begin with discussion of Gargett, Osborn + Nasmyth 1984

then to discussion of

Scalar Spectra: $\theta, S, O_2 \ldots$

reconsider balance previously discussed.

$$-\overline{\theta u_j}\frac{\partial \Theta}{\partial x_j} = \gamma \overline{\frac{\partial \theta}{\partial x_j}\frac{\partial \theta}{\partial x_j}}$$

gradient production = thermal dissipation of $\overline{\theta^2}$

define a thermal Taylor microscale $\overline{\left(\frac{\partial \theta}{\partial x_j}\right)^2} = \frac{\overline{\theta^2}}{\lambda_\theta^2}$

scale $\theta u \frac{\theta}{l} \sim \gamma \frac{\theta^2}{\lambda_\theta^2}$

using: $\left(l/\lambda\right)^2 \sim \frac{ul}{\nu} \rightarrow \frac{\lambda}{\lambda_\theta} \sim \left(\frac{\nu}{\gamma}\right)^{1/2} = \sqrt{Pr}$

to get at dissipative scales.

recall balance eq. for $\overline{\omega_i \omega_i}$ $\quad \overline{\omega_i \omega_j \Delta_{ij}} = \nu \overline{\frac{\partial \omega_i}{\partial x_j}\frac{\partial \omega_i}{\partial x_j}}$

if $\omega_i \sim u/\lambda$, $\Delta_{ij} \sim u/\lambda$

then the only way LHS = RHS, is if $\frac{\partial \omega_i}{\partial x_j} \sim \frac{u}{\lambda \eta}$ T+L P92

apply same argument to the temperature gradient variance eqn. $\overline{\frac{\partial\theta}{\partial x_j}\frac{\partial\theta}{\partial x_j}}$

$$-\overline{\frac{\partial\theta}{\partial x_j}\frac{\partial\theta}{\partial x_j}\Delta_{ij}} = \gamma\overline{\frac{\partial^2\theta}{\partial x_j \partial x_j}\frac{\partial^2\theta}{\partial x_j \partial x_j}}$$

scaling $\frac{\theta^2}{\lambda_\theta^2}\frac{u}{\lambda} \sim \gamma\frac{\theta^2}{\lambda_\theta^2\eta_\theta^2}$ ← smallest fluctuations in θ

$\epsilon = \nu\overline{\Delta_{ij}\Delta_{ij}} \sim \nu u^2/\lambda^2 \Rightarrow u/\lambda \sim (\epsilon/\nu)^{1/2}$ ← strain rate

$\eta = (\nu^3/\epsilon)^{1/4} \rightarrow \eta_\theta = \left(\frac{\gamma^2\nu}{\epsilon}\right)^{1/4}$

↑ Batchelor scale.

$$\eta/\eta_\theta \sim \sqrt{\nu/\gamma} = \sqrt{P_r}$$

what are the small scales of scalar fluctuations

$\eta = (\nu^3/\epsilon)^{1/4}$

$\eta_\theta = (\gamma^2\nu/\epsilon)^{1/4}$ $K_B = 1/\eta_\theta$ ← Batchelor wave #

smallest salinity $\eta_s = \left(\frac{\beta^2\nu}{\epsilon}\right)^{1/4}$ β - molecular diffusivity for salt in water

in the ocean $\gamma \sim 1.4\times 10^{-7}\, m^2/s$

$\beta \sim (0.7 - 1.7)\times 10^{-9}\, m^2/s$

$$\nu \sim (0.5 - 1.7) \times 10^{-6}\ m^2/s$$

$Pr = \nu/\gamma \sim 4 - 12$ (c.f. 0.7 in air)

Schmidt # $= \nu/\beta \sim 300 - 2400$

$\gamma/\beta \sim 100$

$\eta/\eta_\theta = \sqrt{Pr} \sim 3$; $\eta/\eta_s = \sqrt{Sc} \sim 30$

where $\eta \sim 1cm$, $\eta_\theta \sim$ few mms, $\eta_s \sim$ few $\frac{1}{10}$ mms.

Scalar Spectra

passive scalars – they are not dynamically significant
heat → buoyancy ⇝ that buoyancy is small enough not to be a factor

define a spectrum s.t. $\frac{1}{2}\overline{\theta^2} = \int_0^\infty E_\theta(k)\,dk$

if Re is large enough so there exists an equilibrium range in the spectrum, then there should also be one in the spectrum of θ variance, since it is the turbulent motions that mix $\overline{\theta^2}$

defined $\chi \equiv 2\gamma \overline{\frac{\partial\theta}{\partial x_j}\frac{\partial\theta}{\partial x_j}}$ ⇝ T+L $N \equiv \gamma \overline{\frac{\partial\theta}{\partial x_j}\frac{\partial\theta}{\partial x_j}}$.

Equilibrium range

$E_\theta(k)$ should scale with whatever scales
the $\rightarrow \epsilon, \nu$
plus should scale with whatever scales
$\overline{\theta^2} \rightarrow \chi, \gamma$

$$E_\theta(k) = E_\theta(k, \epsilon, \nu, \gamma, \chi)$$

a way to combine variables is

$$E_\theta(k) = \underline{\chi \epsilon^{-1/3} k^{-5/3}}\ \underline{f(k\eta, \sigma)} \qquad \underline{\sigma = \nu/\gamma}$$

Pr

presence of σ suggests a different form
of $E_\theta(k)$ for $\sigma > 1$, $\sigma < 1$

Inertial-Convective Subrange IC range:

if Re large enough for the inertial subrange
and γ small enough to not affect scales in inertial
subrange $\rightarrow$ then θ is simply advected around by
the turbulent eddies
convected

$\rightarrow$ should be independent of γ, ν

$$E_\theta(k) = E_\theta(k, \chi, \epsilon)$$

dimensionally, $E_\theta(k) = \beta \chi \epsilon^{-1/3} k^{-5/3} \rightarrow \beta \sim 0.5$

what happens at smaller scales is determined by σ

$$-\frac{\partial\theta}{\partial x}\frac{\partial\theta}{\partial x_j}\Delta_{ij} = \gamma\,\overline{\frac{\partial^2\theta}{\partial x_j\partial x_j}\,\frac{\partial^2\theta}{\partial x_j\partial x_j}}$$

stretching/squeezing of $\partial\theta/\partial x_j$

diffusion proceeds rapidly

$\sigma > 1$, $\gamma < \nu$ — thermal diffusion weaker than momentum diffusion $\rightarrow$ diffusion of $\overline{\theta^2}$ occurs at scales $< \eta$

$\overline{\theta^2}$ field is exposed to entire spectrum of strain rate fluctuations $\Delta_{ij} \sim (\epsilon/\nu)^{1/2}$

$\sigma < 1$, $\gamma > \nu$ — smallest fluctuations of $\overline{\theta^2}$ not exposed to the full spectrum of Δ_{ij}

in that case ν doesn't matter

only way to scale $\Delta_{ij} \sim (\epsilon/\gamma)^{1/4}$

use these limits on Δ_{ij} in $\overline{\frac{\partial\theta}{\partial x_j}\frac{\partial\theta}{\partial x_j}}$, scale

$\sigma > 1$, $\eta_\theta = \left(\frac{\gamma^2\nu}{\epsilon}\right)^{1/4}$ ← we did this above

$\sigma < 1$, $\eta_\theta = \left(\gamma^3/\epsilon\right)^{1/4}$

$\sigma < 1$, $\eta_\theta > \eta$ $\rightarrow$ Hg, $\sigma \sim .02$
liquid metals

$\sigma > 1$, water, $\sigma \sim 7$, Si oil, $\sigma \sim 200$

$\sigma < 1 \rightarrow$ Inertial - Diffusive Subrange.
if $\sigma \ll 1$ then there is a large scale range
between η, η_θ
spectral transfer of tke is ϵ
net effect is a narrow IC range

$\sigma > 1$ 2 new ranges

① Viscous - convective Subrange VC range.

$\eta > k^{-1} > \eta_\theta$ Scales of $\overline{\theta^2}$ are reduced by s_{ij} but thermal diffusion slow until $k \gg \eta$

$$E_\theta = E_\theta(k, \chi, (\epsilon/\nu)^{1/2})$$

dimensionally $E_\theta = c\,\chi\,(\nu/\epsilon)^{1/2} k^{-1}$ — VC form

② Viscous - Diffusion Range

diffusion is important and spectrum of $\overline{\theta^2}$

rolls off — attenuates ←