Rational Polynomials: Graphing and Asymptotes

Find the intercepts, if there are any.

Step 1: Set the numerator to 0 to solve for horizontal intercepts.

Step 2: Set the x to 0 to solve for vertical intercept.

Step 3: Set the denominator to 0 to solve for vertical asymptotes.

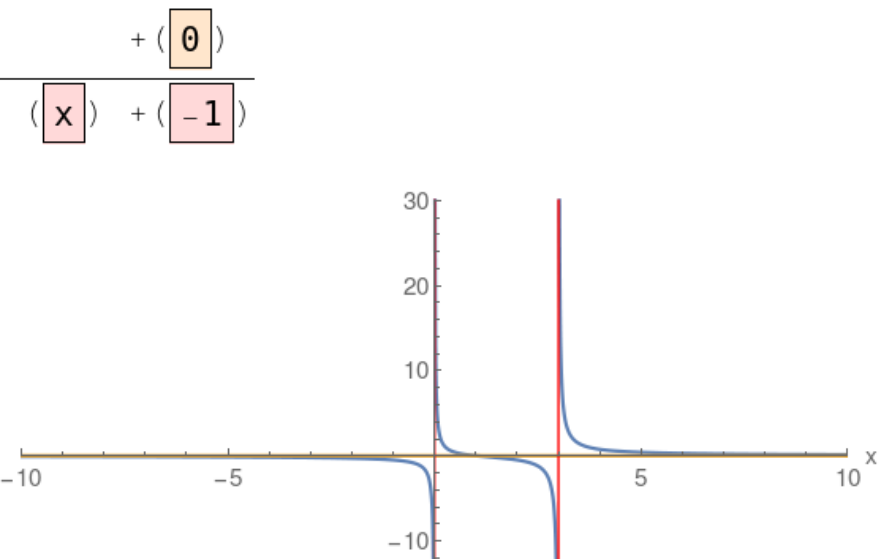
Step 4: Perform a long division to find the quotient which specifies the oblique asymptote.

Note: Blue curve the actual Rational function.
Red and Gold asymptotes.

Example: Horizontal Asymptote

$\frac{x-4}{3x-5}$

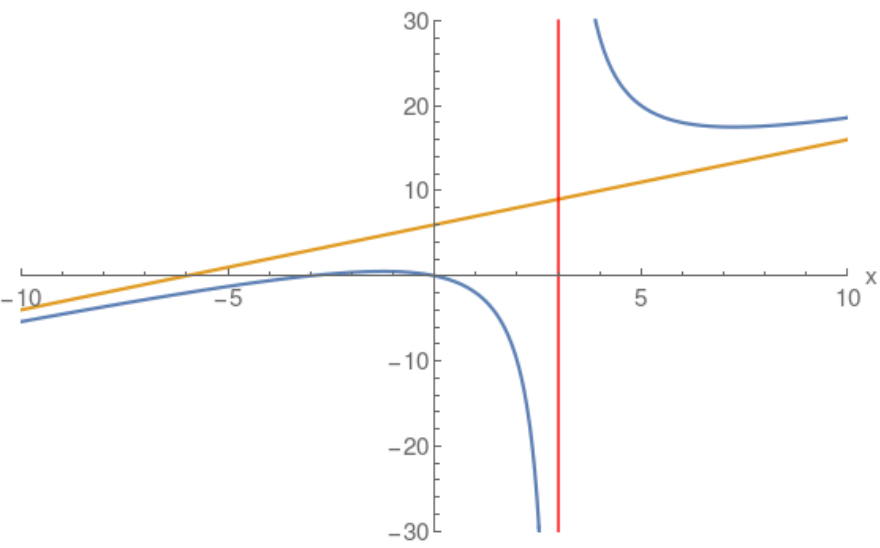
$$
\begin{array}{r|l}
 & +\left(\frac{1}{3}\right) \\
\hline
3x-5 & (1)\,x \;+(-4) \\
 & \left(x\right) \;+\left(-\frac{5}{3}\right) \\
 & +\left(-\frac{7}{3}\right)
\end{array}
$$



Example: Oblique Linear Asymptote

$\frac{x\,(x+3)}{x-3}$

$$
\begin{array}{r|l}
 & +(x) \qquad +(6) \\
\hline
x-3 & (1)\,x^2 \;+(3)\,x \\
 & \left(x^2\right) \;+(-3x) \\
 & +(6)\,x \\
 & +(6x) \qquad +(-18) \\
 & +(18)
\end{array}
$$



Example: Multiple Vertical Asymptotes

$\frac{x-1}{(x-3)\,x}$

$$
\begin{array}{r|l}
 & +(0) \\
\hline
(x) & +(-1)
\end{array}
$$