

# 模拟退火应用举例

- 例题 已知背包的装载量为 $c=8$ ，现有 $n=5$ 个物品，它们的重量和价值分别是 $(2, 3, 5, 1, 4)$ 和 $(2, 5, 8, 3, 6)$ 。试使用模拟退火算法求解该背包问题，写出关键的步骤。
- 求解：假设问题的一个可行解用0和1的序列表示，例如 $i=(1010)$ 表示选择第1和第3个物品，而不选择第2和第4个物品。用模拟退火算法求解关键过程如图所示：

# 模拟退火应用举例

已知：

物体个数：  $n=5$

背包容量：  $c=8$

重量  $w = (2, 3, 5, 1, 4)$

价值  $v = (2, 5, 8, 3, 6)$

第一步：初始化。假设初始解为  $i=(11001)$ ，初始温度为  $T=10$ 。计算  $f(i)=2+5+6=13$ ，最优解  $s=i$

第三步：降温，假设温度降为  $T=9$ 。如果没有达到结束标准，则返回第二步继续执行

假设在继续运行的时候，从当前解  $i=(10110)$  得到一个新解  $j=(00111)$ ，这时候的函数值为  $f(j)=8+3+6=17$ ，这是一个全局最优解。可见上面过程中接受了劣解是有好处的。

第二步：在  $T$  温度下局部搜索，直到“平衡”，假设平衡条件为执行了3次内层循环。

(2-1) 产生当前解  $i$  的一个邻域解  $j$  (如何构造邻域根据具体的问题而定，这里假设为随机改变某一位的0/1值或者交换某两位的0/1值)，假设  $j=(11100)$   
要注意产生的新解的合法性，要舍弃那些总重量超过背包装载量的非法解

(2-2)  $f(j) = 2+5+8=15 > 13=f(i)$ ，所以接受新解  $i=j$ ;  $f(i)=f(j)=15$ ; 而且  $s=i$ ;  
要注意求解的是最大值，因此适应值越大越优

(2-3) 返回 (2-1) 继续执行。

(a) 假设第二轮得到的新解  $j=(11010)$ ，由于  $f(j) = 2+5+3=10 < 15=f(i)$ ，所以需要计算接受概率  $P(T)=\exp((f(j)-f(i))/T) = \exp(-0.5) = 0.607$ ，假设  $\text{random}(0,1) > P(T)$ ，则不接受新解

(b) 假设第三轮得到的新解  $j=(10110)$ ，由于  $f(j) = 2+8+3=13 < 15=f(i)$ ，所以需要计算接受概率  $P(T)=\exp((f(j)-f(i))/T) = \exp(-0.3) = 0.741$ ，假设  $\text{random}(0,1) < P(T)$ ，则接受新解  
按照一定的概率接受劣解，也是跳出局部最优的一种手段

(2-4) 这时候， $T$  温度下的“平衡”已达到 (即已经完成了3次的邻域产生)，结束内层循环

# Traveling Salesman Problem

- The **Traveling Salesman Problem (TSP)** is one of the most widely studied combinatorial optimization problems.
- Its statement is deceptively simple: A salesperson **seeks the shortest tour through  $n$  cities**.

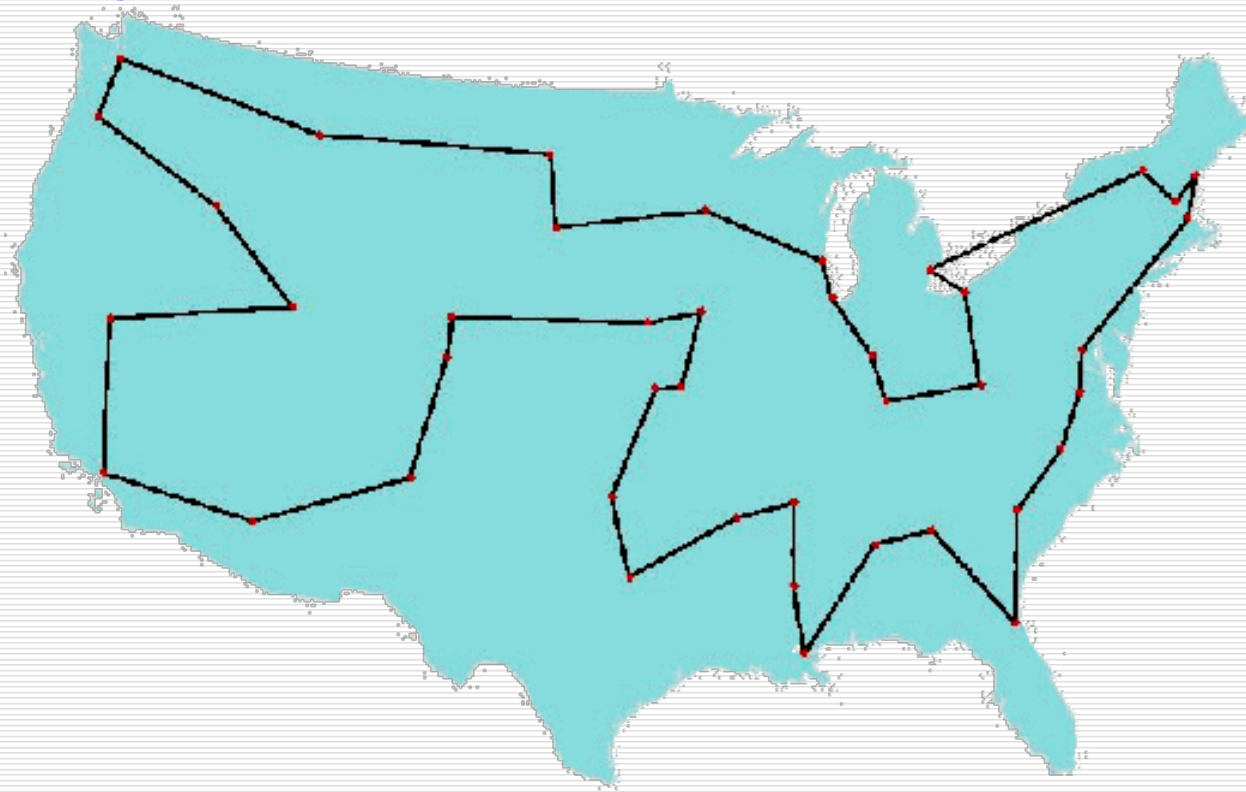


Fig. 1 **George Dantzig, Ray Fulkerson, and Selmer Johnson (1954)**  
a description of a method for solving the TSP :49 cities

# Traveling Salesman Problem

## ❑ Existing Instances

### ■ 49 city problem

- ❑ Dagtzig, G., D. Fulkerson and S. Johnson: “Solution of a large scale traveling salesman problems”, *Operations Research*, vol. 2, pp. 393-410, 1954.

### ■ 120 city problem

- ❑ Grötschel, M.: “On the symmetric traveling salesman problem: solution of a 120 city problem”, *Mathematical Programming Studies*, vol. 12, pp. 61-77, 1980.

### ■ 318 city problem

- ❑ Crowder, H. and M. Padberg: “Solving large scale symmetric traveling salesman problems to optimality”, *Management Science*, vol. 22, pp. 15-24, 1995.

### ■ 532 city problem

- ❑ Padberg, M. and G. Rinaldi: “Optimization of 532 city symmetric traveling salesman problem by branch and cut”, *Operations Research Letters*, vol. 6, pp. 1-7, 1987.

# Traveling Salesman Problem

## ■ 666 city problem

- Grötschel, M. and O. Holland: “Solution of large scale symmetric traveling salesman problems”, *Mathematical Programming Studies*, vol. 51, pp. 141-202, 1991.

## ■ 2392 city problem

- Padberg, M. and G. Rinaldi: “A branch and cut algorithm for the resolution of large scale symmetric traveling salesman problem”, *SIAM Review*, vol. 33, pp. 60-100, 1991.

## ■ The earlier studies using the genetic algorithm to solve TSP

- Grefenstette, J.: *Proceedings of the First International Conference on Genetic Algorithms*, Lawrence Erlbaum Associates, Hillsdale, NJ, 1985.

## ■ TSP has become a target for the genetic algorithm community.

- Michalewicz, Z.: *Genetic Algorithm + Data structure = Evolution Programs*, 2nd ed., Springer-Verlag, New York, 1994.

# Traveling Salesman Problem

## □ Notations

### ■ Indices

$i, j$  : the index of city,  $i, j = 1, 2, \dots, n$

### ■ Parameters

$n$ : the total number of cities

$d_{ij}$  : the distance city  $i$  to city  $j$ , i.e., the distance of route  $(i,j)$ ; the distance matrix  $(d_{ij})$  is symmetric.

### ■ Decision Variables

$x_{ij}$  : the 0,1 decision variable; 1, if route  $(i,j)$  is selected, and 0, otherwise.

# Traveling Salesman Problem

## □ Mathematical Model of TSP

$$\min \quad z = \sum_{i=1}^n \sum_{j=1}^n (d_{ij} x_{ij})$$

$$\text{s.t.} \quad \sum_{i=1}^n x_{ij} = 1, \quad j = 1, 2, 3, \dots, n$$

$$\sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, 3, \dots, n$$

$$x_{ij} = \begin{cases} 1, & \text{if route } (i, j) \text{ is selected} \\ 0, & \text{otherwise} \end{cases}$$

data set for non-directed graph

<i>i</i>	<i>j</i>	<i>d<sub>ij</sub></i>	<i>i</i>	<i>j</i>	<i>d<sub>ij</sub></i>
1	2	8	3	7	7
	3	5		8	12
	4	9		9	12
	5	12		4	5
	6	14		6	17
	7	12		7	10
	8	16		8	7
	9	17		9	15
2	3	9	5	6	8
	4	15		7	10
	5	17		8	6
	6	8		9	15
	7	11		6	7
	8	18		8	14
	9	14		9	8
3	4	7	7	8	8
	5	9		9	6
	6	11		8	9

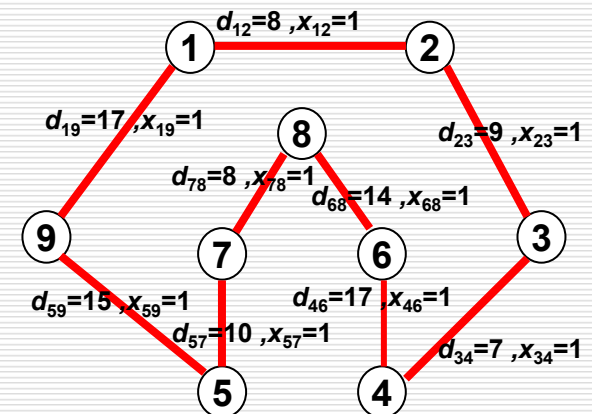


Fig. 2 Simple TSP Model

# 1 Representation

## □ Permutation Representation

- This direct representation is perhaps the most natural representation of a TSP, where cities are listed in the order in which they are visited.

	1	2	3	4	5	6	7	8	9
chromosome	3	2	5	4	7	1	6	9	8

tour list      3 - 2 - 5 - 4 - 7 - 1 - 6 - 9 - 8

- This representation is also called a path representation or order representation.

### procedure: Permutation Encoding

**Input:** city set, total number of cities  $N$

**output:** chromosome  $v$

**begin**

**for**  $j=1$  to  $N$

$v(j) \leftarrow j$ ;

**for**  $i=1$  to  $\lceil n/2 \rceil$

**repeat**

$j \leftarrow \text{random}[1, N]$ ;

$l \leftarrow \text{random}[1, N]$ ;

**until**  $l \neq j$

$\text{swap}(v(j), v(l))$ ;

**output** chromosome  $v$ ;

**end**

### procedure: Permutation Decoding

**Input:** chromosome  $v$ ,

      total number of cities  $N$

**output:** tour list  $L$

**begin**

$L \leftarrow \varphi$ ;

**for**  $i=1$  to  $N$

$L \leftarrow L \cup v(i)$ ;

**output** tour list  $L$ ;

**end**



# 1 Representation

## □ Random Keys Representation

- This indirect representation encodes a solution with random numbers from (0,1).
- These values are used as sort keys to decode the solution.

	1	2	3	4	5	6	7	8	9
chromosome	0.23	0.82	0.45	0.74	0.87	0.11	0.56	0.69	0.78

where position  $i$  in the list represents city  $i$ .

tour list 6 - 1 - 3 - 7 - 8 - 4 - 9 - 2 - 5

**procedure:** Random Keys Encoding

**Input:** city set,

total number of cities  $N$

**output:** chromosome  $v$

**begin**

for  $i = 1$  to  $N$

$v[i] \leftarrow \text{random}[0,1];$

**output** chromosome  $v$ ;

**end**

**procedure:** Random Keys Decoding

**Input:** chromosome  $v$ ,

total number of cities  $N$

**output:** tour list  $L$

**begin**

$L \leftarrow \emptyset;$

for  $i = 1$  to  $N$

$L \leftarrow L \cup i;$

sort  $L$  by  $v[i];$

**output** tour list  $L$ ;

**end**

# 2 Crossover Operators

- During the past decade, several crossover operators have been proposed for permutation representation, such as **partial-mapped crossover (PMX)**, **order crossover (OX)**, **cycle crossover (CX)**, **position-based crossover**, **order-based crossover**, **heuristic crossover**, and so on.
- These operators can be classified into two classes:
  - **Canonical approach**
    - The canonical approach can be viewed as an extension of two-point or multipoint crossover of binary strings to permutation representation.
  - **Heuristic approach**
    - The application of heuristics in crossover intends to generate an improved offspring.

# 2 Crossover Operators

## 1. Partial-Mapped Crossover (PMX)

**procedure** : PMX crossover

**input** : chromosome  $v_1, v_2$ ,  
length of chromosome  $l$

**output** : offspring  $v'_1, v'_2$

**begin**

$R \leftarrow \emptyset$ ;

// step 1: select two positions  
at random

$s \leftarrow \text{random}[1:l-1]$  ;

$t \leftarrow \text{random}[s+1:l]$  ;

// step 2: exchange two substrings

$\leftarrow v_1[1:s-1] \parallel v_2[s:t] \parallel v_1[t+1:l]$  ;

$v'_1 \leftarrow v_2[1:s-1] \parallel v_1[s:t] \parallel v_2[t+1:l]$  ;

// step 3: determine the mapping  
relationship

$R \leftarrow \text{relation}(v_1[s:t], v_2[s:t])$  ;

// step 4: legalize offspring

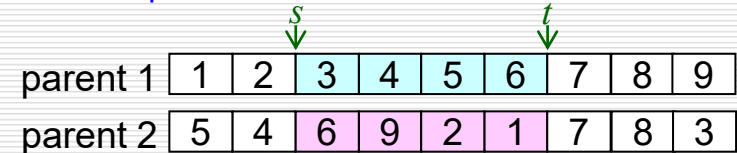
$\text{legalize}(\quad, \quad, R)$ ;

**output** offspring  $\quad, \quad$  ;

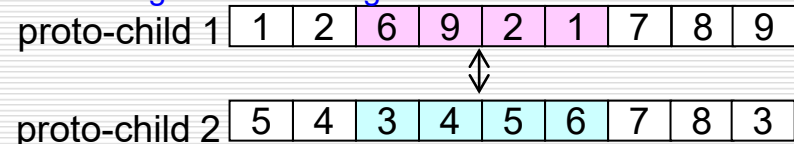
**end**

$v'_1 \quad v'_2$

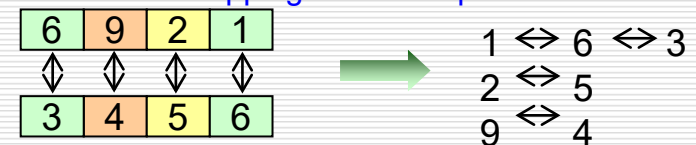
step 1 : select two positions at random



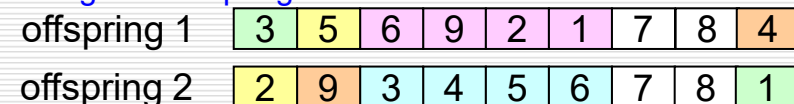
step 2: exchange two substrings



step 3 : determine the mapping relationship



step 4 : legalize offspring



$v_1$  : parent chromosome 1

$v_2$  : parent chromosome 2

$l$  : length of chromosome

$v'_1$  : offspring chromosome 1

$v'_2$  : offspring chromosome 2

$R$  : relationships

$s$  : start position of substring

$t$  : end position of substring

$\text{relation}(v_1, v_2)$  : searching relationship between  $v_1$  and  $v_2$

$\text{legalize}(v_1, v_2, R)$  change genes value of  $v_1, v_2$  based on relationship  $R$

# 2 Crossover Operators

## 2. OX crossover

**procedure** : Order Crossover (OX)

**input** : chromosome  $v_1, v_2$ ,  
length of chromosome  $l$

**output** : offspring  $v'$

**begin**

$w \leftarrow 1$ ;

// step 1: select substring at random

$s \leftarrow \text{random}[1: l-1]$ ;

$t \leftarrow \text{random}[s+1: l]$ ;

// step 2: produce a proto-child by  
copying the substring

$v' \leftarrow v_1[s: t]$ ;

// step 3: produce offspring by filling  
unfixed positions of proto-  
child from parent 2

**for**  $i=1$  **to**  $s-1$

**for**  $j=w$  **to**  $l$

$fg \leftarrow 0$ ;

**for**  $k=s$  **to**  $t$

**if**  $v_2[j] = v_1[k]$  **then**

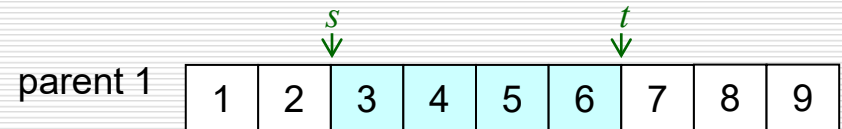
$fg \leftarrow 1$ ; **break**;

**if**  $fg=0$  **then**

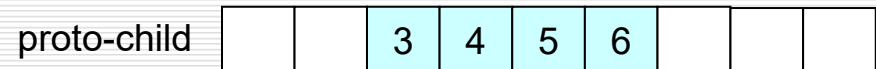
$v'[i] \leftarrow v_2[j]$ ;

$w \leftarrow j + 1$ ; **break**;

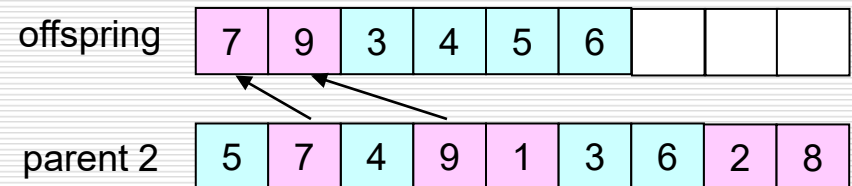
step 1 : select substring at random



step 2 : produce a proto-child by copying the substring



step 3 : produce offspring by filling unfixed positions  
of proto-child from parent 2



$v_1$  : parent chromosome 1

$l$  : length of chromosome

$w$ : working data

$s$  : start position of substring

$v_2$  : parent chromosome 2

$v'$  : offspring chromosome

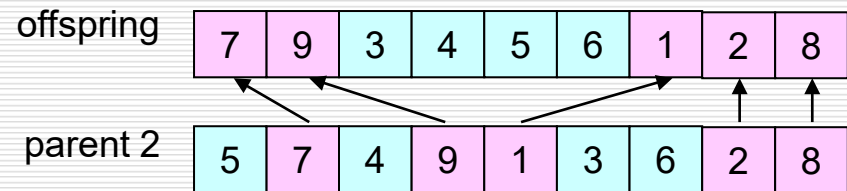
$fg$  : flag

$t$  : end position of substring

# 2 Crossover Operators

```
for  $i=t+1$  to  $l$ 
  for  $j=w$  to  $l$ 
     $fg \leftarrow 0$ ;
    for  $k=s$  to  $t$ 
      if  $v_2[j] = v_1[k]$  then
         $fg \leftarrow 1$ ; break;
    if  $fg=0$  then
       $v'[i] \leftarrow v_2[j]$ ;
       $w \leftarrow j + 1$ ; break ;
```

```
output offspring  $v'$  ;
end;
```



$v_1$ : parent chromosome 1	$v_2$ : parent chromosome 2
$l$ : length of chromosome	$v'$ : offspring chromosome
$w$ : working data	$fg$ : flag
$s$ : start position of substring	$t$ : end position of substring

# 2 Crossover Operators

## 3. Position-based Crossover (PBX)

**procedure** : Position-based Crossover

**input** : chromosome  $v_1, v_2$ ,  
length of chromosome  $l$

**output** : offspring  $v'$

**begin**

$T \leftarrow \emptyset, S \leftarrow \emptyset, w \leftarrow 1;$

// step 1: select a set of positions  
from parent 1 at random

$N \leftarrow \text{random}[1:l];$

// step 2: produce proto-child by  
copying values of  
selected positions

**for**  $i=1$  **to**  $N$

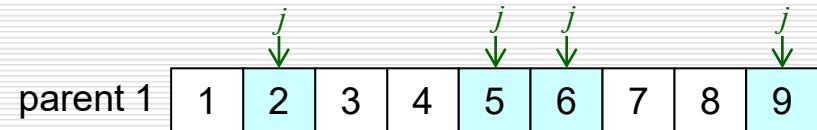
$j \leftarrow \text{random}[1:l];$

$v'[j] \leftarrow v_1[j];$

$T \leftarrow T \cup j;$

$S \leftarrow S \cup v_1[j];$

step 1 : select a set of positions from parent 1 at random



step 2 : produce proto-child by copying  
values of selected positions



$v_1$  : parent chromosome 1       $v_2$  : parent chromosome 2  
 $l$  : length of chromosome       $v'$  : offspring chromosome

$N$  : total number of selected positions

$T = \{t[j]\}, j=1, 2, \dots, N$  : selected positions set

$S = \{s[m]\}, m=1, 2, \dots, N$  : genes value set of selected positions

$fg_1$  : flag 1

$fg_2$  : flag 2

$w$  : working data

# 2 Crossover Operators

// step 3: produce offspring by filing unfixed positions of proto-child from parent 2

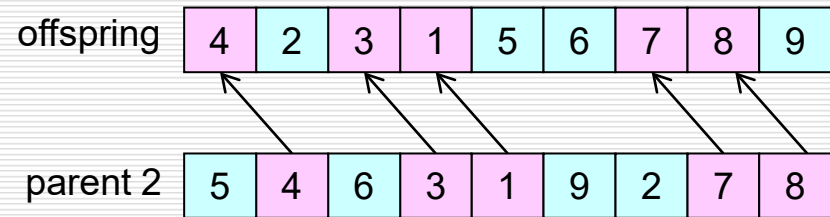
```

for i=1 to l
    fg1 ← 0;
    for j=1 to N
        if i=t[j] then fg1 ← 1;
    if fg1=1 then continue;
    for k=w to l
        fg2 ← 0;
        for m=1 to N
            if v2[k]=s[m] then
                fg2 ← 1; break;
        if fg2=0 then
            v'[i] ← v2[k];
            w ← k + 1 ; break ;

```

**output** offspring v' ;  
**end**

step 3 : produce offspring by filing unfixed positions of proto-child from parent 2



$v_1$  : parent chromosome 1       $v_2$  : parent chromosome 2  
 $l$  : length of chromosome       $v'$  : offspring chromosome  
 $N$  : total number of selected positions  
 $T=\{t[j]\}$  ,  $j=1,2,\dots, N$  : selected positions set  
 $S=\{s[m]\}$ ,  $m=1,2,\dots, N$  : genes value set of selected positions  
 $fg_1$  : flag 1       $fg_2$  : flag 2  
 $w$  : working data

# 2 Crossover Operators

## 4. Order-Based Crossover (OBX)

**procedure** : Order-Based Crossover

**input** : chromosome  $v_1, v_2$ ,  
length of chromosome  $l$

**output** : offspring  $v'$

**begin**

$S \leftarrow \emptyset, T \leftarrow \emptyset, w \leftarrow 1;$

// step 1: select a set of positions  
from parent 1 at random

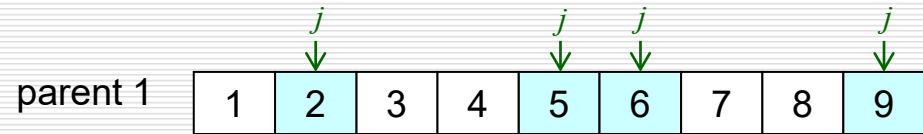
$N \leftarrow \text{random}[1:l];$

**for**  $i=1$  **to**  $N$

$j \leftarrow \text{random}[1:l];$

$S \leftarrow S \cup v_1[j];$

step 1 : select a set of positions  
from parent 1 at random



$v_1$  : parent chromosome 1

$v_2$  : parent chromosome 2

$l$  : length of chromosome

$v'$  : offspring chromosome

$N$  : total number of selected positions

$T = \{t[j]\}, j=1,2,\dots, N$  : positions set of assigned genes from  $v_2$  to  $v'$

$S = \{s[i]\}, i=1,2,\dots, N$  : genes value set of selected positions

$fg_1$  : flag 1

$fg_2$  : flag 2

$w$  : working data



# 2 Crossover Operators

// step 2: produce proto-child by copying  
non-selected value from parent 2

for  $i=1$  to  $l$

$fg_1 \leftarrow 0$ ;

for  $j=1$  to  $N$

if  $v_2[i] = s[j]$  then

$fg_1 \leftarrow 1$ ; break;

if  $fg_1=0$  then

$v'[i] \leftarrow v_2[i]$ ,

$T \leftarrow T \cup i$ ;

// step 3: produce offspring by copying selected  
values from parent 1

for  $i=1$  to  $l$

$fg_2 \leftarrow 0$ ;

for  $j=1$  to  $N$

if  $i = t[j]$  then  $fg_2 \leftarrow 1$ ;

if  $fg_2=1$  then continue;

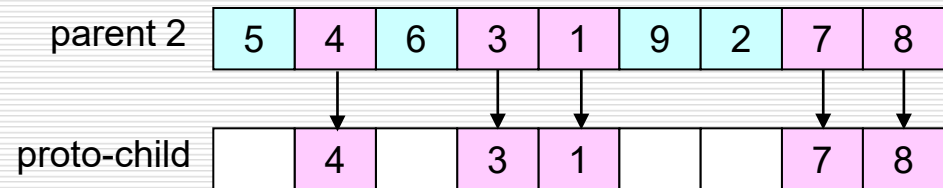
$v'[i] \leftarrow s[w]$ ;

$w \leftarrow w+1$ ;

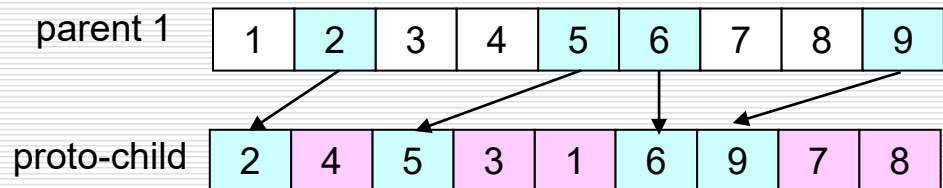
output offspring  $v'$ ;

end

step 2: produce proto-child by copying non-selected value from parent 2



step 3 : produce offspring by copying selected values from parent 1



$v_1$  : parent chromosome 1

$v_2$  : parent chromosome 2

$l$  : length of chromosome

$v'$  : offspring chromosome

$N$  : total number of selected positions

$T = \{t[j]\}$ ,  $j=1, 2, \dots, N$ : positions set of assigned genes from  $v_2$  to  $v'$

$S = \{s[i]\}$ ,  $i=1, 2, \dots, N$  : genes value set of selected positions

$fg_1$  : flag 1

$fg_2$  : flag 2

$w$  : working data

# 2 Crossover Operators

## 5. Cycle Crossover (CX)

**procedure** : CX crossover

**input** : chromosome  $v_1, v_2$ ,

length of chromosome  $l$

**output** : offspring  $v'$

**begin**

$S \leftarrow \emptyset, T \leftarrow \emptyset, w \leftarrow 1;$

// step 1: find the cycle between parents

$C \leftarrow \text{cy}(v_1, v_2);$

// step 2: produce proto-child by copying  
gene values in cycle from parent 1

**for**  $i=1$  **to**  $l$

**for**  $j=1$  **to**  $N-1$

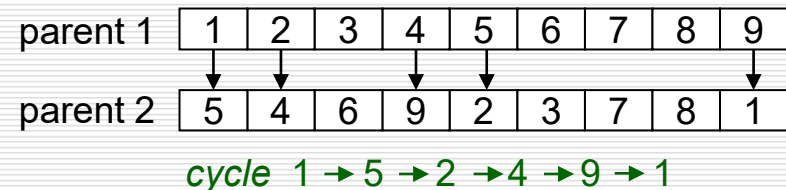
**if**  $v_1[i] = c[j]$  **then**

$v'[i] \leftarrow v_1[i];$

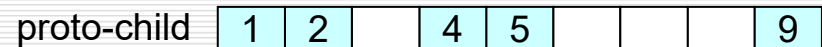
$T \leftarrow T \cup i;$

$S \leftarrow S \cup v_1[i];$

step 1 : find the cycle between parents



step 2 : produce proto-child by copying gene values  
in cycle from parent 1



$v_1$  : parent chromosome 1       $v_2$  : parent chromosome 2  
 $l$  : length of chromosome       $v'$  : offspring chromosome  
 $N$  : total number of values in cycle

$T = \{t[n]\}, n=1, 2, \dots, N-1$ : positions set of  $S$  in proto-child

$S = \{s[k]\}, k=1, 2, \dots, N-1$ : proto-child genes value set in cycle

$C = \{c[j]\}, j=1, 2, \dots, N-1$ : value set of cycle

$fg_1$  : flag 1

$fg_2$  : flag 2

$w$  : working data

$\text{cy}(v_1, v_2)$ : finding the cycle which is defined  
by the corresponding positions between parents

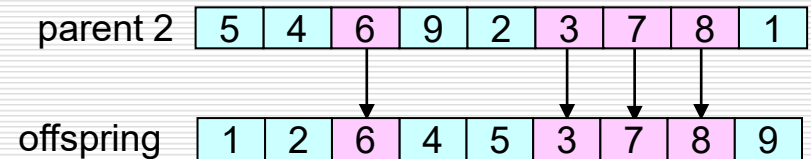
# 2 Crossover Operators

// step 3: produce offspring by filling  
unfixed position from parent 2

```

for  $i=1$  to  $l$ 
     $fg_1 \leftarrow 0$ ;
    for  $n=1$  to  $|T|$ 
        if  $i=t[n]$  then  $fg_1 \leftarrow 1$ ;
    if  $fg_1=1$  then continue;
    for  $j=w$  to  $l$ 
         $fg_2 \leftarrow 0$ ;
        for  $k=1$  to  $|S|$ 
            if  $v_2[j]=s[k]$  then
                 $fg_2 \leftarrow 1$ ; break;
        if  $fg_2=0$  then
             $v'[i] \leftarrow v_2[j]$ ;
             $w \leftarrow j+1$ ; break;
    output offspring ;
end
    
```

step 3 : produce offspring by filling unfixed position from parent 2



$v_1$  : parent chromosome 1       $v_2$  : parent chromosome 2  
 $l$  : length of chromosome       $v'$  : offspring chromosome  
 $N$  : total number of values in cycle

$T=\{t[n]\}$  ,  $n=1,2,\dots, N-1$ : positions set of  $S$  in proto-child  
 $S=\{s[k]\}$ ,  $k=1,2,\dots, N-1$ : proto-child genes value set in cycle  
 $C=\{c[j]\}$ ,  $j=1,2,\dots, N-1$ : value set of cycle  
 $fg_1$  : flag 1       $fg_2$  : flag 2  
 $w$  : working data  
 $\text{cycle}(v_1, v_2)$ : searching cycle between  $v_1$  and  $v_2$

# 2 Crossover Operators

## 6. Subtour Exchange Crossover

**procedure** : Subtour Exchange Crossover

**input** : chromosome  $v_1, v_2$ ,  
length of chromosome  $l$

**output** : offspring  $v'_1, v'_2$

**begin**

$S \leftarrow \emptyset, T \leftarrow \emptyset;$

// step 1: select subtours in parents

$s \leftarrow \text{random}[1:l-1];$

$t \leftarrow \text{random}[s+1:l];$

**for**  $i=1$  **to**  $l$

**for**  $j=s$  **to**  $t$

**if**  $v_2[i]=v_1[j]$  **then**

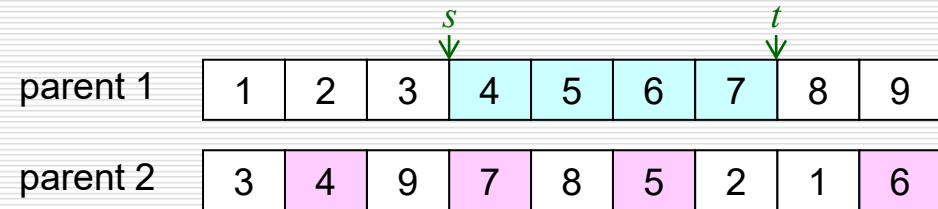
$S \leftarrow S \cup v_2[i];$

**else**

$v'_2[i] \leftarrow v_2[i],$

$T \leftarrow T \cup i;$

step 1 : select subtours in parents



$v_1$  : parent chromosome 1       $v_2$  : parent chromosome 2  
 $l$  : length of chromosome       $v'_1$  : offspring chromosome 1  
 $v'_2$  : offspring chromosome 2  
 $N$  : total number of values in subtour  
 $S = \{s[i]\}, i=1,2,\dots, N$ : value set of subtour  
 $T = \{t[j]\}, j=1,2,\dots, N$ : positions set of  $S$   
 $s$  : start position of substring in  $v_1$      $t$  : end position of substring  $v_1$   
 $fg$  : flag

# 2 Crossover Operators

// step 2: exchange subtours

$v'_1 \leftarrow v_1[1:s-1] \text{ // } S \text{ // } v_1[t+1:l];$

$k \leftarrow s;$

**for**  $i=1$  **to**  $l$

$fg \leftarrow 0;$

**for**  $j=1$  **to**  $|T|$

**if**  $i=t[j]$  **then**  $fg \leftarrow 1; \text{break};$

**if**  $fg=1$  **then** continue;

$v'_2[i] \leftarrow v_1[k];$

$k \leftarrow k+1;$

**output** offspring  $v'_1, v'_2;$

**end**

step 2 : exchange subtours

offspring 1	1	2	3	4	7	5	6	8	9
offspring 2	3	4	9	5	8	6	2	1	7

$v_1$  : parent chromosome 1

$v_2$  : parent chromosome 2

$l$  : length of chromosome

$v'_1$  : offspring chromosome 1

$v'_2$  : offspring chromosome 2

$N$  : total number of values in subtour

$S = \{s[l]\}, i=1,2,\dots, N$ : value set of subtour

$T = \{t[j]\}, j=1,2,\dots, N$ : positions set of  $S$

$s$  : start position of substring in  $v_1$   $t$  : end position of substring  $v_1$

$fg$  : flag

# 3 Mutation Operators

## 1. Inversion Mutation

**procedure** : Inversion Mutation

**input** : chromosome  $v_1, v_2$ ,  
length of chromosome  $l$

**output** : offspring  $v'$

**begin**

// step 1: select subtour at random

$s \leftarrow \text{random}[1:l-1]$  ;

$t \leftarrow \text{random}[s+1:l]$  ;

// step 2: produce offspring by

copying inverse string of

substring

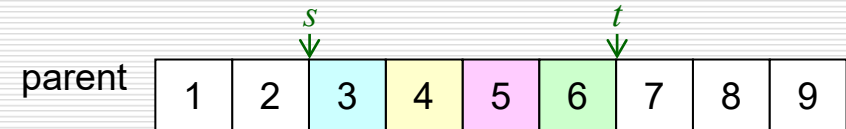
$S \leftarrow \text{invert}(v[s:t])$  ;

$v' \leftarrow v[1:s-1] \parallel S \parallel v[t+1:l]$  ;

**output** offspring  $v'$ ;

**end**

step 1: select subtour at random



step 2 : produce offspring by copying inverse string of substring



$v$  : parent chromosome

$v'$  : offspring chromosome

$t$  : end position of substring

$\text{invert}(string)$  : inversely changing order of  $string$

$l$  : length of chromosome

$s$  : start position of substring

$S$  : inverse string of substring

# 3 Mutation Operators

## 2. Insertion Mutation

**procedure** : Insertion Mutation

**input** : chromosome  $v_1, v_2$ ,  
length of chromosome  $l$

**output** : offspring  $v'$

**begin**

// step 1 : select a position in  
parent 1 at random

$i \leftarrow \text{random}[1:l]$  ;

// step 2: insert selected value in  
randomly selected  
position parent 2

$j \leftarrow \text{random}[1:l-1]$  ;

$W \leftarrow v[1:i-1] \parallel v[i+1:l]$  ;

$v' \leftarrow W[1:j-1] \parallel v[i] \parallel W[j:l-1]$  ;

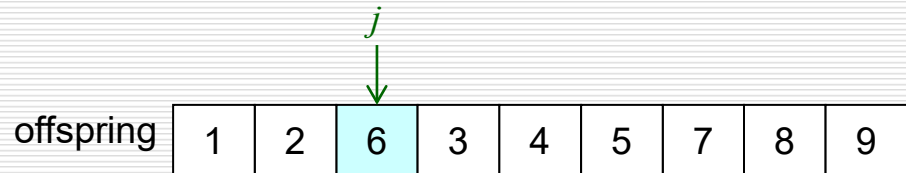
**output** offspring  $v'$  ;

**end**

step 1 : select a position in parent 1 at random



step 2: insert selected value in randomly  
selected position of parent 2



$v$  : parent chromosome                       $l$  : length of chromosome  
 $v'$  : offspring chromosome                 $i$  : selected position in parent 1  
 $j$  : selected position in parent 2     $W$  : working data set

# 3 Mutation Operators

## 3. Displacement Mutation

**procedure** : Displacement Mutation

**input** : chromosome  $v_1, v_2$ ,  
length of chromosome  $l$

**output** : offspring  $v'$

**begin**

// step 1: select subtour

$s \leftarrow \text{random}[1:l-1]$ ;

$t \leftarrow \text{random}[s+1:l]$ ;

// step 2: insert subtour in a  
random position

$n \leftarrow t - (s - 1)$ ;

$i \leftarrow \text{random}[1:l-n]$ ;

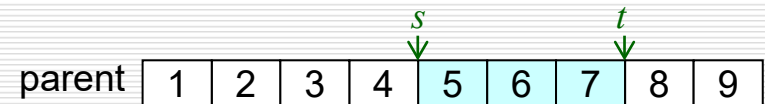
$W \leftarrow v[1:s-1] \parallel v[t+1:l]$ ;

$v' \leftarrow W[1:i-1] \parallel v[s:t] \parallel W[i:l-n]$ ;

**output** offspring  $v'$ ;

**end**

step 1 : select subtour



step 2 : insert subtour in a random position



$v$  : parent chromosome       $l$  : length of chromosome  
 $v'$  : offspring chromosome       $s$  : start position of substring  
 $t$  : end position of substring       $n$  : length of subtour  
 $i$  : insert position  
 $W$  : working data set



# 3 Mutation Operators

## 4. Swap Mutation

**procedure** : Swap Mutation

**input** : chromosome  $v_1, v_2$ ,  
length of chromosome  $l$

**output** : offspring  $v'$

**begin**

// step 1: select two position at random

$i \leftarrow \text{random}[1:l-1]$  ;

$j \leftarrow \text{random}[i+1:l]$  ;

// step 2: produce offspring by swapping

selected positions

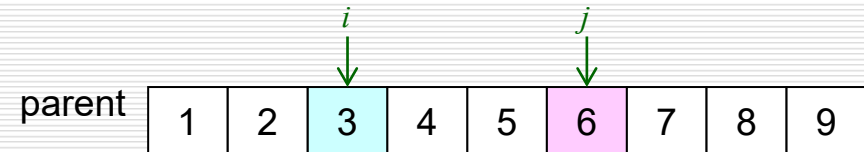
$v' \leftarrow v[1:i-1] // v[j] // v[i+1:j-1] // v[i] // v[j+1:]$

$l$ ];  $v'$

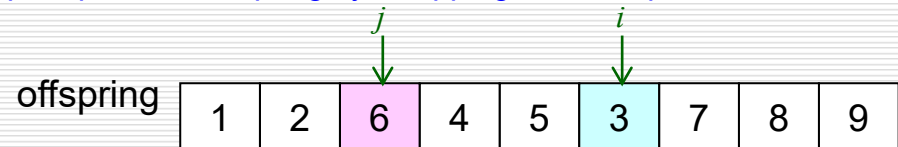
**output** offspring ;

**end**

step 1: select two position at random



step 2 : produce offspring by swapping selected positions



$v$  : parent chromosome

$l$  : length of chromosome

$v'$  : offspring chromosome

$i$  : selected position

$j$  : selected position

# 3 Mutation Operators

## 5. Heuristic Mutation

**procedure** : Heuristic Mutation

**input** : chromosome  $v_1, v_2$ ,  
length of chromosome  $l$

**output** : offspring  $v'$

**begin**

$P \leftarrow \emptyset$  ;

// step 1: select positions and  
produce neighbors

**for**  $i=1$  **to**  $m$  {

$r \leftarrow \text{random}[1:l]$  ;

$P \leftarrow P \cup \text{nb}(v[r])$  ;

}

// step 2: produce offspring by  
evaluating neighbors

$w \leftarrow F(p[1])$ ;

**for**  $i=2$  **to**  $|P|$

**if**  $w > F(p[i])$  **then**

$w \leftarrow F(p[i]), n \leftarrow i$ ;

$v' \leftarrow p[n]$  ;

**output** offspring  $v'$  ;

**end;**

step 1 : select positions and produce neighbors

parent	1	2	3	4	5	6	7	8	9	$F(p[i])$
			$r \downarrow$			$r \downarrow$		$r \downarrow$		32
proto-child 1	1	2	3	4	5	8	7	6	9	27
proto-child 2	1	2	8	4	5	3	7	6	9	54
proto-child 3	1	2	8	4	5	6	7	3	9	45
proto-child 4	1	2	6	4	5	8	7	3	9	23
proto-child 5	1	2	6	4	5	3	7	8	9	56

step 2 : produce offspring by evaluating neighbors

offspring	1	2	6	4	5	8	7	3	9
-----------	---	---	---	---	---	---	---	---	---

$v$  : parent chromosome  
 $v'$  : offspring chromosome  
 $r$  : selected position  
 $w$  : working data  
 $n$  : position of chromosome with best fitness value in  $P$   
 $P\{p[i]\}, i=1,2,\dots,N$  : neighbor chromosome set  
 $\text{nb}(v[r])$  : searching neighbors of  $r$ th gene  
 $F(p[i])$  : fitness value of  $p[i]$   
 $l$  : length of chromosome  
 $m$  : total number of selected positions  
 $N$  : total number of neighbor chromosomes

# 4 Overall Algorithm

## □ GA procedure for Traveling Salesperson Problem

**procedure:** GA for Traveling Salesperson Problem (TSP)

**Input:** TSP data set, GA parameters

**output:** best tour route

**begin**

$t \leftarrow 0$ ;

initialize  $P(t)$  by permutation encoding or random keys encoding;

fitness  $eval(P)$  by permutation decoding or random keys decoding;

**while** (not termination condition) **do**

    crossover  $P(t)$  to yield  $C(t)$  by partial-mapped crossover;

    mutation  $P(t)$  to yield  $C(t)$  by swap mutation;

    fitness  $eval(C)$  by permutation decoding or random keys decoding;

    select  $P(t+1)$  from  $P(t)$  and  $C(t)$ ;

$t \leftarrow t+1$ ;

**end**

**output** best tour route;

**end**