Vertex of the Quadratic

Given a quadratic $p(z) = a\,z^2 + b\,z + c$ compute its value at

$z_1 = -\frac{b}{2a}$ namely $p(z_1) = c - \frac{b^2}{4a}$

Now compute the same quadratic at $z_1 + h$, namely

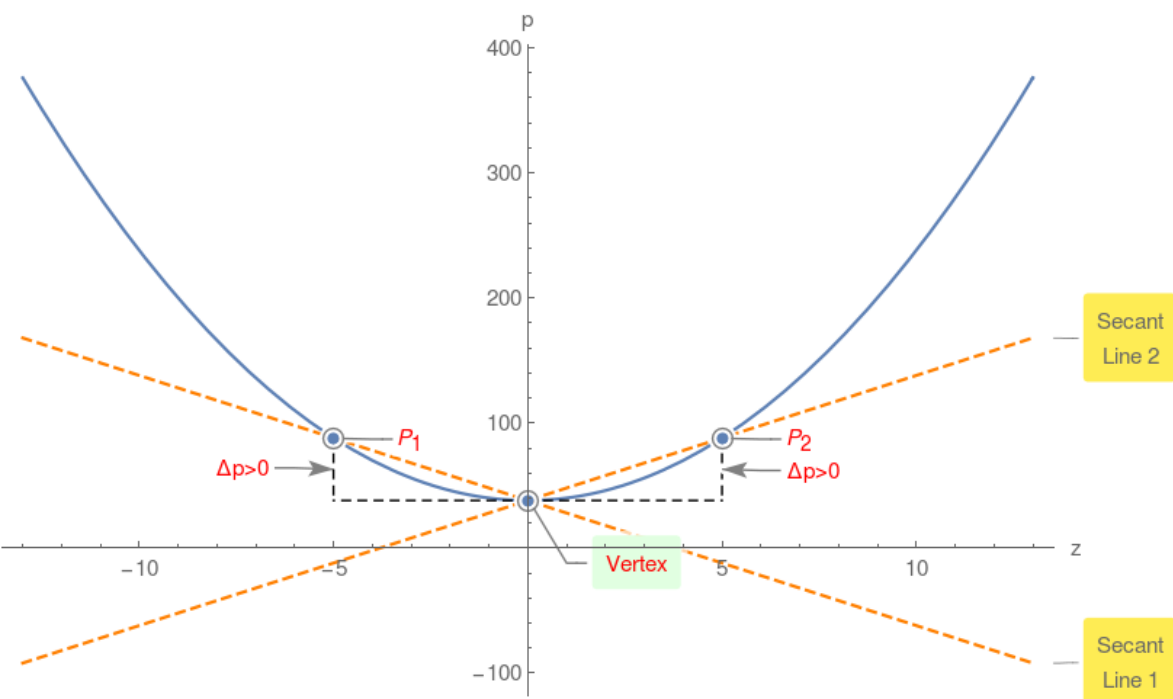$p(z_1 + h) = -\frac{b^2}{4a} + a\,h^2 + c$

Compute $\triangle = p(z_1 + h) - p(z_1) = a\,h^2$

Since $h^2 > 0$, therefore if $\boxed{a > 0}$ then $\triangle > 0$ or vertex is the global minimum!

Example 1.
$p(z) = 2\,z^2 + 38$

However if $\boxed{a < 0}$ then $\triangle < 0$ or vertex is the global maximum!

Example 2.
$p(z) = -3\,z^2 - 18\,z + 55$