Similarity Soln – WAKE FLOW

$$f = \frac{U_\infty - U}{U_s} \qquad g = \frac{-\overline{u'v'}}{U_s^2}$$

momentum Constraint $\longrightarrow \dfrac{g}{f'} = -\dfrac{\nu_T}{U_s l} = -\dfrac{1}{R_T}$; $R_T = \dfrac{U_s l}{\nu_T}$

$U_s\, l = \text{const.}$ if $\nu_T \rightarrow \text{const.}$ $R_T \rightarrow \text{const.}$

$$g = -f'/R_T \qquad g' = \frac{-f''}{R_T}$$

$$\underbrace{R_T\left(\tfrac{1}{2}\tfrac{B}{A}U_\infty\right)}_{\substack{\text{parameter}\\ =\alpha}}(f + \xi f') + f'' = 0$$

$$f'' + \alpha \xi f' + \alpha f = 0$$

$\xi = 0$ (℄) $f = \dfrac{U_\infty - U_{℄}}{\underbrace{U_\infty - U_{℄}}_{U_s}} = 1$

$\xi = \infty$ $f \rightarrow 0$

Soln Form : $f = e^{(-\frac{1}{2}\alpha \xi^2)}$

as $\alpha \uparrow$ ($R_T \uparrow$) greater relative length scale

$\hookrightarrow$ $U_s l / \nu_T \rightarrow$ greater spreading.

$\hookrightarrow \rightarrow \sim$ const.

Alternative Scaling

- $u_* = (\overline{u_1' u_2'})^{1/2}$
- Strain Rate $\longrightarrow$ length scale:

$f'_{max} \longrightarrow$ max strain rate

$$f' = \frac{l}{U_s}\frac{dU}{dy} \longrightarrow f'_{max} = \frac{l}{U_s}\left(\frac{dU}{dy}\right)_{MAX}$$

$$f'_{max} = \frac{l}{l_*}$$ — length scale associated with strain rate

$l_* \downarrow$: $f'_{max} \uparrow$

$$\nu_T = \frac{u_*^2}{(\partial U/\partial y)} = u_*^2 \Big/ \left(\frac{U_s}{l_*}\right)$$

$$\frac{u_*^2}{U_s^2} = \frac{\nu_T}{U_s l_*} = \left(\frac{1}{R_T}\frac{l}{l_*}\right)$$

if $\delta'_{max} = \text{const} \rightarrow \frac{l}{l_*} \rightarrow \text{const.}$

$\frac{u_*}{U_s} \rightarrow \text{const}$ then: $\frac{\nu_T}{U_s l_*} \cdot \frac{U_s}{u_*} = \frac{\nu_T}{u_* l_*} = \text{const}$

$$\left(\frac{u_* l_*}{\nu_T}\right) \rightarrow \text{const.}$$

Wake the budget:

$$\underset{①}{U \frac{\partial}{\partial x_1}\left(\tfrac{1}{2}q^2\right)} + \underset{②}{\overline{uv}\frac{\partial U}{\partial x_2}} + \underset{③}{\frac{\partial}{\partial x_2}\left(\overline{v\left(\tfrac{1}{2}q^2 + \tfrac{p}{\rho}\right)}\right)} = \underset{④}{\varepsilon}$$

$q^2 = u_i u_i$

① advection ② Production ③ Diffusion ④ dissipation

1 — assume $P \sim$ dissipation $\sim u^3/l$

balance ② — ④

then ① & ③ should balance

Advection: $\frac{U}{L}\left(\frac{1}{2}q^2\right) \sim \frac{U}{L}u^2$; $\frac{U}{L} \sim \frac{u}{l}$

term : $\left(u^3/l\right)$

Diffusion : (3) $\frac{\partial}{\partial x_2}\left(\overline{v\left(\tfrac{1}{2}q^2 + \tfrac{p}{\rho}\right)}\right)$

$$\overline{v\left(\tfrac{1}{2}q^2\right)} \sim \overline{vu^2} \sim \nu_T \frac{\partial}{\partial x_2}\left(\tfrac{1}{2}q^2\right)$$

$$\frac{\partial}{\partial x_2}\left(\nu_t \frac{\partial}{\partial x_2}\left(\tfrac{1}{2}q^2\right)\right) \sim \frac{1}{l}\left((ul)\frac{1}{l}u^2\right) = u^3/l$$

model

expect each term significant contribution