Vertex of the Quadratic

Given a quadratic $d(w)=a\,w^2+b\,w+c$ compute its value at $w_1=-\frac{b}{2\,a}$ namely $d(w_1)=c-\frac{b^2}{4\,a}$

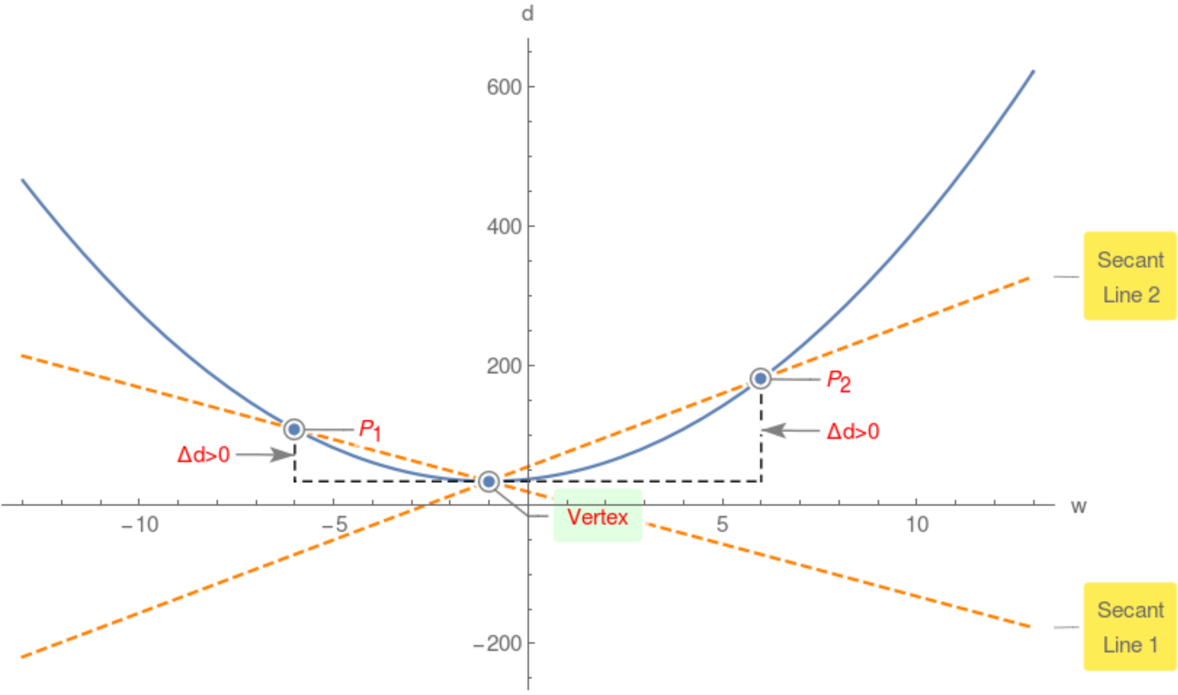
Now compute the same quadratic at w_1+h, namely $d(w_1+h)=-\frac{b^2}{4\,a}+a\,h^2+c$

Compute $\Delta=d(w_1+h)-d(w_1)=a\,h^2$

Since $h^2>0$, therefore if $a>0$ then $\Delta>0$ or vertex is the global minimum!

Example 1.

$d(w)=3\,w^2+6\,w+36$



However if $a<0$ then $\Delta<0$ or vertex is the global maximum!

Example 2.

$d(w)=-3\,w^2+18\,w-63$