Congressional Districting Project: New Mexico

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Course: Operations Research (IEM 4013)

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Executive Summary

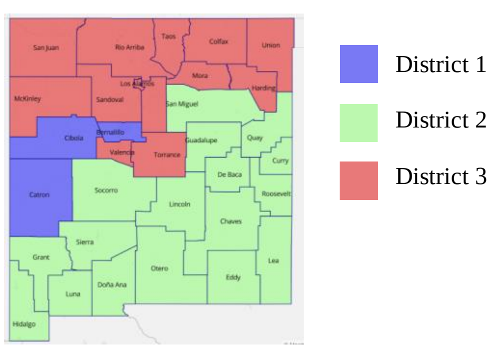
The purpose of this project is to create the most optimal congressional map of New Mexico using Python and Gurobi while following specific federal and state redistricting criteria. Some of the federal and state criteria to be followed are creating 3 contiguous districts without considering political affiliation, with a deviation of less than 1%, minimizing compactness, and taking into account county boundaries and minority representation.

To achieve this goal, our team developed two different Python models using the 2020 Federal census data. The first model minimized the cut edges, which refers to the boundary between two counties of different districts and is a way to measure compactness. The second model was slightly different, as it minimized the moment of inertia of the districts, which can be used to measure how spread out a district is. It used the population as mass and the distance between counties to reach compactness. Of course, both models followed the aforementioned criteria.

Our models will be represented in three different ways: a summary of the math and concepts behind our models, an integer program explaining each constraint in both words and math, and the Python code. Upon obtaining solutions, both models will be compared in different areas (population deviation, compactness, minority representation, etc.) to obtain the best solution.

Upon calculation, both models produced a solution. Overall, our optimal redistricting program provides a solution with a population deviation of .101% (2,142 people), following all other criteria as well. This model has many advantages in terms of compactness, minority representation, and the use of whole counties. These advantages can be quite useful in the political world, ensuring political equality and freedom. Considering the importance of districting maps and the impact they can have, our project can be used to illustrate how complex and important redistricting truly is in today's world.

Shown below is an image of our proposed model.



I. Introduction

Every ten years in the United States, the United States Census Bureau conducts a census, designed to count every resident in the United States. This census is mandated by Article I Section 2 of the United States Constitution. According to the U.S. Census Bureau, "[The census] is also used to draw the lines of legislative districts and reapportion the seats each State holds in Congress" (US Census Bureau). This census is at the core of our democracy, ensuring that each citizen is equally represented and cared for. After each decennial census, congressional district lines are redrawn for each state, according to the new apportionment of states' electoral votes.

These maps are supposed to ensure each citizen receives a fair vote and a voice in the future of the nation. However, politicians often take advantage of the system, by approving congressional maps that exploit the system. According to Fair Vote, "Redistricting encourages manipulation of our elections by allowing incumbent politicians to help partisan allies, hurt political enemies and choose their voters before the voters choose them." The website continues to explain that the process we use now allows boundaries to protect incumbents and reduce competition, rather than ensuring fair representation.

Considering this political coercion, it is vital to ensure that congressional maps are drawn to accurately represent each state's population, diverse communities, and their unique perspectives. As such, our group has been tasked with creating a congressional redistricting map for the state of New Mexico.

Current Maps:

Following the 2010 census, New Mexico had three congressional districts, and a new map was drawn. This map (shown below) is the current map used by New Mexico.

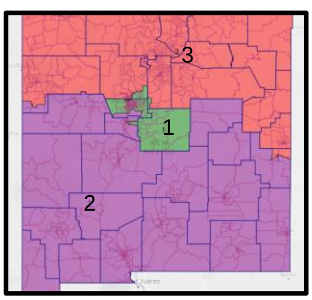


Figure 1.1: 2010 New Mexico Congressional Map

At the time, the state's population was 2,059,179 people, meaning that the ideal population for each district was 686,393. For this map, each district actual had the ideal population. ("DRA 2020"). This means that for the 2010 map, there was a 0.0% population deviation.

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Following the 2020 census, the total population of New Mexico was found to be 2,117,522 people, and the state continued to have three congressional districts. Shown is an image from Dave's Redistricting of the proposed updated congressional district map following the census.

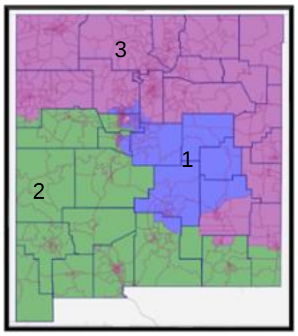

Figure 1.2: 2021 New Mexico Congressional Map

This map has the following deviations: District 1 (blue) by a population of -9, District 2 (green) by a population of 5, and District 3 (purple) by a population of 3. This yields a total deviation of 14 people, or 0.00066%. Governor Michelle Lujan Grisham signed and approved this map on December 16, 2022. She believes it establishes a "reasonable baseline for competitive federal elections, in which no one party or candidate may claim any undue advantage" (Lee). This map shows contiguous precincts but is less compact than the 2010 map. It has strong minority representation, as the 2nd District has a large Hispanic population.

II. Criteria

For a congressional districting map to be considered valid, it must adhere to a set of both federal and state criteria. These criteria can be found on the National Conference of State Legislatures (NCSL) website. Criteria are outlined in the next following sections.

United States Federal Criteria:

All states must adhere to the requirements outlined in the constitution relating to population and anti-discrimination. According to NCSL, The Apportionment Clause of Article I Section 2 of the U.S. Constitution requires that all districts be as nearly equal in population as practicable, meaning they must have as close to the same number of people as possible. Additionally, if the variance is higher than easily acceptable, it must be justified by a consistent state policy. It is also required that the congressional districts be based on population counts that include the total number of people in each state and are as recent as available. Additionally, the Equal Protection Clause of the 14th Amendment to the U.S. Constitution requires that districts be substantially equal. This clause states that, "No State shall make or enforce any law which shall abridge the privileges or immunities of citizens of the United States." Because of this clause, states must ensure that their congressional districts are as close to equal as possible to avoid scrutiny.

In addition to population equality, Section 2 of the Voting Rights Act of 1965 requires adequate minority representation among congressional districts. This clause states,

> "No voting qualification or prerequisite to voting or standard, practice, or procedure shall be imposed or applied by any State or political subdivision in a manner which results in a denial or abridgement of the right of any citizen of the United States to vote on account of race or color."

Although compactness and contiguity is not required by federal criteria, it is highly encouraged at this level. Left up to the states, a narrow majority has written some type of contiguity regulations in their respective state constitutions or statues. Other principles adopted by many states include preservation of counties or other political subdivisions, preservation of communities of interest, preservation of cores of prior districts, avoidance of paired incumbents.

New Mexico State Criteria:

According to the National Conference of State Legislatures, it is required by New Mexico that the congressional districts be: "compact, contiguous, preserve political subdivisions, and preserve communities of interest." The NCSL also states that New Mexico is allowed to "preserve cores of prior districts and avoid pairing incumbents."

The New Mexico State Legislative Council Service has published guidelines for the development of state and congressional redistricting plans. These guidelines are as follows:

1. Congressional districts shall be as equal in population as practicable.
2. State districts shall be substantially equal in population; no plans for state office will be considered that include any district with a total population that deviates more than plus or minus five percent from the ideal.
3. The legislature shall use 2020 federal decennial census data generated by the United States bureau of the census.
4. Since the precinct is the basic building block of a voting district in New Mexico, proposed redistricting plans to be considered by the legislature shall not be comprised of districts that split precincts.
5. Plans must comport with the provisions of the Voting Rights Act of 1965, as amended, and federal constitutional standards. Plans that dilute a protected minority's voting strength are unacceptable. Race may be considered in developing redistricting plans but shall not be the predominant consideration. Traditional race-neutral districting principles (as reflected in paragraph seven) must not be subordinated to racial considerations.

6. All redistricting plans shall use only single-member districts.
7. Districts shall be drawn consistent with traditional districting principles. Districts shall be composed of contiguous precincts, and shall be reasonably compact. To the extent feasible, districts shall be drawn in an attempt to preserve communities of interest and shall take into consideration political and geographic boundaries. In addition, and to the extent feasible, the legislature may seek to preserve the core of existing districts, and may consider the residence of incumbents.

III. Problem Statement

Our objective is to create a congressional district map that maximizes compactness, while adhering to both state and federal criteria. These models shall contain three contiguous districts, drawn without considering political affiliation of region or incumbency to avoid bias. Upon producing our models, we plan to evaluate the minority representation within the district, to ensure that all parties are rightly represented. Additionally, since New Mexico is one of the few states in the US that can be divided into valid districts by keeping counties whole, we plan to use whole counties in our model. Counties are an important geographic division, so keeping them whole can help heighten people's identity as a member of their respective county.

IV. OR Model (in words)

Our team decided to construct two different models. One such model will minimize cut edges and the other will minimize moment of inertia. Both models impose the necessary corresponding constraints for population deviation and contiguity. These models are summarized and explained below. Relative advantages and disadvantages of each solution will be analyzed in later sections.

Minimize Cut Edges (Model 1):

The objective of Model 1 is to minimize cut edges. In this case, an "edge" is a boundary in which one county neighbors another county. More generally, an edge is a connection between any two nodes. For example, edge (1,2) would be the boundary where node 1 neighbors node 2. A "cut edge" is a boundary across which two nodes are separated into two different regions.

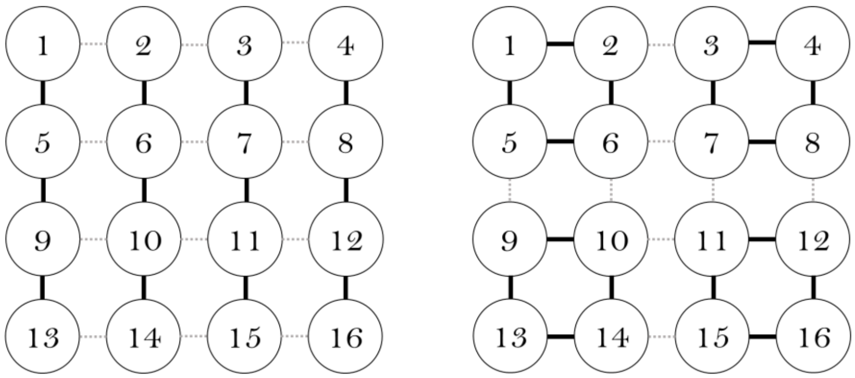

Figure 4.1: Cut Edges Diagram

For example, Figure 4.1 depicts 16 nodes, each having 24 edges between them. The gray, dashed-lines represent a cut edge. If we want to divide the 16 nodes into four different regions, there are several different strategies to do so. The diagram on the left shows the nodes divided into four columns. In this scenario, there are 12 cut edges, splitting the 16 nodes into their respective regions. In the diagram on the right, the nodes are divided into four squares, and there are only eight cut edges. Notice that the diagram on the right has much more compact regions. Generally speaking, when a set of nodes has fewer cut edges, that produces more compact regions (Buchanan).

In congressional districting, a cut edge is a boundary between two counties of a different district. Using principles of cut edges, we can form compact congressional districts by minimizing the number of cut edges.

In order to create a valid model, it must adhere to a certain set of constraints. For example, each county must belong to one and only one district. Additionally, the districts must have a relatively low population deviation. Ideally, each district would have an equal population. For New Mexico, this would mean the target value of each district is one-third of the state's population because New Mexico has three districts. Oftentimes, an acceptable population deviation is considered to be at most 1%. This means that each district must be within ± 0.5% of the target value. Therefore, we will impose constraints saying that each district's population must fall within this bound.

Additionally, we will apply contiguity constraints which ensure that every district is one, continuous, uniform body where every county of that district neighbors another county within the district. To accomplish this, we will define each district as having flow that originates in a district center or "root." This flow can travel across any edge that is not cut. If the districts are continuous, then every county in the district should be able to consume some flow. We will set constraints to ensure that every county in each congressional district is consuming flow from the root.

Minimize Moment of Inertia (Model 2):

In our second model, the objective is to minimize the moment of inertia. As explained by the OpenStax textbook:

> "The moment of inertia is the quantitative measure of rotational inertia, just as in translational motion, and mass is the quantitative measure of linear inertia—that is, the more massive an object is, the more inertia it has, and the greater is its resistance to change in linear velocity."

In reference to "Two MIP's for redistricting," the moment of inertia model treats the each district as a rigid body, each with their corresponding properties. The 'mass' of each body is the population of each district, p, and the distance from county i to county j is denoted as d in the model shown in section 5. The higher the population of each county, the higher the 'mass' and therefore, the higher the moment of inertia. The distance between districts is also a factor when calculating the moment of inertia; demonstrating a linear relationship between said moment of inertia and distance between counties.

For the actual calculation of moment of inertia, we take the sum of population of county i, multiplied by the distance between counties i and j, squared. As mentioned earlier, the objective is to minimize this, while also keeping true to our other constraints; each county belonging to only one district, keeping the population of each district between bounds, each district is continuous, etc. These constraints follow the same general concepts as outlined in the previous section (Minimize Cut Edges: Model 1).

V. OR Model (in math)

Shown are the Integer Program (IP) Models for minimizing cut edges and minimizing the moment of inertia.

Integer Program for Minimize Cut Edges (Model 1):

Sets:

E is the set of Edges where two counties share a border
V is the set of counties
N(i) is he set of Neighbors of county i

Indices:

i is a county ($i \in V$)
j is a congressional district ($j = 1, 2, \ldots, k$)
e is an edge ($e \in E$)

Parameters:

p_i is the population of county i
U is the Upper bound on the population of each district (target +0.5%)
L is the Lower bound on the population of each district (target -0.5%)
k is the number of districts
M is equal to n-k+1

Variables:

$$x_{ij} = \begin{cases} 1 \text{ if county i belongs to district j} \\ 0 \text{ Otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1 \text{ if county i belongs to district j} \\ 0 \text{ Otherwise} \end{cases}$$

$$r_{ij} = \begin{cases} 1 \text{ if county i is the root of district j} \\ 0 \text{ Otherwise} \end{cases}$$

f_{ij} = is the flow associated with each edge $\{i,j\} \in E$, where flow is from i to j

(A)	$\min \sum_{e \in E} y_e$	
(B)	s.t. $x_{uj} - x_{vj} \leq y_e$	$\forall e = \{u, v\} \in E, \forall j \in \{1,2, \ldots, k\}$
(C)	$\sum_{j=1}^{k} x_{ij} = 1$	$\forall i \in V$
(D)	$L \leq \sum_{i \in V} p_i x_{ij} \leq U$	$\forall j \in \{1,2, \ldots, k\}$
(E)	$\sum_{i \in V} r_{ij} = 1$	$\forall j \in \{1,2, \ldots, k\}$
(F)	$r_{ij} \leq x_{ij}$	$\forall i \in V, \forall j \in \{1,2, \ldots, k\}$
(G)	$\sum_{u \in N(i)} (f_{ui} - f_{iu}) \geq 1 - M \sum_{j=1}^{k} r_{ij}$	$\forall i \in V$
(H)	$f_{ij} + f_{ji} \leq M(1 - y_e)$	$\forall e = \{i, j\} \in E$
(I)	$x_{ij} \in \{0,1\}$	$\forall i \in V, \forall j \in \{1,2, \ldots, k\}$
(J)	$y_e \in \{0,1\}$	$\forall e \in E$
(K)	$f_{ij}, f_{ji} \geq 0$	$\forall \{i, j\} \in E$
(L)	$r_{ij} \in \{0,1\}$	$\forall i \in V, \forall j \in \{1,2, \ldots, k\}$.

(A) Our objective is to minimize the number of cut edges
(B) An edge is a cut edge if one vertex belongs to a different district than the other vertex
(C) Every county is assigned to one and only one district
(D) Every district has a population between the upper (U) and lower (L) bound
(E) Every district has one and only one root
(F) A county cannot serve as the root of a district if it does not belong to said district
(G) Every county must consume some flow if it is not the root of its district; if it is the root of the district, the county can only send out (so much) flow
(H) There cannot be flow across cut edges
(I) x_{ij} binary (county either is or is not in a district)
(J) y_e binary (edge either is or is not cut)
(K) Flow must be nonnegative
(L) r_{ij} binary (county either is or is not the root of its district)

Integer Program for Minimize Moment of Inertia (Model 2):

Sets:
- E is the set of Edges where two counties share a border
- V is the set of counties
- N(i) is he set of Neighbors of county i

Indices:
- i and j denote counties ($\in V$)
- v denotes a county that serves as its district's center

Parameters:
- p_i is the population of county i
- d_{ij} is the distance, in miles, between counties i and j
- U is the Upper bound on the population of each district (target +0.5%)
- L is the Lower bound on the population of each district (target -0.5%)
- k is the number of districts

Variables:

$$x_{ij} = \begin{cases} 1 \text{ if vertex i is assigned to (the district centered at) vertex j} \\ 0 \text{ Otherwise} \end{cases}$$

f_{ij}^{v} = is the amount of flow, originating at district center v, that is sent across edge {i,j} ∈ E (from i to j)

(A)	$\min \sum_{i \in V} \sum_{j \in V} (p_i d_{ij}^2)\, x_{ij}$	
(B)	$\sum_{j \in V} x_{ij} = 1$	$\forall i \in V$
(C)	$\sum_{j \in V} x_{jj} = k$	
(D)	$\mathrm{L}x_{jj} \le \sum_{i \in V} p_i x_{ij} \le U x_{jj}$	$\forall j \in V$
(E)	$x_{ij} \le x_{jj}$	$\forall i, j \in V$
(F)	$\sum_{u \in N(i)} \left(f_{ui}^{j} - f_{iu}^{j}\right) = x_{ij}$	$\forall i \in V \backslash \{j\}, \forall j \in V$
(G)	$\sum_{u \in N(i)} f_{ui}^{j} \le (n-1) x_{ij}$	$\forall i \in V \backslash \{j\}, \forall j \in V$
(H)	$\sum_{u \in N(i)} f_{uj}^{j} = 0$	$\forall j \in V$
(I)	$x_{ij} \in \{0,1\}$	$\forall i, j \in V$
(J)	$f_{ij}^{v}, f_{ji}^{v} \ge 0$	$\forall \{i,j\} \in E, \forall v \in \mathrm{V}$

(A) Our objective is to minimize the moment of inertia, calculated as the sum of population times distance squared. By definition, in physics, moment of inertia is always classified as being proportional to mass of an object times radius squared. This treats each district as a physical body with centroid j, surrounded by counties i, at a distance of d_{ij}, each with a mass of p_i.
(B) Every county is assigned to one and only one district
(C) k districts (and thus k district centers) are chosen
(D) Every district has a population between the upper (U) and lower (L) bound
(E) If i is assigned to j, then j must be a center to its district
(F) Every county must consume 1 unit of flow if it is not the root of its district; if it is the root of the district, the county does not consume any flow
(G) County I can only receive flow of type j if i is assigned to center j
(H) Prevent flow circulations (flow that originated at j should never flow back into j)
(I) x_{ij} binary (county either is or is not in the district with center j)
(J) Flow must be nonnegative

VI. Experiments

Both models were run on a 2021 Windows Surface Laptop 3. This computer had the following attributes:

- OS: Microsoft Windows 10 Pro
- System Type: 64-bit operating system, x64-based processor
- Processor Speed: 1.30 GHz
- RAM: 16.0 GB

The optimization solver used for both models was Gurobi Optimizer Version 10.0.0, build v10.0.0rc2 (win64). This optimization solver was used within Python, and the code was run using Visual Studio Code.

Minimize Cut Edges (Model 1):
Objective Value of Optimization Model: 19 cut edges
Time Required: 0.7s

Minimize Moment of Inertia (Model 2):
Objective Value of Optimization Model: 12654737015.702263 (people•mi^2)
Time Required: 0.6s

VII. Plans and Maps

Minimize Cut Edges (Model 1):

District 1

- Counties – Curry, San Juan, Guadalupe, McKinley, Rio Arriba, Santa Fe, Quay, Union, San Miguel, Taos, Colfax, Sandoval, Harding, Los Alamos, Mora
- Population – 702,632 people

District 2

- Counties – Bernalillo, Catron, Cibola
- Population – 707,195 people

District 3

- Counties – Roosevelt, Lincoln, Torrance, Luna, Lea, Chaves, Sierra, Valencia, De Baca, Otero, Grant, Doña Ana, Hidalgo, Socorro, Eddy
- Population – 707,695

Total Population Deviation – 5,063 people (0.239%)

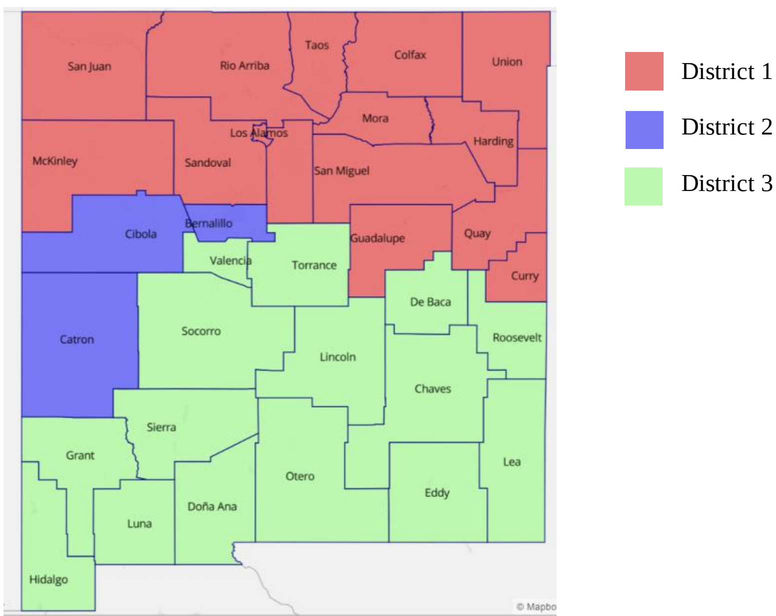

Figure 7.1: Model 1 Map

Minimize Moment of Inertia (Model 2):

District 1

- Counties – Bernalillo, Catron, Cibola
- Population – 707,195 people

District 2

- Counties - Curry, Roosevelt, Lincoln, Guadalupe, Luna, Lea, Chaves, Quay, Sierra, De Baca, Otero, San Miguel, Grant, Doña Ana, Hidalgo, Socorro, Eddy
- Population – 705,274 people

District 3

- Counties – San Juan, Torrance, McKinley, Rio Arriba, Santa Fe, Valencia, Union, Taos, Colfax, Sandoval, Harding, Los Alamos, Mora
- Population – 705,053

Total Population Deviation – 2,142 people (.101%)

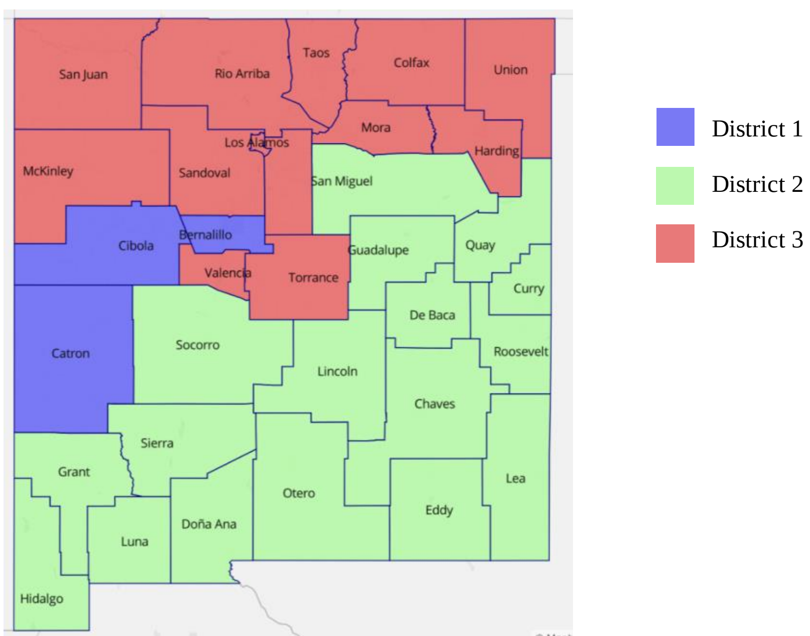

Figure 7.2: Model 2 Map

VIII. Evaluation of Plans

For a plan to adhere to all criteria, it must also adhere to minority representation rule. As far as not splitting minority populations, the two biggest minority populations in the state of New Mexico are Hispanics and Native Americans. The Hispanic population in New Mexico accounts for 47.5% of the total population and Native Americans account for 10.9%, according to the New Mexico Indian Affairs department.

More specifically, the Hispanic population is about 989,000, with Bernalillo County and Dona Ana being the most populous, (having populations of 328,000 and 143,000, respectively). Other counties have much lower populations of around 35,000 and under. These two districts are on opposite sides of the map of New Mexico and wouldn't be able to be kept together, contiguously. However, ensuring that the counties are kept whole helps preserve minority groups across the two separate regions. It should also be considered that the previous passed congressional district maps divided districts in a similar manner. This implies that all criteria are met by both plans presented in Section VII, including federal adherence to the Voting Rights Act of 1965.

The first model produced a plan that maximized district compactness while keeping counties whole, adhered to both state and federal criteria, and kept said districts contiguous. Along with adherence to all constraints, the plan produced a total population deviation of 0.293%, or 5,063 people. In comparison to the 2020 approved map, the total population is higher but still below the necessary 1%. The second model also adheres to all criteria while delivering a much smaller total population deviation of 0.101%, or 2,142 people. This makes the second plan much more favorable than the first, although there is a slight increase in the amount of cut edges.

Additionally, it is worth noting that the second model has less political bias than the first one. Shown below are maps that depict the political affiliation of New Mexico's counties (left) and precincts (right). Blue depicts Democratic regions, while red depicts Republican. If the first map were selected, Districts 1 and 2 would contain entirely Democratic counties, while District 3 would be entirely Republican counties. This division is unfavorable because it would lead to less political variation for each district.

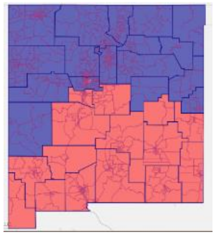

Figure 8.1: Political Affiliation by County

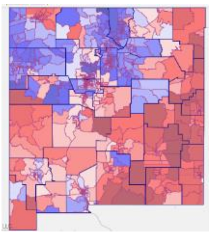

Figure 8.2: Political Affiliation by Precinct

IX. Conclusions

Upon in-depth plan analysis, both presented plans adhere to all federal and state criteria. The plan our team recommends for future congressional districting is the second plan, the model based on moment of inertia. This plan splits the counties into three districts, with populations of 707,195, 705,274, and 705,053 people, with a total population deviation of 0.101%, or 2,142 people. This model is favorable because it has a relatively low population deviation, compactness, contiguous counties, preservation of minorities, and political variety. Hopefully, with this plan, New Mexico could use redistricting as an opportunity to provide its citizens with a fair vote and a strong voice in the future of the nation.

X. Repository

For more information on our project, please visit our GitHub Repository:

https://github.com/loryn-grace/New-Mexico-Redistricting.git

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