Buoyancy Effects on Turbulence: 2

$$\frac{\partial q^2}{\partial t} = P + J_b - \epsilon$$

Simple shear flow, only $\partial U_1/\partial x_3 \neq 0$

now it's stratified → gradient of density, ρ

component eq'ns: $\overline{u_1 u_3}\, \partial U_1/\partial x_3$

$$\frac{\Delta \overline{u_1^2}}{\Delta t} = P - \epsilon/3 + \overline{p/\rho \frac{\partial u_1}{\partial x_1}}$$

$$\frac{\Delta \overline{u_2^2}}{\Delta t} = -\epsilon/3 + \overline{p/\rho \frac{\partial u_2}{\partial x_2}}$$

$$\frac{\Delta \overline{u_3^2}}{\Delta t} = J_b - \epsilon/3 + \overline{p/\rho \frac{\partial u_3}{\partial x_3}}$$

P - source $\quad \epsilon, J_b$ - sink

pressure-work terms redistribute energy between $\overline{u_1^2}, \overline{u_2^2}, \overline{u_3^2}$

since J_b acts only on vertical term

we might expect that $\overline{u_3^2}$ is preferentially suppressed at large scales — evidence from obs → revisit when we discuss spectra.

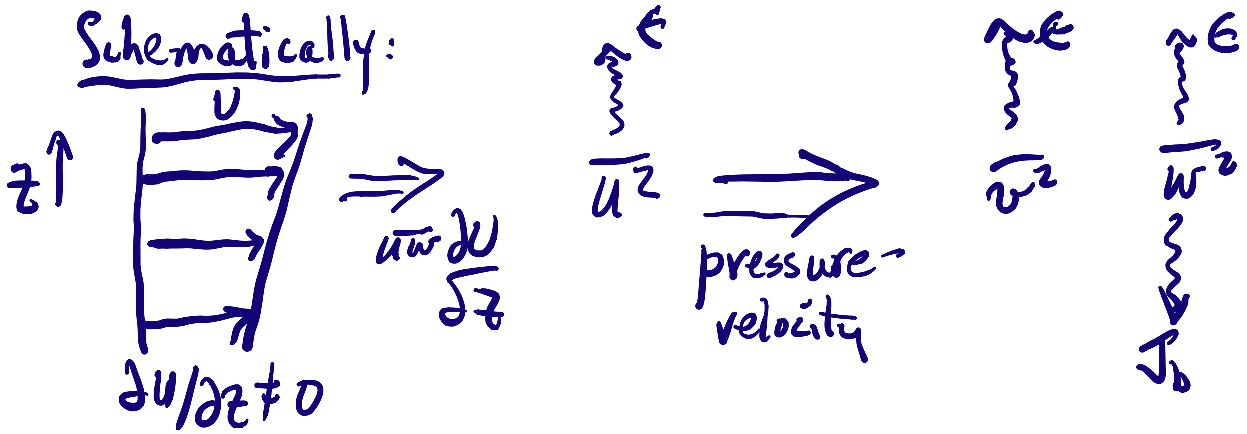
Convection: heating from below, not sheared

$\partial U_i/\partial x_j = 0$ all comp.

Then J_b is a source of tke → generates $\overline{u_3^2}$

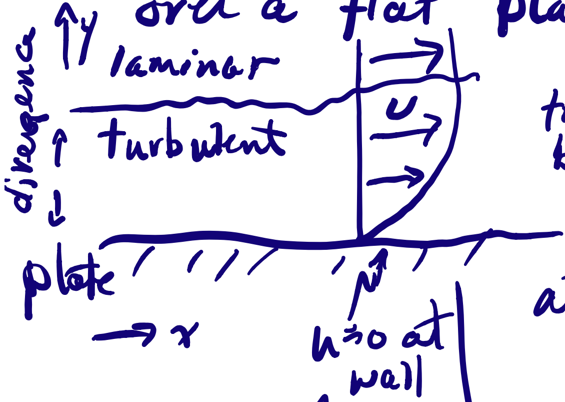
and pressure-velocity redistribute to $\overline{u_1^2}, \overline{u_2^2}$

Schematically:



consider transport (T), P, ϵ but not J_b yet

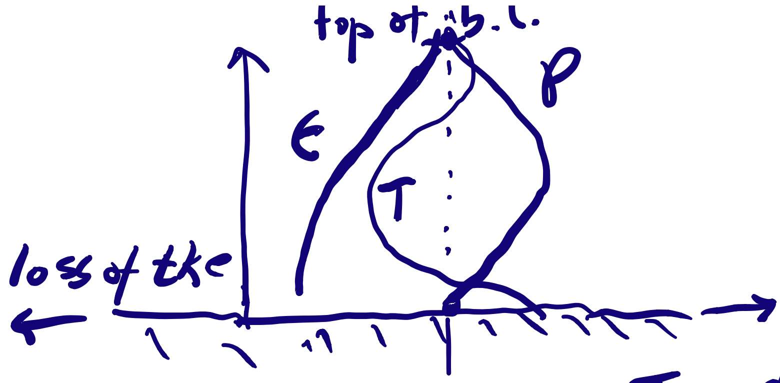
consider example (schematically) of boundary layer over a flat plate → more detail in Ch. 5



turbulent boundary layer → $U_j \frac{\partial \overline{q^2}}{\partial x_j}$

at top of b.c, $\frac{\partial U}{\partial y} = 0$, $P \to 0$ $\quad \overline{uv}\,\frac{\partial U}{\partial y}$

$\overline{uv} = 0$ at wall, viscous sublayer, $\overline{uv}\,\frac{\partial U}{\partial Y} \to 0$



TKE terms across the boundary layer.

T redistributes TKE spatially.

$$\int_0^{\delta} T\,dy = 0$$

$P = \epsilon$ not true at a point

$$\int_0^{\delta} P\,dy = \int_0^{\delta} \epsilon\,dy$$

Buoyancy length scale l_b, L_o ← Ozmidov

consider a fluid parcel of dimension l, velocity u in a stratified fluid

inertial force per unit mass → $F_I = \hat{u}\cdot\nabla\tilde{u} \sim u^2/l$ ← NL

if $u \sim (\epsilon l)^{1/3}$, $F_I = \dfrac{(\epsilon l)^{2/3}}{l} = \left(\dfrac{\epsilon^2}{l}\right)^{1/3}$

we integrate over the fluid parcel with volume $\sim l^3$, density ρ_o

$$F_I^V = \rho_o \int F_I\,dV \sim \rho_o l^3 \left(\frac{\epsilon^2}{l}\right)^{1/3} = \rho_o \epsilon^{2/3} l^{8/3}$$

buoyant force: $F_b = \delta\rho V g$ — $\delta\rho$ is the difference in ρ between parcel & surround.

define $N^2 = -\dfrac{g}{\rho}\dfrac{\partial \rho}{\partial z}$ ← a measure of stratification units s^{-2} → squared frequency

N → frequency → natural frequency of oscillation of internal wave

$N^2 \sim \frac{g}{\rho_0}\frac{\delta\rho}{l}$ → $\delta\rho = \frac{\rho_0 l N^2}{g}$

so $F_b \sim \rho_0 \frac{l N^2}{g} \cdot l^3 \cdot g = \rho_0 N^2 l^4$

This gives 2 scale-dependent estimates for F_b, F_I

in case buoyant forces = inertial forces

$$F_b = F_I$$

$$\rho_0 N^2 l^4 = \rho_0 \epsilon^{2/3} l^{8/3}$$

$$\boxed{l_b = (\epsilon / N^3)^{1/2}}$$ ← buoyancy length scale

at l_b in a stratified turbulence, buoyant forces ~~on~~ damp motion

scales >> l_b, turbulence is damped by F_B

<< l_b, should be free of buoyant damping

↳ locally isotropic turbulence

This scaling can be derived on dimensional grounds if ϵ, N are the only factors that determine u, ℓ ← energy-containing scales.

then $u_b = (\epsilon/N)^{1/2}$, $k_b = \frac{2\pi}{\ell_b} = (N^3/\epsilon)^{1/2}$

we will use ℓ_b to help in gaining empirical-theoretical solutions to time-dependent tke equation →

→ will arise later when we discuss spectra.