Boundary Layer flow viz from efluids gallery

http://www.efluids.com/efluids/gallery/gallery_pages/eddies_page.jsp

Wall Bounded Flow → new scaling requirements needed.

→ Solid surface interactions

→ NO-SLIP : $V_{surface} \to 0$: $\frac{\partial U}{\partial y}\Big|_{wall}$

near : τ_w dominates flow conditions

Surface : flat / curvature / 

$\frac{dP}{dx} \to f(x)$: vs. τ_w

$\tau_w \longrightarrow$ reducing mom. across entire flow.

↳ scale ?

Strain Rate → $S_{12} = f(x,y)$  $S_{12} = \frac{\partial U}{\partial y}$

· Generating vorticity at wall →

Chapt: 2 (2.5) → friction velocity : $\rho u_*^2 = T_{12}$ (wall)

dominated by $(-\rho \overline{u_1' u_2'})$

$$-\overline{u_1' u_2'} = u_*^2 \longrightarrow u_* = l \frac{dU_1}{dy}$$

length scale = ?

$l \sim y$

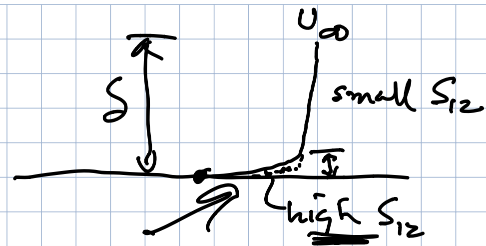
$$\frac{dU_1}{u_*} = \frac{dy}{l = y}$$

Const. → u_*

$$\boxed{\frac{U_1}{u_*} = \frac{1}{\kappa} \ln y + C}$$

Flow Conditions :

- no slip
- $U_1 \longrightarrow U_\infty$ ("far" from surface)
- turbulence supressed at "near wall"
- $\overline{u_1' u_2'}$ → effective mixer of mean Momentum.
  → S_{12} fairly small



"outer" $\frac{\partial U}{\partial y} \sim \frac{U_s}{\delta}$

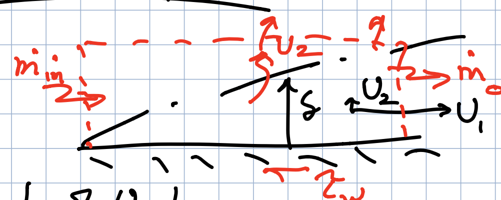
"inner" $\frac{\partial U}{\partial y} \sim \frac{U_s}{(\nu/U_s)} = \frac{U_s^2}{\nu}$

$$\frac{S_{inner}}{S_{outer}} = \left(\frac{U\delta}{\nu}\right) = Re_\delta \quad \text{as } Re_\delta \uparrow$$

- Nearly parallel flow: $U_1 >> U_2$
  $\frac{\partial U_1}{\partial x} << \frac{\partial U_1}{\partial y} \qquad \frac{\partial}{\partial x} << \frac{\partial}{\partial y}$
- Large Re_δ

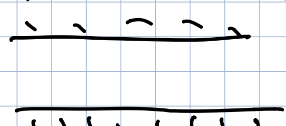
→ Two regions need to work together
"overlap region"

Test case:



nearly parallel

Re ↑ relative cont. of U_2 ↓



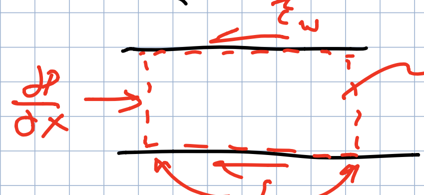
parallel plates

parallel flow $\frac{\partial U_2}{\partial x} = 0$

$U_2 = 0 \qquad \frac{\partial U_2}{\partial y} = 0$

Fully developed

N-S steady:
$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{\partial(\overline{u'v'})}{\partial y} + \nu\frac{\partial^2 U}{\partial y^2}$$
press — Re stress — visc. stress



CV: $2\tau_w(L\delta) = \Delta P H \delta$

Force Balance → $\boxed{\frac{\Delta P}{L} = \frac{2\tau_w}{H}}$

how does turb. affect needed ΔP to drive flow

→ $Re_\delta \uparrow$ then $\tau_w \uparrow$ because $S_{12_{inner}} \uparrow$?

T (total stress) distribution across flow

$\longrightarrow$ linear distribution

Scaling:

length: δ, $\dfrac{\nu}{U_S}$

outer, inner scaling

$U_S \rightarrow u_*$: related to τ_w : $u_* = \sqrt{\dfrac{\tau_w}{\rho}}$

Outer Region: S_{12} small

$(U_\infty - U) \rightarrow$ vel. deficit.

proposed $\Rightarrow$ $\boxed{\left(\dfrac{U_\infty - U}{u_*}\right) = f\left(\dfrac{y}{\delta}\right)}$

Closer to wall: "const. stress layer"

$$u_*^2 = \frac{\tau_w}{\rho} = -\overline{u_1' u_2'} = \nu_T \frac{\partial U}{\partial y}$$

$$\nu_T = \kappa\, u_*\, y$$

$$u_*^2 = \kappa u_* y \frac{\partial U}{\partial y} \rightarrow \boxed{\frac{U}{u_*} = \frac{1}{\kappa} \ln y + C}$$

κ = von Karman Const. $\rightarrow$ 0.41

use U (not deficit).