Vertex of the Quadratic

Given a quadratic $n(u) = a\, u^2 + b\, u + c$ compute its value at

$u_1 = -\frac{b}{2a}$ namely $n(u_1) = c - \frac{b^2}{4a}$
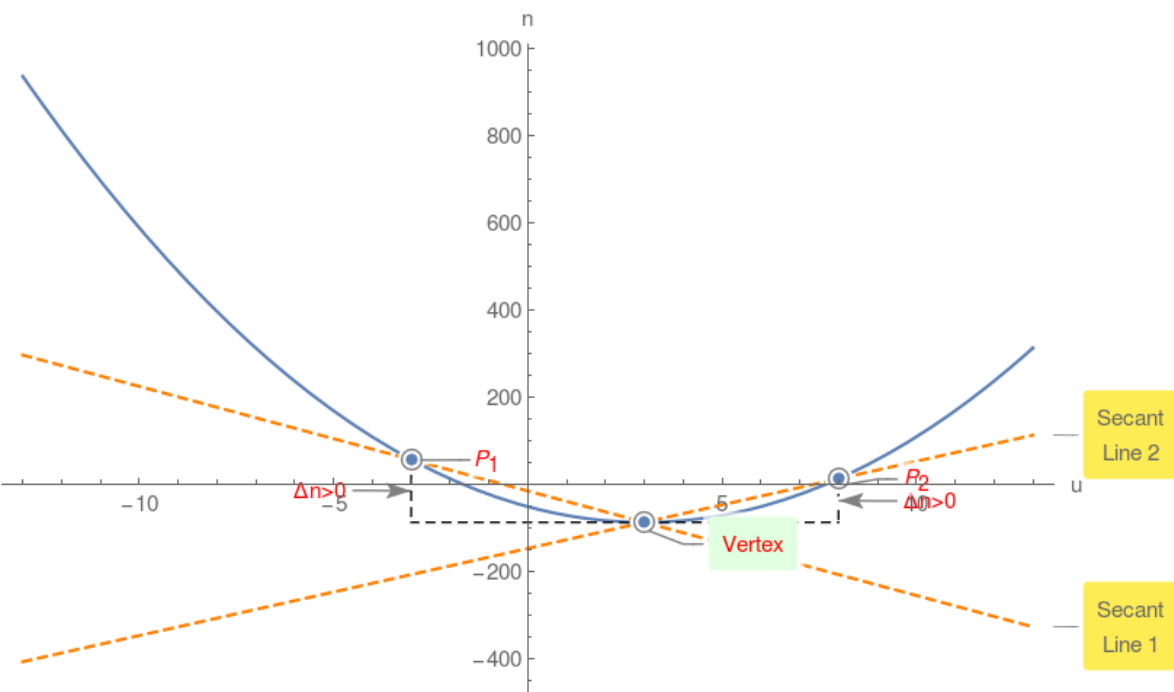
Now compute the same quadratic at $u_1 + h$, namely

$n(u_1 + h) = -\frac{b^2}{4a} + a\, h^2 + c$

Compute $\triangle = n(u_1 + h) - n(u_1) = a\, h^2$

Since $h^2 > 0$, therefore if $\boxed{a > 0}$ then $\triangle > 0$ or vertex is the global minimum!

Example 1.

$n(u) = 4\, u^2 - 24\, u - 52$

However if $\boxed{a < 0}$ then $\triangle < 0$ or vertex is the global maximum!

Example 2.

$n(u) = -u^2 - 8\, u + 64$