

Data Structures

Homework #2

Due: Nov 1, 2019

1. Give a recursive algorithm to compute the product of two positive integers, n and m , using only addition and subtraction.
2. Recall the **selection sort** we discussed in class to illustrate how to have the pseudo-code. The illustrated pseudo-code for selection sort is iterative as below.

ITERATIVE_SELECTION_SORT(A)

Input: An input array A of size n

Output: A sorted array // Sort the array $A[1 : n]$ in a nondecreasing order.

```
1  for  $i = 1$  to  $n$ 
2       $j = i$ 
3      for  $k = (i + 1)$  to  $n$ 
4          if ( $A[k] < A[j]$ )
5               $j = k$ 
6       $t = A[i]$ 
7       $A[i] = A[j]$ 
8       $A[j] = t$ 
9  return  $A$ 
```

- (a) Count the total number of primitive operations used in ITERATIVE_SELECTION_SORT(A).
 - (b) With the same idea, please provide a recursive version for the selection sort, RECURSIVE_SELECTION_SORT(A), with pseudo-code.
 - (c) Please try to derive the total number of primitive operations used in the recursive version you provided in 2b.
3. Show that for any real constants a and b , where $b > 0$, $(n + a)^b = \Theta(n^b)$.
 4. Consider the Fibonacci function $F(n)$, $F(1) = 1, F(2) = 2, F(3) = 3, F(4) = 5, \dots, F(n) = F(n - 1) + F(n - 2)$. Show by induction that $F(n)$ is $\Omega((3/2)^n)$.
 5. Let $f(n)$ be an asymptotically positive functions. Prove or disprove each of the following conjectures.
 - (a) $f(n) = O((f(n))^2)$.
 - (b) $f(n) = \Theta(f(n/2))$.
 6. **(Programming problem 1)**
Consider the recursive approach in above problem 1.
 - (a) Implement the approach as a function named as **product_rec** using Python.

- (b) Please have an iterative version for the approach and write a function for this version with function name `product_ite`.
- (c) Compare these two function with the same input in terms of running time and write what you have observed.

Note that we will use an in-built python library `timeit` and the module function `timeit.timeit()` for measuring the running time.

7. (Programming problem 2)

Consider the iterative version of **selection sort** in problem 2.

- (a) Implement the iterative approach using Python and name the function as `select_sort_ite`.
- (b) Please implement the recursive version you provide in problem 2(b) with the function name of `select_sort_rec`.
- (c) Compare these two function with the same input in terms of running time and write what you have observed.

About submitting this homework

1. For problem 1, 2, 3, 4 and 5, Please
 - (1) write all of your solutions on the papers of size A4,
 - (2) leave you name and student ID on the first page, and
 - (3) hand in your solutions for problem 1, 2, 3, 4, and 5 to me **before** class.
2. For problem 6 and 7,
 - (1) please finish each problem right after the problem description in the HW2.ipynb file provided on the i-school *ischool* (<http://www.ischool.ntut.edu.tw/>) platform; and
 - (2) please upload the completed .ipynb file with the filename as HW2_studentID.ipynb to *ischool* platform.
3. **Late work** is not acceptable. Remember, the **deadline** is the midnight of **Nov 1**, 2019.
4. **Honest Policy**: We encourage students to discuss their work with the peer. However, each student should write the program or the problem solutions on her/his own. Those who copy others work will get 0 on the homework grade.