Collaborative Filtering

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CENTRALE NANTES

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Course website: https://github.com/mathieulagrange/datasim

Outline

What is Collaborative Filtering?

Group of users

Group of items

What is Collaborative Filtering?

Group of users

Group of items

- Observe some user-item preferences
- Predict new preferences:

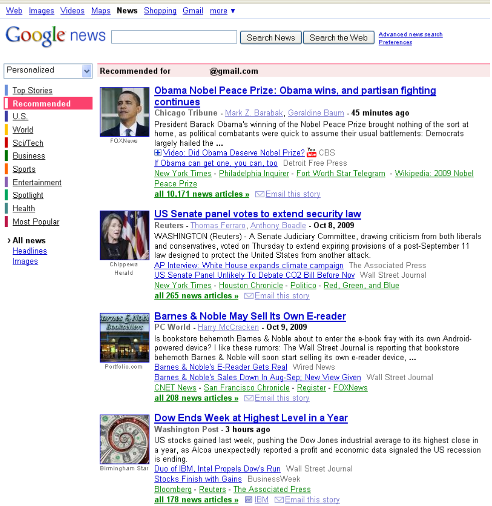
Does Bob like strawberries???

Collaborative Filtering in the Wild...

Amazon.com recommends products based on purchase history

Linder et al., 2003

Collaborative Filtering in the Wild...



- **Google News** recommends new articles based on click and search history
- Millions of users, millions of articles

Das et al., 2007

Collaborative Filtering in the Wild...

Netflix predicts other "Movies You'll ♡" based on past numeric ratings (1-5 stars)

- Recommendations drive 60% of Netflix's DVD rentals
- Mostly smaller, independent movies (Thompson 2008)

http://www.netflix.com

Collaborative Filtering in the Wild...

- **Netflix Prize:** Beat Netflix recommender system, using Netflix data → Win \$1 million
- **Data:** 480,000 users 18,000 movies 100 million observed ratings = only 1.1% of ratings observed

"The Netflix Prize seeks to substantially improve the accuracy of predictions about how much someone is going to love a movie based on their movie preferences."

What is Collaborative Filtering?

Insight: Personal preferences are correlated

- If Jack loves A and B, and Jill loves A, B, and C, then Jack is more likely to love C

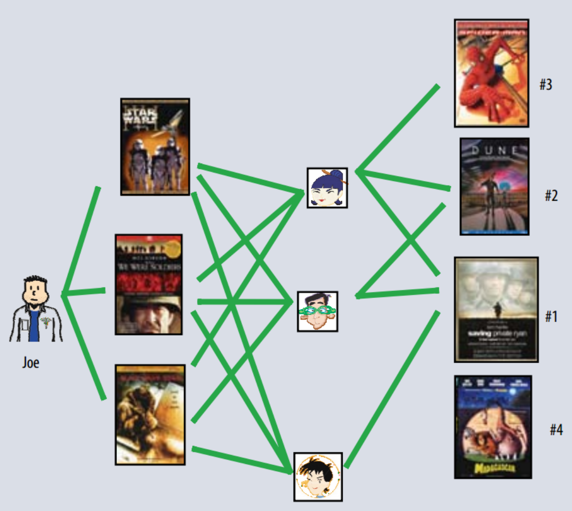
Collaborative Filtering Task

- Discover patterns in observed preference behavior (e.g. purchase history, item ratings, click counts) across community of users
- Predict new preferences based on those patterns

Does not rely on item or user attributes (e.g. demographic info, author, genre)

- Content-based filtering: complementary approach

What is Collaborative Filtering?



What is Collaborative Filtering?

Given:

- Users $u \in \{1, \ldots, U\}$
- Items $i \in \{1, \ldots, M\}$
- Training set $\mathcal{T}$ with observed, real-valued preferences r_{ui} for some user-item pairs (u, i)
  - r_{ui} = e.g. purchase indicator, item rating, click count ...

Goal: Predict unobserved preferences

- Test set $\mathcal{Q}$ with pairs (u, i) not in $\mathcal{T}$

View as matrix completion problem

- Fill in unknown entries of sparse preference matrix

$$\mathbf{R} = \underbrace{\left.\begin{bmatrix} ? & ? & 1 & \ldots & 4 \\ 3 & ? & ? & \ldots & ? \\ ? & 5 & ? & \ldots & 5 \end{bmatrix}\right\} U \text{ users}}_{M \text{ items}}$$

What is Collaborative Filtering?

Measuring success

- Interested in error on unseen test set Q, not on training set
- For each (u,i) let r_{ui} = true preference, $\hat{r}_{ui}$ = predicted preference
- Root Mean Square Error
  - $\text{RMSE} = \sqrt{\frac{1}{|Q|} \sum_{(u,i)\in Q} (r_{ui} - \hat{r}_{ui})^2}$
- Mean Absolute Error
  - $\text{MAE} = \frac{1}{|Q|} \sum_{(u,i)\in Q} |r_{ui} - \hat{r}_{ui}|$
- Ranking-based objectives
  - e.g. What fraction of true top-10 preferences are in predicted top 10?

Centering Your Data

- What?
  - Remove bias term from each rating before applying CF methods: $\tilde{r}_{ui} = r_{ui} - b_{ui}$
- Why?
  - Some users give systematically higher ratings
  - Some items receive systematically higher ratings
  - Many interesting patterns are in variation around these systematic biases
  - Some methods assume mean-centered data
    - Recall PCA required mean centering to measure variance around the mean

Centering Your Data

- What?
  - Remove bias term from each rating before applying CF methods: $\tilde{r}_{ui} = r_{ui} - b_{ui}$
- How?
  - Global mean rating
    - $b_{ui} = \mu := \frac{1}{|\mathcal{T}|} \sum_{(u,i)\in\mathcal{T}} r_{ui}$
  - Item's mean rating
    - $b_{ui} = b_i := \frac{1}{|R(i)|} \sum_{u\in R(i)} r_{ui}$
    - $R(i)$ is the set of users who rated item i
  - User's mean rating
    - $b_{ui} = b_u := \frac{1}{|R(u)|} \sum_{i\in R(u)} r_{ui}$
    - $R(u)$ is the set of items rated by user u
  - Item's mean rating + user's mean deviation from item mean
    - $b_{ui} = b_i + \frac{1}{|R(u)|} \sum_{i\in R(u)} (r_{ui} - b_i)$

Shrinkage

- What?
  - Interpolating between an estimate computed from data and a fixed, predetermined value
- Why?
  - Common task in CF: Compute estimate (e.g. a mean rating) for each user/item
  - Not all estimates are equally reliable
  - Some users have orders of magnitude more ratings than others
  - Estimates based on fewer datapoints tend to be noisier

$$\mathbf{R} = \begin{array}{lcccccc c} & A & B & C & D & E & F & \text{User mean} \\ \textit{Alice} & 2 & 5 & 5 & 4 & 3 & 5 & 4 \\ \textit{Bob} & 2 & ? & ? & ? & ? & ? & 2 \\ \textit{Craig} & 3 & 3 & 4 & 3 & ? & 4 & 3.4 \end{array}$$

- Hard to trust mean based on one rating

Shrinkage

- What?
  - Interpolating between an estimate computed from data and a fixed, predetermined value
- How?
  - e.g. Shrunk User Mean:

$$\tilde{b}_u = \frac{\alpha}{\alpha + |R(u)|} * \mu + \frac{|R(u)|}{\alpha + |R(u)|} * b_u$$

  - μ is the global mean, α controls degree of shrinkage
  - When user has many ratings, $\tilde{b}_u \approx$ user's mean rating
  - When user has few ratings, $\tilde{b}_u \approx$ global mean rating

$\mathbf{R} =$

	A	*B*	*C*	*D*	*E*	*F*	User mean	Shrunk mean
Alice	2	5	5	4	3	5	4	3.94
Bob	2	?	?	?	?	?	2	2.79
Craig	3	3	4	3	?	4	3.4	3.43

Global mean $\mu = 3.58$, $\alpha = 1$

Classification/Regression for CF

Interpretation: CF is a set of M classification/regression problems, one for each item

- Consider a fixed item i
- Treat each user as incomplete vector of user's ratings for all items except i: $\vec{r}_u = (3,?,?,4,?,5,?,1,3)$
- Class of each user w.r.t. item i is the user's rating for item i (e.g. $1,2,3,4,$ or 5)
- Predicting rating $r_{ui} \equiv$ Classifying user vector $\vec{r}_u$

Classification/Regression for CF

Approach:

- Choose your favorite classifier/regression algorithm
- Train separate predictor for each item
- To predict r_{ui} for user u and item i, apply item i's predictor to vector of user u's incomplete ratings vector

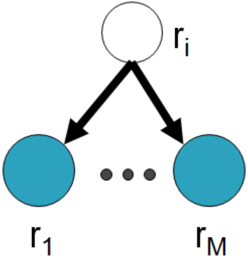
Pros:

- Reduces CF to a well-known, well-studied problem
- Many good prediction algorithms available

Cons:

- Predictor must handle missing data (unobserved ratings)
- Training M independent predictors can be expensive
- Approach may not take advantage of problem structure
  - Item-specific subproblems are often related

Naive Bayes Classifier



- Treat distinct rating values as classes
- Consider classification for item i
- Main assumption
  - For any items $j \neq k \neq i$, r_j and r_k are conditionally independent given r_i
  - When we know rating r_{ui} all of a user's other ratings are independent
- Parameters to estimate
  - Prior class probabilities: $P(r_i = v)$
  - Likelihood: $P(r_j = w | r_i = v)$

Naive Bayes Classifier

Train classifier with all users who have rated item i

- Use counts to estimate prior and likelihood

$$P(r_i = v) = \frac{\sum_{u=1}^{U} \mathbf{1}(r_{ui} = v)}{\sum_{w=1}^{V} \sum_{i=1}^{U} \mathbf{1}(r_{ui} = w)}$$

$$P(r_j = w | r_i = v) = \frac{\sum_{u=1}^{U} \mathbf{1}\left(r_{ui} = v, r_{uj} = w\right)}{\sum_{z=1}^{V} \sum_{u=1}^{U} \mathbf{1}\left(r_{ui} = v, r_{uj} = z\right)}$$

- Complexity
  - $O(\sum_{u=1}^{U} |R(u)|^2)$ time and $O(M^2 V^2)$ space for all items

Predict rating for (u, i) using posterior

$$P(r_{ui} = v | r_{u1}, \ldots, r_{uM}) = \frac{P(r_{ui} = v) \prod_{j \neq i} P(r_{uj} | r_{ui} = v)}{\sum_{w=1}^{V} P(r_{ui} = w) \prod_{j \neq i} P(r_{uj} | r_{ui} = w)}$$

Naive Bayes Summary

Pros:

- Easy to implement
- Off-the-shelf implementations readily available

Cons:

- Large space requirements when storing parameters for all M predictors
- Makes strong independence assumptions
- Parameter estimates will be noisy for items with few ratings
  - E.g. $P(r_j = w | r_i = v) = 0$ if no user rated both i and j

Addressing cons:

- Tie together parameter learning in each item's predictor
- Shrinkage/smoothing is an example of this

K Nearest Neighbor Methods

Most widely used class of CF methods

- Flavors: Item-based and User-based
- Represent each item as incomplete vector of user ratings: $\vec{r}_{.i} = (3,?,?,4,?,5,?,1,3)$
- To predict new rating r_{ui} for query user u and item i:
  1. Compute similarity between i and every other item
  2. Find K items rated by u most similar to i
  3. Predict weighted average of similar items' ratings
- Intuition: Users rate similar items similarly.

KNN: Computing Similarities

How to measure similarity between items?

- Cosine similarity

$$S(\vec{r}_{.i}, \vec{r}_{.j}) = \frac{\langle \vec{r}_{.i}, \vec{r}_{.j} \rangle}{\|\vec{r}_{.i}\| \|\vec{r}_{.j}\|}$$

- Pearson correlation coefficient

$$S(\vec{r}_{.i}, \vec{r}_{.j}) = \frac{\langle \vec{r}_{.i} - \mathrm{mean}(\vec{r}_{.i}), \vec{r}_{.j} - \mathrm{mean}(\vec{r}_{.j}) \rangle}{\|\vec{r}_{.i} - \mathrm{mean}(\vec{r}_{.i})\| \|\vec{r}_{.j} - \mathrm{mean}(\vec{r}_{.j})\|}$$

- Inverse Euclidean distance

$$S(\vec{r}_{.i}, \vec{r}_{.j}) = \frac{1}{\|\vec{r}_{.i} - \vec{r}_{.j}\|}$$

Problem: These measures assume complete vectors

Solution: Compute over subset of users rated by both items

Complexity: $O(\sum^{U} |R(u)|^2)$ time

KNN: Choosing K neighbors

How to choose K nearest neighbors?

- Select K items with largest similarity score to query item i

Problem: Not all items were rated by query user u

Solution: Choose K most similar items rated by u

Complexity: $O(min(KM, M \log M))$

Herlocker et al., 1999

KNN: Forming Weighted Predictions

Predicted rating for query user u and item i

- $N(i;u)$ is the *neighborhood* of item i for user u
  - i.e. the K most similar items rated by u
- $\hat{r}_{ui} = b_{ui} + \sum_{N(i;u)} w_{ij}(r_{uj} - b_{uj})$

How to choose weights for each neighbor?

- Equal weights: $w_{ij} = \frac{1}{|N(i;u)|}$
- Similarity weights: $w_{ij} = \frac{S(i,j)}{\sum_{j \in N(i;u)} S(i,j)}$ (Herlocker et al., 1999)
- Learn optimal weights for each user (Bell and Koren, 2007)
- Learn optimal global weights (Koren, 2008)

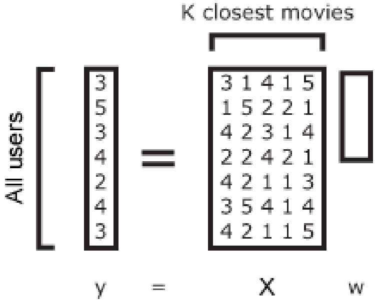
Complexity: $O(K)$

KNN: User Optimized Weights

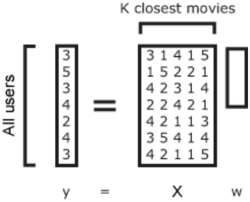
Intuition: For a given query user u and item i, choose weights that best predict other known ratings of item i using only $N(i;u)$:

$$\min_{\mathbf{w}_{i.}} \sum_{s \in R(i), s \neq u} \left(r_{si} - \sum_{j \in N(i;u)} w_{ij} r_{sj} \right)^2$$

With no missing ratings, this is a linear regression problem:



KNN: User Optimized Weights



Bell and Koren, 2007

- Optimal solution: $w = A^{-1}b$ for $A = X^TX, b = X^Ty$
- Problem: X contains missing entries
  - Not all items in $N(i;u)$ were rated by all users
- Solution: Approximate A and b

$$
\begin{aligned}
\hat{A}_{jk} &= \frac{\sum_{s \in R(j) \cap R(k)} r_{sj} r_{sk}}{|R(j) \cap R(k)|} \\
\hat{b}_k &= \frac{\sum_{s \in R(i) \cap R(k)} r_{si} r_{sk}}{|R(i) \cap R(k)|} \\
\hat{w} &= \hat{A}^{-1}\hat{b}
\end{aligned}
$$

- Estimates based on users who rated each pair of items

KNN: User Optimized Weights

Benefits

- Weights optimized for the task of rating prediction
  - Not just borrowed from the neighborhood selection phase
- Weights not constrained to sum to 1
  - Important if all nearest neighbors are dissimilar
- Weights derived simultaneously
  - Accounts for correlations among neighbors
- Outperforms KNN with similarity or equal weights
- Can compute entries of $\hat{A}$ and $\hat{b}$ offline in parallel

Drawbacks

- Must solve additional *KxK* system of linear equations per query

Bell and Koren, 2007

KNN: Globally Optimized Weights

Consider the following KNN prediction rule for query (u,i):

$$\hat{r}_{ui} = b_{ui} + |N(i;u)|^{-\frac{1}{2}} \sum_{j \in N(i;u)} w_{ij}(r_{uj} - b_{uj})$$

Could learn a single set of KNN weights w_{ij}, shared by all users, that minimize regularized MSE:

$$E = \frac{1}{|\mathcal{T}|} \sum_{(u,i)\in\mathcal{T}} \frac{1}{2}(\hat{r}_{ui} - r_{ui})^2 + \lambda \sum_{i=1}^{M} \sum_{j=1}^{M} \frac{1}{2} w_{ij}^2 = \frac{1}{|\mathcal{T}|} \sum_{(u,i)\in\mathcal{T}} E_{ui}$$

Optimize objective using **stochastic gradient descent**:

- For each example $(u,i) \in \mathcal{T}$, update w_{ij} $\forall j \in N(i;u)$

$$\begin{aligned} w_{ij}^{t+1} &= w_{ij}^t - \gamma \frac{\partial}{\partial w_{ij}} E_{ui} \\ &= w_{ij}^t - \gamma(|N(i;u)|^{-\frac{1}{2}}(\hat{r}_{ui} - r_{ui})(r_{uj} - b_{uj}) + \lambda w_{ij}^t) \end{aligned}$$

KNN: Globally Optimized Weights

Benefits

- Weights optimized for the task of rating prediction
  - Not just borrowed from the neighborhood selection phase
- Weights not constrained to sum to 1
  - Important if all nearest neighbors are dissimilar
- Weights derived simultaneously
  - Accounts for correlations among neighbors
- Outperforms KNN with similarity or equal weights

Drawbacks

- Must solve global optimization problem at training time
- Must store $O(M^2)$ weights in memory

Koren, 2008

KNN: Summary

Pros

- Intuitive interpretation
- When weights not learned...
  - Easy to implement
  - Zero training time
- Learning prediction weights can greatly improve accuracy for little overhead in space and time

Cons

- When weights not learned...
  - Need to store all item (or user) vectors in memory
  - May redundantly recompute similarity scores at test time
  - Similarity/equal weights not always suitable for prediction
- When weights learned...
  - Need to store $O(M^2)$ or $O(U^2)$ parameters
  - Must update stored parameters when new ratings occur

Low Dimensional Matrix Factorization

Matrix Completion

- Filling in the unknown ratings in a sparse $U \times M$ matrix R

$$\mathbf{R} = \begin{bmatrix} ? & ? & 1 & \dots & 4 \\ 3 & ? & ? & \dots & ? \\ ? & 5 & ? & \dots & 5 \end{bmatrix}$$

Low dimensional matrix factorization

- Model R as a product of two lower dimensional matrices

- A is $U \times K$ “user factor” matrix, $K \ll U, M$
- B is $M \times K$, “item factor” matrix
- Learning A and B allows us to reconstruct all of R

Low Dimensional Matrix Factorization

Interpretation: Rows of A and B are low dimensional feature vectors a_u and b_i for each user u and item i

Motivation: Dimensionality reduction

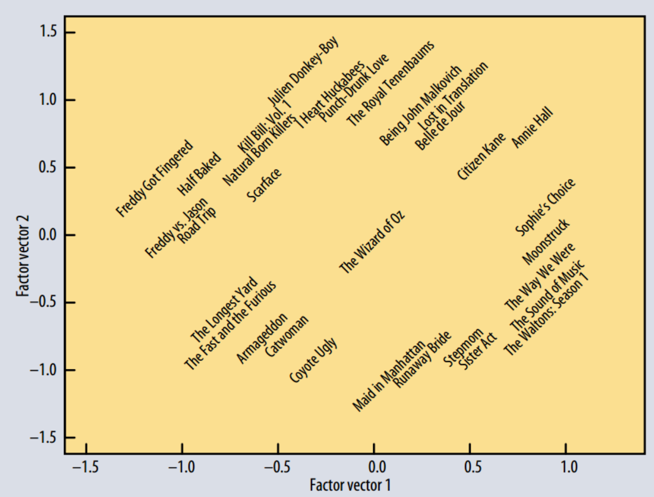
- Compact representation: only need to learn and store $UK + MK$ parameters
- Matrices can often be adequately represented by low rank factorizations

Low Dimensional Matrix Factorization

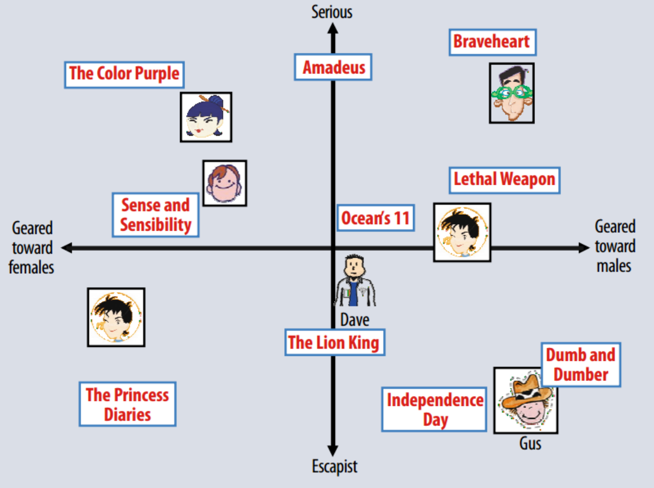
Very general framework that encapsulates many ML methods

- Singular value decomposition
- Clustering
  - A can represent cluster centers
  - B probabilities of belonging to each cluster
- Factor Analysis/Probabilistic PCA

IProjecting movies



Projecting users and movies



Singular Value Decomposition

Squared error objective for MF

$$\operatorname*{argmin}_{A,B} \|R - AB^T\|_2^2 = \operatorname*{argmin}_{A,B} \sum_{u=1}^{U} \sum_{i=1}^{M} (r_{ui} - \langle a_u, b_i \rangle)^2$$

- Reasonable objective since RMSE is our error metric

When all of R is observed, this problem is solved by singular value decomposition (SVD)

- **SVD:** $R = H \Sigma V^T$
  - H is $U \times U$ with $H^T H = I_{U \times U}$
  - V is $M \times M$ with $V^T V = I_{M \times M}$
  - Σ is $U \times M$ and diagonal
- **Solution:** Take first K pairs of singular vectors
  - Let $A = H_{U \times K} \Sigma_{K \times K}$ and $B = V_{M \times K}$

SVD with Missing Values

Weighted SE objective

$$\underset{A,B}{\operatorname{argmin}} \sum_{u=1}^{U} \sum_{i=1}^{M} W_{ui}(r_{ui} - \langle a_u, b_i \rangle)^2$$

Binary weights

- $W_{ui} = 1$ if r_{ui} observed, $W_{ui} = 0$ otherwise
- Only penalize errors on known ratings

How to optimize?

- Straightforward singular value decomposition no longer applies
- Local minima exist $\Rightarrow$ algorithm initialization is important

SVD with Missing Values

Insight: Chicken and egg problem

- If we knew the missing values in R, could apply SVD
- If we could apply SVD, we could find the missing values in R
- Idea: Fill in unknown entries with best guess; apply SVD; repeat

Expectation-Maximization (EM) algorithm

- Alternate until convergence:
  1. E step: $X = W * R + (1 - W) * \hat{R}$
     (* represents entrywise product)
  2. M step: $[H, \Sigma, V] = SVD(X)$, $\hat{R} = H_{U \times K} \Sigma_{K \times K} V^T_{M \times K}$

Complexity: $O(UM)$ space and $O(UMK)$ time per EM iteration

- What if UM or UMK is very large?
  - $UM = 8.5$ billion for Netflix Prize dataset
- Complete ratings matrix may not even fit into memory!

SVD with Missing Values

Regularized weighted SE objective

$$\underset{A,B}{\operatorname{argmin}} \sum_{u=1}^{U} \sum_{i=1}^{M} W_{ui}(r_{ui} - \langle a_u, b_i \rangle)^2 + \lambda(\sum_{u=1}^{U} \|a_u\|^2 + \sum_{i=1}^{M} \|b_i\|^2)$$

Equivalent form

$$\underset{A,B}{\operatorname{argmin}} \sum_{(u,i) \in \mathcal{T}} (r_{ui} - \langle a_u, b_i \rangle)^2 + \lambda(\sum_{u=1}^{U} \|a_u\|^2 + \sum_{i=1}^{M} \|b_i\|^2)$$

Motivation

- Counters *overfitting* by implicitly restricting optimization space
  - Shrinks entries of A and B toward 0
- Can improve *generalization error*, performance on unseen test data

Low Dimensional MF: Summary

Pros

- Data reduction: only need to store $UK + MK$ parameters at test time
  - $MK + M^2$ needed for Factor Analysis
- Gradient descent and ALS procedures are easy to implement and scale well to large datasets
- Empirically yields high accuracy in CF tasks
- Matrix factors could be used as inputs into other learning algorithms (e.g. classifiers)

Cons

- Missing data MF objectives plagued by many local minima
- Initialization is important
- EM approaches tend to be slow for large datasets

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