Turbulent Dispersion contd

references Ch.7 T+L, Taylor (1921), Garrett (2006)

based on a Lagrangian description of the flow

- rate of dispersion of particles into an already existing turbulent flow.

<u>example</u> - Smoke particles/pollutants injected into the atmos. boundary layer
- purposeful dye injection into ocean

integral time scale based on autocorrelation or de-correlation of a signal with itself at some time lag, Δt

at $\Delta t = 0$, signals are perfectly correlated, $\rho(\Delta t) = 1$

as $\Delta t \uparrow$ the signals de-correlate, $\rho(\Delta t) \to 0$

the integral time scale $T = \int_0^\infty \rho(\Delta t)\, d(\Delta t)$

Taylor's diffusion equation

$$\overline{y^2}(t) = 2\overline{v^2}\int_0^t dt' \int_0^{t'} \rho(\Delta t)\, d(\Delta t)$$

$v(t) = dy/dt$, Lagrangian velocity of a particle at time t

at small t, $\rho \sim 1$ so $\overline{y^2}(t) = 2\overline{v^2}\, t^2/2$

at large t, $\int_0^\infty \rho(\Delta t)\, d(\Delta t) = T$

$$\overline{y^2}(t) = 2\overline{v^2}\,T t$$

$$y_{rms} = v\sqrt{2Tt} \qquad\qquad y_{rms} = vt$$

rapid initial dispersion of a fluid parcel injected into the turbulence when the scales of the parcel are small c.f. turbulence. later times, scales are $\simeq$ slower

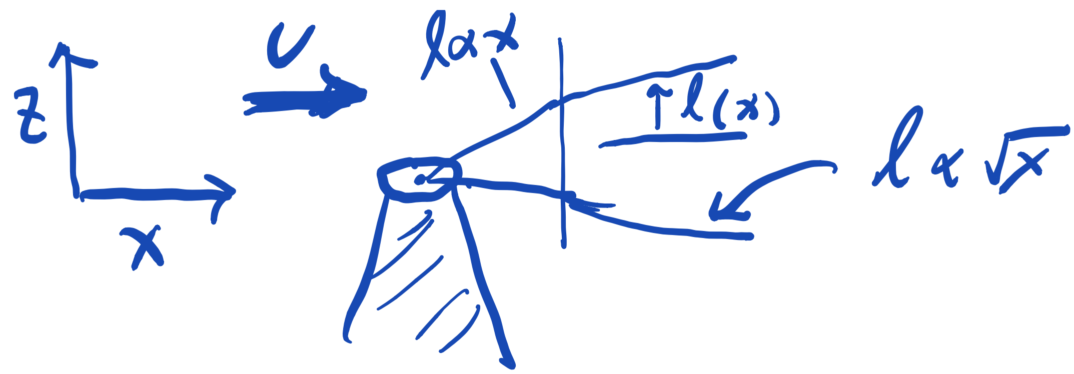
Note: we can define an integral length scale L via correlation of a velocity with itself separated in space

when Δx is smaller than L, turbulent eddies are large c.f. fluid parcel

when Δx is $\gtrsim L$ parcel is the size of large eddies

Smoke Plume in a Wind consider dispersion of smoke plume in a mean wind U that is also turbulent

by Taylor's frozen flow hypothesis $x \sim Ut$



Estimate for Diffusivity

$$K_m \equiv \frac{1}{2}\frac{d\overline{y^2}}{dt}$$

this is like the spreading rate of y

small $t \rightarrow K_m \sim \frac{1}{2}\frac{d}{dt}(v'^2 t^2) = v'^2 t \,, \quad t \ll T$

large $t \rightarrow K_m \sim \frac{1}{2}\frac{d}{dt}(2v'^2 T t) = v'^2 T \,, \quad t \gg T$

a constant at long times

at short times, an initial burst of diffusion

The Spectrum of Turbulence

- practical importance - statistical tool to quantify turbulence
- by scaling spectra → tke, ϵ, χ, Re stress, ...
- start with a discussion energy cascade
- define different forms of energy/dissipation spectra

- Kolmogorov's hypotheses → dimensional reasoning
  physical interpretation + how they lead to a useful form of energy spectrum
- Experimental verification of Kolmogorov spectrum
- Buoyancy effects on the spectrum.
- Scalar spectra → θ high $Pr = \nu/\gamma$, T, S in water

  low Pr, Hg, liquid metals

Role of Vorticity

when we discussed vorticity dynamics we learned
1.) vortex stretching causes a scale change
2) vortex stretching ↑ $\overline{\omega^2}$ overall, dominated by increased $\overline{\omega^2}$ at smallest scales
3) vortex stretching ↑ tke at small scales

net effect ⇒ scale change to small scales where tke ↑ + cascade from large to small scales

If we consider a spectrum of eddies of size $2\pi/k_m$

⇒ eddies only experience strain rate of larger eddies.

Strain rate of large eddies $\sim u/l$

Strain rate of smaller eddies $\sim u/\lambda$ ← def'n of Taylor μscale.

$$u/\lambda \gg u/l$$

So we anticipate that the strain rate spectrum $\uparrow \; \overline{w} \; k$

if $E(k) \propto \phi_{uu}(k) \propto k^{-5/3}$.

turb spectrum

then $\Delta(k) = \phi_{u_x u_x}(k) \propto \phi_{\Delta_{ij}\Delta_{ij}}(k)$

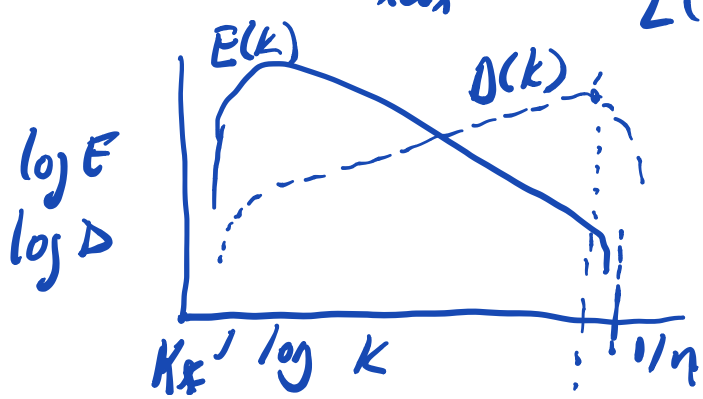
$$= k^2 \phi_{uu}(k) \propto k^{+1/3}$$

$$F_k\left[\frac{\partial g}{\partial x_j}(\tilde{x})\right] = i k_j g(\tilde{k})$$ ← property of Fourier transform

dimensionally: $\phi_{uu}(k) \rightarrow [(m/s)^2/cpm]$

$$k^2 \phi_{uu}(k) \rightarrow [(1/s)^2/cpm]$$

$$\phi_{u_x u_x}(k) \rightarrow [(1/s)^2/cpm]$$



K_F - largest scale determined by forcing, boundaries

$K_{\eta} / \frac{1}{\eta} \propto R_e^{-3/4}$

Eddies of a particular size are strained by larger eddies only

Since the highest strain is associated with smallest scales, we expect that it is the strain rate of eddies which are larger, but close in size that have biggest influence.

That is, most of the energy crossing a given k comes from eddies of slightly smaller k and goes to eddies of slightly larger k

Analogy to CASCADE ~ or a series of waterfalls

This also implies that neither largest nor smallest scales has a direct effect on energy transfer at intermediate scales.

Turbulence Spectra → Definitions

Turbulent flows vary randomly in space/time

To fully describe the flow → 4D spectrum
3 wave#, frequency

Experiments usually time series at a point
or spatial series

examples → Sonic anemometer on a tower in the atmospheric b.l. – fixed

↳ frequency spectrum

↳ Taylor's ff → k spectrum

$k = V/f$

spatial series → array of probes at various spacing

If turbulence is stationary (for a time series) or homogeneous (spatial series), then we can derive a frequency or 1D waveno. spectrum.

We need to interpret this in terms of a 3D spectrum