Vertex of the Quadratic

Given a quadratic $x(c) = a\,c^2 + b\,c + c$ compute its value at $c_1 = -\frac{b}{2\,a}$ namely $x(c_1) = c - \frac{b^2}{4\,a}$

Now compute the same quadratic at $c_1 + h$, namely

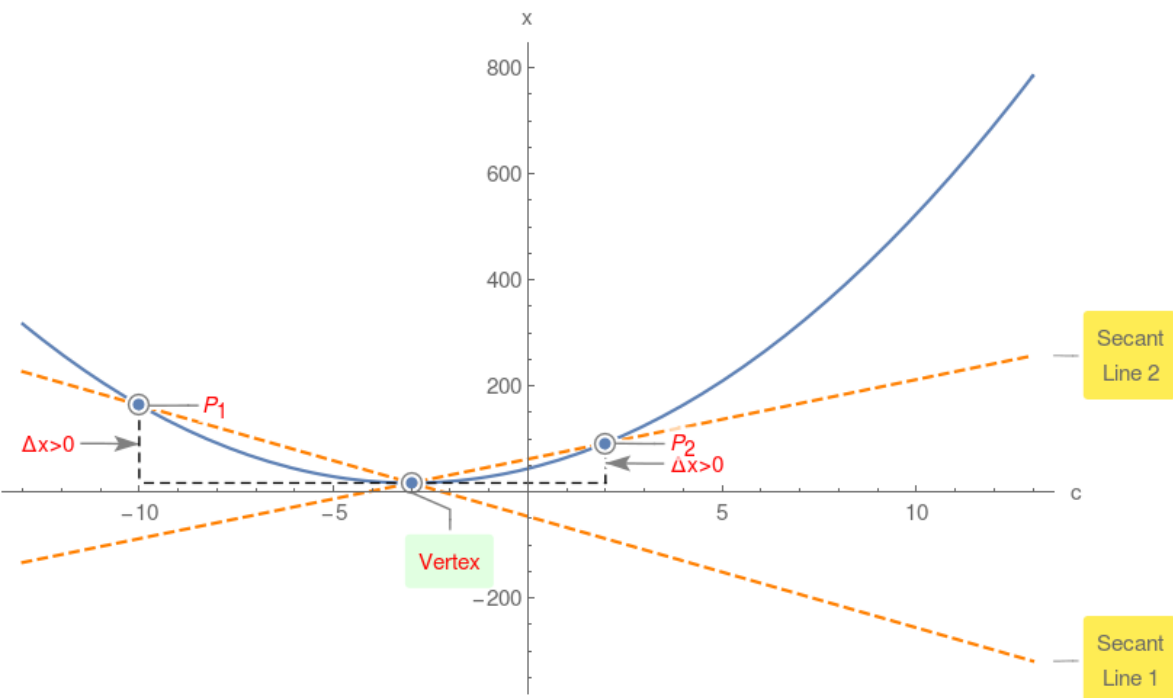
$x(c_1 + h) = -\frac{b^2}{4\,a} + a\,h^2 + c$

Compute $\triangle = x(c_1 + h) - x(c_1) = a\,h^2$

Since $h^2 > 0$, therefore if $a > 0$ then $\triangle > 0$ or vertex is the global minimum!

Example 1.

$x(c) = 3\,c^2 + 18\,c + 44$



However if $a < 0$ then $\triangle < 0$ or vertex is the global maximum!

Example 2.

$x(c) = -2\,c^2 - 16\,c - 61$