Vertex of the Quadratic

Given a quadratic $k(u) = a\,u^2 + b\,u + c$ compute its value at $u_1 = -\frac{b}{2\,a}$ namely $k(u_1) = c - \frac{b^2}{4\,a}$

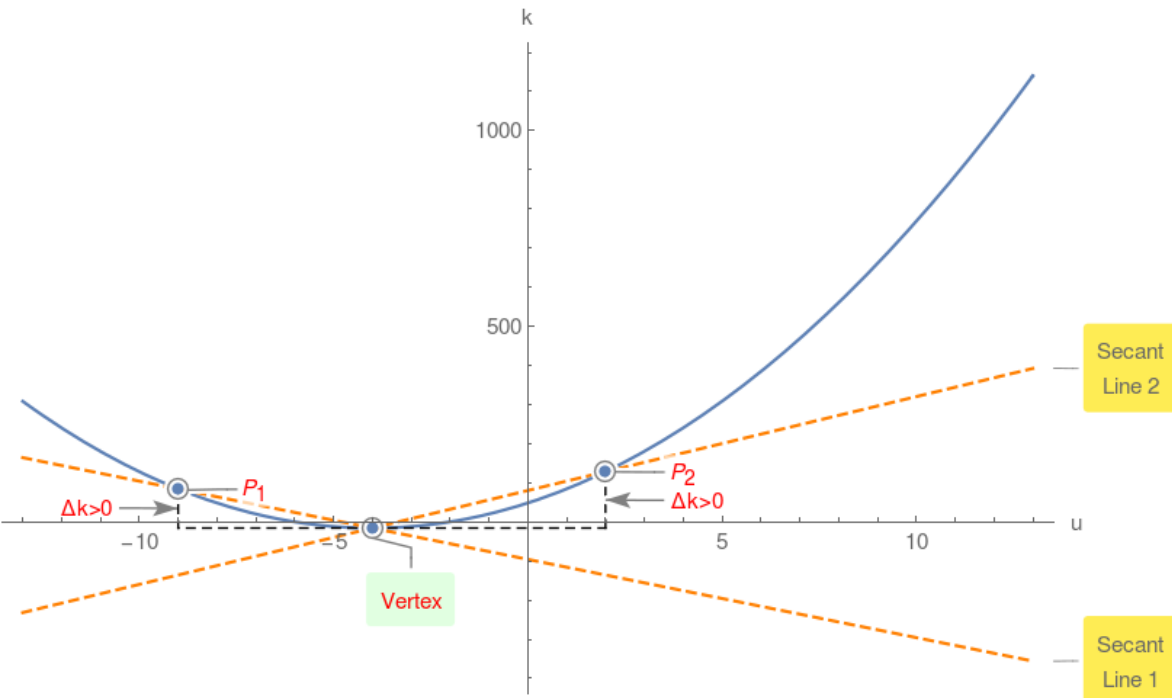
Now compute the same quadratic at $u_1 + h$, namely $k(u_1 + h) = -\frac{b^2}{4\,a} + a\,h^2 + c$

Compute $\triangle = k(u_1 + h) - k(u_1) = a\,h^2$

Since $h^2 > 0$, therefore if $a > 0$ then $\triangle > 0$ or vertex is the global minimum!

Example 1.

$k(u) = 4\,u^2 + 32\,u + 48$



However if $a < 0$ then $\triangle < 0$ or vertex is the global maximum!

Example 2.

$k(u) = -4\,u^2 - 16\,u + 36$