4.

4. Since the fences enclose a rectangular plot without one side, two sides have length b and one side has length a:

$a + b + b = 1678 \implies a = 1678 - 2b$

the area:

$area = a \times b = (l - 2b)\, b = 1678\, b - 2\, b^2$

Note that the function is for parabola. Now solve for the vertex:

$vertex = (\frac{839}{2}\ feet\ ,\ \frac{703\,921}{2}\ feet^2\ )$

Square the unit for vertex's vertical coordinate since it is a quadratic.