Rational Polynomials: Graphing and Asymptotes

Find the intercepts, if there are any.

Step 1: Set the numerator to 0 to solve for horizontal intercepts.

Step 2: Set the x to 0 to solve for vertical intercept.

Step 3: Set the denominator to 0 to solve for vertical asymptotes.

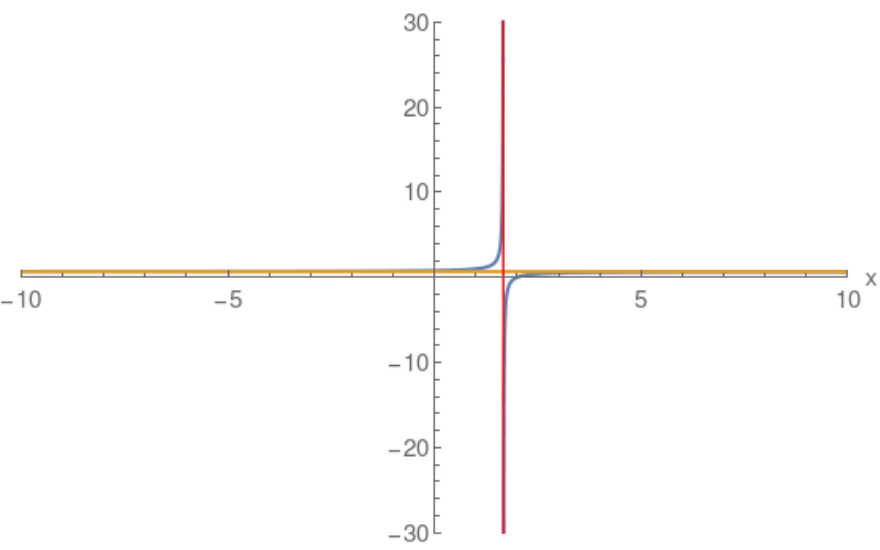
Step 4: Perform a long division to find the quotient which specifies the oblique asymptote.

Note: Blue curve the actual Rational function.
Red and Gold asymptotes.

Example: Horizontal Asymptote

$$\frac{2x-4}{3x-5}$$

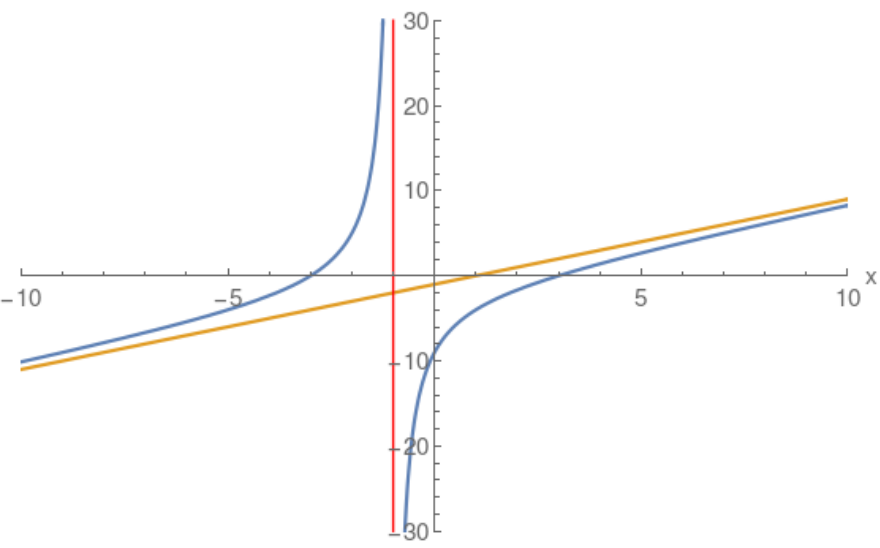
$$\begin{array}{l|ll} & & +\left(\boxed{\frac{2}{3}}\right) \\ \hline \boxed{3x-5} & (2)\,x & +(-4) \\ & \left(\boxed{2x}\right) & +\left(\boxed{-\frac{10}{3}}\right) \\ & & +\left(\boxed{-\frac{2}{3}}\right) \end{array}$$



Example: Oblique Linear Asymptote

$$\frac{(x-3)\,(x+3)}{x+1}$$

$$\begin{array}{l|ll} & & +\left(\boxed{x}\right) \quad +\left(\boxed{-1}\right) \\ \hline \boxed{x+1} & (1)\,x^2 & +(-9) \\ & \left(\boxed{x^2}\right) & +\left(\boxed{x}\right) \\ & & +(-1)\,x \quad +(-9) \\ & & +\left(\boxed{-x}\right) \quad +\left(\boxed{-1}\right) \\ & & +\left(\boxed{-8}\right) \end{array}$$



Example: Multiple Vertical Asymptotes

$$\frac{x+3}{(x-3)\,(x+1)}$$

$$\begin{array}{l|ll} & & +\left(\boxed{0}\right) \\ \hline \left(\boxed{x}\right) & +\left(\boxed{3}\right) & \end{array}$$