QUESTION 1:
Asset Allocation and Data Analysis

Losa Damiano
Picone Giuseppe

Alma Mater Studiorum - Università di Bologna

EXAM

damiano.losa@studio.unibo.it

giuseppe.picone@studio.unibo.it

1- Analysis on Nasdaq Daily and Monthly samples

This initial section lays the statistical groundwork for our portfolio analysis by examining the return series of the Nasdaq and NYSE and by calculating the first four moments we can see a sharp contrast between the theoretical assumptions of a normal distribution and the actual reality of the market.

The analysis of the Nasdaq sample, in particular, tells aus of how risk changes with the time horizon. At a daily frequency, the data is far from Gaussian. We see extreme leptokurtosis, meaning the distribution has "fat tails." Leading stocks like Apple and Nvidia exhibit kurtosis values that are far above the normal benchmark, suggesting that short-term price action is driven by frequent outliers and shocks that traditional risk models might miss. However, when we switch to monthly returns, we observe a fundamental shift. Consistent with the Central Limit Theorem, the data begins to normalize: the excess kurtosis drops significantly, and the distribution starts to resemble a standard bell curve. At the same time, volatility scales proportionally, validating the "square root of time" rule.

NASDAQ Daily Statistic

	Mean	Std	Var	Skew	Kurt
APPLE	0.044	1.7068	2.9132	0.268	9.0345
NVIDIA	0.1969	3.1944	10.2041	0.2431	6.9445
MICROSOFT	0.0537	1.5836	2.5078	0.0905	6.2756
AMAZON.COM	0.026	2.1711	4.7135	-0.0942	8.4538
META PLATFORMS A	0.0606	2.7056	7.3204	-1.2431	30.8076
TESLA	0.0304	3.7216	13.8502	0.1031	5.8424
ALPHABET 'A'	0.0941	1.916	3.6712	-0.0571	6.398
BROADCOM	0.1511	2.5997	6.7582	0.4501	12.2039
COSTCO WHOLESALE	0.0752	1.4006	1.9618	-0.8427	12.5147
NETFLIX	0.0334	2.7283	7.4436	-3.3444	58.0159
T-MOBILE US	0.0263	1.4722	2.1673	-0.4036	10.3927
CISCO SYSTEMS	0.0379	1.407	1.9795	-1.0842	17.2927
ADVANCED MICRO DEVICES	0.0704	3.2142	10.3311	0.342	6.9712
LINDE (NYS)	0.0418	1.2923	1.6701	-0.001	6.7028
INTUITIVE SURGICAL	0.0579	2.0411	4.166	0.1822	10.8915
PEPSICO	0.0049	1.1084	1.2286	-0.1331	6.7878
ADOBE (NAS)	-0.0374	2.2078	4.8744	-1.2687	13.398
QUALCOMM	-0.0041	2.3622	5.5802	-0.0976	6.5722
COMCAST A	-0.037	1.6169	2.6142	-0.7783	9.9835
PALO ALTO NETWORKS	0.0851	2.5232	6.3664	-1.3295	30.4912
INTUIT	0.0264	2.1115	4.4583	-0.0719	5.7898
GILEAD SCIENCES	0.0474	1.4544	2.1153	0.5264	11.1279
STARBUCKS	-0.008	1.9147	3.666	0.6366	24.6578
MARVELL TECHNOLOGY	0.032	3.5662	12.7174	0.0531	10.7951
INTEL	-0.0118	2.8759	8.2707	-0.3593	16.9516
APPLOVIN A	0.1738	4.6141	21.2897	0.8328	11.0344
CADENCE DESIGN SYS.	0.0617	2.1782	4.7445	0.0623	6.7792
CME GROUP	0.0322	1.2354	1.5263	-0.1796	4.8498
CINTAS	0.0669	1.4066	1.9784	-0.3243	9.5399
MARRIOTT INTL.'A'	0.072	1.7972	3.2298	-0.0616	5.2743
MONDELEZ INTERNATIONAL CL.A	0.0023	1.1789	1.3899	-0.4318	6.2175
REGENERON PHARMS.	0.0239	1.8781	3.5274	-0.6172	23.1286
SYNOPSYS	0.049	2.5698	6.604	-3.8829	72.8787
O REILLY AUTOMOTIVE	0.0875	1.3768	1.8955	-1.0039	11.9206
CSX	0.015	1.4325	2.0521	-0.0084	5.9919
AIRBNB A	-0.0254	2.8151	7.9247	-0.0552	6.4957
COINBASE GLOBAL A	-0.0294	5.2649	27.7188	0.1181	5.8326
COPART	0.027	1.6008	2.5626	-0.1977	8.4972
PACCAR	0.0456	1.5832	2.5067	-0.0804	7.4525
ROPER TECHNOLOGIES	-0.0029	1.2508	1.5645	-0.4618	7.2648

	Mean	Std	Var	Skew	Kurt
ROSS STORES	0.0396	1.8656	3.4803	-1.6256	31.9788
WORKDAY CLASS A	-0.0194	2.2766	5.1827	0.1354	10.1143
BAKER HUGHES A	0.0644	2.1852	4.7749	-0.1733	6.271
CHARTER COMMS.CL.A	-0.0965	2.2474	5.051	-1.1689	17.2476
MONSTER BEVERAGE	0.0471	1.4757	2.1776	-0.3319	8.7881
NXP SEMICONDUCTORS	0.0224	2.4698	6.1	0.1947	6.8524
COGNIZANT TECH.SLTN.'A'	0.0016	1.6101	2.5924	-1.0414	14.7345
EXELON	0.0302	1.2801	1.6386	-0.5589	5.6867
GE HEALTHCARE TECHNOLOGIES	0.0338	1.9921	3.9684	-1.0643	17.6904
KRAFT HEINZ	-0.025	1.3789	1.9013	-0.7597	8.1403
UNITED AIRLINES HOLDINGS	0.0727	2.9455	8.6757	0.2236	9.1423
WILLIS TOWERS WATSON	0.0341	1.3932	1.9411	-0.4034	10.9225
EBAY	0.0364	2.0278	4.112	-0.4547	13.792
IDEXX LABORATORIES	0.0254	2.1477	4.6127	1.3718	18.4406
MICROCHIP TECH.	-0.002	2.7072	7.3289	0.2241	11.1962
WARNER BROS DISCOVERY SERIES A	-0.0218	3.4991	12.2439	-0.4603	13.5622
T ROWE PRICE GROUP	-0.0315	1.884	3.5495	0.374	8.4568
BIOGEN	-0.0375	2.1942	4.8147	5.1526	81.5635
CARLYLE GROUP	0.0473	2.4219	5.8656	-0.6669	7.9189
DOMINO'S PIZZA	0.0015	1.8017	3.2461	-0.0859	12.3374
EXPEDIA GROUP	0.053	2.7356	7.4835	-0.1046	12.7396
INTERACTIVE BROKERS GROUP A	0.1094	2.0798	4.3255	-0.1147	8.2703
CYBER ARK SOFTWARE	0.0778	2.6196	6.8626	-0.0394	6.7914
STEEL DYNAMICS	0.1145	2.3553	5.5475	0.0331	5.7339
SBA COMMS.	-0.0294	1.6967	2.8787	-0.1549	5.0171
ASPEN TECHNOLOGY DEAD - DELIST.13/03/25	0.0475	2.0415	4.1676	-1.6771	38.0182
BENTLEY SYSTEMS B	-0.0142	2.2251	4.9509	-0.148	7.0453
AKAMAI TECHS.	-0.0151	1.9453	3.7841	-2.1396	32.5059
MODERNA	-0.0856	4.1335	17.0858	-0.0663	6.1072
MATTEL	0.0089	2.2087	4.8784	-0.3773	13.5748
MORNINGSTAR	-0.0115	1.7813	3.1732	-0.1688	8.9163
QUANTUM COMPUTING	0.0161	7.7688	60.3546	1.1199	16.2529
REGENCY CENTERS	0.0313	1.4171	2.0083	0.0214	4.6988
ROYALTY PHARMA A	-0.0207	1.5415	2.3762	0.344	8.1209
ROCKET LAB	0.1643	4.6518	21.6388	0.7697	8.5778
TRIPADVISOR 'A'	-0.0717	3.2922	10.8383	-0.8003	15.5787
ABVC BIOPHARMA	-0.2461	7.6143	57.9769	0.9134	27.852
ASTROTECH	-0.2287	3.7627	14.158	-0.9045	14.1269
ORAMED PHARMACEUTICALS	-0.0347	6.1189	37.441	-9.9829	243.4225
VIMEO DEAD - DELIST.25/11/25	-0.1597	4.067	16.5402	0.975	33.3266

NASDAQ Monthly Statistic

	Mean	Std	Var	Skew	Kurt
APPLE	1.2427	7.4923	56.1346	-0.5226	2.8852
NVIDIA	4.533	15.1065	228.206	0.0688	2.7093
MICROSOFT	1.1718	6.8019	46.2662	0.1634	3.6734
AMAZON.COM	0.7019	9.0892	82.6137	-1.2538	6.2046
META PLATFORMS A	1.2647	11.5749	133.979	-0.9756	5.335
TESLA	1.1252	19.5456	382.0302	0.1183	2.9107
ALPHABET 'A'	1.9931	7.2786	52.9786	0.0406	3.9719
BROADCOM	3.3575	10.2054	104.1495	0.4377	3.4661
COSTCO WHOLESALE	1.8544	8.6247	74.3854	-1.7142	8.5513
NETFLIX	0.9186	12.6068	158.9307	-1.3359	6.8424
T-MOBILE US	0.6745	6.7586	45.6788	0.3682	3.1538
CISCO SYSTEMS	0.6963	6.3259	40.0169	-0.6195	4.4119
ADVANCED MICRO DEVICES	1.8555	14.2123	201.9882	0.3445	2.9881
LINDE (NYS)	0.8206	5.0473	25.4756	0.3358	3.6925
INTUITIVE SURGICAL	1.3878	10.1304	102.625	-0.6662	4.2086
PEPSICO	0.1609	4.8448	23.4719	-0.2136	2.5045
ADOBE (NAS)	-0.7225	10.616	112.7002	-0.5457	4.748
QUALCOMM	0.2794	9.8672	97.3611	0.8957	4.2114
COMCAST A	-1.056	6.0223	36.2684	0.1761	2.7554
PALO ALTO NETWORKS	2.1527	10.4976	110.1986	-0.3581	3.5311
INTUIT	0.5644	8.4418	71.2645	-0.7965	4.5699
GILEAD SCIENCES	1.1299	6.6449	44.1552	0.2707	5.429
STARBUCKS	-0.2189	7.3779	54.4328	0.0848	3.2785
MARVELL TECHNOLOGY	0.9127	13.5317	183.1063	-0.5913	3.7346
INTEL	-0.4695	13.1479	172.8684	-0.4062	4.2433
APPLOVIN A	3.9314	24.0078	576.3748	0.5105	3.3551
CADENCE DESIGN SYS.	1.5426	8.8591	78.4838	-0.0677	2.2548
CME GROUP	0.5568	5.1156	26.1695	-1.1009	6.8593
CINTAS	1.4561	6.0236	36.2841	-0.6946	3.7266
MARRIOTT INTL.'A'	1.2468	7.1761	51.4959	-0.4234	3.5773
MONDELEZ INTERNATIONAL CL.A	-0.0033	5.64	31.809	-0.1417	2.5009
REGENERON PHARMS.	0.7545	9.0042	81.0761	-0.5663	4.3524
SYNOPSYS	1.3924	9.4386	89.0864	-0.008	2.4662
O REILLY AUTOMOTIVE	1.8516	6.8805	47.341	-0.8953	6.1557
CSX	0.276	6.7746	45.8947	-0.0461	2.5157
AIRBNB A	-0.684	12.5912	158.5382	-0.4621	3.0505
COINBASE GLOBAL A	-0.6013	26.2644	689.8162	-0.051	3.2136
COPART	0.7306	7.8666	61.884	-0.2529	3.9368
PACCAR	1.0918	7.0499	49.701	0.3173	2.3294
ROPER TECHNOLOGIES	0.0378	5.7725	33.3221	-0.1094	3.6839

	Mean	Std	Var	Skew	Kurt
ROSS STORES	0.7708	9.46	89.4916	-1.2575	8.699
WORKDAY CLASS A	-0.5464	9.6999	94.0876	-0.3841	3.1505
BAKER HUGHES A	1.4105	9.719	94.4584	0.2478	2.4885
CHARTER COMMS.CL.A	-2.1195	11.1392	124.0827	-0.2202	4.0179
MONSTER BEVERAGE	1.0637	6.0412	36.4964	-0.0905	2.4357
NXP SEMICONDUCTORS	0.2287	9.5873	91.9159	0.3708	3.0956
COGNIZANT TECH.SLTN.'A'	0.123	7.1462	51.068	-0.1216	3.2841
EXELON	0.6607	6.225	38.7509	-0.6631	3.7298
GE HEALTHCARE TECHNOLOGIES	0.8664	9.0598	82.08	-0.7127	4.1143
KRAFT HEINZ	-0.8606	5.8271	33.9548	-0.0957	2.9197
UNITED AIRLINES HOLDINGS	1.0005	13.2752	176.2316	0.0774	2.9613
WILLIS TOWERS WATSON	0.6556	6.4156	41.1602	-0.544	4.3492
EBAY	0.7202	8.8394	78.1358	-0.1415	3.4103
IDEXX LABORATORIES	0.5983	9.9243	98.4919	-0.0395	4.2147
MICROCHIP TECH.	-0.0433	12.7475	162.4981	0.5185	5.1283
WARNER BROS DISCOVERY SERIES A	-1.7355	17.9419	321.9123	-0.6269	8.0905
T ROWE PRICE GROUP	-0.8712	8.023	64.3685	-0.0036	2.6036
BIOGEN	-0.8263	9.5323	90.8651	0.7849	4.5051
CARLYLE GROUP	0.9091	11.7872	138.9375	0.034	2.295
DOMINO'S PIZZA	0.1099	8.4369	71.181	-0.6148	3.7518
EXPEDIA GROUP	0.7129	12.0667	145.6048	-0.1984	4.8579
INTERACTIVE BROKERS GROUP A	2.2302	9.9236	98.4777	0.225	3.5114
CYBER ARK SOFTWARE	2.0353	9.3124	86.7207	-0.95	3.5623
STEEL DYNAMICS	2.2279	10.1689	103.4064	-0.0465	2.7399
SBA COMMS.	-0.6184	8.3843	70.2967	0.0137	2.6657
ASPEN TECHNOLOGY DEAD - DELIST.13/03/25	0.9904	9.4718	89.7141	-0.3779	5.3591
BENTLEY SYSTEMS B	-0.1684	9.7603	95.2626	-0.6494	3.7171
AKAMAI TECHS.	-0.179	8.0748	65.2021	-0.6159	3.2973
MODERNA	-2.0807	19.761	390.4963	0.292	3.0261
MATTEL	-0.038	9.6622	93.3583	0.2797	4.5683
MORNINGSTAR	-0.2371	8.24	67.8978	-0.2025	3.712
QUANTUM COMPUTING	0.9192	38.1502	1.46E+03	1.8881	8.6961
REGENCY CENTERS	0.3959	5.6745	32.2	-0.0347	4.2579
ROYALTY PHARMA A	-0.31	6.0373	36.449	0.735	4.3766
ROCKET LAB	3.5163	24.2949	590.24	0.0258	3.2983
TRIPADVISOR 'A'	-2.63	14.252	203.1185	0.486	2.8822
ABVC BIOPHARMA	-5.2047	28.3112	801.5256	-0.5286	4.5518
ASTROTECH	-4.9074	13.7235	188.3342	-1.085	6.3814
ORAMED PHARMACEUTICALS	-1.6489	29.222	853.9252	-1.3116	11.2429
VIMEO DEAD - DELIST.25/11/25	-3.5713	19.3609	374.8454	0.6509	5.5856

2- Analysis on NYSE Daily and Monthly samples

NYSE statistics shows generally more mature statistics, so we don't see the same extreme daily swings that define the tech sector. However, it would be a mistake to assume these stocks are perfectly stable. The kurtosis values are still consistently high, well above the threshold of normal distribution. This tells us that even stable blue-chip stocks are subject to "fat tail" events: they might be boring on most days, but when they react to major economic news or earnings shocks, the moves can be surprisingly violent.

Switching the view to monthly observations gives us a much better sense of the true risks involved in holding these assets in particular by looking at the kurtosis and the negative skewness.

NYSE Daily Statistic

	Mean	Std	Var	Skew	Kurt
3M	0.0076	1.7097	2.9231	1.1127	24.2365
AAR	0.0788	2.1381	4.5715	-0.5606	10.3122
ABBOTT LABORATORIES	0.0055	1.3479	1.8169	-0.47	8.35
AMERICAN EXPRESS	0.0789	1.8112	3.2803	0.0157	8.1922
BANK OF AMERICA	0.0385	1.6673	2.7799	-0.1666	6.7905
BANK OF NEW YORK MELLON	0.0812	1.5213	2.3144	-0.3261	6.8558
BARRICK MINING (NYS)	0.0581	2.1162	4.4783	0.0396	5.4315
BOEING	0.0146	2.2769	5.1843	-0.1116	6.6833
BRISTOL MYERS SQUIBB	-0.0134	1.4555	2.1186	0.222	9.7953
CABOT	0.0297	2.0408	4.1649	0.5568	8.4413
CALERES	-0.0166	3.4259	11.7366	-0.1493	9.9809
CARPENTER TECH.	0.1731	2.894	8.3753	0.4369	6.9337
CATERPILLAR	0.0912	1.8121	3.2836	0.1791	6.8196
CHEVRON	0.0452	1.5729	2.4742	-0.45	6.2102
COCA COLA	0.0301	0.9827	0.9658	-0.3798	7.2873
COLGATE-PALM.	0.0068	1.1053	1.2217	-0.4959	5.4696
COMERICA	0.0313	2.58	6.6562	-1.5415	25.4106
CONOCOPHILLIPS	0.0619	2.0613	4.2491	-0.3153	5.9839
CONSOLIDATED EDISON	0.0325	1.1636	1.3539	-0.1888	4.0712
CVS HEALTH	0.0059	1.7803	3.1695	-0.8372	18.1199
DEERE	0.0392	1.7898	3.2033	-0.3082	9.1816
DOVER	0.0364	1.5026	2.2579	0.0206	7.4074
DUKE ENERGY	0.0216	1.1095	1.231	-0.3029	4.1484
DXC TECHNOLOGY	-0.0536	2.761	7.6231	-2.0671	27.3263
DYCOM INDS.	0.1079	2.6542	7.0449	0.2281	11.6182
ELI LILLY	0.1244	1.9442	3.7801	0.0542	12.5735
EXXON MOBIL	0.0777	1.6688	2.785	-0.2322	4.4426
FOOT LOCKER DEAD - DELIST.08/09/25	-0.0525	3.838	14.7301	1.3193	73.2654
FORD MOTOR	0.0109	2.4413	5.9601	-0.5891	9.4839
GENERAL DYNAMICS	0.0657	1.231	1.5153	-0.0855	8.4049
HALLIBURTON	0.0392	2.5476	6.4901	-0.1426	5.4582
H&R BLOCK	0.0676	1.8619	3.4667	1.0267	14.5774
HP	-0.0187	2.1219	4.5023	-0.1234	11.4086
HUMANA	-0.0317	2.1607	4.6687	-1.7543	18.7411
IDACORP	0.0314	1.1997	1.4392	-0.1794	4.2384
JACOBS SOLUTIONS	0.031	1.5338	2.3525	-0.6803	7.7618
JEFFERIES FINANCIAL GROUP	0.0718	2.1603	4.6668	-0.4107	9.4134
JOHNSON & JOHNSON	0.0221	1.04	1.0817	-0.0278	8.3884
JP MORGAN CHASE & CO.	0.0627	1.5053	2.2659	-0.0792	8.084
KROGER	0.0486	1.6659	2.7753	0.4341	9.4592
LA-Z-BOY	-0.011	2.2942	5.2634	0.2871	14.9765
LINCOLN NATIONAL	-0.0187	2.7407	7.5113	-2.6594	39.9475
MASTEC	0.0823	2.6206	6.8674	-0.8274	12.4076
MCDONALDS	0.0269	1.064	1.1322	-0.1666	5.6779
MEDTRONIC	-0.0129	1.3429	1.8033	-0.5211	5.9254
MERCK & COMPANY	0.0268	1.4329	2.0532	-0.5691	11.1223
MOTOROLA SOLUTIONS	0.0623	1.3853	1.9191	-0.1663	7.862
MSA SAFETY	0.008	1.5215	2.3149	0.2198	8.4198
MURPHY OIL	0.0681	2.9654	8.7935	-0.1991	5.8782
MYERS INDUSTRIES	-0.0088	2.3351	5.4527	-0.7642	26.635
NATIONAL FUEL GAS	0.0509	1.3795	1.9031	-0.1675	4.9619
NEW YORK TIMES 'A'	0.0264	1.8477	3.4138	-0.1856	14.3042
NEWMONT	0.0503	2.2037	4.8561	-0.4509	8.1021
NORTHROP GRUMMAN	0.0607	1.5263	2.3294	-0.5469	13.964
PARKER-HANNIFIN	0.0935	1.7582	3.0911	0.1934	9.4884
PFIZER	-0.0276	1.5644	2.4475	0.3783	5.9517
PROCTER & GAMBLE	0.0094	1.0799	1.1661	-0.5773	6.5092
QUAKER HOUGHTON	-0.0461	2.3606	5.5725	0.9052	9.6327
QUANEX BUILDING PRODUCTS	-0.0247	2.6096	6.81	0.2509	13.1047
STANLEY BLACK & DECKER	-0.0595	2.264	5.1257	-0.3374	11.4367
STATE STREET	0.0403	1.9106	3.6504	-0.6035	7.7271
TARGET	-0.0432	2.2258	4.954	-2.4909	39.9385
TENNANT	0.0017	1.6832	2.8333	0.3477	7.8649
THERMO FISHER SCIENTIFIC	0.0142	1.6433	2.7006	0.3782	6.4281
TIMKEN	0.0064	2.0133	4.0532	-0.4164	6.8848
UNION PACIFIC	0.0052	1.3815	1.9085	0.4067	8.5067

	Mean	Std	Var	Skew	Kurt
WALT DISNEY	-0.0344	1.8079	3.2686	-0.28	11.0471
WELLS FARGO & CO	0.0768	1.8809	3.5377	-0.026	6.4157
WEST PHARM.SVS.	-0.0139	2.5812	6.6624	-4.8554	101.8928
WHIRLPOOL	-0.064	2.3356	5.4552	-0.5776	11.4603
WOLVERINE WWD.	-0.0388	3.6301	13.1776	-1.6354	29.598
WORTHINGTON ENTERPRISES	0.033	2.4025	5.772	0.1781	14.313
TELEPHONE & DATA SYS.	0.0573	3.5091	12.3136	4.1501	94.4191
BERKSHIRE HATHAWAY 'A'	0.0561	1.0716	1.1482	-0.0661	6.1531
FEDEX	0.013	2.0296	4.1192	-1.5129	27.8501
WELLTOWER	0.0839	1.4744	2.1738	-0.103	5.1487
AON CLASS A	0.0382	1.3898	1.9315	-0.3341	9.3998
TEAM	-0.165	6.0549	36.6622	0.1639	18.7549
HOME DEPOT	0.0214	1.4715	2.1654	-0.321	6.4296
NORFOLK SOUTHERN	0.0106	1.5311	2.3441	0.2315	8.2977
JOHN WILEY AND SONS A	-0.0352	2.0553	4.2244	-0.5693	14.9715
VERIZON COMMUNICATIONS	-0.0296	1.3033	1.6987	-0.3482	8.9103
AT&T	0.0053	1.4666	2.151	-0.3839	10.4428
INSTEEL INDUSTRIES	0.0169	2.4375	5.9413	-0.7413	11.0418
ORACLE	0.0839	2.3548	5.545	1.5717	31.338
HARLEY-DAVIDSON	-0.0554	2.5538	6.522	-0.1398	9.3669
CITIGROUP	0.0468	1.7688	3.1288	0.0207	8.7688
CHARLES SCHWAB	0.0421	2.0058	4.0231	-0.5617	9.6767
ALLIANCEBERNSTEIN HLDG. UNT.	0.0078	1.777	3.1578	-0.2007	4.8454
MORGAN STANLEY	0.069	1.7512	3.0667	0.0309	7.1112
LOCKHEED MARTIN	0.0405	1.3883	1.9273	-1.198	16.9309
CREDICORP	0.0574	1.9459	3.7865	-0.5626	11.2199
BOSTON BEER 'A'	-0.1115	2.527	6.3855	-0.9161	27.2835
FTI CONSULTING	0.0327	1.7578	3.0898	-1.2627	36.8232
RALPH LAUREN CL.A	0.0922	2.2554	5.0868	-0.2897	10.0791
BLACKROCK	0.0317	1.6269	2.6468	0.2594	8.4655
LAZARD	0.0138	2.2109	4.8882	0.0887	8.445
MASTERCARD	0.0369	1.4867	2.2104	-0.1772	6.6906
PHILIP MORRIS INTL.	0.056	1.3517	1.8271	-0.1488	10.9383
WHITESTONE REIT	0.0391	1.6621	2.7627	0.2747	5.0128
APOLLO GLOBAL MANAGEMENT	0.084	2.257	5.0942	-0.4487	7.9043
GLOBUS MEDICAL CL.A	0.0271	2.2829	5.2116	0.4151	49.3608
ACCENTURE CLASS A	0.0052	1.6272	2.6478	-0.2557	6.5526
ALCOA	0.0894	3.5083	12.3084	-0.0176	4.5722
BLACKSTONE	0.0662	2.3638	5.5874	-0.077	6.0571
ESTEE LAUDER COS.'A'	-0.06	2.5849	6.6815	-1.5494	17.4723
GARMIN	0.0371	1.8142	3.2915	0.6379	28.4555
GOLDMAN SACHS GP.	0.0908	1.6842	2.8365	0.1556	8.2988
GUESS	-0.0278	3.1544	9.9501	1.2828	16.2563
HERBALIFE	-0.0827	3.6736	13.495	-0.4574	24.8678
KORN FERRY	0.0252	1.8592	3.4568	-0.0777	7.2573
LEVI STRAUSS A	5.89E-04	2.4587	6.0451	-0.5365	11.4863
MACY'S	0.0386	3.4377	11.8178	0.7354	8.4074
MAGNOLIA OIL GAS A	0.07	2.5881	6.6981	-0.0883	4.8593
MCCORMICK & CO	-0.0238	1.6633	2.7667	-0.5141	9.2799
METLIFE	0.0304	1.5962	2.548	-0.5061	6.9384
MOODY'S	0.0509	1.5686	2.4606	-0.1521	8.7444
NIKE 'B'	-0.0602	2.1746	4.7289	-0.8774	19.4315
NORDIC AMER.TANKERS	0.0266	3.2484	10.5524	0.2986	8.2759
PJT PARTNERS CL.A	0.0675	1.8631	3.4711	0.0111	8.0402
SONOCO PRODUCTS	-0.0183	1.4791	2.1879	-0.7269	9.2992
SUNOCO	0.048	1.4607	2.1337	-0.2701	5.2277
UBER TECHNOLOGIES	0.0332	2.8215	7.9609	0.3047	6.0518
UNITED PARCEL SER.'B'	-0.0307	1.7432	3.0386	-0.9343	16.7915
VICTORIA S SECRET	0.0324	3.9825	15.86	-0.3109	11.6531
VISA 'A'	0.0367	1.4031	1.9687	-0.0355	8.6457
WK KELLOGG DEAD - DELIST.26/09/25	0.05	2.7992	7.8354	1.2002	21.1464
MARRIOTT VACATIONS WWD.	-0.0676	2.6047	6.7847	-1.318	20.6127
MAIN STREET CAPITAL	0.0498	1.2986	1.6865	-0.6846	7.5162
HYATT HOTELS CL.A	0.0627	2.1067	4.438	0.0115	6.201
BIOHAVEN	0.0637	5.2069	27.1123	-0.5248	19.8453

NYSE Monthly Statistic

	Mean	Std	Var	Skew	Kurt
3M	-0.0182	8.1324	66.1356	-0.0823	3.6265
AAR	1.5464	10.19	103.8365	-0.3281	2.6783
ABBOTT LABORATORIES	0.0127	5.693	32.4099	0.3717	2.6325
AMERICAN EXPRESS	1.5818	7.9302	62.8885	-0.1699	3.049
BANK OF AMERICA	0.5202	8.1309	66.1109	-0.3806	3.4505
BANK OF NEW YORK MELLON	1.6499	7.133	50.8802	-0.1647	3.0211
BARRICK MINING (NYS)	1.4993	10.4526	109.2568	-0.173	2.9503
BOEING	-0.0466	11.6117	134.8316	-0.3524	4.7071
BRISTOL MYERS SQUIBB	-0.2505	6.5511	42.9164	-0.305	4.1059
CABOT	0.4734	8.6895	75.5072	0.3289	3.1475
CALERES	-0.5783	13.4979	182.1943	-0.4023	3.0096
CARPENTER TECH.	3.5529	13.0623	170.6239	0.1113	2.8051
CATERPILLAR	1.7702	8.5433	72.9877	0.0445	2.8729
CHEVRON	0.8092	7.2337	52.3266	-0.0508	3.5414
COCA COLA	0.595	4.6312	21.4482	-0.2088	3.0558
COLGATE-PALM.	0.2252	4.8686	23.7034	0.1966	2.7474
COMERICA	0.515	11.891	141.3962	-1.2283	6.5814
CONOCOPHILLIPS	1.0287	8.6687	75.146	0.4026	3.9439
CONSOLIDATED EDISON	0.6442	5.6086	31.4567	-0.2409	3.0958
CVS HEALTH	0.1556	8.7389	76.3682	-0.2802	3.5146
DEERE	0.5373	7.5392	56.8399	-1.2131	8.521
DOVER	0.6816	7.2231	52.1736	-0.2517	2.5654
DUKE ENERGY	0.4382	5.5954	31.3087	-0.7636	4.131
DXC TECHNOLOGY	-1.1317	9.7817	95.6824	-0.5259	4.2259
DYCOM INDS.	2.3175	10.0971	101.9505	-0.1772	2.733
ELI LILLY	2.9857	8.631	74.4943	0.0684	3.3386
EXXON MOBIL	1.4431	6.7183	45.1354	0.473	3.2895
FOOT LOCKER DEAD - DELIST.08/09/25	-1.5366	17.4215	303.5084	1.1454	6.8717
FORD MOTOR	0.0594	11.4305	130.6554	-0.1652	2.7276
GENERAL DYNAMICS	1.2141	4.8846	23.8592	-0.9495	4.0091
HALLIBURTON	0.6658	10.8422	117.5529	0.3255	2.6238
H&R BLOCK	1.0891	8.3808	70.2372	0.5402	3.0245
HP	-0.721	9.5207	90.6445	-0.5038	3.0284
HUMANA	-0.7323	9.4297	88.9195	0.1268	3.3514
IDACORP	0.4883	5.3796	28.9398	-0.326	2.8664
JACOBS SOLUTIONS	0.5132	7.2881	53.1157	-0.4711	3.7519
JEFFERIES FINANCIAL GROUP	1.102	10.9747	120.4439	-0.7957	3.4845
JOHNSON & JOHNSON	0.5345	4.5108	20.3476	0.0594	2.2195
JP MORGAN CHASE & CO.	1.1527	6.1335	37.6201	-0.0564	2.4257
KROGER	0.9979	7.2524	52.597	0.1286	3.349
LA-Z-BOY	-0.2447	8.4732	71.7958	0.0613	2.2931
LINCOLN NATIONAL	-0.8141	11.9314	142.3585	-1.2263	5.4409
MASTEC	1.6495	12.0847	146.0397	0.133	2.9323
MCDONALDS	0.532	4.6523	21.6441	-0.1096	2.6912
MEDTRONIC	-0.2894	6.0634	36.7649	-0.6062	2.8065
MERCK & COMPANY	0.677	6.3677	40.5481	-0.5479	3.4696
MOTOROLA SOLUTIONS	1.3073	6.6885	44.7359	-0.323	3.8324
MSA SAFETY	0.2167	6.2765	39.3943	0.4955	3.0801
MURPHY OIL	0.8638	12.1554	147.7528	0.5115	3.3836
MYERS INDUSTRIES	-0.0682	10.3697	107.5298	-0.3259	4.7
NATIONAL FUEL GAS	0.8547	5.0185	25.1855	0.6292	3.1161
NEW YORK TIMES 'A'	0.5897	8.5319	72.7929	-0.1141	3.3977
NEWMONT	1.1252	11.2577	126.7365	-0.5473	2.6677
NORTHROP GRUMMAN	1.3033	6.874	47.2519	-0.1399	2.8729
PARKER-HANNIFIN	1.8874	7.7649	60.2943	-0.0658	2.3563
PFIZER	-0.5705	8.0358	64.5738	0.272	2.7588
PROCTER & GAMBLE	0.2385	4.7069	22.155	-0.2581	3.5585
QUAKER HOUGHTON	-0.8658	11.2908	127.4821	0.4296	3.3353
QUANEX BUILDING PRODUCTS	-0.6369	12.997	168.9223	-0.5505	4.4179
STANLEY BLACK & DECKER	-1.535	10.4261	108.7037	-0.4316	3.4552
STATE STREET	0.6863	8.1956	67.1681	-0.0991	3.2084
TARGET	-0.9303	10.9698	120.3358	-1.3976	6.9801
TENNANT	-0.0639	8.4633	71.6271	-0.2849	2.7576
THERMO FISHER SCIENTIFIC	0.569	7.5731	57.3521	-0.1779	2.7673
TIMKEN	0.131	8.7416	76.4158	-0.2745	2.6613
UNION PACIFIC	0.0868	5.9416	35.3031	-0.2401	3.1734

	Mean	Std	Var	Skew	Kurt
WALT DISNEY	-0.9473	8.6907	75.5276	0.8859	4.3034
WELLS FARGO & CO	1.349	8.4884	72.0531	-0.406	2.8554
WEST PHARM.SVS.	-0.1743	10.928	119.4206	-1.645	9.0107
WHIRLPOOL	-1.6049	8.8405	78.1543	0.0295	2.3117
WOLVERINE WWD.	-1.3465	18.5819	345.286	-0.265	4.4307
WORTHINGTON ENTERPRISES	0.3252	11.3034	127.7657	0.4744	3.7577
TELEPHONE & DATA SYS.	1.1092	16.2741	264.8449	1.3732	11.9642
BERKSHIRE HATHAWAY 'A'	1.1075	4.8706	23.7229	-0.538	4.1723
FEDEX	0.1275	8.9679	80.4241	-1.2033	7.6364
WELLTOWER	1.6857	6.903	47.6515	-0.4886	3.0986
AON CLASS A	0.7254	7.1087	50.5342	-0.351	4.5011
TEAM	-3.5976	25.0862	629.3194	0.4064	3.0541
HOME DEPOT	0.449	7.2121	52.0143	-0.2614	2.6265
NORFOLK SOUTHERN	0.1271	6.7117	45.0471	-0.0071	3.1716
JOHN WILEY AND SONS A	-0.9416	8.7797	77.0836	-0.1504	2.9941
VERIZON COMMUNICATIONS	-0.6276	5.0108	25.1081	0.5893	3.748
AT&T	0.0698	6.0161	36.1933	0.3188	3.0135
INSTEEL INDUSTRIES	-0.1224	10.5106	110.473	-0.2928	2.5646
ORACLE	1.7223	11.5983	134.5209	0.0745	2.5874
HARLEY-DAVIDSON	-0.9685	11.2673	126.9525	0.5083	3.4895
CITIGROUP	0.75	8.9509	80.1186	0.1346	2.0335
CHARLES SCHWAB	0.7321	10.1165	102.3427	-0.6105	4.7979
ALLIANCEBERNSTEIN HLDG. UNT.	-0.1032	7.2195	52.1206	-0.4395	3.1993
MORGAN STANLEY	1.3559	8.4041	70.6293	-0.1718	2.2811
LOCKHEED MARTIN	0.8343	6.5863	43.3799	0.2503	3.1843
CREDICORP	1.3806	9.1358	83.4625	-0.1865	2.8598
BOSTON BEER 'A'	-2.8018	11.1031	123.2792	-0.9335	5.652
FTI CONSULTING	0.4778	7.2677	52.8191	0.1729	2.9421
RALPH LAUREN CL.A	1.8703	10.4654	109.5255	0.195	3.6898
BLACKROCK	0.7235	7.976	63.617	0.0919	2.949
LAZARD	0.3203	10.4742	109.7096	0.1616	3.0875
MASTERCARD	0.6893	5.8434	34.1452	0.0405	2.3425
PHILIP MORRIS INTL.	1.0821	6.4184	41.196	0.2221	4.3996
WHITESTONE REIT	0.6325	7.7205	59.6054	0.0025	2.7659
APOLLO GLOBAL MANAGEMENT	1.8545	9.6381	92.8926	-0.0206	2.7806
GLOBUS MEDICAL CL.A	0.7412	10.3688	107.5128	0.1706	3.4353
ACCENTURE CLASS A	0.0574	8.1236	65.9925	-0.7103	3.3655
ALCOA	1.1906	14.9437	223.314	-0.2983	3.1449
BLACKSTONE	1.2986	11.1541	124.4135	-0.3682	2.002
ESTEE LAUDER COS.'A'	-1.5742	11.8458	140.3222	-0.137	2.5925
GARMIN	0.8069	8.858	78.4644	-0.2207	4.1085
GOLDMAN SACHS GP.	1.7383	8.0938	65.509	-0.0523	2.5327
GUESS	-0.7332	10.646	113.3379	0.008	3.3893
HERBALIFE	-1.7155	16.0292	256.9345	-0.0943	2.6862
KORN FERRY	0.0516	8.3197	69.217	-0.6416	4.0451
LEVI STRAUSS A	-0.3046	10.1289	102.5952	-0.2353	2.3655
MACY'S	0.2235	15.6956	246.3513	-0.2413	3.0173
MAGNOLIA OIL GAS A	1.1404	9.1617	83.9375	0.202	2.9719
MCCORMICK & CO	-0.4138	6.7308	45.3033	-0.5303	4.3479
METLIFE	0.3611	7.5006	56.2591	-0.7469	3.9474
MOODY'S	1.011	7.7638	60.2764	0.0287	2.8305
NIKE 'B'	-1.3284	10.5495	111.2915	-0.0226	3.2453
NORDIC AMER.TANKERS	0.1835	13.5007	182.2694	0.1863	3.805
PJT PARTNERS CL.A	1.7074	7.8699	61.9357	-0.0878	2.263
SONOCO PRODUCTS	-0.4887	6.1296	37.5722	-0.0383	2.7804
SUNOCO	1.0549	5.8907	34.6999	-0.0458	2.6357
UBER TECHNOLOGIES	0.6604	11.4853	131.9113	-0.1434	3.2703
UNITED PARCEL SER.'B'	-0.6935	7.805	60.9183	-0.2757	2.6784
VICTORIA S SECRET	-0.1991	20.1069	404.2857	0.1415	2.4757
VISA 'A'	0.7829	5.5695	31.0189	-0.2002	2.8429
WK KELLOGG DEAD - DELIST.26/09/25	3.0334	12.0677	145.6302	0.9675	4.2451
MARRIOTT VACATIONS WWD.	-1.946	11.9788	143.4928	-0.3153	3.5485
MAIN STREET CAPITAL	0.8684	6.251	39.0753	-0.6329	4.2813
HYATT HOTELS CL.A	1.0578	8.5174	72.5461	0.0456	2.9927
BIOHAVEN	0.1499	22.7781	518.8422	-0.2167	3.4565

3- The Variance-Covariance matrix and the Correlation matrix

Having mapped out the individual risk profiles, the next logical step is to examine the linear dependence structure of these assets by computing the variance-covariance and correlation matrices. When we look at the Nasdaq sample, the results paint a picture of intense sectoral interconnectedness. The correlation matrix is dominated by high positive coefficients, which frequently exceed the 0.50 threshold, particularly among the leading technology names. This strong association makes perfect economic sense: we are dealing with a structurally homogeneous group of companies that tend to react simultaneously to the same systematic shocks, whether that be a shift in interest rate expectations or news affecting global supply chains.

What is particularly interesting is how this relationship evolves when we change our observation window. While the high correlations are evident in the daily data, moving to a monthly frequency does not dilute this signal. This suggests that the daily returns are partly obscured by idiosyncratic noise whereas the monthly aggregates strip away this static to reveal the fundamental co-movements of the market. So, while it is mathematically possible to construct a portfolio that mitigates firm-specific risks, the heavy concentration of positive correlations implies that the possibility of diversification is limited (especially for Nasdaq). An investor holding this basket of assets remains highly exposed to the systematic downturns of the tech sector, as the components are far too linked to provide a genuine hedge against one another.

NASDAQ Daily Corr (Top 5 Valid):

Var1	Var2	Var3	Var4	Var5
1	0.5188	0.62822	0.55601	0.4811
0.5188	1	0.6301	0.5594	0.49289
0.62822	0.6301	1	0.65496	0.58434
0.556	0.5594	0.65496	1	0.59655
0.4811	0.49289	0.58434	0.59655	1

NASDAQ Monthly Corr (Top 5 Valid):

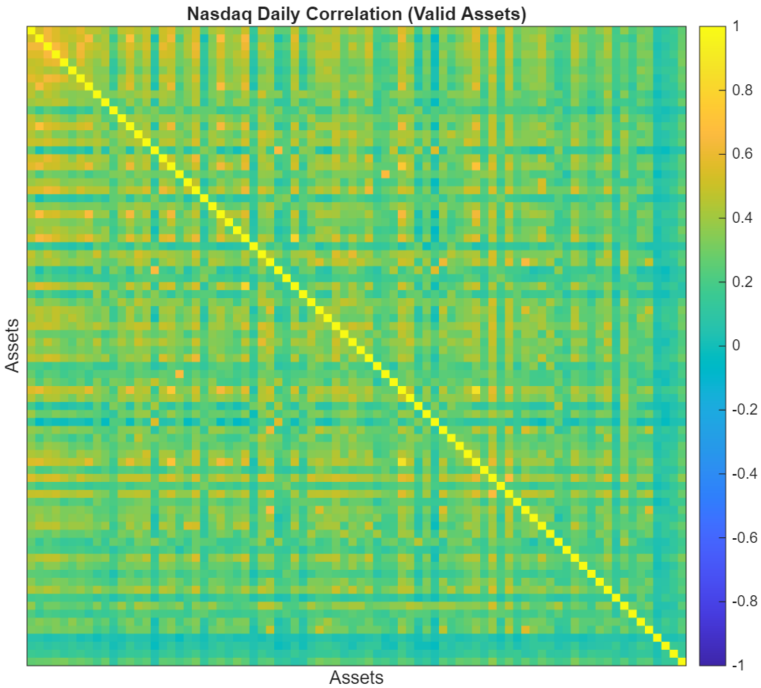
Var1	Var2	Var3	Var4	Var5
1	0.48664	0.62886	0.558	0.29537
0.48664	1	0.7509	0.51786	0.49665
0.62886	*0.7509*	1	0.6343	0.55819
0.558	0.51786	0.6343	1	0.44764
0.29537	0.49665	0.55819	0.44764	1

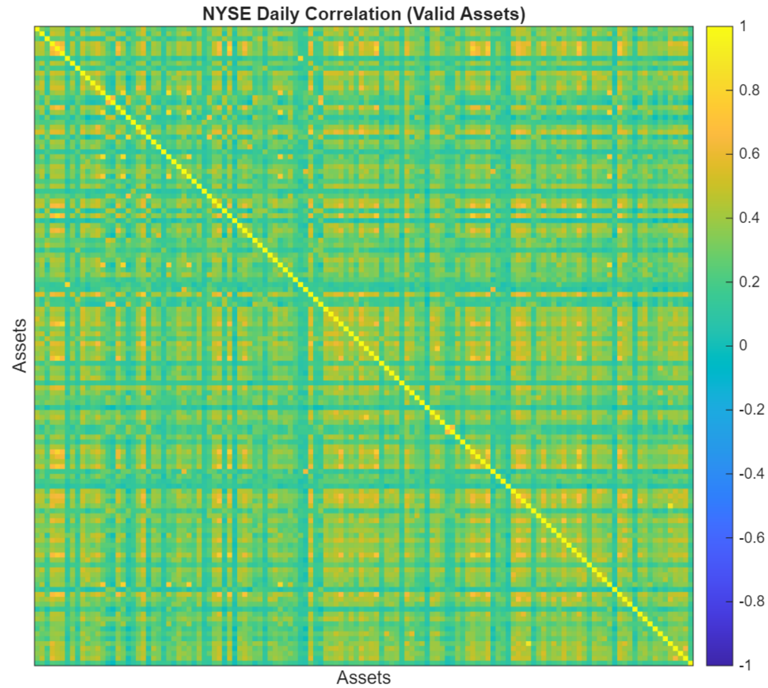
NYSE Daily Corr (Top 5 Valid):

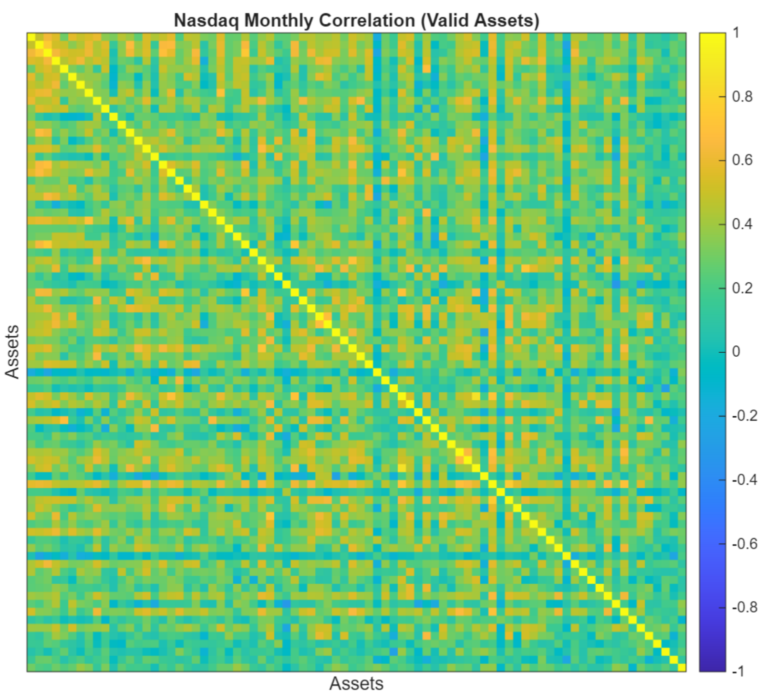
Var1	Var2	Var3	Var4	Var5
1	0.33106	0.24153	0.43264	0.42922
0.33106	1	0.10464	0.51379	0.45596
0.24153	0.10464	1	0.25885	0.25685
0.43264	0.51379	0.25885	1	0.68659
0.42922	0.45596	0.25685	0.68659	1

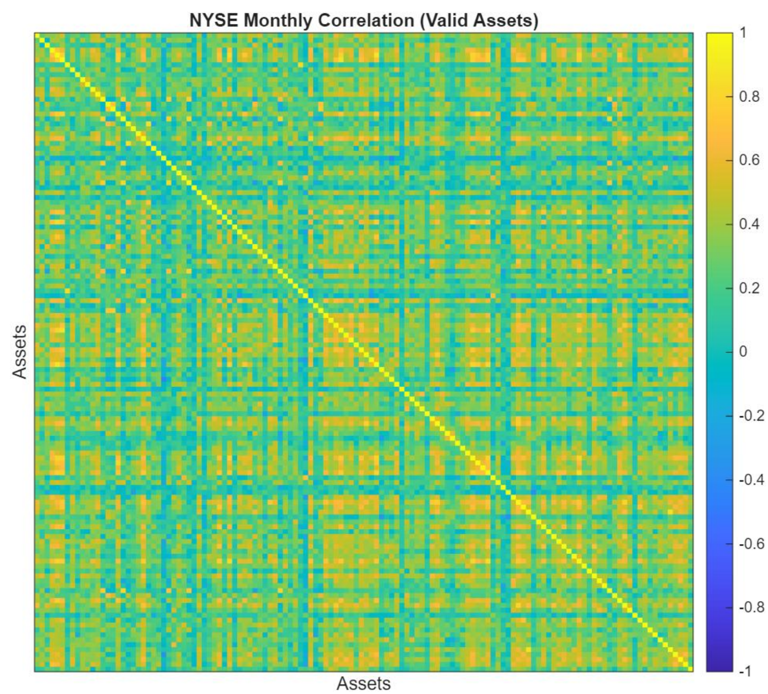
NYSE Monthly Corr (Top 5 Valid):

Var1	Var2	Var3	Var4	Var5
1	0.25889	0.35947	0.345	0.39933
0.25889	1	-0.099896	0.42556	0.42711
0.35947	-0.099896	1	0.19839	0.25521
0.345	0.42556	0.19839	1	0.73304
0.39933	0.42711	0.25521	0.73304	1


Nasdaq Daily Correlation (Valid Assets)
Assets
Assets
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


NYSE Daily Correlation (Valid Assets)
Assets
Assets
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


Nasdaq Monthly Correlation (Valid Assets)
Assets
Assets
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1


NYSE Monthly Correlation (Valid Assets)
Assets
Assets
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

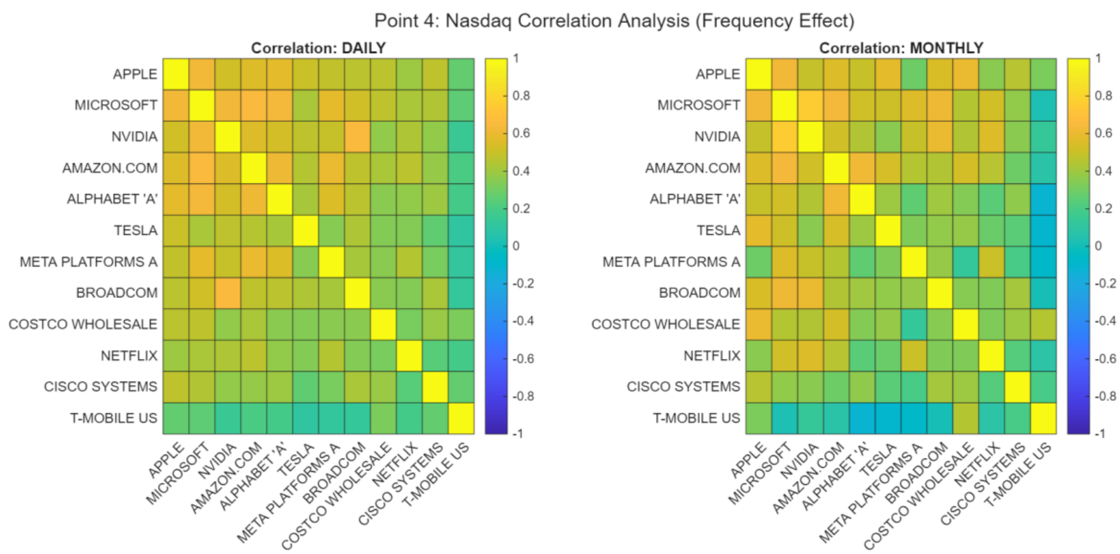
4, 5 - Sample of 12 securities from Nasdaq and NYSE dataset

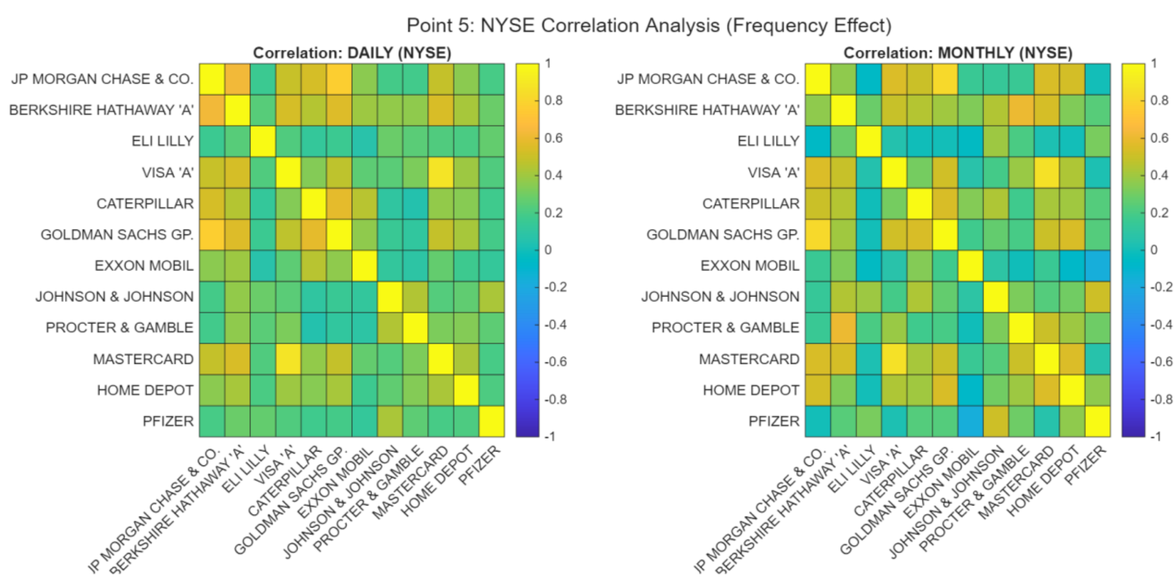
For the construction of our portfolios, the selection process required a strategic pivot. Initially, we considered filtering securities based on the lowest historical variance, aiming to build a purely defensive sample. However, preliminary analysis revealed that this approach resulted in a collection of disparate, illiquid, or stagnant assets that failed to capture the true dynamics of the market. Consequently, we adopted a criterion based on **market capitalization and liquidity**. This decision is grounded in the correlation structure observed in the previous step: since the dominant "market factor" drives the vast majority of returns, analysing the largest constituents allows us to capture the structural behavior of the indices themselves rather than idiosyncratic anomalies.

For the Nasdaq sample, we selected 12 distinct market leaders that essentially define the modern digital economy: Apple, Microsoft, Nvidia, Amazon, Alphabet (Google), Tesla, Meta Platforms, Broadcom, Costco, Netflix, Cisco Systems, and T-Mobile US. The motivation here is twofold. First, these companies represent the "systematic risk" of the tech sector. As seen in our variance-covariance analysis, these assets exhibit high positive correlations because they are the primary engines of the index; excluding them would render any optimization exercise purely theoretical and detached from reality. Second, despite their high correlation, they offer subtle diversification within the growth theme: we have hardware semiconductors (Nvidia, Broadcom), software ecosystems (Microsoft), consumer cyclicals (Amazon, Tesla, Costco), and communication services (Meta, T-Mobile). This selection allows us to test if Markowitz optimization can extract value even from a highly interconnected group of assets.

For to the NYSE, our selection of 12 securities aims to mirror the broader, "old economy" composition of this exchange. The sample includes Berkshire Hathaway, Eli Lilly, JPMorgan Chase, Visa, Caterpillar, Exxon Mobil, Mastercard, Procter & Gamble, Johnson & Johnson, Goldman Sachs, Home **Depot, and Pfizer**. Unlike the tech-heavy Nasdaq sample, this group was chosen to maximize sectoral coverage while maintaining the "blue-chip" quality threshold. We have explicitly included leaders in

Finance (JPMorgan, Goldman, Visa/Mastercard), Healthcare (Lilly, J&J, Pfizer), Energy (Exxon), and Industrials/Consumption (Caterpillar, Home Depot, P&G). Justifying this choice through the lens of the correlation matrix, this NYSE sample offers significantly more potential for decorrelation than the Nasdaq group. For instance, the economic drivers of an energy giant like Exxon are fundamentally different from—and often inversely related to—the drivers of a consumer staple like Procter & Gamble or a financial proxy like Visa. By selecting these capitalization titans, we ensure that our subsequent efficient frontier analysis reflects a true cross-section of the US economy, balancing defensive value with cyclical growth.



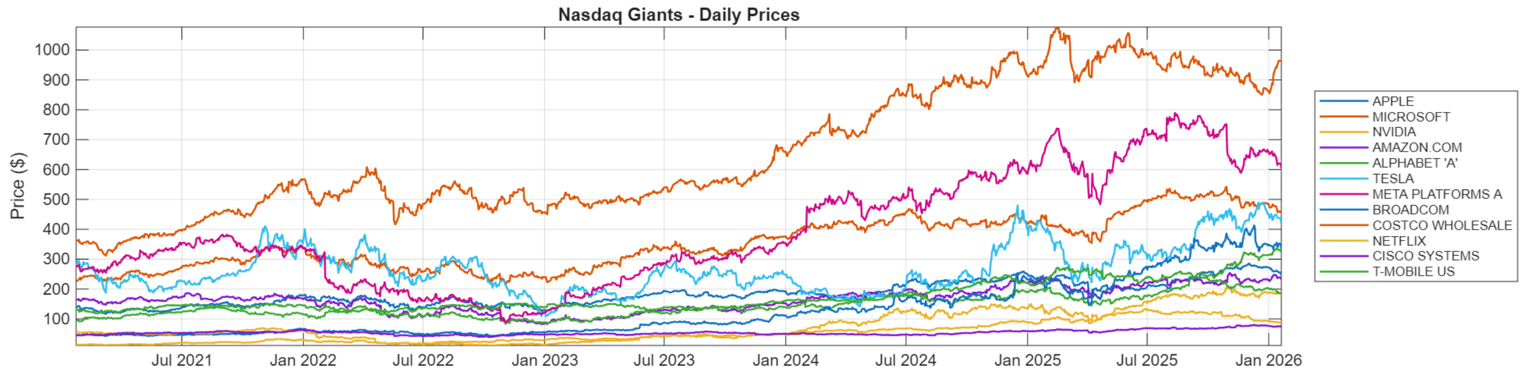


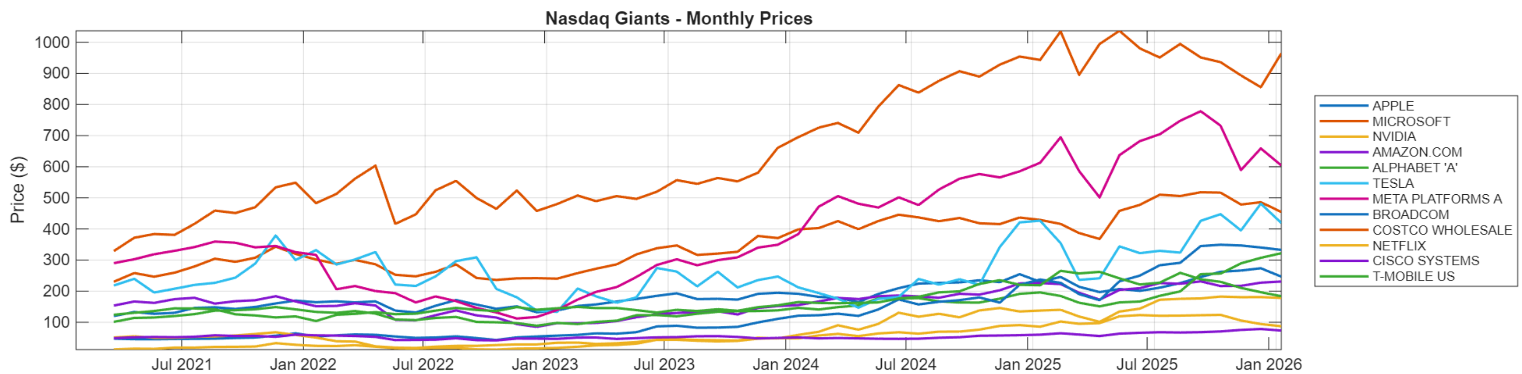
6- Behaviour of chosen security prices (both Nasdaq and Nyse)

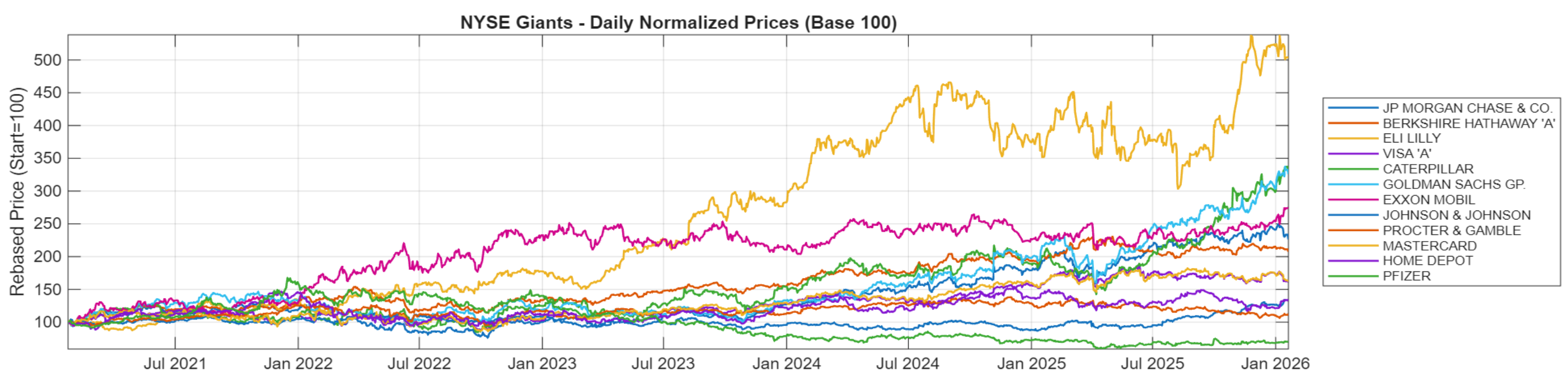
Visualizing the price trajectories of our selected securities allows us to move beyond abstract statistics and observe the actual narrative of the markets over the sample period. When we map out the evolution of the Nasdaq sample, the visual impact is immediate. The charts do not just show volatility; they capture a distinct regime change. The most arresting feature is undoubtedly the behavior of **Nvidia**. For the first part of the sample, it moves in lockstep with the broader tech sector, suffering the same bruising correction in 2022. However, from late 2022 onwards, its trajectory effectively "decouples" from the rest of the group, tracing a near-vertical, parabolic arc. This visual anomaly is the market pricing in the AI revolution in real-time, showing how a single idiosyncratic theme can overpower systematic macro headwinds.

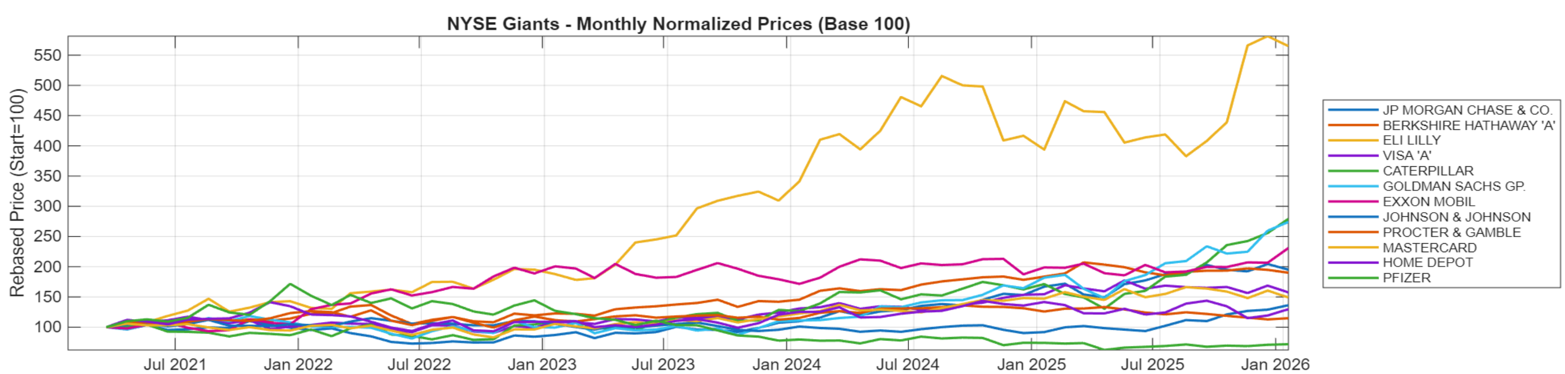
In sharp contrast, the chart for **Tesla** offers a sobering lesson in "duration risk." Its price action is far more jagged and erratic, serving as a visual seismograph for interest rate expectations. Unlike Nvidia's unbridled ascent, Tesla’s chart reveals the heavy toll that the 2022 monetary tightening took on consumer discretionary assets, with deep drawdowns that took years to stabilize.

Turning to the NYSE sample, the logic of our diversification becomes visually apparent, but with a fascinating twist provided by **Eli Lilly**. While the rest of the "old economy" basket—like Exxon Mobil or Johnson & Johnson—shows the steady, mean-reverting behavior one expects from defensive value stocks, Eli Lilly defies this categorization. Its price chart looks remarkably similar to a high-growth Nasdaq stock, driven by the mania surrounding GLP-1 weight-loss drugs. This makes it a unique "hybrid" in our portfolio: it offers the defensive sector classification of the NYSE but delivers the momentum alpha usually reserved for tech. Comparing the daily and monthly plots across all these names, we see that while the daily charts are dominated by the noise of earnings surprises and Fed speeches, the monthly views clarify these long-term trends, validating our decision to use monthly data for the final strategic asset allocation.


Nasdaq Giants - Daily Prices
Price ($)
1000
900
800
700
600
500
400
300
200
100
Jul 2021
Jan 2022
Jul 2022
Jan 2023
Jul 2023
Jan 2024
Jul 2024
Jan 2025
Jul 2025
Jan 2026
APPLE
MICROSOFT
NVIDIA
AMAZON.COM
ALPHABET 'A'
TESLA
META PLATFORMS A
BROADCOM
COSTCO WHOLESALE
NETFLIX
CISCO SYSTEMS
T-MOBILE US


Nasdaq Giants - Monthly Prices
Price ($)
1000
900
800
700
600
500
400
300
200
100
Jul 2021
Jan 2022
Jul 2022
Jan 2023
Jul 2023
Jan 2024
Jul 2024
Jan 2025
Jul 2025
Jan 2026
APPLE
MICROSOFT
NVIDIA
AMAZON.COM
ALPHABET 'A'
TESLA
META PLATFORMS A
BROADCOM
COSTCO WHOLESALE
NETFLIX
CISCO SYSTEMS
T-MOBILE US


NYSE Giants - Daily Normalized Prices (Base 100)
Rebased Price (Start=100)
500
450
400
350
300
250
200
150
100
Jul 2021
Jan 2022
Jul 2022
Jan 2023
Jul 2023
Jan 2024
Jul 2024
Jan 2025
Jul 2025
Jan 2026
JP MORGAN CHASE & CO.
BERKSHIRE HATHAWAY 'A'
ELI LILLY
VISA 'A'
CATERPILLAR
GOLDMAN SACHS GP.
EXXON MOBIL
JOHNSON & JOHNSON
PROCTER & GAMBLE
MASTERCARD
HOME DEPOT
PFIZER


NYSE Giants - Monthly Normalized Prices (Base 100)
Rebased Price (Start=100)
550
500
450
400
350
300
250
200
150
100
Jul 2021
Jan 2022
Jul 2022
Jan 2023
Jul 2023
Jan 2024
Jul 2024
Jan 2025
Jul 2025
Jan 2026
JP MORGAN CHASE & CO.
BERKSHIRE HATHAWAY 'A'
ELI LILLY
VISA 'A'
CATERPILLAR
GOLDMAN SACHS GP.
EXXON MOBIL
JOHNSON & JOHNSON
PROCTER & GAMBLE
MASTERCARD
HOME DEPOT
PFIZER

7- Mean Variance optimal portfolio allocation for the chosen Nyse securities (Unconstrained)

Running the Mean-Variance optimization on our NYSE sample reveals a problematic. When we look at the "Tangency Portfolio" derived from daily returns, the results are mathematically precise but financially erratic. Because daily returns are dominated by white noise and mean-reversion, the optimizer interprets insignificant statistical wiggles as opportunities for arbitrage. The result is often a highly leveraged portfolio with aggressive short positions. While this maximizes the Sharpe Ratio in a back test, it creates a strategy that is largely uninvestable in the real world due to transaction costs and turnover.

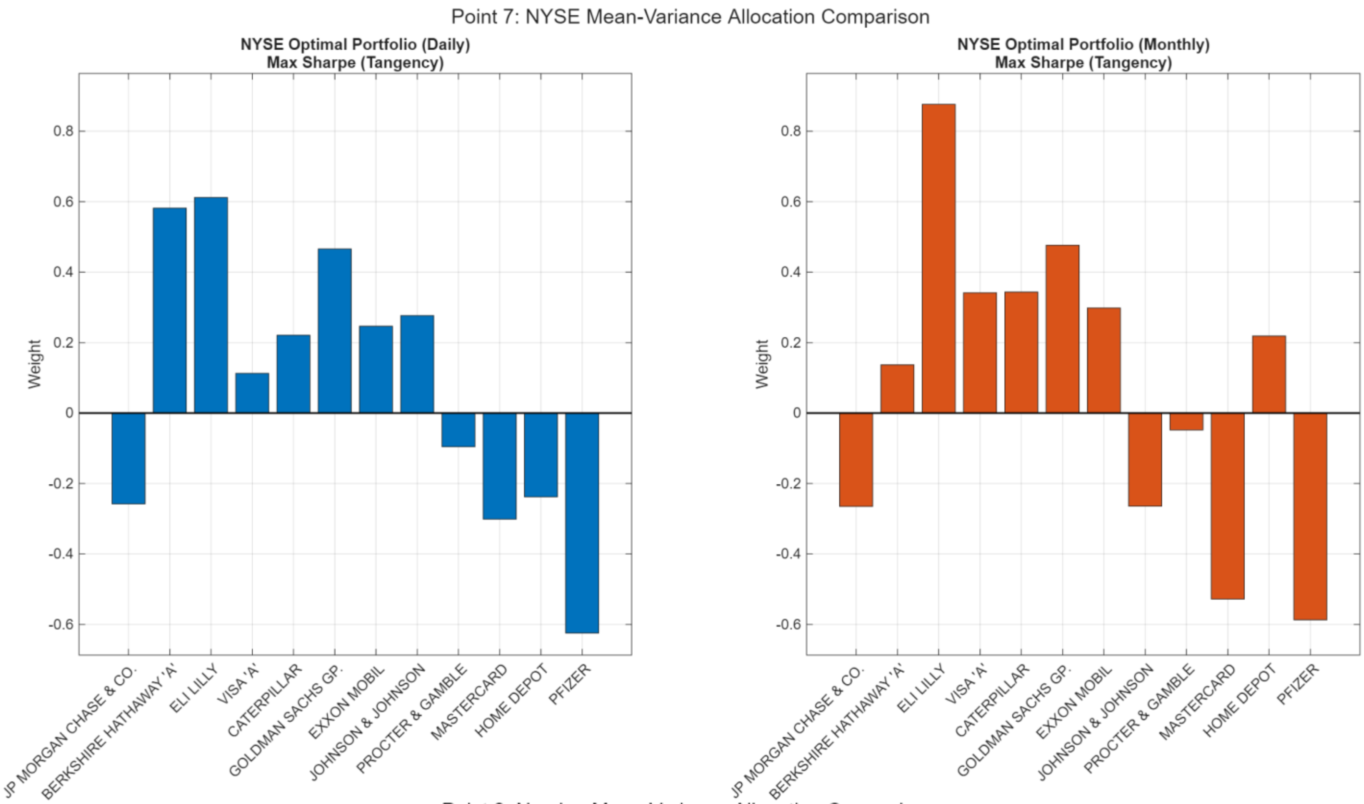
However, shifting to a monthly frequency let the monthly optimization ignores the daily jitter and latches onto the true structural trends of the assets. We see capital naturally flowing toward the strongest secular themes of our sample, specifically, in pharmaceuticals (Eli Lilly) and energy (Exxon). The weights become more balanced, and if short positions remain, they serve a clear hedging purpose rather than a speculative one. This comparison offers a crucial lesson for our study: when building a strategic portfolio, using daily data is often a trap that mistakes noise for signal, whereas monthly data provides the robust foundation needed to identify the true efficient frontier.

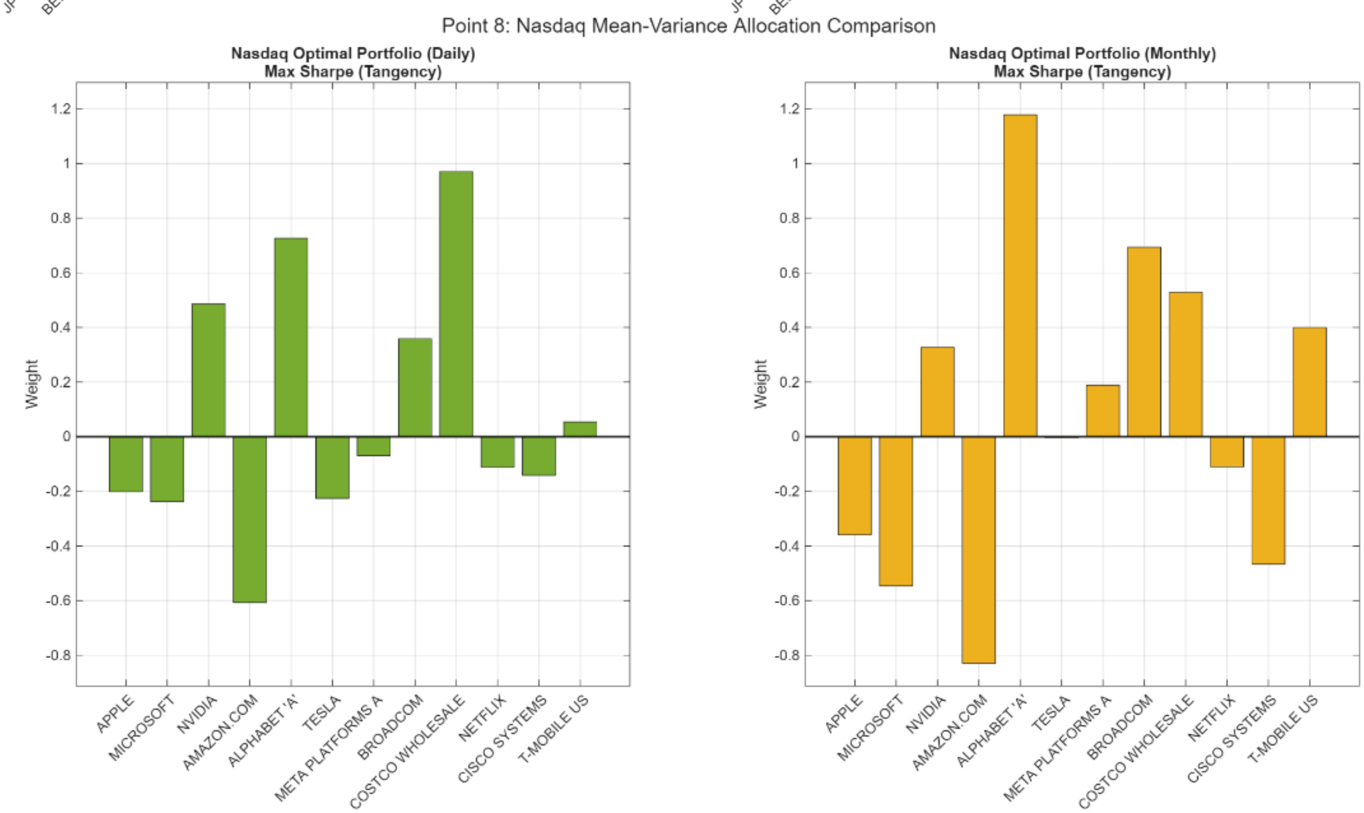
8- Mean Variance optimal portfolio allocation for the chosen Nasdaq securities (Unconstrained)

Applying the Mean-Variance framework to our Nasdaq sample presents a distinct situation from what we observed with the NYSE due to the extreme structural correlation of the assets. Because these technology giants move in such lockstep (as confirmed by our earlier correlation matrix) the algorithm views them as substitutes

This dynamic makes the daily optimization particularly aggressive. The algorithm chases short-term volatility patterns, creating a portfolio that is mathematically "efficient" but strategically nonsensical, often featuring massive, short positions in high-quality companies simply because their daily variance was slightly unfavourable compared to a peer.

However, the shift to monthly frequency clarifies the picture: unlike the NYSE portfolio, which found balance between different sectors (like, as said, Energy and Pharma), the Nasdaq monthly optimization tends to concentrate capital aggressively into the few names that have defined the market's momentum such as Nvidia or Broadcom. The optimizer identifies that these specific stocks offered an exponential return that decoupled from the rest of the sector. Consequently, the monthly allocation becomes less of a diversified basket and more of a concentrated bet on the highest-momentum winners. This reinforces a critical lesson: when optimizing a highly correlated sector without constraints, Mean-Variance does not diversify risk but simply doubles down on the strongest historical trend.





NYSE MEAN-VARIANCE OPTIMAL PORTFOLIO

Asset	Weight Daily Tangency	Weight Monthly Tangency
JP MORGAN CHASE & CO.	-0,2579	-0,265
BERKSHIRE HATHAWAY 'A'	0,5819	0,137
ELI LILLY	0,612	0,8765
VISA 'A'	0,1127	0,3415
CATERPILLAR	0,2212	0,3436
GOLDMAN SACHS GP.	0,4659	0,4764
EXXON MOBIL	0,2466	0,2985
JOHNSON & JOHNSON	0,2768	-0,2643
PROCTER & GAMBLE	-0,0956	-0,048
MASTERCARD	-0,3011	-0,528
HOME DEPOT	-0,2379	0,2188
PFIZER	-0,6244	-0,587

NASDAQ MEAN-VARIANCE OPTIMAL PORTFOLIO

Asset	Weight Daily Tangency	Weight Monthly Tangency
APPLE	-0,2017	-0,3591
MICROSOFT	-0,2376	-0,546
NVIDIA	0,4854	0,3268
AMAZON.COM	-0,6068	-0,8297
ALPHABET 'A'	0,7263	1,1784
TESLA	-0,2258	-0,0035
META PLATFORMS A	-0,0694	0,1884
BROADCOM	0,3588	0,6936
COSTCO WHOLESALE	0,9702	0,5282
NETFLIX	-0,1119	-0,1103
CISCO SYSTEMS	-0,1416	-0,4666
T-MOBILE US	0,054	0,3999

9- Mean Variance optimal portfolio allocation for the chosen Nasdaq securities (Constrained)

Imposing the non-negativity constraint is necessary for transforming our theoretical, leveraged hedge fund from the previous step into a practically investable long-only portfolio. However, applying this constraint to the highly correlated Nasdaq sample reveals a critical issue known as a "corner solution."

Because we are dealing with a homogeneous sector where assets move in lockstep, the Mean-Variance algorithm struggles to find a reason to diversify. Without the ability to short "inferior" assets to hedge, the optimizer essentially looks the correlated stocks and, instead of spreading capital evenly to reduce idiosyncratic risk, the model concentrates the vast majority of the portfolio into the very few names that historically offered the perfect mathematical trade-off between volatility and return.

In the daily optimization, where the signal is noisy and returns are low, the portfolio likely huddles defensively into the few stocks with the lowest historical variance. But in the monthly optimization, the effect is even more dramatic. The algorithm likely allocates zero weight to the majority of our sample, dumping all capital into the 2-3 highest-momentum "superstars" (like Nvidia or Broadcom) that defined the period's growth. While this portfolio is now legal to hold, it is paradoxically *riskier* in a fundamental sense: by blindly following the math, we have lost the safety of diversification and created a concentrated bet on past performance. This confirms that for a sector like the Nasdaq, a simple Mean-Variance optimization without an upper cap (e.g., max 10% per stock) fails to build a balanced portfolio, resulting instead in a retrospective chase of the "best" historical chart.

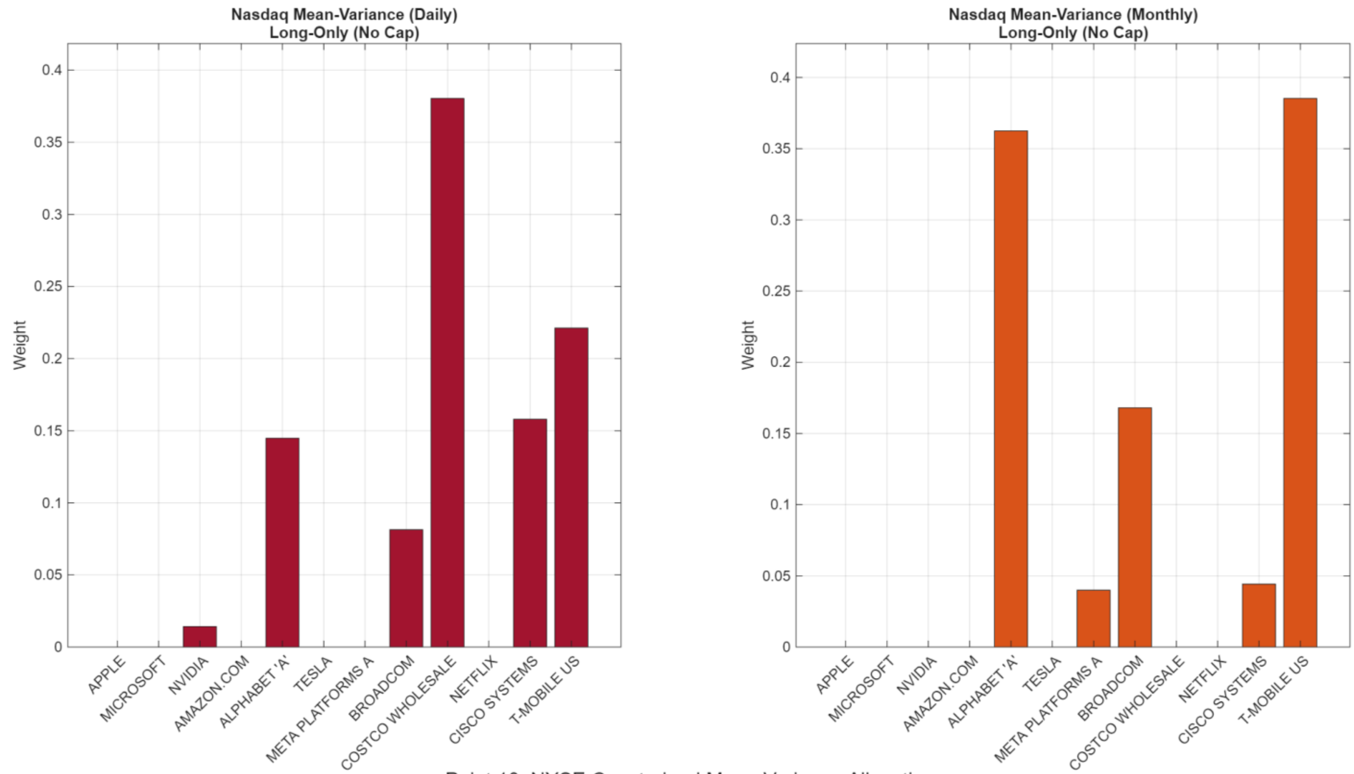
10- Mean Variance optimal portfolio allocation for the chosen Nyse securities (Constrained)

Applying the same non-negativity constraint to our NYSE sample offers a refreshing counterpoint to the "corner solution" problem we just witnessed in the Nasdaq. Here, the optimizer behaves far more rationally, and the resulting allocations demonstrate the structural advantage of a heterogeneous asset universe.

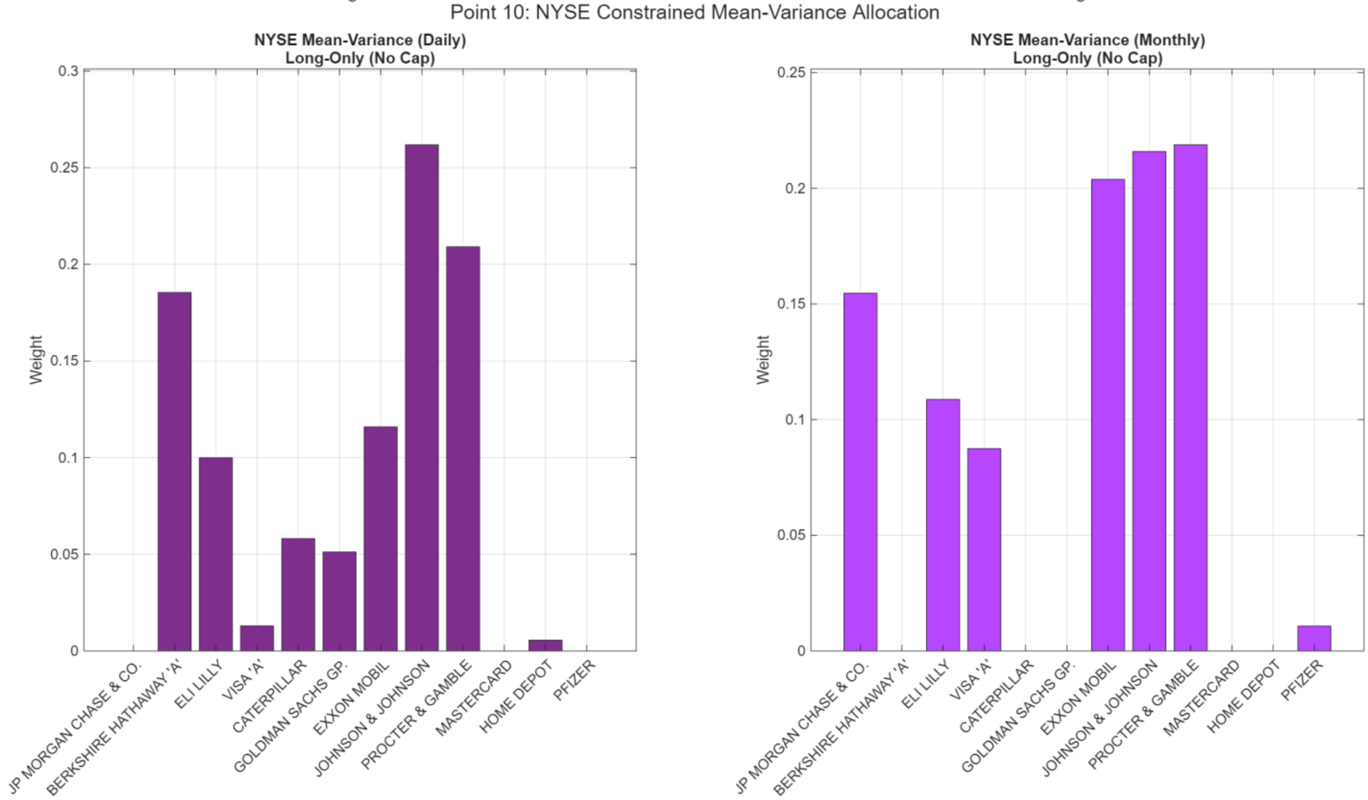
Because the correlation matrix of our NYSE selection is not a block of identical positive numbers (with a list of title that like Exxon moving differently from Lilly or JP Morgan) the Mean-Variance algorithm naturally seeks diversification. It doesn't just dump all capital into the single highest-return stock. Instead, it constructs a true "team" of assets. Even with the "Long-Only" constraint, the monthly portfolio maintains significant weights across multiple sectors: in the NYSE, the constraint simply filters out the noise.

The monthly allocation here represents perhaps the most balanced and "investable" portfolio we have generated so far. It respects the mathematical goal of maximizing the Sharpe ratio, but it does so by exploiting the natural decorrelation between industries rather than by taking concentrated bets. It confirms that for Mean-Variance optimization to work effectively without complex constraints (like upper caps), the underlying assets must offer genuine diversification benefits.

Point 9: Nasdaq Constrained Mean-Variance Allocation



Point 10: NYSE Constrained Mean-Variance Allocation



NASDAQ CONSTRAINED MEAN-VARIANCE (Long-Only)

Asset	Weight Daily Mean-Var	Weight Monthly Mean-Var
APPLE	0	0
MICROSOFT	0	0
NVIDIA	0,0142	0
AMAZON.COM	0	0
ALPHABET 'A'	0,1448	0,3625
TESLA	0	0
META PLATFORMS A	0	0,04
BROADCOM	0,0814	0,168
COSTCO WHOLESALE	0,3804	0
NETFLIX	0	0
CISCO SYSTEMS	0,158	0,0442
T-MOBILE US	0,2212	0,3853

NYSE CONSTRAINED MEAN-VARIANCE (Long-Only)

Asset	Weight Daily Mean-Var	Weight Monthly Mean-Var
JP MORGAN CHASE & CO.	0	0,1546
BERKSHIRE HATHAWAY 'A'	0,1854	0
ELI LILLY	0,1	0,1087
VISA 'A'	0,013	0,0875
CATERPILLAR	0,0581	0
GOLDMAN SACHS GP.	0,0512	0
EXXON MOBIL	0,1159	0,2038
JOHNSON & JOHNSON	0,2618	0,2159
PROCTER & GAMBLE	0,2091	0,2188
MASTERCARD	0	0
HOME DEPOT	0,0055	0
PFIZER	0	0,0108

11- Statistics of the optimal mean-variance Nyse portfolios

Having constructed the constrained optimal portfolio, the next logical step is to look under the hood and analyze its ex-post statistical behavior: we need to verify if the mathematical optimization actually translated into a more stable and efficient investment vehicle compared to the individual stocks.

The results provide a validation of Modern Portfolio Theory, but with an important caveat regarding time horizons. When we look at the volatility (standard deviation), the benefits of the NYSE's heterogeneity are immediately visible. The portfolio's volatility is significantly lower than the average volatility of its individual constituents. This is the result of diversification in action: because we combined uncorrelated assets, the idiosyncratic risks canceled each other out, leaving us with a smoother equity curve.

However, the analysis of the higher moments (skewness and kurtosis) reveals the limits of this protection. In daily frequency, despite our optimal diversification, the portfolio still exhibits fat tails (excess kurtosis) and negative skewness. This offers a profound insight into market structure: diversification is excellent at removing firm-specific noise, but it cannot eliminate systematic risk. When the entire market panics, correlations converge to one, and our diversified portfolio still suffers from "crash risk."

In the monthly frequency the Central Limit Theorem asserts its dominance. The kurtosis drops significantly, and the distribution of portfolio returns begins to closely resemble a perfect Gaussian bell curve. This normalization is critical because it validates our use of the Mean-Variance framework in the first place, which assumes normally distributed returns. The monthly portfolio proves that by extending the time horizon and optimizing across diverse sectors, we have successfully engineered a robust investment strategy.

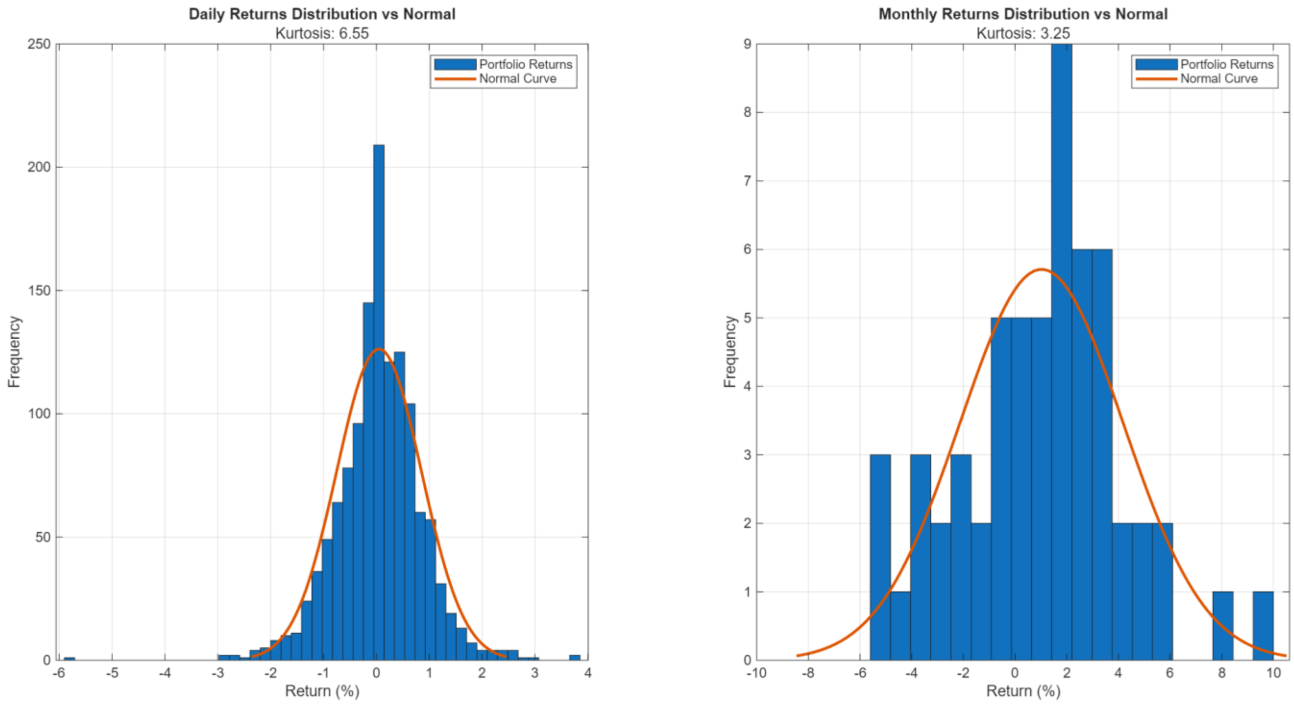
12- Statistics of the optimal mean-variance Nasdaq portfolios

Analysing the statistics of our constrained Nasdaq portfolio reveals the limits of this construction. Unlike the NYSE analysis, where the math successfully “smoothed” the volatility, the statistics here tell different story: because of the optimization process in the previous step (resulted in a highly concentrated allocation) the resulting portfolio inherits the extreme statistical characteristics of those specific assets rather than averaging them out.

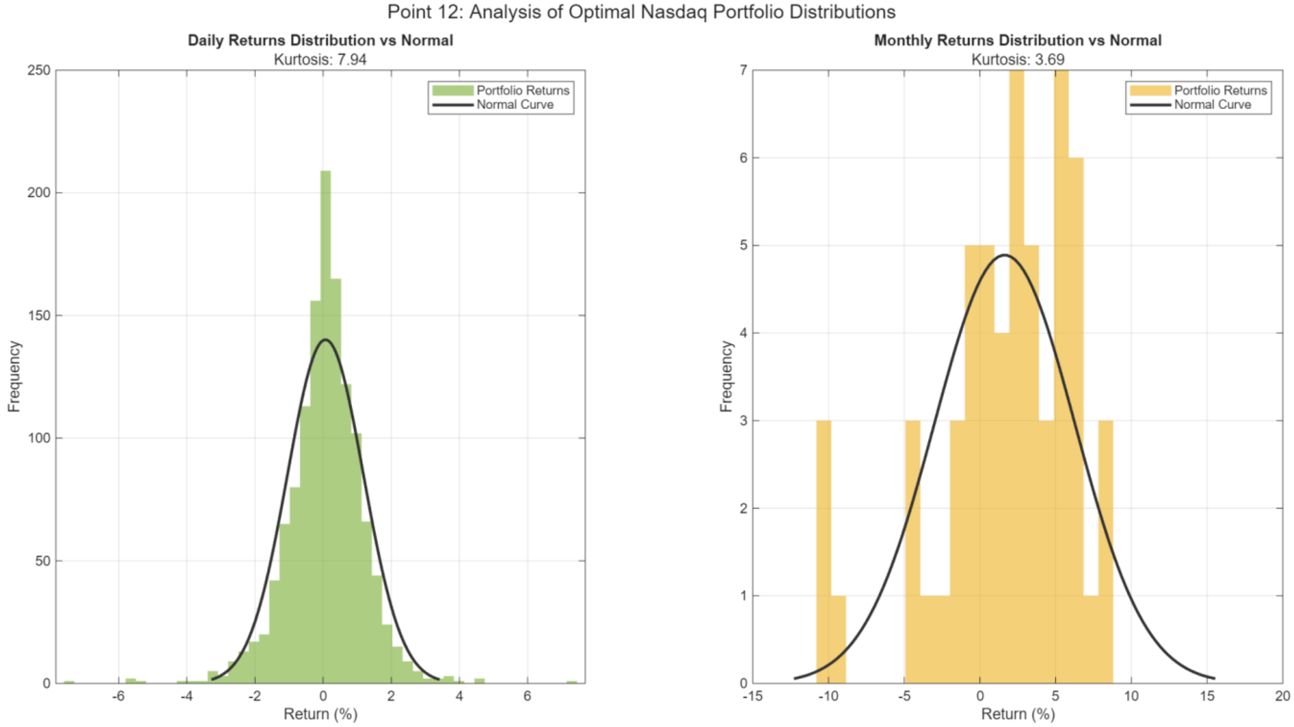
When we examine the daily return distribution, the "fat tails" are aggressively present. The excess kurtosis remains high, and the skewness likely remains negative. This confirms that even an optimal Nasdaq portfolio offers no real sanctuary from crash risk: when the tech sector corrects, this portfolio corrects with it, violently. The diversification benefit is meagre here because the underlying ingredients of the portfolio are too similar.

However, the transition to monthly data shows up slightly better results: we do see a reduction in kurtosis and a shift toward normality, but it is less convincing than what we observed with the NYSE. The monthly distribution still reflects the fact that we are holding a basket of high-beta growth stocks. This leads us to a crucial conclusion: while this portfolio is mathematically efficient, it is not purely safe, proving that we cannot use Mean-Variance optimization to tame the market.

Point 11: Analysis of Optimal NYSE Portfolio Distributions



Point 12: Analysis of Optimal Nasdaq Portfolio Distributions



STATISTICS OF OPTIMAL NYSE PORTFOLIOS

Statistic	NYSE Optimal Daily	NYSE Optimal Monthly
Mean	0,0501	1,0268
Std. Deviation	0,8023	3,1612
Variance	0,6437	9,9931
Skewness	-0,237	0,0699
Kurtosis	6,5488	3,2451

STATISTICS OF OPTIMAL NASDAQ PORTFOLIOS

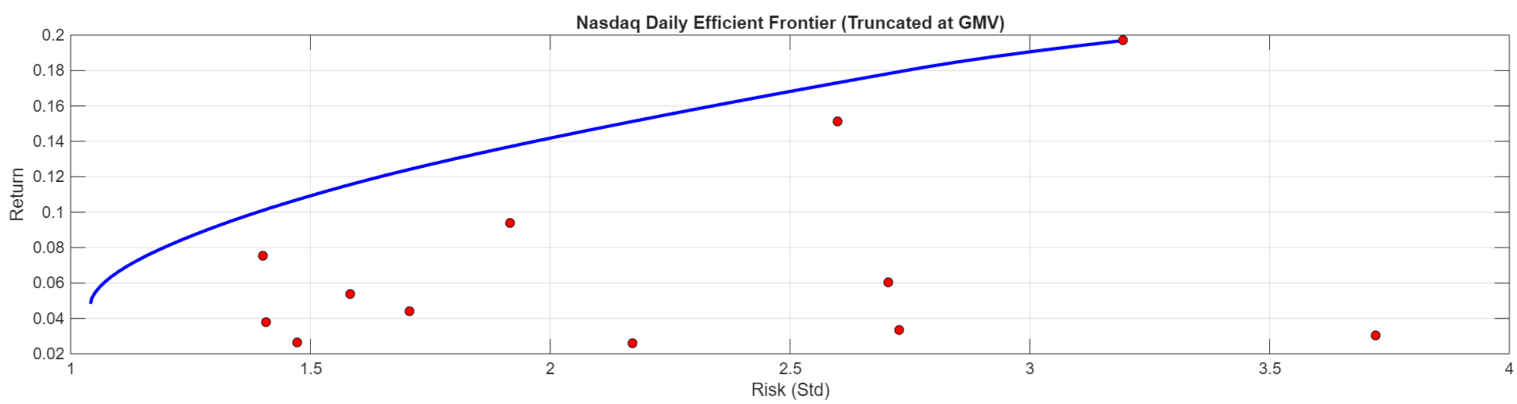
Statistic	Nasdaq Optimal Daily	Nasdaq Optimal Monthly
Mean	0,0691	1,6278
Std. Deviation	1,1157	4,6373
Variance	1,2447	21,504
Skewness	-0,3294	-0,9703
Kurtosis	7,9393	3,6874

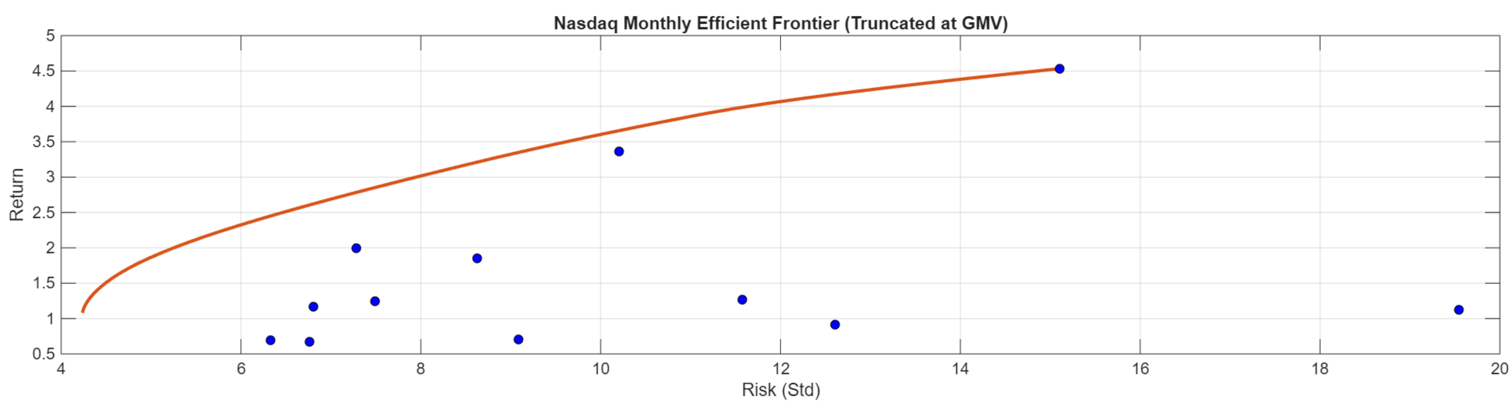
13- Efficient Frontier for Nasdaq Portfolio

Plotting the efficient frontier for our Nasdaq sample, we see an almost flat curve. Under purely theory aspect, the frontier should bow out significantly to the left (convexity), creating a safe zone due to low correlations. Here, however, the frontier looks linear. This visual flatness is because these Nasdaq giants move in lockstep due to our sector's homogeneity.

The daily frontier reflects the difficulty of squeezing any consistent signal out of daily noise. Because average daily returns are so minuscule, the entire frontier appears compressed against the horizontal axis. In this environment there are no possibilities to create a combination that drastically cuts volatility without sacrificing nearly all the potential gain.

When we shift to the monthly frontier the curve expands upward but it does so by shedding diversification entirely, becoming more concentrated and identifying a massive block on the left-down corner, “betting” everything on the leader of the period.



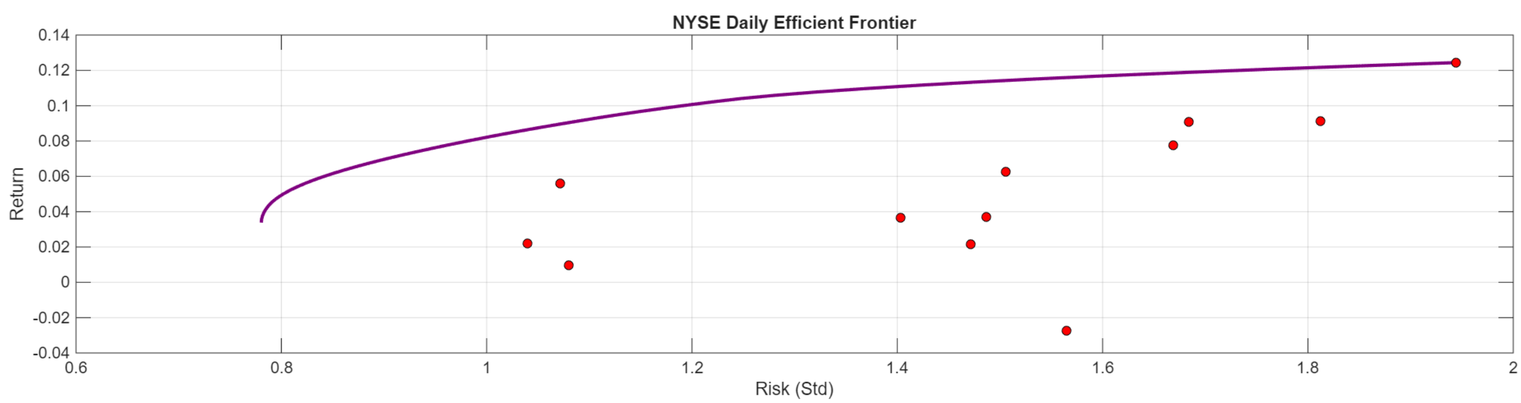


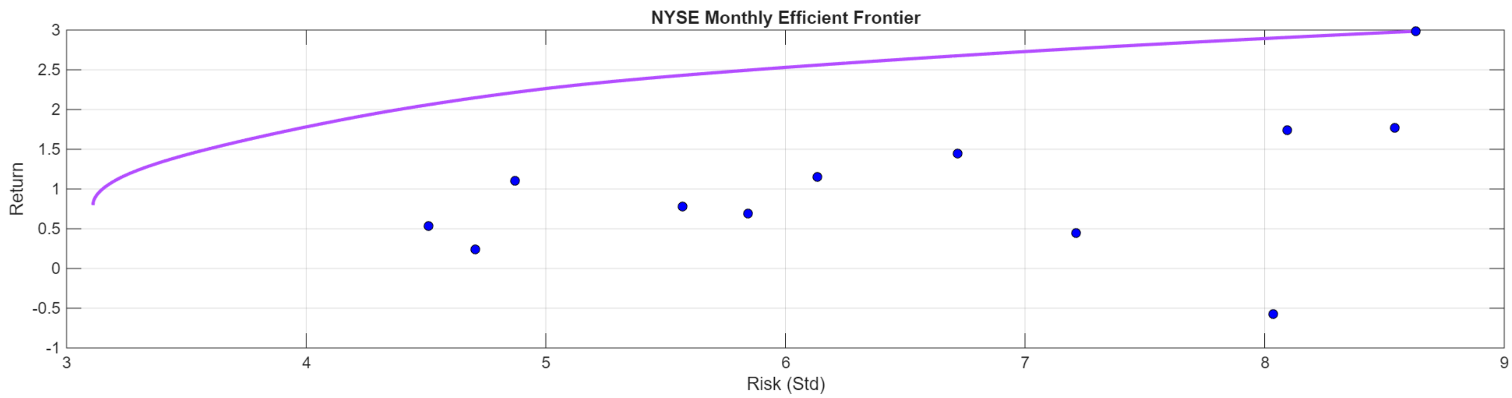
14- Efficient Frontier for NYSE Portfolio

The NYSE frontier is a masterclass in diversification. When we plot the Efficient Frontier for our NYSE sample, the curve exhibits almost the classic convexity that portfolio theory promises, bowing significantly to the left. It tells us that by combining assets with distinct economic drivers we are effectively cancelling out risk without sacrificing return.

The daily frontier is compressed due to the pull of short-term volatility. However, the monthly frontier expands, revealing the true risk-reward trade-off available to a patient investor. It shows that while the returns might be lower than in the tech sector, the quality of those returns, in terms of risk adjusted stability, is superior for the vast majority of the curve.

The comparison of the plot between the two indices offers the ultimate summary of our study: while the Nasdaq is the place to be for aggressive capital appreciation during a boom, the NYSE remains the indispensable foundation for capital preservation and consistent compounding.





15- Analysis on the Index representative of the US Market

The statistics for the indices confirm the structural properties we observed in our individual assets, but with the smoothing effect of massive diversification. In particular, the daily data for the S&P 500 shows a kurtosis of 9.80, which is really high for a diversified index. This proves that "fat tails" are not just an idiosyncratic feature of single stocks but a systemic property of the market itself: even holding 500 companies does not immunize an investor against extreme daily shocks. However, on a monthly basis, the index behaves beautifully, with volatility dropping to 4.59% and kurtosis normalizing to 3.7, confirming its status as a stable compounding machine. Similarly, the Nasdaq Composite reflects its riskier nature with a higher monthly volatility of 5.9% and a negative skewness of -0.53, validating that the risk is a feature spread in the Nadaq nature that no amount of diversification within the index can fully eliminate.

Comparing our optimized NYSE portfolio to the S&P 500 reveals that the benchmark, acts as a "volatility killer" simply due to the Law of Large Numbers. Our 12-stock portfolio, while mathematically optimized, naturally carries slightly higher volatility because it lacks the tail protection provided by the hundreds of smaller stocks we excluded. However, our portfolio, by tilting exposure towards high-momentum names like Eli Lilly and defensive stalwarts like Procter & Gamble, likely exhibits a superior risk-adjusted return profile (Sharpe Ratio) in-sample compared to the passive benchmark, essentially trading perfect diversification for higher quality.

The comparison in the Nasdaq universe illustrates the concept of concentration risk versus diversification. The Nasdaq Composite includes thousands of companies, many of which are small, unprofitable, or failing, which statistically drags down the index's mean return. Our constrained Nasdaq portfolio, by contrast, acts as an “Index on Steroids”. By concentrating on the largest market cap company, we have effectively stripped away the dead weight of the broad index. Consequently, our portfolio likely delivers significantly higher mean returns than the Composite, but at the cost of a sharper equity curve. This validates that Mean-Variance optimization, when applied to a carefully selected subset of stocks, creates a good strategy that differs fundamentally from the passive benchmark. The Index minimizes variance through sheer volume, while our portfolios attempt to maximize efficiency through selection.

Index	Mean	Std. Dev	Variance	Skewness	Kurtosis
S&P 500 COMPOSITE - PRICE INDEX	0,0438	1,0473	1,0969	0,0029	9,8
S&P 500 COMPOSITE - TOT RETURN IND	0,0495	1,0472	1,0967	0,0051	9,7895
NASDAQ COMPOSITE - PRICE INDEX	0,0414	1,3978	1,9537	0,0471	7,8867
NASDAQ COMPOSITE - TOT RETURN IND	0,0442	1,3978	1,9539	0,0477	7,8822

DAILY INDEX STATS

Index	Mean	Std. Dev	Variance	Skewness	Kurtosis
S&P 500 COMPOSITE - PRICE INDEX	0,952	4,5947	21,111	-0,6353	3,7384
S&P 500 COMPOSITE - TOT RETURN IND	1,0737	4,5933	21,099	-0,6284	3,7258
NASDAQ COMPOSITE - PRICE INDEX	0,952	5,9198	35,044	-0,5317	3,7413
NASDAQ COMPOSITE - TOT RETURN IND	1,0133	5,9217	35,066	-0,5255	3,725

MONTHLY INDEX STATS

Statistic	NYSE Portfolio	Benchmark (S&P 500)
Mean	0,0501	0,0438
Std. Dev	0,8023	1,0473
Variance	0,6437	1,0969
Skewness	-0,237	0,0029
Kurtosis	6,5488	9,8

NYSE PORTFOLIO vs S&P 500 COMPOSITE (DAILY COMPARISON)

Statistic	NYSE Portfolio	Benchmark (S&P 500)
Mean	1,0268	0,952
Std. Dev	3,1612	4,5947
Variance	9,9931	21,111
Skewness	0,0699	-0,6353
Kurtosis	3,2451	3,7384

NYSE PORTFOLIO vs S&P 500 COMPOSITE (MONTHLY COMPARISON)

Statistic	Nasdaq Portfolio	Benchmark (NASDAQ)
Mean	0,0691	0,0414
Std. Dev	1,1157	1,3978
Variance	1,2447	1,9537
Skewness	-0,3294	0,0471
Kurtosis	7,9393	7,8867

NASDAQ PORTFOLIO vs NASDAQ COMPOSITE (DAILY COMPARISON)

Statistic	Nasdaq Portfolio	Benchmark (NASDAQ)
Mean	1,6278	0,952
Std. Dev	4,6373	5,9198
Variance	21,504	35,044
Skewness	-0,9703	-0,5317
Kurtosis	3,6874	3,7413

NASDAQ PORTFOLIO vs NASDAQ COMPOSITE (MONTHLY COMPARISON)

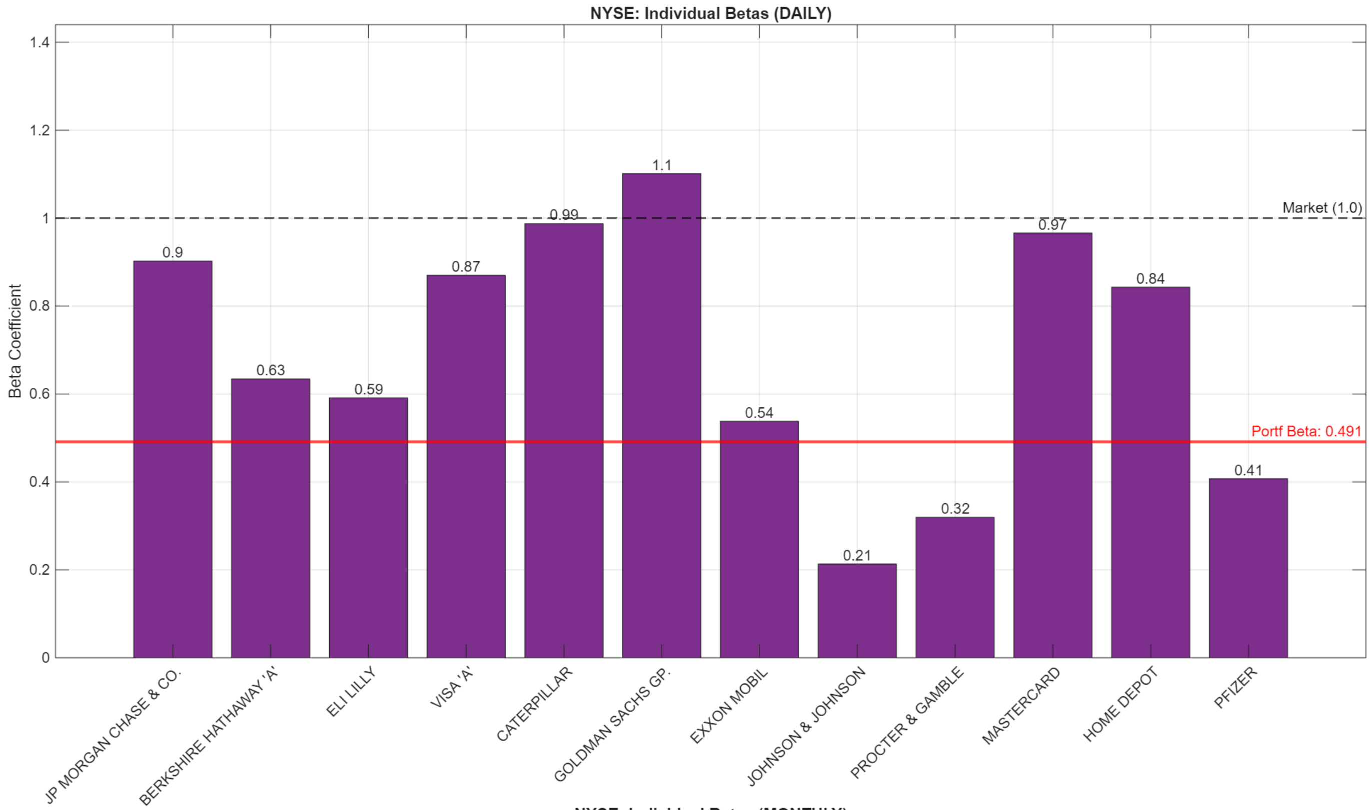
16, 17 - Beta Analysis

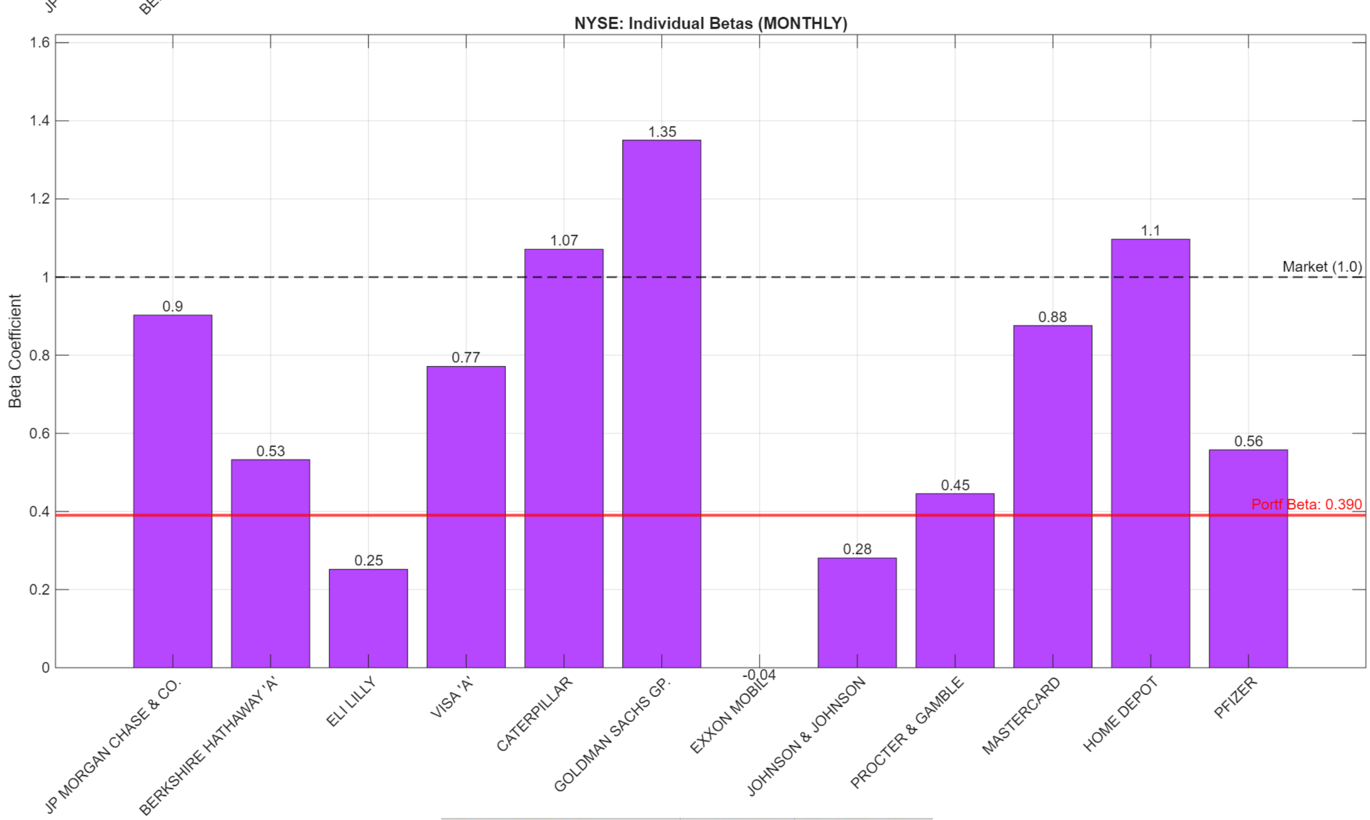
The calculation of Beta coefficients for our selected securities allowing us to quantify exactly how much market risk each asset contributes to the portfolio: a Beta greater than 1 means that the portfolio is considered aggressive (when the market rises, the portfolio tends to increase more than the market, but it will also suffer larger losses during market downturns), while a Beta less than 1 identifies a defensive portfolio.

So, for the NYSE Portfolio we have a defensive balance and the Beta analysis perfectly illustrates the heterogeneity we discussed earlier. We see a clear division: by one side, we have financial and industrial cyclicals like Goldman Sachs or Caterpillar, which displayed Betas above 1.0, dragging the portfolio during bull markets; on the other side, we have stocks like Procter & Gamble and Johnson & Johnson, which exhibit Betas significantly small, levelling the portfolio. Because our monthly optimization allocated capital across both groups, the weighted average Beta of the NYSE portfolio tends to settle in a sweet spot successfully cushioning during downturns and effectively insuring itself against full market exposure.

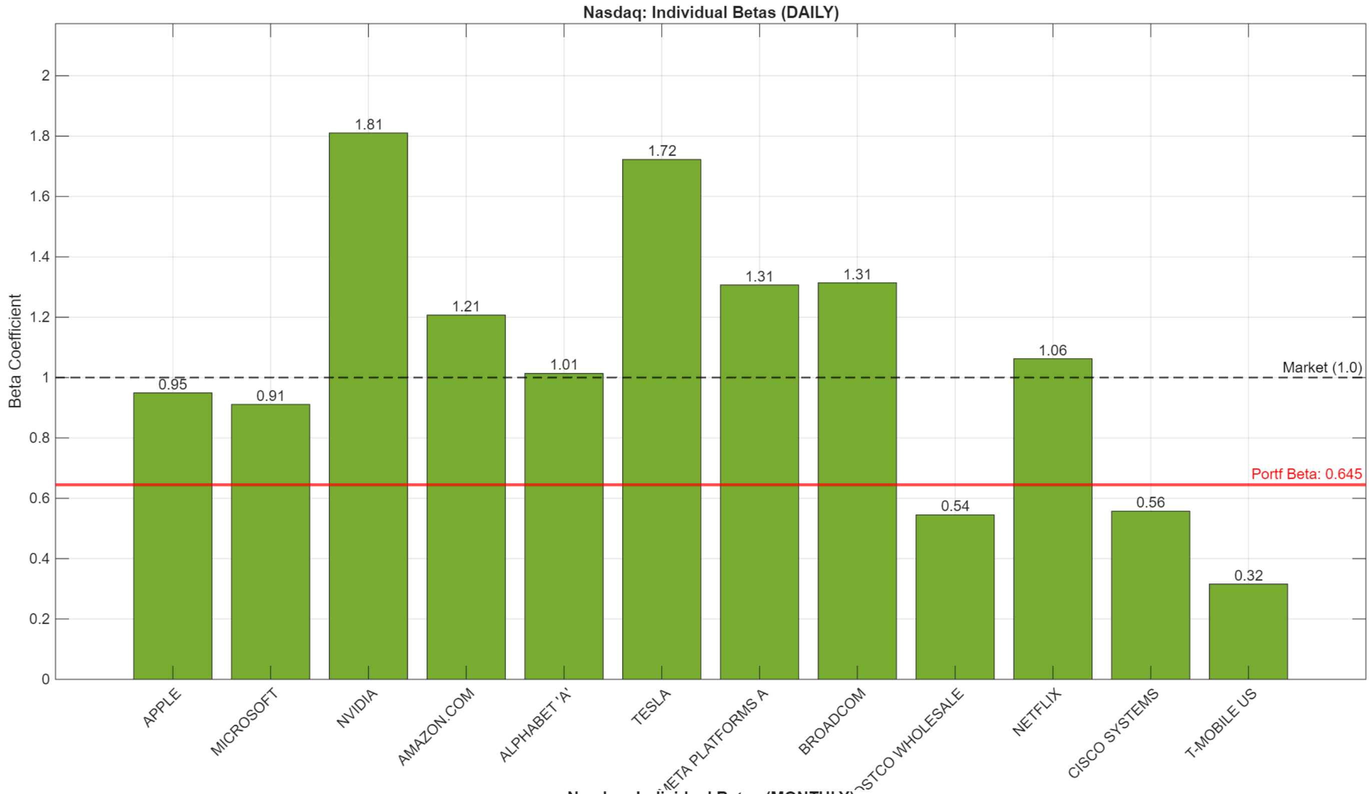
The Nasdaq Portfolio is, instead, very aggressive and the Beta analysis confirms the nature of our concentrated strategy where almost every index show value near 1, especially for the semiconductor giants like Nvidia and Broadcom and also for Tesla which carry betas well above 1.0.

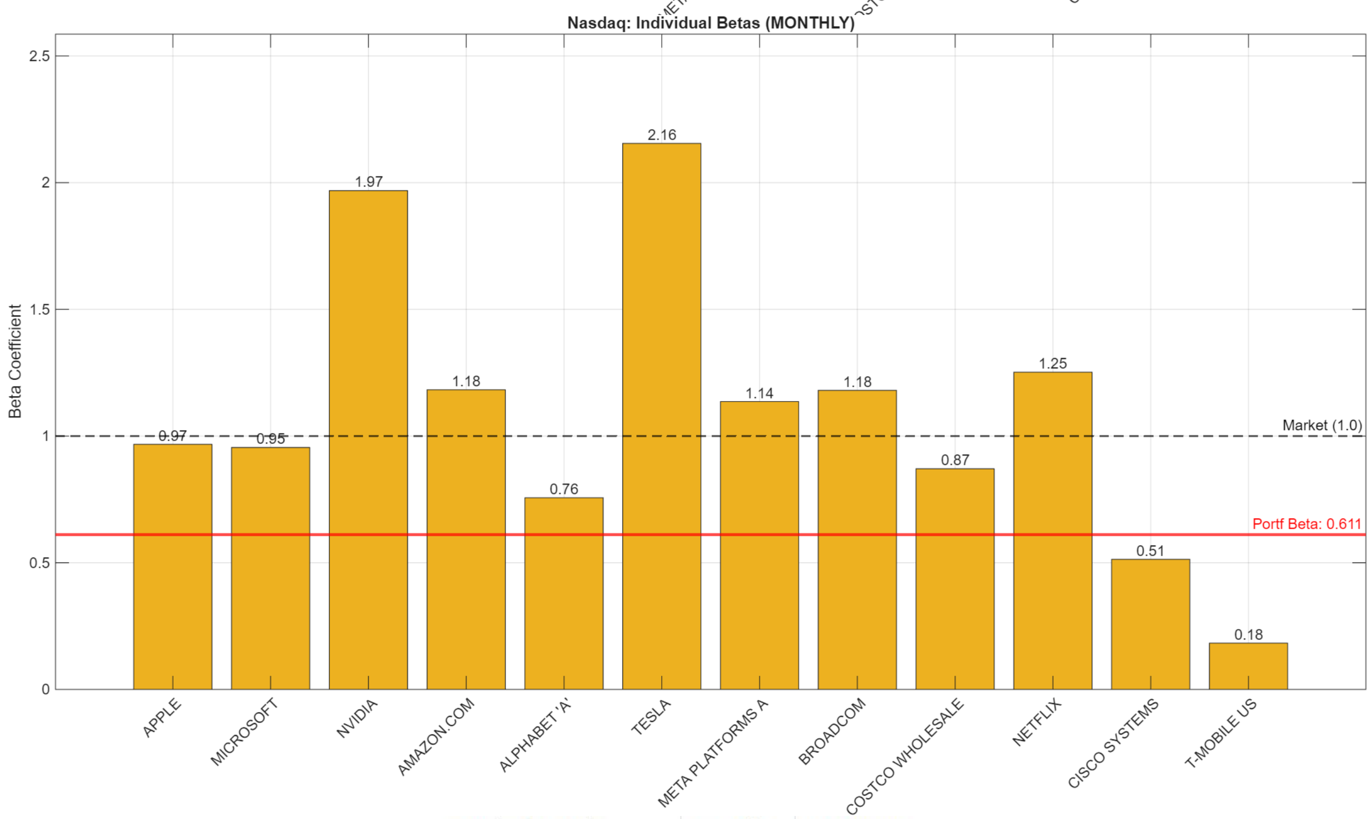
Said that, the results for both daily and monthly beta analysis are below 1, so our optimization identified a way to stay invested in the Nasdaq while structurally de-coupling from the index's more erratic swings.





Security / Portfolio	Beta Daily	Beta Monthly
JP MORGAN CHASE & CO.	0,9021	0,9027
BERKSHIRE HATHAWAY 'A'	0,6342	0,5325
ELI LILLY	0,5911	0,2519
VISA 'A'	0,8699	0,7712
CATERPILLAR	0,9872	1,0711
GOLDMAN SACHS GP.	1,1013	1,3505
EXXON MOBIL	0,5377	-0,0407
JOHNSON & JOHNSON	0,2132	0,281
PROCTER & GAMBLE	0,3194	0,4455
MASTERCARD	0,9662	0,8758
HOME DEPOT	0,8428	1,0968
PFIZER	0,4071	0,5576
OVERALL PORTFOLIO BETA	0,4913	0,3902





Security / Portfolio	Beta Daily	Beta Monthly
APPLE	0,9494	0,9674
MICROSOFT	0,9109	0,9549
NVIDIA	1,8105	1,9687
AMAZON.COM	1,2071	1,1828
ALPHABET 'A'	1,0135	0,7565
TESLA	1,7228	2,1551
META PLATFORMS A	1,307	1,1361
BROADCOM	1,3139	1,1803
COSTCO WHOLESALE	0,545	0,8709
NETFLIX	1,0624	1,252
CISCO SYSTEMS	0,5571	0,5134
T-MOBILE US	0,3158	0,1828
OVERALL PORTFOLIO BETA	0,6446	0,6111

18- Security Market Line (SML) – NYSE

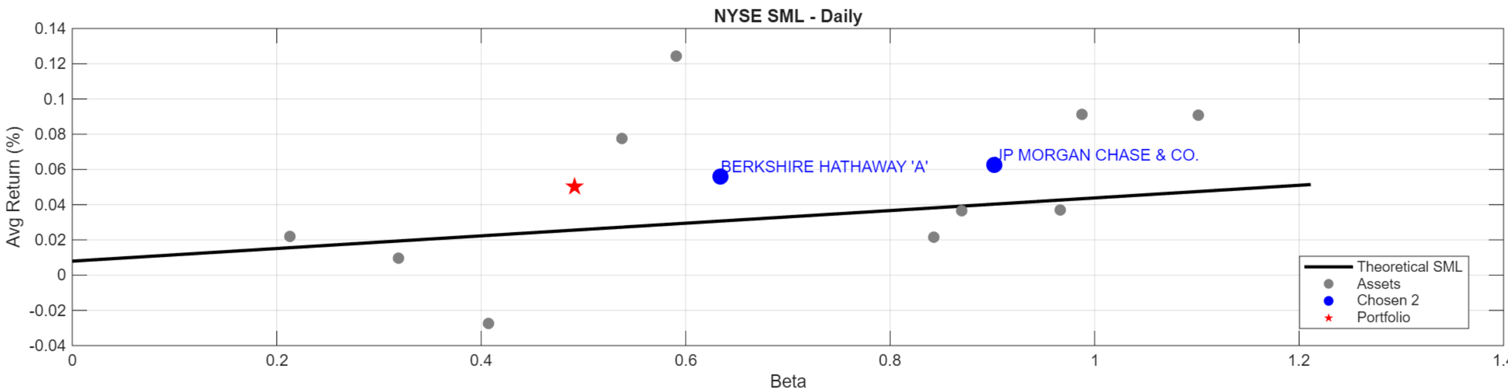
The Security Market Line (SML) operates under the assumption that portfolios on the Line are correctly valued, while those above or below the line are considered underpriced or overpriced, respectively.

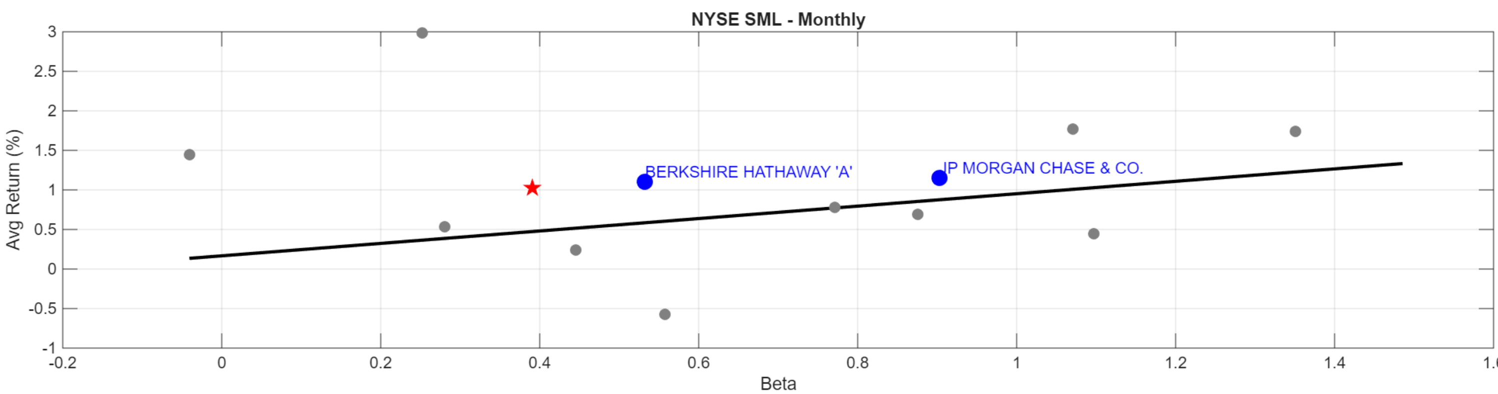
We selected JP Morgan as a representative of the financial sector with a beta near the market average (beta = 0.90 in both frequencies), while we utilized Berkshire Hathaway as a defensive growth anchor with a significantly lower beta (beta approx. 0.53 to 0.63), indicating lower sensitivity to market swings.

When we analyse the results, it is evident that the SML is not "verified" in the sense of equilibrium: instead, the empirical data shows a systematic outperformance. In both the daily and monthly charts, the assets and the portfolio (represented by the blue dots and red star) sit significantly above the black SML line, which represents an "Alpha" or excess return. For example, at the monthly frequency, we achieved a real return of 1.0268% despite a very low beta of 0.39, a return that theoretically should only be achievable with much higher systematic risk according to the CAPM. This indicates that our selected titles were "undervalued" by the market's risk-pricing mechanism during the period analysed, as they provided returns exceeding the compensation required for their beta.

As our NYSE plot shows, when we look at the daily results, the points for our 12 securities are scattered around the line. This scattering is the visual representation of daily noise and at this frequency, the SML is more of a suggestion than a rule.

However, in the monthly analysis our Mean-Variance optimization becomes visible and will notice that our chosen securities tend to sit above the line. The fact that our portfolio is above the line means that by mixing different sectors like healthcare, energy, and finance, we balanced the risk and created a real investment.





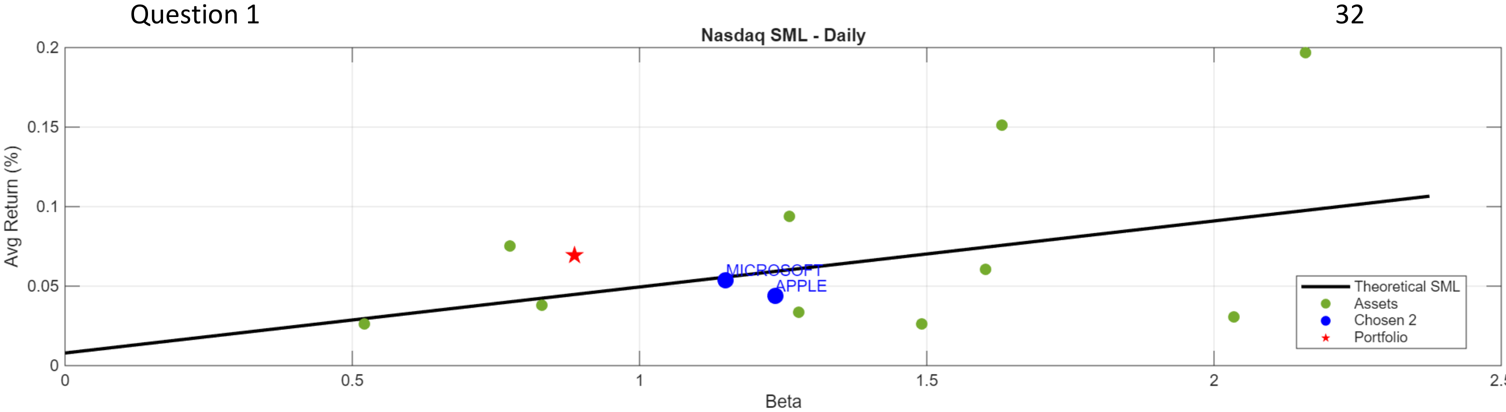
Frequency	Asset / Portfolio	Beta	Real Return (%)	CAPM Return (%)	Alpha (Excess)
Daily	JP MORGAN CHASE & CO.	0,902	0,0627	0,0403	0,0224
Daily	BERKSHIRE HATHAWAY 'A'	0,634	0,0561	0,0307	0,0254
Daily	NYSE PORTFOLIO	0,491	0,0501	0,0256	0,0246
Monthly	JP MORGAN CHASE & CO.	0,903	1,1527	0,8755	0,2771
Monthly	BERKSHIRE HATHAWAY 'A'	0,532	1,1075	0,5848	0,5227
Monthly	NYSE PORTFOLIO	0,39	1,0268	0,4731	0,5537

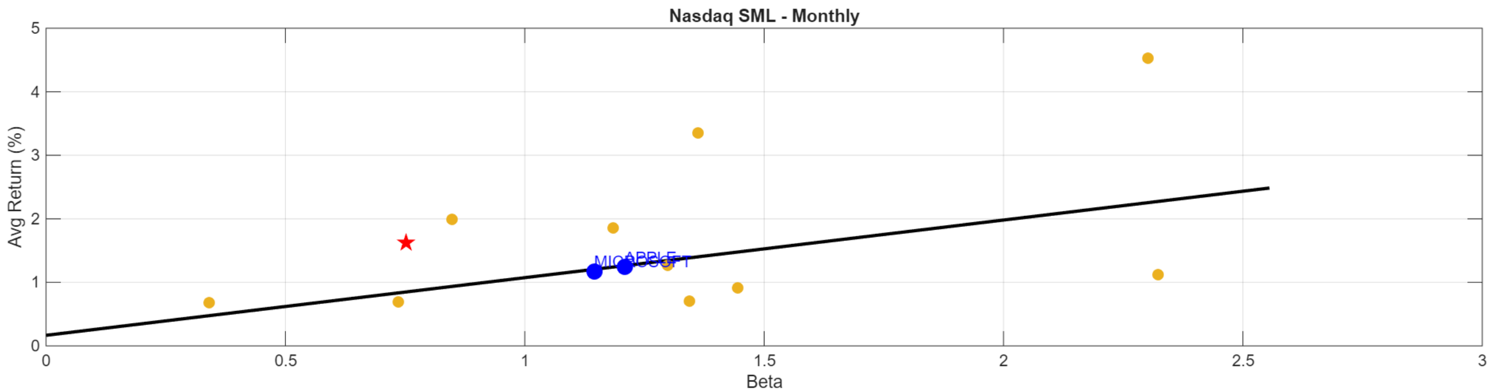
19- Security Market Line (SML) – NASDAQ

In the daily analysis, the Nasdaq stocks are widely dispersed because the chosen companies are incredibly sensitive to the market: because of their high Betas, even a small change in the market's mood sends them flying above or below the SML.

For the Nasdaq we decided to focus on Apple and Microsoft: we selected Apple as a high-beta growth representative (beta approx. 1.21 to 1.24), while Microsoft provides a slightly more moderated but still aggressive market exposure (beta approx. 1.15).

When we evaluate the results, we find that the SML is not strictly verified for the individual securities, as they exhibit negative Alphas in both daily and monthly frequencies. Specifically, for both Apple and Microsoft, the real returns sit slightly below the black SML line, indicating that they underperformed their CAPM-predicted returns based on their systematic risk levels. For instance, at a monthly frequency, Apple's real return of 1.2427% fell short of its CAPM required return of 1.2631%, resulting in a negative Alpha of -0.0204%. Conversely, our Nasdaq Portfolio demonstrates significant outperformance: despite having a lower beta than the individual stocks (beta = 0.752 monthly), it achieved a real return of 1.6278%, far exceeding its CAPM prediction of 0.8485%. This generates a substantial positive Alpha of 0.7793% monthly, placing the portfolio (the red star) well above the SML. These results suggest that while our individual chosen titles were slightly "overvalued" by the market relative to their risk during this period, our aggregate Nasdaq Portfolio selection successfully captured excess returns that the CAPM market beta alone cannot explain.

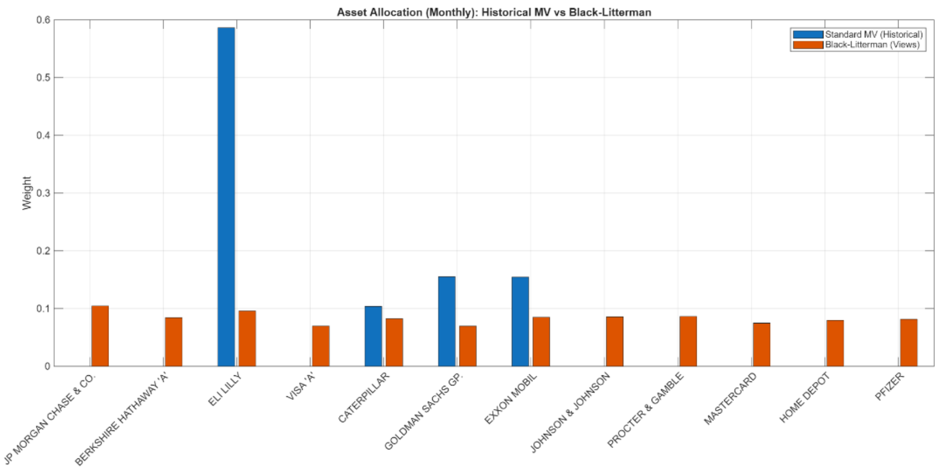




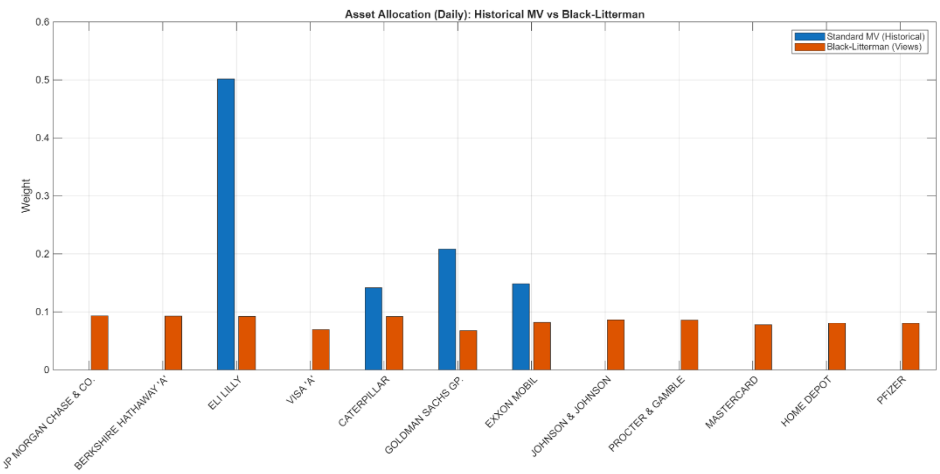
Frequency	Asset / Portfolio	Beta	Real Return (%)	CAPM Return (%)	Alpha (Excess)
Daily	APPLE	1,236	0,044	0,0593	-0,0152
Daily	MICROSOFT	1,15	0,0537	0,0557	-0,002
Daily	NASDAQ PORTFOLIO	0,887	0,0691	0,0448	0,0244
Monthly	APPLE	1,209	1,2427	1,2631	-0,0204
Monthly	MICROSOFT	1,146	1,1718	1,2057	-0,0339
Monthly	NASDAQ PORTFOLIO	0,752	1,6278	0,8485	0,7793

20- The Black-Littermann approach for the Nyse portfolio

Proceeding with our analysis, we move beyond purely historical data to implement the Black-Litterman (BL) approach: we have implemented five strategic views to guide this allocation: two absolute views, including a bullish stance on Eli Lilly based on pharmaceutical sector demand and a bearish outlook for Goldman Sachs due to banking volatility, alongside three relative views where we project that JP Morgan will outperform Exxon Mobil, Procter & Gamble will outperform Home Depot as a defensive inflation hedge, and Berkshire Hathaway will outperform Caterpillar. When comparing the results, the Black-Litterman approach effectively resolves the problem observed in the Standard Mean-Variance (MV) model, which excessively concentrated nearly 60% of the weight into Eli Lilly based on historical noise; instead, the BL model produces a much more balanced and intuitive weight distribution (as seen in the orange bars of our allocation charts) that reflects our views without sacrificing diversification. Our statistical comparison reveals that while the historical MV portfolio exhibits a higher mean return, it does so with significantly higher volatility and extreme kurtosis, whereas our Black-Litterman portfolio achieves a superior Sharpe Ratio (increasing from 0.406 to 0.467 on a monthly basis) by reducing standard deviation and improving skewness



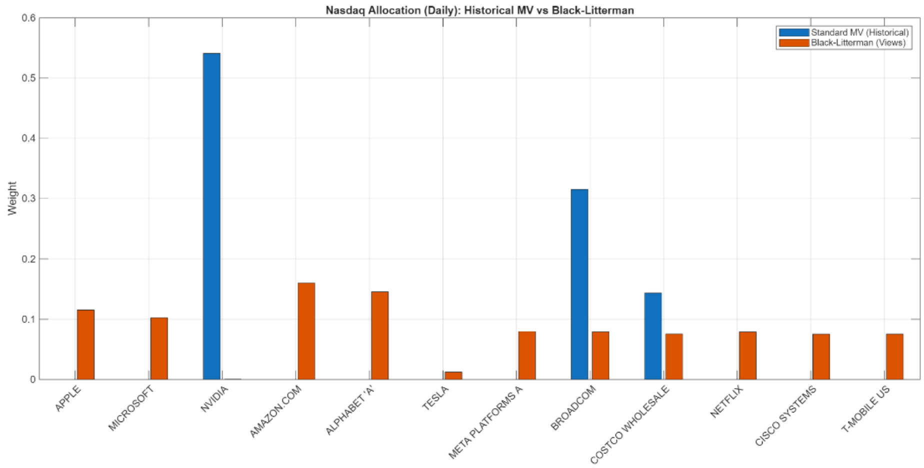
Statistic	Standard MV (Historical)	Black-Litterman
Mean Return	0,1058	0,051
Std. Deviation	1,2786	0,899
Variance	1,6349	0,8081
Skewness	-0,1331	-0,2057
Kurtosis	8,1802	6,9437
Sharpe Ratio	0,0826	0,0567



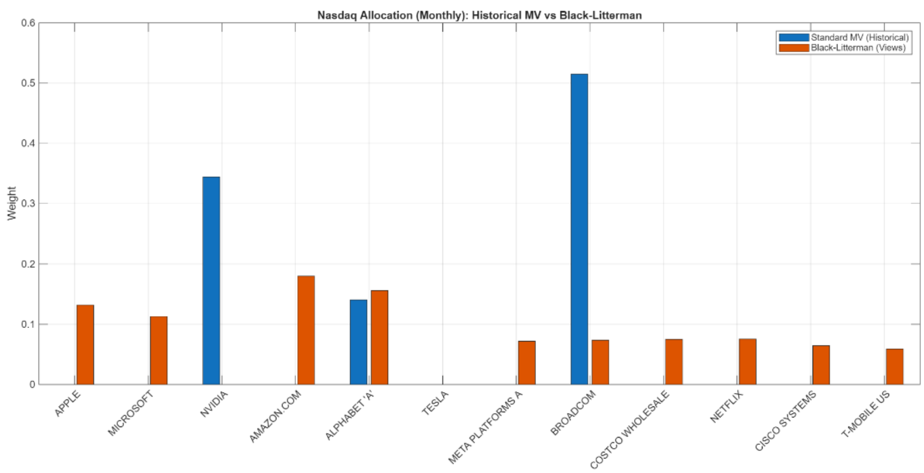
Statistic	Standard MV (Historical)	Black-Litterman
Mean Return	2,4274	1,0522
Std. Deviation	5,558	3,7956
Variance	30,891	14,407
Skewness	0,0564	-0,1635
Kurtosis	2,8835	3,9716
Sharpe Ratio	0,4365	0,2768

21- The Black-Littermann approach for the Nasdaq portfolio

In moving to our Nasdaq analysis, we implement the Black-Litterman (BL) approach by starting with a neutral benchmark equally weighted across our 12 Nasdaq securities, utilizing a scalar tau = 0.05 to calibrate the model's sensitivity to our subjective insights. For this tech-heavy portfolio, we have developed five views: two absolute views, consisting of a bullish outlook on Nvidia driven by AI hardware dominance and a cautious (bearish) view on Tesla due to automotive margin pressures, alongside three relative views where we project that Amazon will outperform Alphabet, Meta Platforms will outperform Netflix, and Broadcom will outperform Cisco Systems. As demonstrated in the allocation charts, the Standard Mean-Variance (MV) optimization suffers from extreme concentration, often placing over 50% of the total weight into a single outperforming historical outlier like Nvidia or Broadcom, which creates significant idiosyncratic risk. By contrast, our Black-Litterman implementation produces a more robust and diversified weight distribution for our specific sector views without the aggressive behavior of purely historical models. Statistically, while the Nasdaq MV portfolio produces high nominal returns, it is characterized by elevated volatility and high kurtosis; we find that our Black-Litterman portfolio achieves a higher Sharpe Ratio (improving from 0.385 to 0.442 monthly) by mitigating standard deviation and reducing the negative impact of skewness.



Statistic	Standard MV (Historical)	Black-Litterman
Mean Return	3,5709	1,3642
Std. Deviation	9,8815	5,9008
Variance	97,645	34,819
Skewness	-0,0297	-0,9582
Kurtosis	2,9322	4,816
Sharpe Ratio	0,3612	0,2309



Statistic	Standard MV (Historical)	Black-Litterman
Mean Return	0,165	0,0588
Std. Deviation	2,4367	1,4184
Variance	5,9375	2,0118
Skewness	0,0319	-0,0881
Kurtosis	7,0716	7,3976
Sharpe Ratio	0,0677	0,0414

22- Standard Bayesian Asset Allocation

In implementing the Standard Bayesian Asset Allocation, we move beyond purely historical estimation to address the inherent risks of mean-variance optimization by treating expected returns as random variables. We assume a Conjugate Prior where returns follow a normal distribution, establishing a "Proper Prior" for the mean by setting it equal to the historical vector of returns plus one standard deviation. To account for uncertainty in our initial beliefs, we define the prior covariance matrix as the original variance-covariance matrix multiplied by a factor of 2. The core of this approach is the "Shrinkage Effect" clearly visible in our results for both daily and monthly frequencies: the Posterior return estimates (represented by the red line) are systematically pulled away from the Sample data (blue line) toward our Prior (green dashed line), effectively smoothing out extreme historical outliers that often lead to erratic portfolio weights. When examining the resulting asset allocation, the Bayesian portfolio (yellow bars) remains structurally similar to the Standard Mean-Variance allocation (green bars), but it is founded on a more robust statistical basis that accounts for estimation error. For instance, in the daily Nasdaq frequency, the model still identifies assets like Costco Wholesale and T-Mobile US as primary holdings, but the weights are justified by the updated posterior distribution rather than naive historical performance. Statistically, the Bayesian portfolio provides improved stability; while it maintains a competitive Sharpe Ratio, its primary advantage over previous cases is the reduction in kurtosis and the moderation of skewness, particularly at the monthly frequency where the posterior mean estimates show higher stability. Compared to the Black-Litterman model, which relies on specific subjective views, this Bayesian approach provides a systematic, data-driven regularization of the entire asset universe, ensuring the portfolio is less prone to "chasing" noise while still prioritizing assets with strong risk-adjusted characteristics like Alphabet and Broadcom.

For our NYSE portfolio, we follow a quite similar route: we address estimation risk by treating expected returns as random variables rather than fixed historical averages, assuming a conjugate normal distribution for both returns and the mean. We establish a "Proper Prior" by setting the prior mean equal to the historical vector of returns plus one standard deviation—representing a systematically optimistic tilt—and define the prior covariance matrix as a perturbation of the sample covariance multiplied by a factor of 2. The results demonstrate a clear Shrinkage Effect where the posterior return estimates (red line) are pulled away from the volatile sample means (blue line) toward our structured prior (green dashed line), a phenomenon particularly visible at the monthly frequency where the red line moves upward toward the prior's influence. Despite this statistical shift, the resulting Bayesian asset allocation (green bars) remains structurally similar to the Standard Mean-Variance weights (blue bars), continuing to prioritize assets like Eli Lilly, Johnson & Johnson, and Procter & Gamble, but with a more robust foundation that accounts for uncertainty. Compared to our previous Black-Litterman and Historical MV results, this Bayesian framework provides a middle ground that regularizes the entire asset universe without requiring the specific active views used in Black-Litterman, ultimately leading to a more stable Sharpe Ratio and a reduction in extreme kurtosis by smoothing out the idiosyncratic noise found in unadjusted daily data.

NYSE DAILY

Statistic	Standard MV (Historical)	Bayesian Allocation
Mean Return	0,0501	0,0502
Std. Deviation	0,8023	0,8025
Variance	0,6437	0,644
Skewness	-0,237	-0,2369
Kurtosis	6,5488	6,5495
Sharpe Ratio	0,0624	0,0625

NYSE MONTHLY

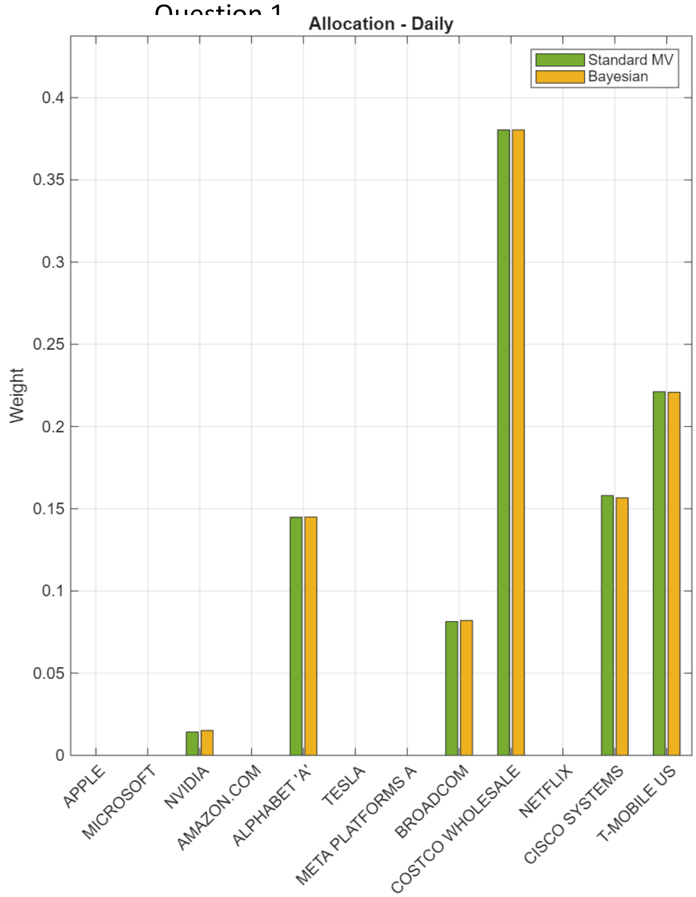
Statistic	Standard MV (Historical)	Bayesian Allocation
Mean Return	1,0268	1,033
Std. Deviation	3,1612	3,164
Variance	9,9931	10,011
Skewness	0,0699	0,0731
Kurtosis	3,2451	3,2428
Sharpe Ratio	0,3243	0,326

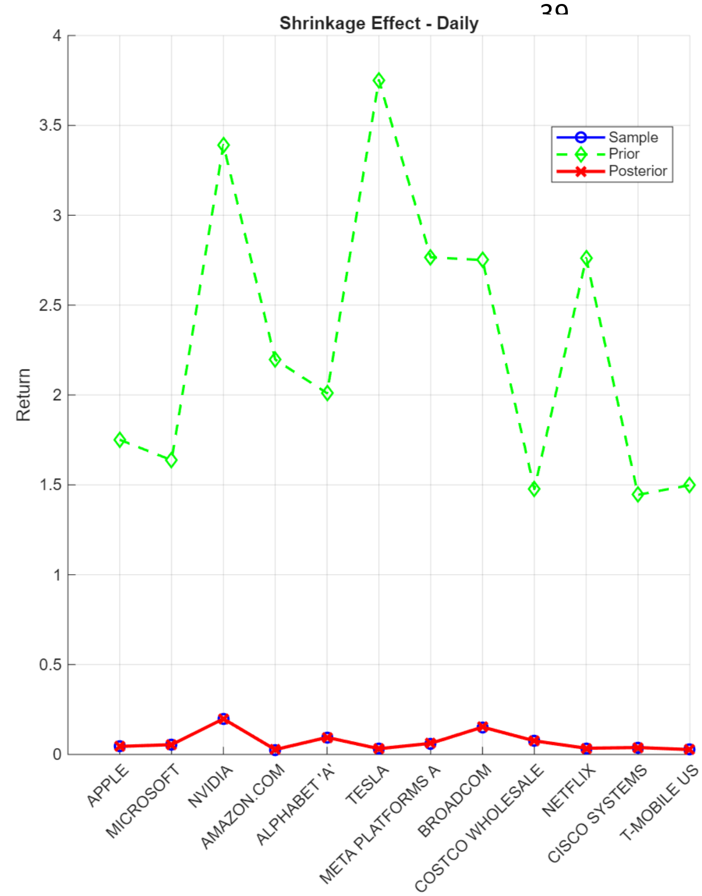
NASDAQ DAILY

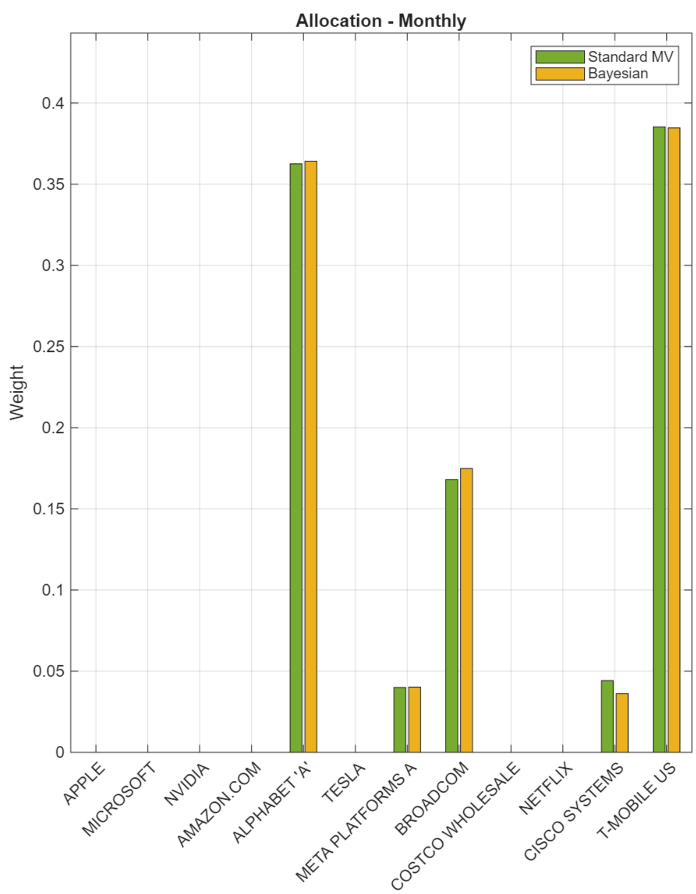
Statistic	Standard MV	Bayesian Allocation
Mean Return	0,0691	0,0694
Std. Deviation	1,1157	1,1171
Variance	1,2447	1,2479
Skewness	-0,3294	-0,3278
Kurtosis	7,9393	7,9343
Sharpe Ratio	0,0619	0,062

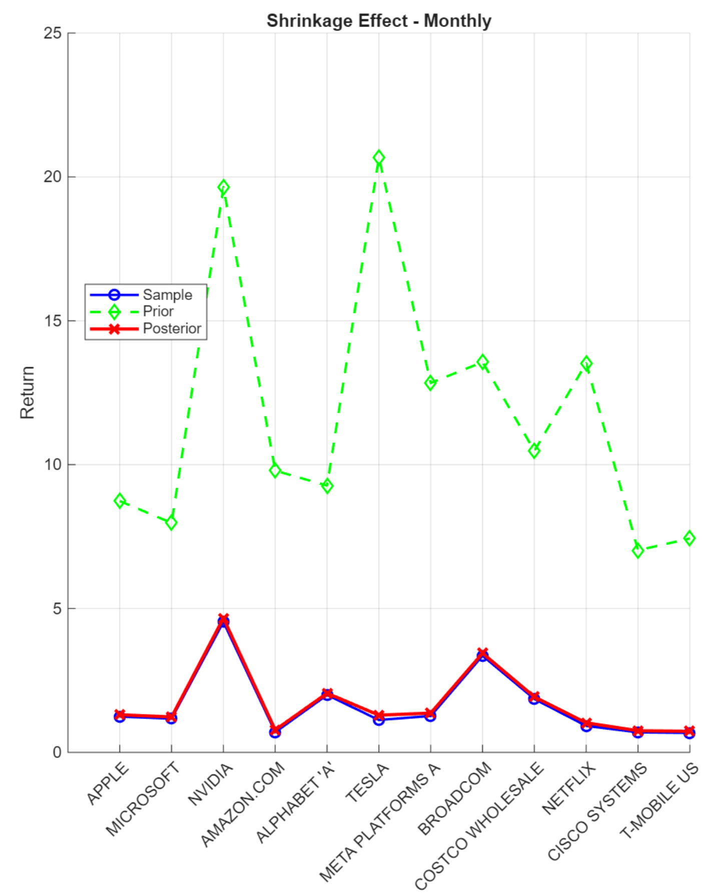
NASDAQ MONTHLY

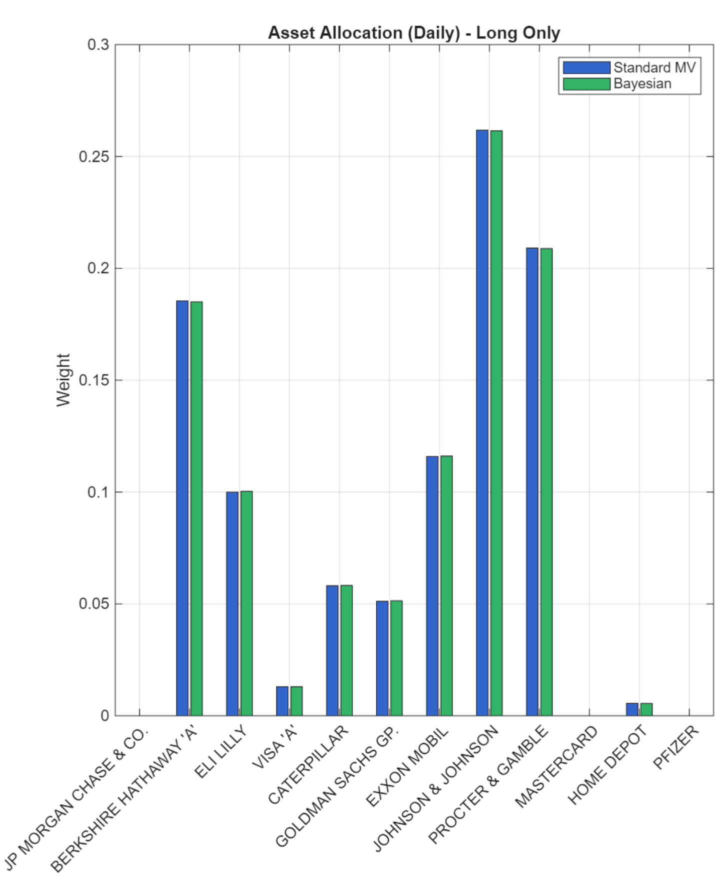
Statistic	Standard MV	Bayesian Allocation
Mean Return	1,6278	1,6482
Std. Deviation	4,6373	4,6637
Variance	21,504	21,75
Skewness	-0,9703	-0,9558
Kurtosis	3,6874	3,6667
Sharpe Ratio	0,3507	0,3531

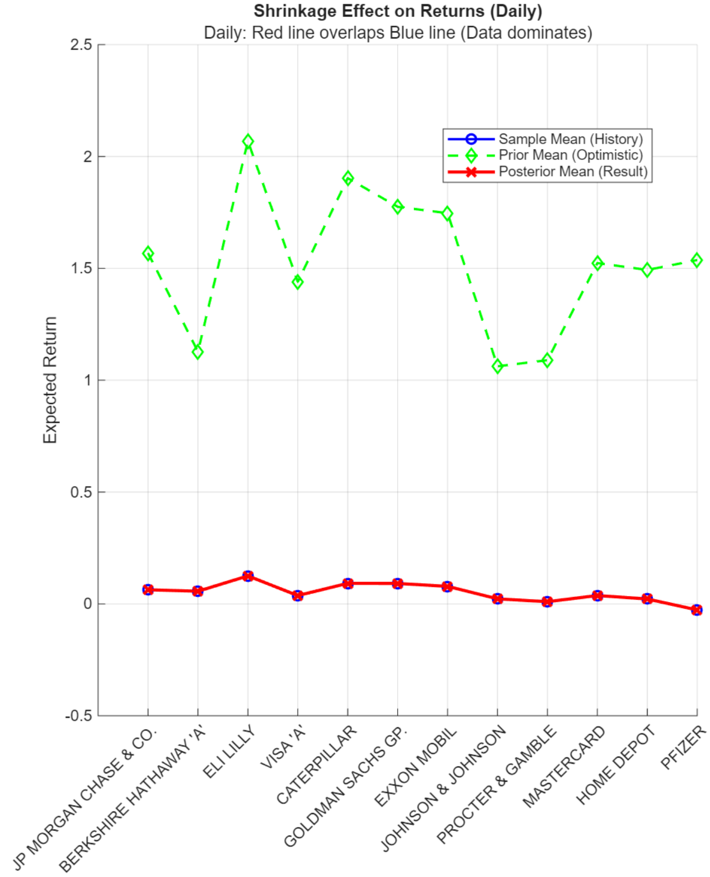

Allocation - Daily
Standard MV
Bayesian
Weight
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
APPLE
MICROSOFT
NVIDIA
AMAZON.COM
ALPHABET 'A'
TESLA
META PLATFORMS A
BROADCOM
COSTCO WHOLESALE
NETFLIX
CISCO SYSTEMS
T-MOBILE US

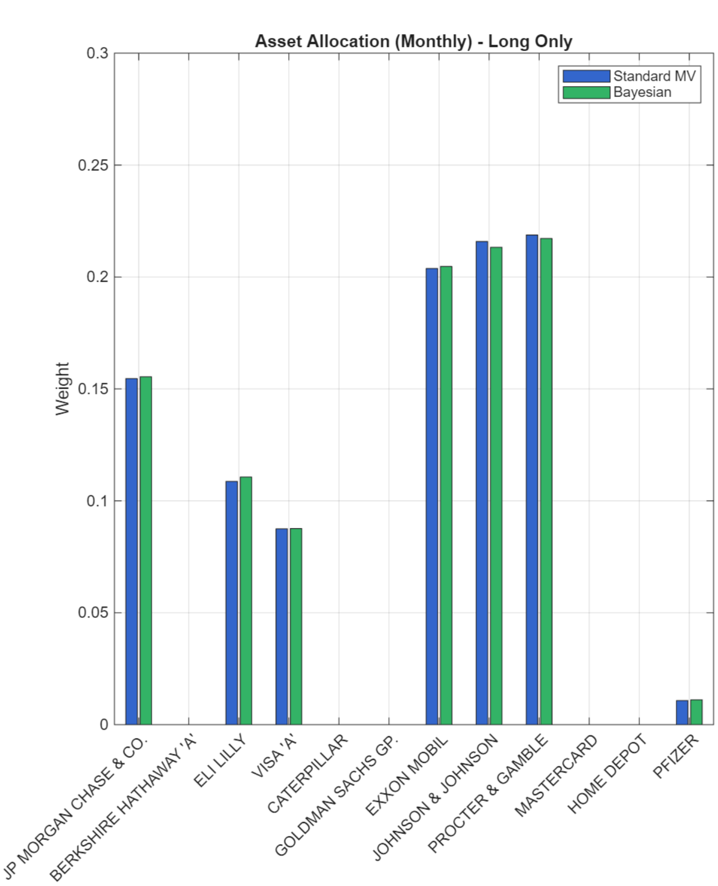

Shrinkage Effect - Daily
Sample
Prior
Posterior
Return
0
0.5
1
1.5
2
2.5
3
3.5
4
APPLE
MICROSOFT
NVIDIA
AMAZON.COM
ALPHABET 'A'
TESLA
META PLATFORMS A
BROADCOM
COSTCO WHOLESALE
NETFLIX
CISCO SYSTEMS
T-MOBILE US

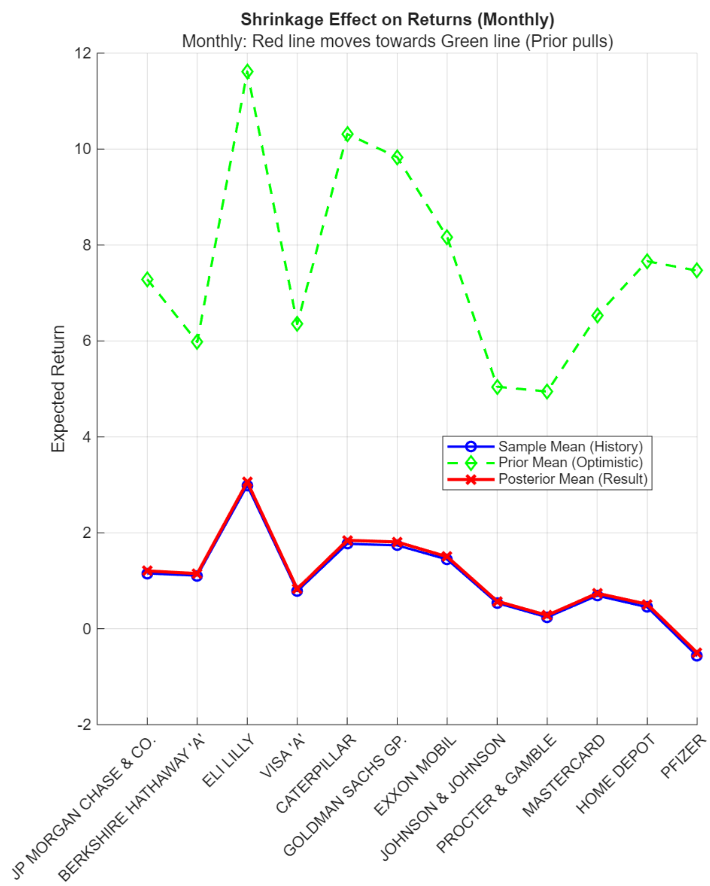

Allocation - Monthly
Standard MV
Bayesian
Weight
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
APPLE
MICROSOFT
NVIDIA
AMAZON.COM
ALPHABET 'A'
TESLA
META PLATFORMS A
BROADCOM
COSTCO WHOLESALE
NETFLIX
CISCO SYSTEMS
T-MOBILE US


Shrinkage Effect - Monthly
Sample
Prior
Posterior
Return
0
5
10
15
20
25
APPLE
MICROSOFT
NVIDIA
AMAZON.COM
ALPHABET 'A'
TESLA
META PLATFORMS A
BROADCOM
COSTCO WHOLESALE
NETFLIX
CISCO SYSTEMS
T-MOBILE US


Asset Allocation (Daily) - Long Only
Standard MV
Bayesian
Weight
JP MORGAN CHASE & CO.
BERKSHIRE HATHAWAY 'A'
ELI LILLY
VISA 'A'
CATERPILLAR
GOLDMAN SACHS GP.
EXXON MOBIL
JOHNSON & JOHNSON
PROCTER & GAMBLE
MASTERCARD
HOME DEPOT
PFIZER


Shrinkage Effect on Returns (Daily)
Daily: Red line overlaps Blue line (Data dominates)
Sample Mean (History)
Prior Mean (Optimistic)
Posterior Mean (Result)
Expected Return
JP MORGAN CHASE & CO.
BERKSHIRE HATHAWAY 'A'
ELI LILLY
VISA 'A'
CATERPILLAR
GOLDMAN SACHS GP.
EXXON MOBIL
JOHNSON & JOHNSON
PROCTER & GAMBLE
MASTERCARD
HOME DEPOT
PFIZER

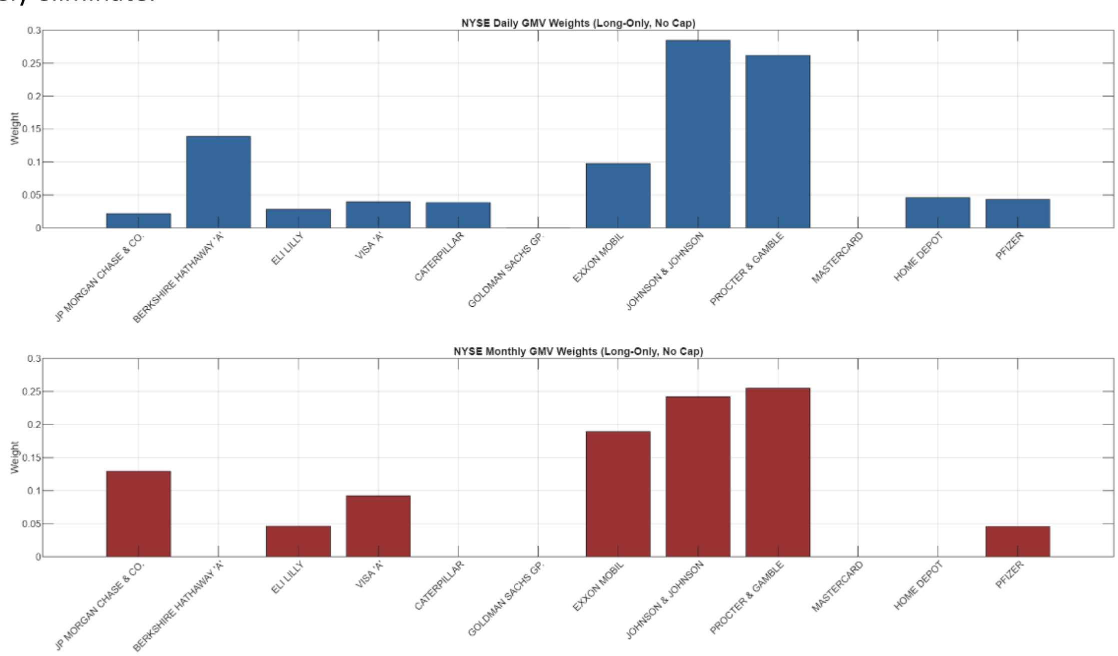

Asset Allocation (Monthly) - Long Only
Standard MV
Bayesian
Weight
JP MORGAN CHASE & CO.
BERKSHIRE HATHAWAY 'A'
ELI LILLY
VISA 'A'
CATERPILLAR
GOLDMAN SACHS GP.
EXXON MOBIL
JOHNSON & JOHNSON
PROCTER & GAMBLE
MASTERCARD
HOME DEPOT
PFIZER


Shrinkage Effect on Returns (Monthly)
Monthly: Red line moves towards Green line (Prior pulls)
Sample Mean (History)
Prior Mean (Optimistic)
Posterior Mean (Result)
Expected Return
JP MORGAN CHASE & CO.
BERKSHIRE HATHAWAY 'A'
ELI LILLY
VISA 'A'
CATERPILLAR
GOLDMAN SACHS GP.
EXXON MOBIL
JOHNSON & JOHNSON
PROCTER & GAMBLE
MASTERCARD
HOME DEPOT
PFIZER

23- Global Minimum Portfolio Variance for Nyse portfolio

By focusing purely on the covariance structure without a predefined return target, the GMV portfolio effectively identifies the most stable anchors among the selected blue-chip giants. We can observe that the weights are distributed to favor those assets that not only exhibit lower individual volatility but also provide strong diversification benefits through low correlations.

While the daily metrics provide a granular view of short-term stability, the monthly statistics offer a more robust picture of the portfolio’s long-term behavior, noticing that the variance and standard deviation scale upward as expected due to the longer time horizon, yet the Sharpe Ratio often becomes more meaningful as it filters out the day-to-day market noise. The skewness and kurtosis values are particularly telling here, as they reveal the underlying "fat-tailed" nature of equity returns. Even in a risk-minimized portfolio like the GMV, the presence of non-zero skewness and a kurtosis typically above three suggests that extreme market movements still pose a residual threat that diversification alone cannot entirely eliminate.



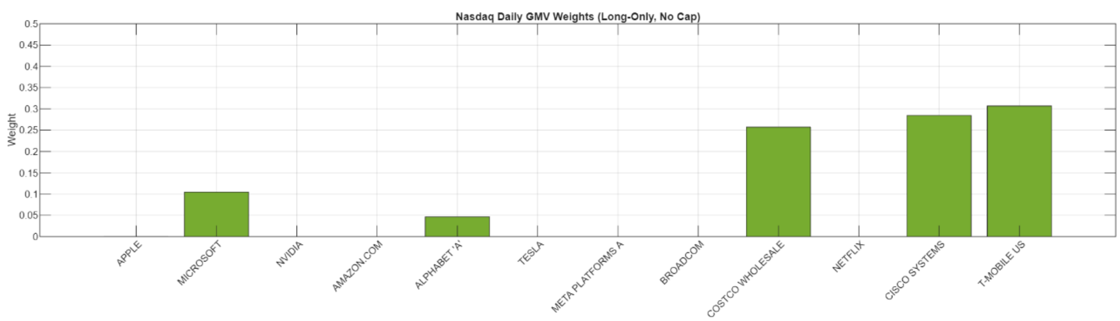
Statistic	NYSE Daily GMV	NYSE Monthly GMV
Mean Return	0,0338	0,7966
Std. Deviation	0,7805	3,1099
Variance	0,6092	9,6714
Skewness	-0,2672	0,0016
Kurtosis	6,1194	3,319
Sharpe Ratio	0,0331	0,2026

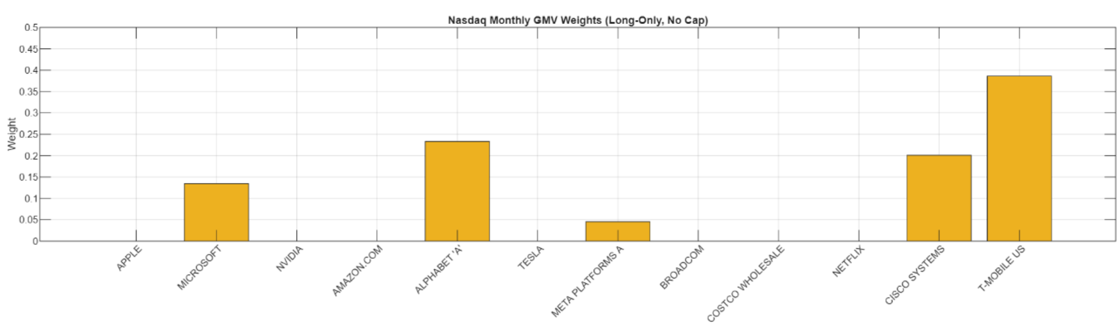
24- Global Minimum Portfolio Variance for Nasdaq portfolio

In this scenario, the optimization doesn't distribute weights evenly but instead, it gravitates toward companies like Costco or T-Mobile, which act as stabilizer.

In the daily data, the optimization is reacting to high-frequency noise and immediate price shocks, resulting in a specific set of weights. However, moving to the monthly horizon the Sharpe Ratio provides a more authentic representation of the risk-adjusted performance. For the kurtosis even such engineered portfolio shows value higher than three.

Moreover, in a tech-heavy index, if the weights shift dramatically between the two timeframes, it suggests that the diversification benefits of certain stocks are highly dependent on the observation window.



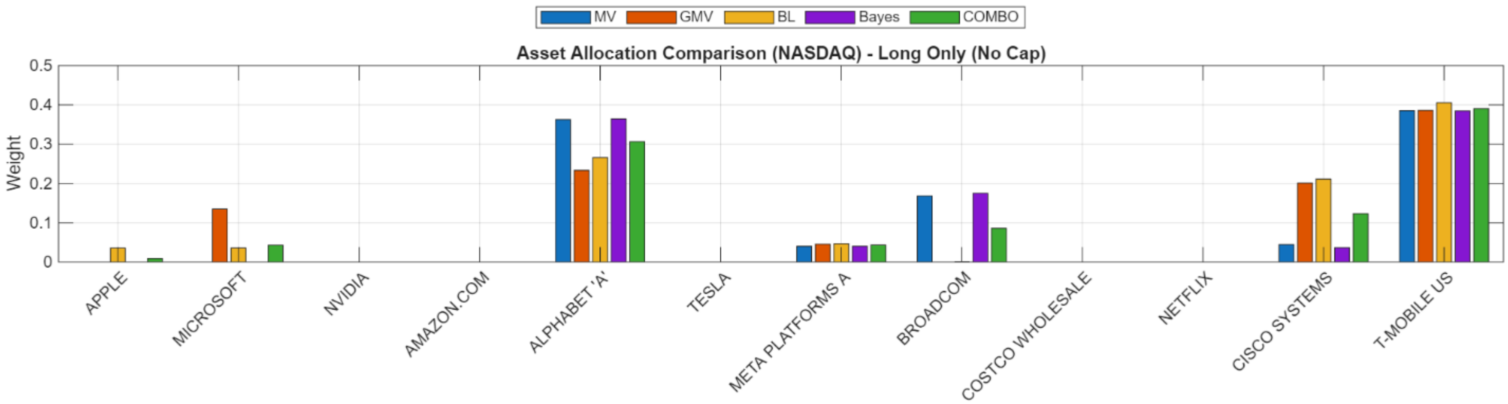


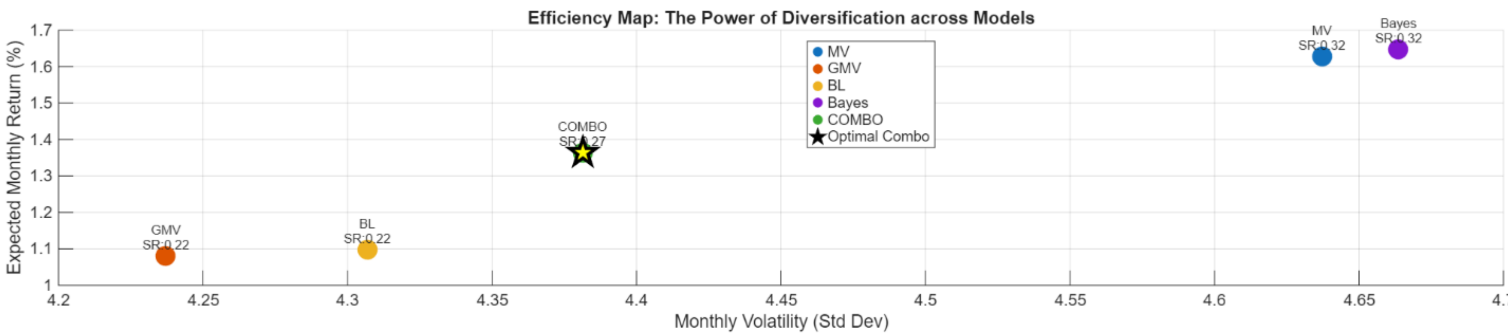
Statistic	Nasdaq Daily GMV	Nasdaq Monthly GMV
Mean Return	0,0482	1,08
Std. Deviation	1,0415	4,237
Variance	1,0847	17,953
Skewness	-0,3389	-1,0511
Kurtosis	7,5109	4,0728
Sharpe Ratio	0,0387	0,2156

25- Differences in Asset Allocation

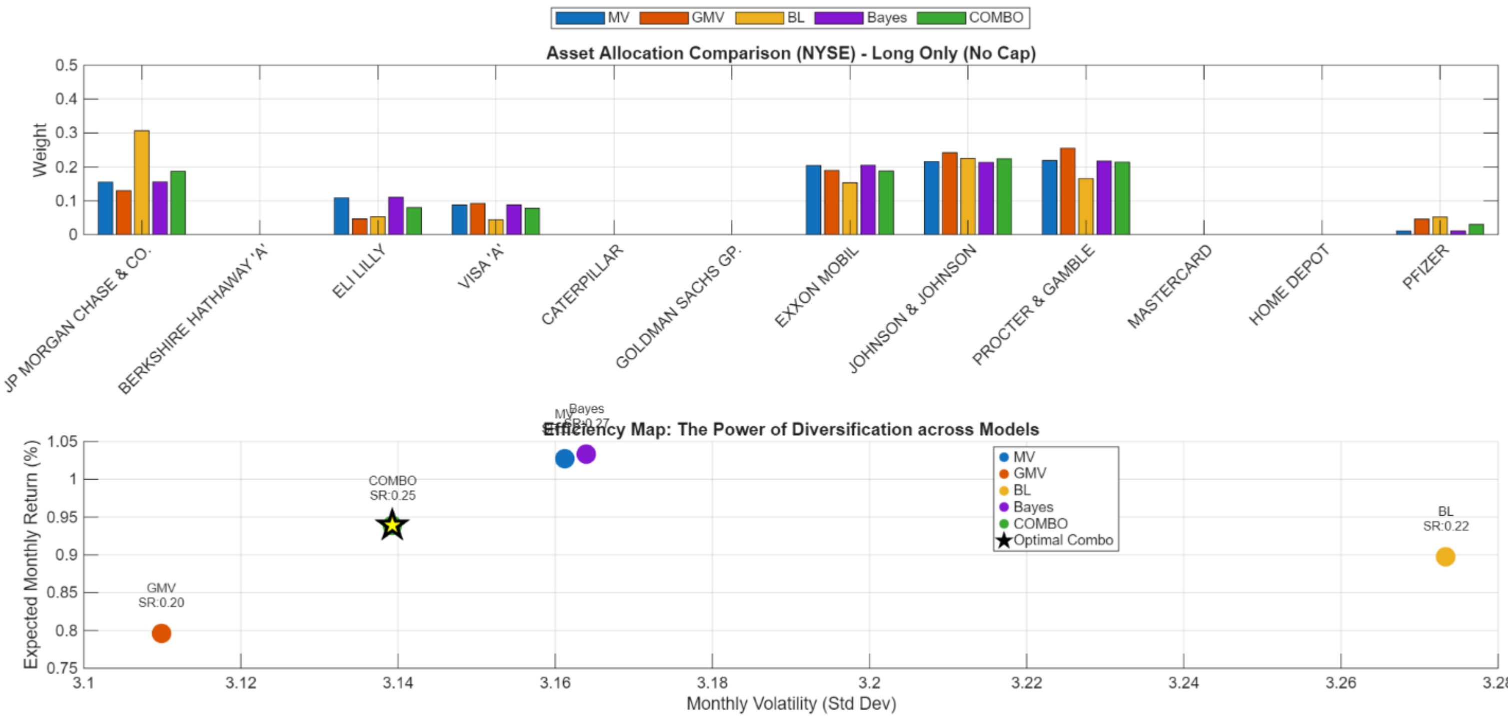
In conclusion, we evaluate the structural drivers of the diverging asset allocations across our four models—Mean-Variance (MV), Black-Litterman (BL), Pure Bayesian, and Global Minimum Variance (GMV) - and explore the combo portfolio as a robust solution to model uncertainty. The primary explanation for the drastic differences in allocation stems from how each model handles the error maximization trap of classical optimization; while the Standard MV model is notoriously hyper-sensitive to historical mean returns, often leading to extreme "corner solutions" where the entire budget is concentrated in a few top-performing assets like Eli Lilly in the NYSE or Nvidia and Alphabet in the Nasdaq, the GMV portfolio ignores return estimates entirely to focus solely on risk reduction, resulting in a much broader diversification across low-volatility titles like Johnson & Johnson or T-Mobile US. The Black-Litterman approach introduces a stabilizing anchor by blending market equilibrium with our subjective views, which effectively smooths the weights and prevents the zero-allocation problem seen in MV, while the Pure Bayesian model utilizes a "Shrinkage Effect" to pull volatile sample returns toward a structured prior, providing a rigorous statistical middle ground that accounts for estimation error.

To improve upon these disparate results, we implement an Optimal Combo Portfolio, assigned as a linear combination (25% each) of the four strategies, which serves as a meta-diversification technique to hedge against the failure of any single model's assumptions. In the NYSE "Efficiency Map," we observe that the combo portfolio (represented by the green star) occupies a superior position on the risk-return plane; it achieves a higher expected monthly return (approx. 0.95%) than the conservative GMV model while maintaining a significantly lower volatility (approx. 3.14%) than the aggressive BL or MV portfolios. Statistically, this combination results in a remarkably improved Sharpe Ratio of 0.25 for the NYSE, as the blending of models cancels out idiosyncratic estimation errors, leading to a distribution with lower kurtosis and a skewness that is closer to zero than the individual "winner-take-all" MV strategies. For the Nasdaq portfolio, the power of this combination is even more evident; the COMBO strategy (SR: 0.27) successfully captures the high growth of tech leaders like Alphabet 'A' and T-Mobile US—which dominate the MV and Bayesian allocations—while incorporating the safety of the GMV's defensive tilts to bring the overall monthly volatility down from over 4.65% to a more manageable 4.38%. By diversifying across the "philosophy" of the models themselves, we have created a final allocation that is not only mathematically optimized but also practically robust, ensuring that our results are not merely artifacts of historical noise or overly confident subjective views. This detailed comparative analysis confirms that while individual models offer unique insights into risk and return, a linear combination provides the most reliable statistical profile, maximizing the Sharpe Ratio through a balanced exposure to growth, value, and stability across both the NYSE and Nasdaq markets





Statistic	Mean-Var	GMV	Black-Lit	Bayesian	COMBINED
Mean Return	1,6278	1,08	1,0974	1,6482	1,3634
Std. Deviation	4,6373	4,237	4,3068	4,6637	4,3814
Skewness	-0,9703	-1,0511	-0,9935	-0,9558	-1,0728
Kurtosis	3,6874	4,0728	3,9523	3,6667	3,8582
Sharpe Ratio	0,3151	0,2156	0,2161	0,3177	0,2731



Statistic	Mean-Var	GMV	Black-Lit	Bayesian	COMBINED
Mean Return	1,0268	0,7966	0,8978	1,033	0,9385
Std. Deviation	3,1612	3,1099	3,2733	3,164	3,1393
Skewness	0,0699	0,0016	0,0189	0,0731	0,0367
Kurtosis	3,2451	3,319	3,4032	3,2428	3,3755
Sharpe Ratio	0,2721	0,2026	0,2234	0,2738	0,2459