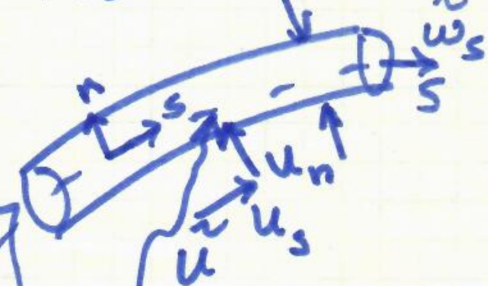
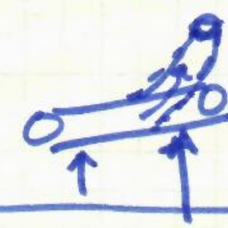
~~Defm~~ Deformation terms.

$\tilde{\omega} \rightarrow$ vector $\quad \tilde{\omega}_s \dfrac{\partial \tilde{u}}{\partial s}$



inviscid: $\boxed{\dfrac{D\tilde{\omega}_s}{Dt} = \tilde{\omega}_s \dfrac{\partial \tilde{u}_s}{\partial s}}$ $\leftarrow$ stretching/comp

$$\frac{D\tilde{\omega}_n}{Dt} = \tilde{\omega}_s \frac{\partial \tilde{u}_n}{\partial s}$$



$\tilde{\omega}_j \dfrac{\partial \tilde{u}_i}{\partial x_j} \quad ; \quad \dfrac{\partial \tilde{u}_i}{\partial x_j} = \tilde{S}_{ij} + \tilde{r}_{ij}$

$$\tilde{\omega}_j (\tilde{s}_{ij} + \tilde{r}_{ij}) = \tilde{\omega}_j \hat{S}_{ij} + \tilde{\omega}_j \tilde{r}_{ij},$$

$$\tilde{r}_{ij} = -\tfrac{1}{2} \epsilon_{ijk} \omega_k$$

$$-\tfrac{1}{2} \underbrace{\epsilon_{ijk}}_{\text{antisy.}} \underbrace{\tilde{\omega}_j \tilde{\omega}_k}_{\text{sym.}} \rightarrow 0 \equiv$$

$$\boxed{\tilde{\omega}_j \frac{\partial \tilde{u}_i}{\partial x_j} = \tilde{\omega}_j \tilde{S}_{ij}}$$

need a strainrate to "deform" $\underline{\underline{\omega}}$ mag & dir.

Reynold's Decomposition $\hat{\omega}_i = \Omega_i + \omega_i$

mean vorticity (Ω_i) fluct. field (ω_i)

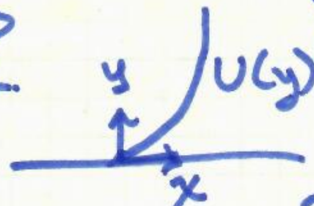
↓

into instan. vorticity

& time ave. ($\Omega_i \neq f(t)$)

$$U_j \cdot \frac{\partial \Omega_i}{\partial x_j} = \underbrace{-\overline{u_j \cdot \frac{\partial \omega_i}{\partial x_j}} + \overline{\omega_j \cdot s_{ij}}}_{\text{turb. contributions to Mean Vorticity}} + \Omega_i S_{ij} + \nu \frac{\partial^2 \Omega_i}{\partial x_j \partial x_j}$$

2D B.L.



$$\Omega = -\frac{\partial U}{\partial y}$$

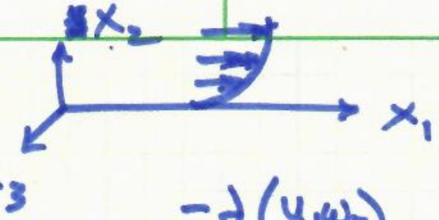
<u>incompressible:</u>

$$-\overline{u_j \frac{\partial \omega_i}{\partial x_j}} = -\frac{\partial (\overline{u_j \cdot \omega_i})}{\partial x_j}$$

<u>N-S</u>: similar to $\underbrace{\text{Reynolds Stress}}_{\text{gradients}}$ $\frac{\partial \overline{u_i \cdot u_j}}{\partial x_j}$

$\overline{\omega_j \cdot s_{ij}}$ → stretching/compress. that affects mean vorticity.

Boundary Layer



1st Turb. term: $-u_j \dfrac{\partial \omega_i}{\partial x_j}$ let $i=3$ $\rightarrow$ $-\dfrac{\partial(u_1\omega_3)}{\partial x_1} - \dfrac{\partial(u_2\omega_3)}{\partial x_2} - \dfrac{\partial(u_3\omega_3)}{\partial x_3}$

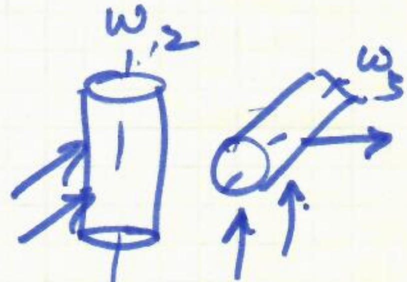
$\dfrac{\partial}{\partial x_3} \approx 0 \quad \dfrac{\partial}{\partial x_1} \approx 0$

so: $\boxed{-\dfrac{\partial(u_2\omega_3)}{\partial x_2}}$ 1st

2nd. term: $\omega_i s_{ij} \rightarrow \omega_2 \dfrac{\partial u_3}{\partial x_2} \rightarrow \boxed{\dfrac{\partial}{\partial x_2}(u_3\omega_2)}$ 2nd

N-S: $\dfrac{\partial(-\overline{u_1 u_2})}{\partial x_2}$

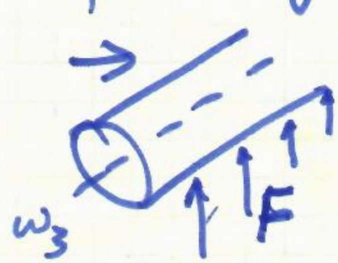
$\left.\begin{array}{l} u_2\omega_3 \\ u_3\omega_2 \end{array}\right\}$ "Forces"



"Gradients of Forces"

$\dfrac{\partial}{\partial x_2}(\overline{u_2\omega_3}) \rightarrow$

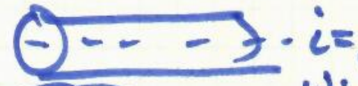
transport by u_2 of ω_3



"Torque to vortex tubes."

Stretching/compress:

$$\omega_j \frac{\partial u_i}{\partial x_j} \qquad i=j \Rightarrow$$



$\cdot\ i=j$

ω_j

$\omega_1 \frac{\partial u_1}{\partial x_1}$ – stretching

1st Term: $\underbrace{\frac{\partial}{\partial x_j}(\overline{\omega_i u_j})}_{\text{in } \Omega_i} \sim \frac{u^2/l}{l} \sim \boxed{u^2/l^2}$

2nd term: $\overline{\omega_j s_{ij}} \rightarrow \frac{\partial(\overline{\omega_j u_i})}{\partial x_j}$

?? scaling.

"dominant scale"

Vorticity $\longrightarrow$ influence ε (dissipation rate).

small scales.

$$\underset{\sim}{\varepsilon} \longrightarrow \nu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_j}{\partial x_i} \longrightarrow \text{expand}$$

$$\longrightarrow \boxed{2\nu\,\overline{s_{ij}s_{ij}}}$$

$$\rightarrow \frac{\partial u_j}{\partial x_i}\frac{\partial u_j}{\partial x_i} = \frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_j}$$

general: $2\,\overline{s_{ij}s_{ij}} - 2\,\overline{r_{ij}r_{ij}} = 4\,\dfrac{\partial^2 \overline{u_i u_j}}{\partial x_i \partial x_j}$

expand RHS : $\overline{\dfrac{\partial u_j}{\partial x_i}\dfrac{\partial u_i}{\partial x_j}}$

Re → large : ~ homogeneous $\dfrac{\partial^2 \overline{u_i u_j}}{\partial x_i \partial x_j} \to 0$

$$2\,\overline{s_{ij}s_{ij}} \simeq 2\,\overline{r_{ij}r_{ij}} = \frac{\varepsilon}{\nu}$$

$$r_{ij} = -\frac{1}{2}\epsilon_{ijk}\,\omega_k$$

$$\boxed{\underset{\sim}{\varepsilon} \simeq \nu\,\overline{\omega_i\omega_i}}$$ at Large Re.

scaling: $\overline{s_{ij}s_{ij}} \sim u^2/\lambda^2$ λ – Taylor microscope.

Re { λ → l, η }   "Measure" → $\underset{=}{u}^*$ → large scale fluct. vel.

$\boxed{\Omega_i\Omega_i}$ & $\boxed{\overline{\omega_i\omega_i}}$ → MKE & tke.

?? "entrophy"

$\Omega_i \Omega_i$ eqn.: Dot Ω_i eqn. with Ω_i

$$U_j \frac{\partial}{\partial x_j}\left(\tfrac{1}{2}\Omega_i\Omega_i\right) = \overset{①}{-\frac{\partial}{\partial x_j}\left(\Omega_i \overline{\omega_i u_j}\right)} + \overset{②}{\overline{u_j \omega_i}\,\frac{\partial \Omega_i}{\partial x_j}}$$

$$+ \overset{③}{\Omega_i \Omega_j S_{ij}} + \overset{④}{\Omega\,\overline{\omega_j s_{ij}}} + \overset{⑤}{\nu \frac{\partial^2}{\partial x_j \partial x_j}\left(\tfrac{1}{2}\Omega_i\Omega_i\right)}$$

$$\overset{⑥}{- \nu \frac{\partial \Omega_i}{\partial x_j}\frac{\partial \Omega_i}{\partial x_j}}$$

① Gradient Transport → redistribution by turbulence

② "Production" → opposite sign in $\overline{\omega_i \omega_i}$

③ Distorted by mean strain

④ Gain/Loss stretching/compression.
by turb. variables $\overline{\omega_j s_{ij}}$

⑤ viscous transport

⑥ viscous loss

$\Omega \to u/\ell$

1 → 4 terms: u^3/ℓ^3 large scale.

5, 6 → scaling ~~u^3/ℓ^3~~ $\nu \frac{1}{\ell^2}\left(\frac{u}{\ell}\right)^2 = \frac{u^3}{\ell^3}\underbrace{\left(\overset{1/Re}{\frac{\nu}{u\ell}}\right)}_{5\ \&\ 6}$

as Re ↑ visc. terms ↓
not important in dynamics of $\boxed{\Omega_i \Omega_i}$

$\overline{\omega_i \omega_i}$!

$$U_j \frac{\partial}{\partial x_j}\Big(\tfrac{1}{2}\underbrace{\overline{\omega_i \omega_i}}_{\text{enstrophy}}\Big) =$$

1. $-\overline{u_j \omega_i}\,\dfrac{\partial \Omega_i}{\partial x_j}$ → Gradient Production { loss by mean / gain by entrophy

2. $-\dfrac{1}{2}\dfrac{\partial}{\partial x_j}\left(\overline{u_j \omega_i \omega_i}\right)$ → Gradient Diffusion

3. → $\overline{\omega_i \omega_j s_{ij}}$ → prod. by stretching.

4. $+\ \overline{\omega_i \omega_j}\, S_{ij}$ → Gain/loss stretching by S_{ij}

5 $+\Omega_j\, \overline{\omega_i s_{ij}}$ → in $\Omega_i \Omega_i$ eqn. ("Mixed Production")

$+\ \nu \dfrac{\partial^2}{\partial x_j \partial x_j}\left(\tfrac{1}{2}\overline{\omega_i \omega_i}\right)$ → visc. diff.

$-\ \nu\, \overline{\dfrac{\partial \omega_i}{\partial x_j}\dfrac{\partial \omega_i}{\partial x_j}}$ → visc. dissipation

---

$$\nu \overline{\omega_i \omega_i} \sim \epsilon \sim \nu \frac{u^2}{\lambda^2}$$

scaling