Vertex of the Quadratic

Given a quadratic $w(x)=a\,x^2+b\,x+c$ compute its value at $x_1=-\frac{b}{2\,a}$ namely $w(x_1)=c-\frac{b^2}{4\,a}$

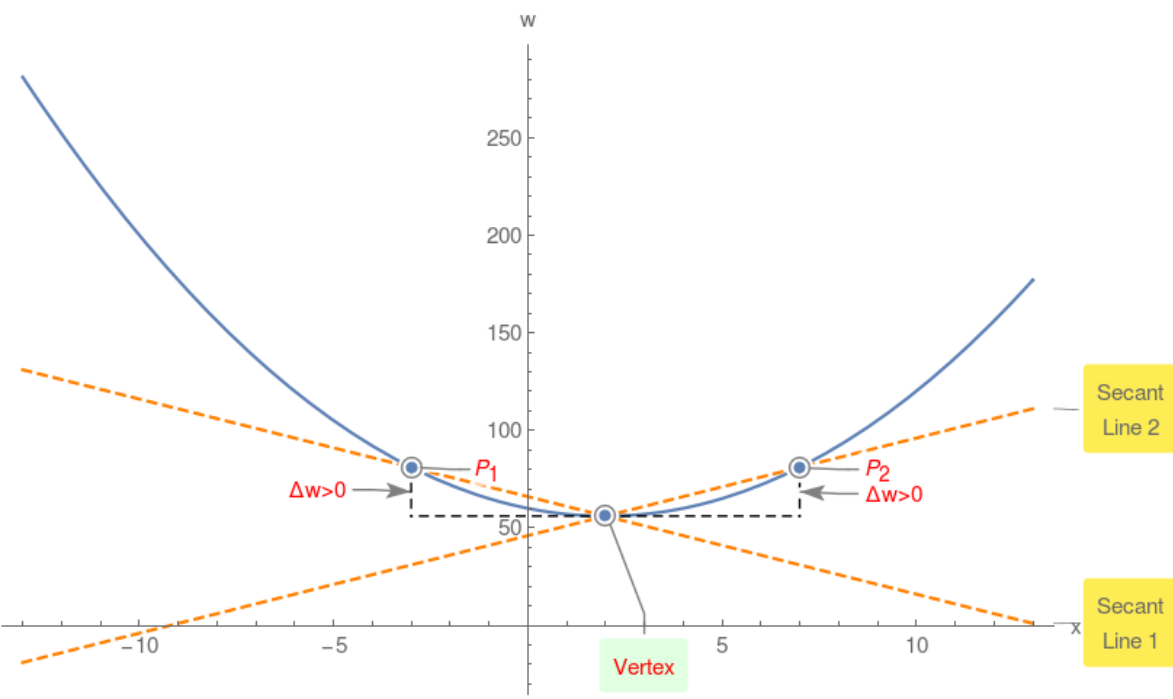
Now compute the same quadratic at x_1+h, namely $w(x_1+h)=-\frac{b^2}{4\,a}+a\,h^2+c$

Compute $\triangle=w(x_1+h)-w(x_1)=a\,h^2$

Since $h^2>0$, therefore if $a>0$ then $\triangle>0$ or vertex is the global minimum!

Example 1.

$w(x)=x^2-4\,x+60$



However if $a<0$ then $\triangle<0$ or vertex is the global maximum!

Example 2.

$w(x)=-x^2-8\,x+43$