Vertex of the Quadratic

Given a quadratic $m(w) = a\,w^2 + b\,w + c$ compute its value at $w_1 = -\frac{b}{2\,a}$ namely $m(w_1) = c - \frac{b^2}{4\,a}$

Now compute the same quadratic at $w_1 + h$, namely

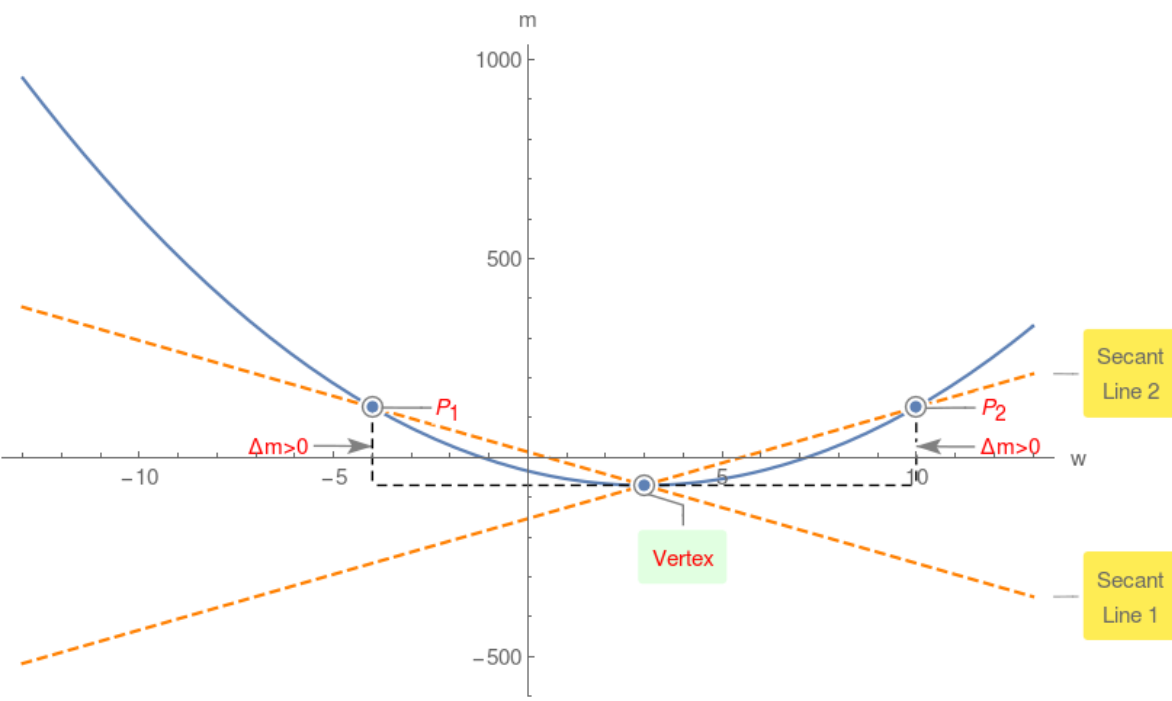
$m(w_1 + h) = -\frac{b^2}{4\,a} + a\,h^2 + c$

Compute $\triangle = m(w_1 + h) - m(w_1) = a\,h^2$

Since $h^2 > 0$, therefore if $a > 0$ then $\triangle > 0$ or vertex is the global minimum!

Example 1.

$m(w) = 4\,w^2 - 24\,w - 34$



However if $a < 0$ then $\triangle < 0$ or vertex is the global maximum!

Example 2.

$m(w) = -2\,w^2 + 4\,w + 34$