Optimisation (CM3): Assignment

Questions 1 and 2 rely on material from lecture 5 (in week 5 of this term), so you may want to start from Questions 3 and 4 (or read ahead, the slides for lecture 5 will be made available in advance). You need to use the freely available PuLP optimisation package, which is an ILP solver that works with Python, for Questions 3 and 4 in this assignment.

You need to submit (via DUO) a zip archive containing a **single pdf** file with your solutions and four files `q3.py, q4a.py, q4b.py, q4c.py` that you used for the solver. Do not expect me to read your `*.py` files, but I might use them to check your claimed results.

The marks available for correct answers to each question are indicated. Partial credit will be given for good attempts.

Question 1 [15 marks]

Find (by hand) an optimal solution of the following LP using the simplex method:

$$\begin{array}{ll} \text{Minimize} & x_1 - 3x_2 \\ \text{subject to} & x_1 - x_2 \leq 1 \\ & x_1 - x_2 \geq -1 \\ & 2x_1 - x_2 \leq 3 \\ & x_1, x_2 \geq 0. \end{array}$$

Include all tableaus that you derive, explaining your choice of pivots. Give the values of x_1, x_2 and the optimum *of the original problem* as a part of your answer.

Question 2 [15 marks]

Consider the LP $\max\{z(x) = c^\top x$ subject to $Ax = b, x \geq \mathbb{0}\}$, where

$$A = \begin{pmatrix} -2 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 2 & -3 & -1 & 0 & 0 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix} \quad c = \begin{pmatrix} 2 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Note that this LP is in canonical form with respect to basis $B = \{4, 5, 6\}$, and that B is feasible. Apply (by hand) the simplex algorithm with Bland's rule to this LP to find either an optimal solution (with a certificate of optimality) or a certificate of unboundedness. Include all tableaus that you derive, explaining your choice of pivots.

Question 3 [20 marks]

A paintshop has a close-down sale. They have 10 gallons of cyan paint, 5 gallons of magenta paint and 11 gallons of yellow paint. Unfortunately, customers are not interested in those colours at the moment. However, the shop could sell red paint (for £10/gallon), green paint (for £15/gallon), blue paint (for £25/gallon), or black paint (for £25/gallon).

Paints can be mixed in the following way:

- 1 unit of yellow and 1 unit of magenta gives 2 units of red
- 1 unit of yellow and 1 unit of cyan gives 2 units of green
- 1 unit of magenta and 1 unit of cyan gives 2 units of blue
- 1 unit each of cyan, magenta, and yellow gives 3 units of black

How should the paints be mixed to achieve maximal income (no more paints of any kind can be bought)?

Formulate a linear program for this problem, *explaining in detail* why your LP exactly captures the problem. Solve this LP using PuLP, and *interpret the solution* given by the solver in terms of the original problem.

Question 4

While on holiday, you bought a number of items. When packing to fly back, you realise that you can take only 20kg of luggage with you. The weights and values of the items are given in the following table:

	A	B	C	D	E	F
Weight (kg)	6	7	4	9	3	8
Value (£)	60	70	40	70	16	100

The problem is decide which items to take with you to maximize the total value while not exceeding the weight requirement (i.e. total weight of selected items is at most 20kg). For example, selecting items A, C, and D gives total value £170 and a total weight of 19kg.

(a) Formulate an *integer* program for the above problem, and then solve it using PuLP. [20 marks]

(b) Assume that taking item C makes sense only if item D is also selected (but D without C is fine). Add constraint(s) to your IP from Question 4a to account for this additional condition and solve the resulting IP using PuLP. [5 marks]

(c) Suppose now that you can exceed the limit of 20kg, but would need to pay the airline £15 for each additional kg. For example, if the total weight of selected items is 22kg then you would need to pay £30 to the airline. You now want to maximise the total value of selected items minus the cost paid to the airline. Modify your IP from Question 4a to an IP that captures this problem, and then solve it using PuLP.

[25 marks]

In each case, *explain in detail* why your IP exactly captures the problem and *interpret the solution* given by PuLP in terms of the original problem.