Vertex of the Quadratic

Given a quadratic $f(u)=a\,u^2+b\,u+c$ compute its value at $u_1=-\frac{b}{2\,a}$ namely $f(u_1)=c-\frac{b^2}{4\,a}$

Now compute the same quadratic at u_1+h, namely

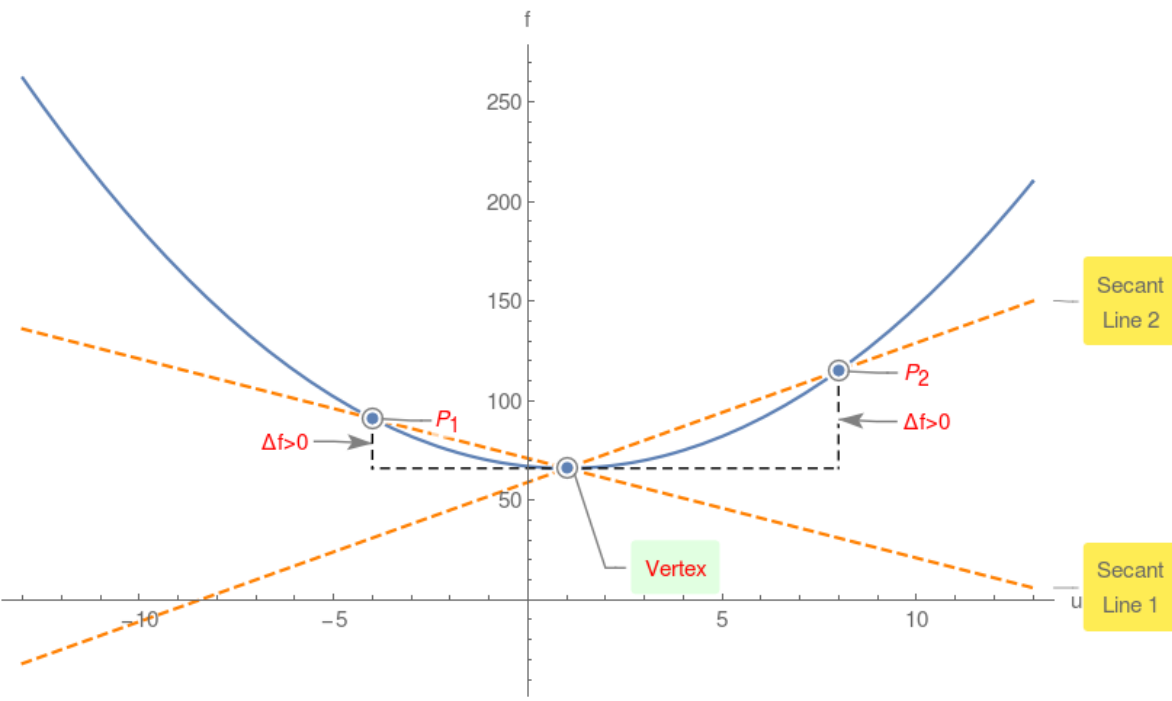
$f(u_1+h)=-\frac{b^2}{4\,a}+a\,h^2+c$

Compute $\triangle=f(u_1+h)-f(u_1)=a\,h^2$

Since $h^2>0$, therefore if $a>0$ then $\triangle>0$ or vertex is the global minimum!

Example 1.

$f(u)=u^2-2\,u+67$



However if $a<0$ then $\triangle<0$ or vertex is the global maximum!

Example 2.

$f(u)=-u^2-8\,u-43$