Scaling the enstrophy eqn. ($\overline{\omega_i \omega_i}$)

1. $\overline{u_i \omega_j} \frac{\partial \Omega_i}{\partial x_j} \sim \left(\frac{u^2}{l}\right)\left(\frac{u}{l^2}\right) = u^3/l^3$

• $\overline{u_i \omega_j}$ is part of large scale effects in Re stress term affecting mean (large) momentum

5. $\overline{\omega_j S_{ij}}$ or $\Omega_j \frac{\partial}{\partial x_j}(\overline{\omega_j u_i})$ (since $\nabla \cdot \omega = 0$ & $\omega_j r_{ij} = 0$)

$= \frac{u}{l}\left(\frac{1}{l}\frac{u}{l}u\right) \sim u^3/l^3$

• term is in Ω_i eqn. & is change of Ω_i by stretching & rotation of ω_i

4. $\overline{\omega_i \omega_j} S_{ij}$ expect $\overline{\omega_i \omega_i} \sim u^2/\lambda^2$ since $\sim \epsilon$. but if $i \neq j$ what is interation with S_{ij} ($\sim \frac{u}{l}$)

$\frac{u^2}{\lambda^2} \cdot \frac{u}{l} = u^3/\lambda^2 l$? but let's try to account for time scale difference of terms: $\overline{\omega_i \omega_j}$: $\tau \sim (\lambda^2/u^2)^{1/2}$ (high freq.)

S_{ij} : $\tau \sim (1/u)$ (Low freq.)

So we reduce the interactions of these two terms by ratio of time scales: $\frac{\lambda}{l}$

new scaling: $\frac{u^3}{\lambda^2 l} \cdot \frac{\lambda}{l} = \frac{u^3}{\lambda l^2}$

3. $\overline{\omega_i \omega_j s_{ij}} \sim \frac{u^2}{\lambda^2} \frac{u}{l} = \frac{u^3}{\lambda^2 l}$ — but again use time scale diff. l/λ:

SCALE $\sim u^3/\lambda^3$

here we have increased the interaction saying that $s_{ij} \sim u/\lambda$ as it interacts with $\overline{\omega_i \omega_j}$

Summary of scales for $\omega_i \omega_i$ terms

1 ~ u^3/l^3

2 ~ neglect since diffusion term

3 ~ u^3/λ^3 small scales interact strongly

4 ~ $\frac{u^3}{\lambda l^2}$

5 u^3/l^3 large scales

6. visc. diffusion neglect

7. $\nu \overline{\frac{\partial \omega_i}{\partial x_j} \frac{\partial \omega_i}{\partial x_j}} \rightarrow ?$

- option: $\sim \left(\frac{u^2}{\lambda^2}\right) \frac{1}{\delta^2}$ where we say $\omega \sim \frac{u}{\lambda}$

but the derivatives scale at some unknown scale δ.

- Since the largest term so far is u^3/λ^3 (#3)

then we may say that viscous term should ~~balance~~ this:

$$\overline{\omega_i \omega_j S_{ij}} \sim \nu \overline{\frac{\partial \omega_i}{\partial x_j} \frac{\partial \omega_i}{\partial x_j}}$$

$$u^3/\lambda^3 \sim \nu \frac{u^2}{\lambda^2 \delta^2} \rightarrow \delta \sim \left(\frac{\nu \lambda}{u}\right)^{1/2}$$

or $\frac{\delta}{\lambda} \sim Re_\lambda^{-1/2}$ (δ is smaller than λ ∴ rather close to η)

Taylor Microscale:

- We will see how this is related to $\overline{u_i u_j}$ Correlation function in Chapt. 6.

but if we assume $\varepsilon \sim 15\nu \underbrace{\overline{s_{ij}s_{ij}}}_{u^2/\lambda^2}$ u - large, λ - ?

and $\varepsilon = u^3/l$

then $u^3/l \sim 15\nu \dfrac{u^2}{\lambda^2}$

$$\boxed{\frac{\lambda}{l} \sim Re_l^{-1/2}}$$

and $\dfrac{\lambda}{\eta} = \dfrac{\lambda}{l} \cdot \dfrac{l}{\eta} \sim Re_l^{-1/2} \, Re_l^{3/4}$

$$\boxed{\frac{\lambda}{\eta} \sim Re_l^{1/4}}$$

$$\boxed{\frac{l}{\eta} \sim Re_l^{3/4}}$$

<u>see progression of powers.</u>

- $\frac{l}{\eta} \to$ greatest separation as $Re \uparrow$
- $\frac{\lambda}{\eta} \to$ not so great separation
- $\frac{l}{\lambda} \to$ medium separation

<u>So</u> for $\overline{\omega_i \omega_i}$ we scale $\omega \to u/\lambda$

<u>Also</u>: $\dfrac{\lambda}{l} \sim Re_\lambda^{-1}$ & $\dfrac{\lambda}{\eta} \sim Re_\lambda^{1/2}$ so $\dfrac{\delta}{\eta} = Re_\lambda^{-1/2} Re_\lambda^{1/2}$

or $\delta \sim \eta$!!

"Philosophy" of scaling:

- try to determine physical mechanism involved (dissipation, interaction with mean flows, etc.)

- look at terms that have similar components in other eqns. & interpret their physics

- Adjust scales based on potential for interactions due to scale imbalances — this can weaken the interactions.