Vertex of the Quadratic

Given a quadratic $h(z)=a\,z^2+b\,z+c$ compute its value at $z_1=-\frac{b}{2\,a}$ namely $h(z_1)=c-\frac{b^2}{4\,a}$

Now compute the same quadratic at z_1+h, namely

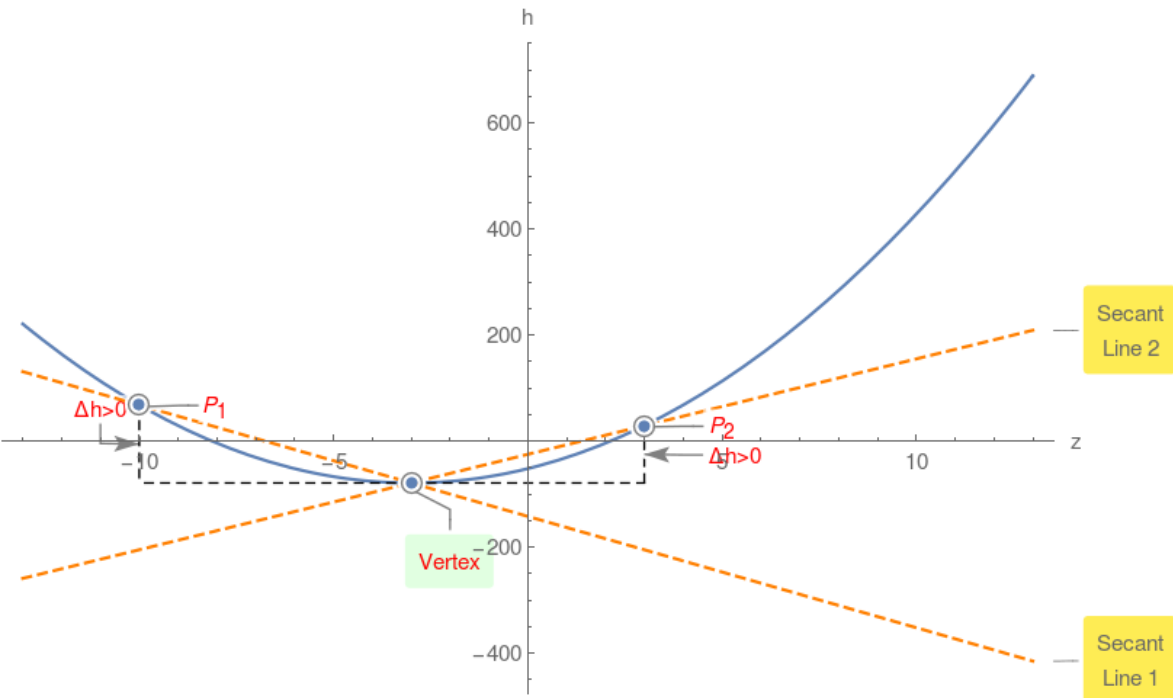
$h(z_1+h)=-\frac{b^2}{4\,a}+a\,h^2+c$

Compute $\triangle=h(z_1+h)-h(z_1)=a\,h^2$

Since $h^2>0$, therefore if $a>0$ then $\triangle>0$ or vertex is the global minimum!

Example 1.

$h(z)=3\,z^2+18\,z-52$



However if $a<0$ then $\triangle<0$ or vertex is the global maximum!

Example 2.

$h(z)=-z^2+2\,z-55$