Vertex of the Quadratic

Given a quadratic $h(n)=a\,n^2+b\,n+c$ compute its value at $n_1=-\frac{b}{2\,a}$ namely $h(n_1)=c-\frac{b^2}{4\,a}$

Now compute the same quadratic at n_1+h, namely

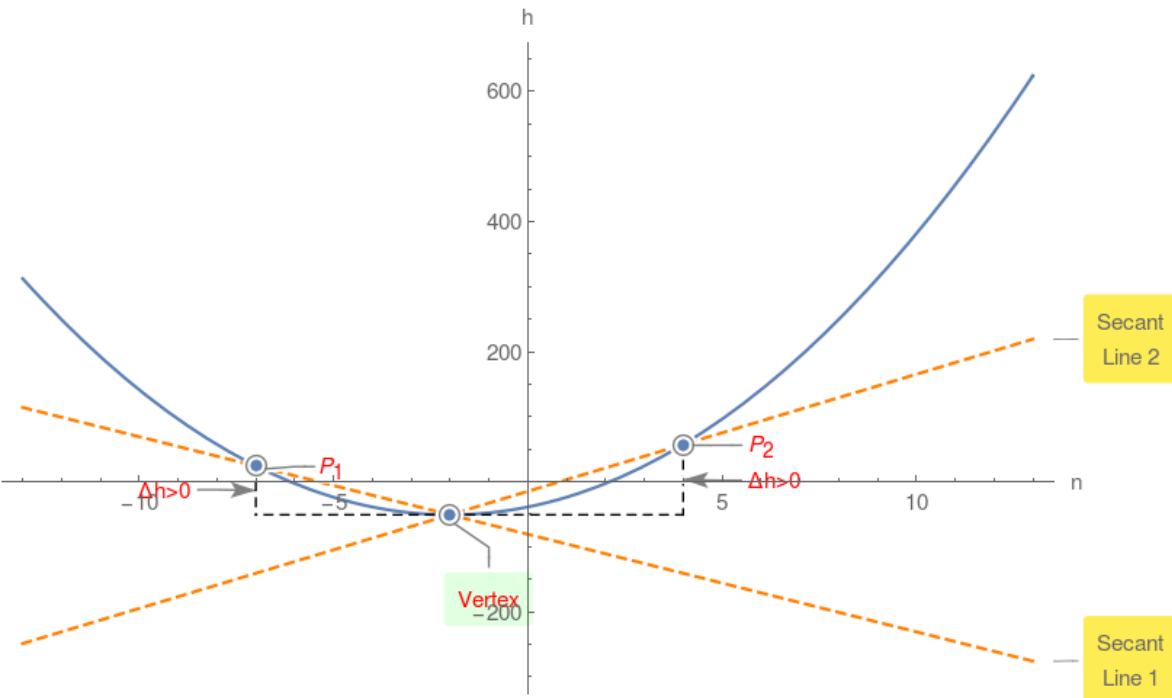
$h(n_1+h)=-\frac{b^2}{4\,a}+a\,h^2+c$

Compute $\triangle=h(n_1+h)-h(n_1)=a\,h^2$

Since $h^2>0$, therefore if $a>0$ then $\triangle>0$ or vertex is the global minimum!

Example 1.

$h(n)=3\,n^2+12\,n-39$



However if $a<0$ then $\triangle<0$ or vertex is the global maximum!

Example 2.

$h(n)=-2\,n^2-12\,n-58$