Vertex of the Quadratic

Given a quadratic $r(y)=a\,y^2+b\,y+c$ compute its value at $y_1=-\frac{b}{2a}$ namely $r(y_1)=c-\frac{b^2}{4a}$

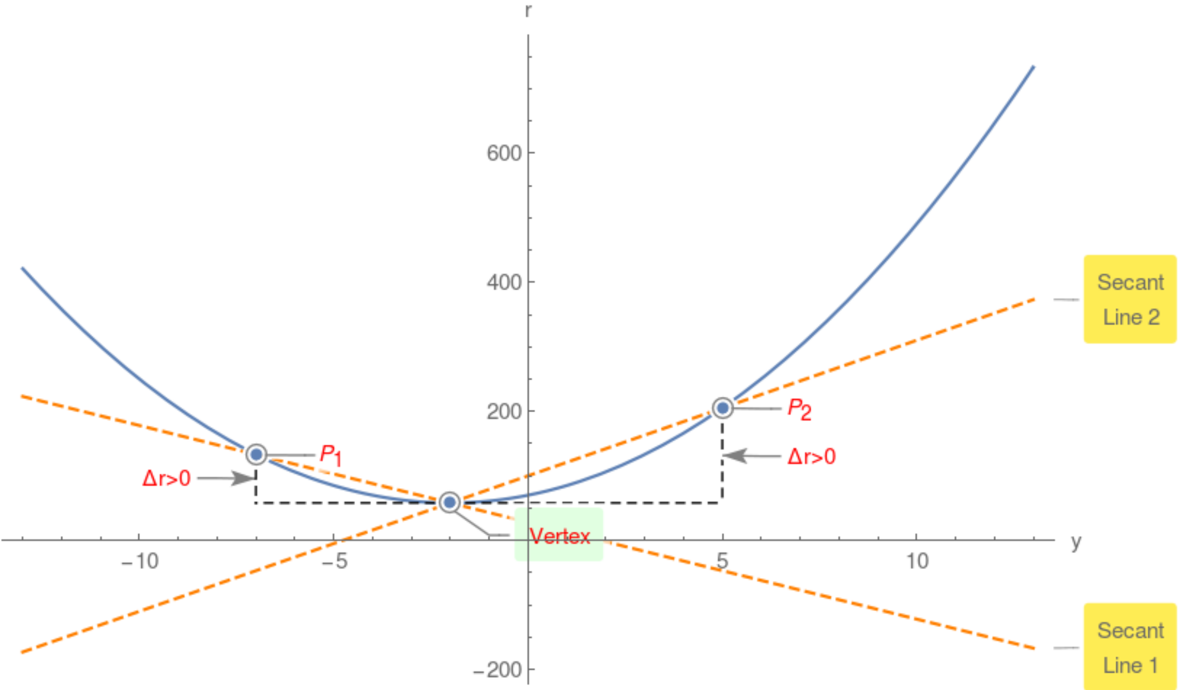
Now compute the same quadratic at y_1+h, namely $r(y_1+h)=-\frac{b^2}{4a}+a\,h^2+c$

Compute $\Delta=r(y_1+h)-r(y_1)=a\,h^2$

Since $h^2>0$, therefore if $a>0$ then $\Delta>0$ or vertex is the global minimum!

Example 1.

$r(y)=3\,y^2+12\,y+70$



However if $a<0$ then $\Delta<0$ or vertex is the global maximum!

Example 2.

$r(y)=-y^2-6\,y-60$