Model Predictive Control 03/11/2025 - Partial Exam

Name: ______________________________

Student ID: ______________________________

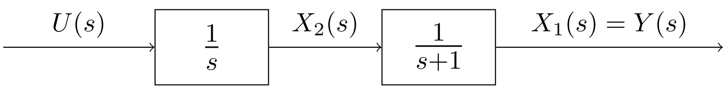
Master Degree: ______________________________

Exam instructions

- The exam consists of three parts: Problem 1, Problem 2, and MATLAB problem.
- In total, the exam includes 45 multiple-choice questions. Most of the questions have only one correct answer, while others may have more than one (this will be specified in the question).
- The duration of the exam is 1 hour and 30 minutes.
- The use of any technological device (including calculators) is not allowed.
- Collaboration or the use of notes/books is not allowed.
- It is recommended to first solve the problems thoroughly and then use the obtained solutions to answer the corresponding questions.

Problem n.1

Given the following system:



- Find the state space representation S(A,B,C,D)
- Find the state feedback gain vector K that allocates the poles in $-1 \pm 2i$
- Find the gain vector L that sets the observer's poles so that they are 3 times faster than the poles of the feedback system. Keep δ constant.

Problem n.2

Given the following system

$$\begin{cases} x_1(k+1) &= x_2(k) \\ x_2(k+1) &= x_3(k) \\ x_3(k+1) &= u(k) + x_3(k) \\ y(k) &= x_1(k) \end{cases}$$

and the following cost function:

$$J(k) = \sum_{i=H_w}^{H_P} \left(r(k+i) - y(k+i)\right)^2 + \sum_{i=0}^{H_u} u^2(k+i)$$

with $H_P = 3, H_u = 2, H_W = 1$

- Compute the control action $u(k)$ that minimizes the given cost function $J(k)$
- Check the system stability when the computed $u(k)$ is applied

After you have solved the problems, go to the answering section on next pages.

Problem n.1

1. The state-space representation S(A,B,C,D) is:

a) $A = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

b) $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$

c) $A = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

d) None of the above

2. What can be said about the open-loop stability of this system?

a) The system is asymptotically stable since all eigenvalues are negative.
b) The system is marginally stable because one eigenvalue is zero.
c) The system is unstable because A is not diagonal.
d) The system is unstable because the determinant of A is zero.
e) The system is stable only if B and C are chosen properly.
f) The system is exponentially unstable since $\mathrm{tr}(A) > 0$.

3. Indicate the Kalman controllability matrix $\mathcal{C}$?

a) $\mathcal{C} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ b) $\mathcal{C} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ c) $\mathcal{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ d) $\mathcal{C} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ e) $\mathcal{C} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ f) None of the above

4. Given the desired closed-loop poles at $-1 \pm 2i$, what is the desired characteristic polynomial of the system?

a) $\lambda^2 + 4\lambda + 5$ b) $\lambda^2 + 2\lambda + 4$ c) $\lambda^2 + \lambda + 5$ d) $\lambda^2 + 5\lambda + 4$ e) $\lambda^2 + 2\lambda + 5$ f) $\lambda^2 + 3\lambda + 9$

5. Compute the gain vector K to allocate poles in $-1 \pm 2i$

a) $K = [1 \ \ 4]$ b) $K = [2 \ \ 3]$ c) $K = [4 \ \ 1]$ d) $K = [3 \ \ 2]$ e) $K = [5 \ \ 0]$ f) None of the above

6. To obtain poles that are three times faster, with the same δ, one must multiply:

a) The real parts of the poles by $\frac{1}{3}$.
b) The imaginary parts of the pole by $\frac{1}{3}$
c) The real parts of the poles by 3.
d) The imaginary parts of the poles by 3.

7. Why do we typically choose observer poles that are faster than the controller poles?

a) So that the observer's state estimate converges quickly and the estimation errors die out rapidly, minimizing their effect on the closed-loop behavior.
b) So that the observer error dominates the closed-loop dynamics.
c) It's not necessary; observer poles are usually chosen equal to controller poles.
d) To ensure the separation principle holds (otherwise it fails).
e) To guarantee closed-loop stability (otherwise the system would be unstable).

8. If the closed-loop controller poles are located at $-1 \pm 2i$, and we want the observer poles to be three times faster, what should their values be?

a) $-2 \pm 4i$ b) $-1 \pm 2i$ c) $-6 \pm 3i$ d) $-3 \pm 2i$ e) $-3 \pm 6i$ f) $-9 \pm 18i$

9. What is the observer gain vector L such that $A - LC$ has the desired eigenvalues?

a) $L = \begin{bmatrix} 45 \\ 5 \end{bmatrix}$ b) $L = \begin{bmatrix} 6 \\ 45 \end{bmatrix}$ c) $L = \begin{bmatrix} 5 \\ 45 \end{bmatrix}$ d) $L = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$ e) $L = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$ f) None of the above

10. The observer estimation error converges if:

a) The eigenvalues of $A - LC$ lie in the right half-plane.
b) The eigenvalues of $A - LC$ have negative real parts.
c) The eigenvalues are purely imaginary.
d) The eigenvalues are positive.
e) $L = 0$.

11. The separation principle states that:

a) The observer design always influences the controller design.
b) The design of the observer and the controller can be carried out independently.
c) The gain L is related to K.

Problem n.2

12. What is the state-space matrix A?

a) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

13. Indicate the correct $y(k+3)$?

a) $y(k+3) = u(k+1) + u(k) + x_3(k)$
b) $y(k+3) = 2u(k+1) + u(k) + x_3(k)$
c) $y(k+3) = x_3(k+3)$
d) $y(k+3) = u(k) + x_3(k)$
e) $y(k+3) = x_2(k+2)$
f) $y(k+3) = u(k+2) + u(k+1) + u(k) + x_3(k)$

14. Indicate the correct $J(k)$?

a) $J = [r(k) - x_1(k)]^2 + [r(k+1) - x_2(k)]^2 + [r(k+2) - x_3(k)]^2 + u^2(k) + u^2(k+1)$
b) $J = \sum_{i=1}^{3} [r(k+i) - y(k+i)] + u^2(k+i)$
c) $J = [r(k+1) - x_2(k)]^2 + [r(k+2) - x_3(k)]^2 + [r(k+3) - u(k) - x_3(k)]^2 + u^2(k) + 2u^2(k+1)$
d) $J = r(k+3) - y(k+3) + u(k)^2 + u(k+1)^2$

15. What condition is imposed to compute $u(k)$?

a) $\frac{\partial J(k)}{\partial u(k)} = 1$
b) $\frac{\partial J(k)}{\partial u(k)} = 0$
c) $J(k) = 0$
d) $u(k+1) = 0$
e) $\frac{\partial J(k)}{\partial u(k+1)} = 0$
f) $J(k+1) = J(k)$

16. The optimal $u(k)$ is:

a) $u(k) = x_3(k) + r(k+3)$
b) $u(k) = \frac{r(k+3) - x_3(k)}{2}$
c) $u(k) = -\frac{x_3(k)}{2} + \frac{r(k+3)}{2}$
d) $u(k) = \frac{x_3(k)}{2} - \frac{r(k+3)}{2}$
e) $u(k) = -x_3(k) + r(k+3)$
f) $u(k) = \frac{r(k+1) - x_3(k)}{2}$

17. The optimal $u(k+1)$ is:

a) $u(k+1) = u(k)$
b) $u(k+1) = 0$
c) $u(k+1) = -x_3(k)$
d) $u(k+1) = \frac{r(k+3)}{2}$
e) $u(k+1) = \frac{u(k)}{2}$
f) $u(k+1) = r(k+3) - x_3(k)$

18. To check closed-loop stability, the reference signal is set equal to:

a) $r(k) = 1(k)$
b) $r(k) = u(k)$
c) $r(k) = 0$
d) $r(k) = x_3(k)$
e) $r(k) = -x_3(k)$
f) $r(k) = u(k+1)$

19. Given $u(k) = -\frac{x_3(k)}{2}$, what is the feedback gain vector K?

a) $[0, 0, 2]$
b) $[0, 0, \frac{1}{2}]$
c) $[\frac{1}{2}, 0, 0]$
d) $[0, 1, \frac{1}{2}]$
e) $[0, 0, -\frac{1}{2}]$
f) $[1, 1, 1]$

20. The closed-loop state matrix $A - BK$ is:

a) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1/2 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1/2 \end{bmatrix}$ e) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$ f) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1/2 \end{bmatrix}$

21. What are the eigenvalues of $A - BK$?

a) $\lambda = 1, 1, 1$ b) $\lambda = 0, \frac{1}{2}, 1$ c) $\lambda = 0, 0, \frac{1}{2}$ d) $\lambda = 0, 0, 1$ e) $\lambda = -1, -1, -\frac{1}{2}$ f) $\lambda = \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

22. What can be said about the closed-loop stability of the system?

a) The system is marginally stable.
b) The system is unstable.
c) The system is asymptotically stable.
d) The system diverges exponentially.

23. What is the size of K in $u(k) = -Kx(k)$,?

a) 1×1
b) 3×1
c) 1×3
d) 3×3
e) 2×3
f) 3×2

24. In the cost function, which term penalizes the control effort?

a) $[r(k+i) - y(k+i)]^2$
b) $u^2(k+i)$
c) $x^2(k)$
d) $[r(k) - u(k)]^2$
e) $y^2(k)$
f) $[r(k+i) - x_3(k)]^2$

25. If H_u=1 which term would disappear from the cost function?

a) $u^2(k+1)$
b) $u^2(k)$
c) $[r(k+2) - x_3(k)]^2$
d) $[r(k+1) - x_2(k)]^2$
e) $[r(k+3) - u(k) - x_3(k)]^2$
f) None, since the cost function is unaffected by H_u.

26. Assume that a weighting coefficient λ is introduced in the cost function $J(k)$ such that:

$$J(k) = \sum_{i=H_w}^{H_p} [r(k+i) - y(k+i)]^2 + \lambda \sum_{i=0}^{H_u} u^2(k+i)$$

How does increasing $\lambda > 1$ affect the controller behavior?

a) It reduces control action and slows the dynamics.
b) It increases control aggressiveness and overshoot.
c) It changes only steady-state error.
d) It eliminates all control effort.
e) It leaves the transient dynamics unchanged.

27. With the given $J(k)$, the condition $\frac{\partial J(k)}{\partial u(k)} = 0$ leads to:

a) A necessary condition for a global minimum
b) A necessary condition for a local maximum
c) A saddle point
d) A necessary and sufficient condition for a local minimum
e) A necessary and sufficient condition for a global minimum

28. If the system is controlled by $u(k) = -Kx(k)$ and one eigenvalue of $A - BK$ equals 1, what can be said about stability?

a) The system is asymptotically stable.
b) The system is marginally stable.
c) The system diverges exponentially.
d) The system converges to a fixed point.
e) Stability cannot be determined.

29. Which statement best describes the role of H_p relative to H_u?

a) $H_p > H_u$ predicts outputs beyond the time when control action becomes constant.
b) H_p affects only steady-state tracking, not transients.
c) H_u defines how far ahead predictions are made.
d) H_p determines the control signal magnitude directly.

MPC Basics in Matlab

30. The command `ss(A,B,C,D)` defines:

a) A state-space model
b) A transfer function
c) A simulation setup
d) A nonlinear plant
e) An MPC object
f) A static gain model

31. The function `mpcmove()`:

a) Defines the plant model
b) Starts the simulation
c) Sets controller weights
d) Builds the prediction matrix
e) Computes the optimal control action and updates the state
f) Clears controller memory

32. Which of the following correctly represents the syntax of the `mpcmove` command?

a) `u = mpcmove(x, y, r, mpcobj);`
b) `mpcmove(u, x, y, r, mpcobj);`
c) `u = mpcmove(mpcobj, r, y, x);`
d) `u = mpcmove(mpcobj, x, y, r);`
e) `[x, y, u] = mpcmove(mpcobj);`
f) `u = mpcmove(mpcobj, r);`

33. You defined a transfer function using `s = tf('s'); G = 1/s; plantC = tf(G);` What is the issue in this approach?
a) The last command is redundant; `G` is already a transfer function.
b) The variable `s` must be declared using `syms s`.
c) The system is unstable because of the integrator.
d) The command `tf(G)` converts it to state-space automatically.
e) MATLAB will interpret `1/s` as matrix division.
f) There is no issue; all commands are strictly required.

34. When creating an MPC controller with `mpc(plant,Ts)`, which of the following errors is most likely if the plant is TC?
a) The command fails unless `Ts=0`.
b) MATLAB will automatically discretize it using default zero-order hold assumptions.
c) The controller is continuous-time by default.
d) The system becomes uncontrollable.
e) The plant is ignored and a default double integrator is used.
f) It causes a simulation warning only when running `sim`.

35. In the extended command `mpc(plant,Ts,Hp,Hc,W,MV,OV,DV)`, which of the following is *not* true?
a) `Hp` represents the prediction horizon.
b) `Hc` represents the control horizon.
c) The vector `W` defines tuning weights.
d) `MV`, `OV`, and `DV` correspond to structure arrays for variable properties.
e) The command can be called without some of these arguments.
f) `Hp` must always be greater than `Hc` by exactly one.

36. Which of the following statements about the properties of the manipulated variables is incorrect?
a) They can include fields such as `Min`, `Max`, and `RateMax`.
b) They can specify scaling factors to normalize variables.
c) They can be indexed if multiple inputs exist.
d) They define output constraints when set in `mpcobj.ManipulatedVariables`.
e) They can be modified after controller creation.
f) They affect how `mpcmove` enforces limits.

37. You wish to limit the maximum variation of the control input between samples. Which field must be set?

a) `mpcobj.ManipulatedVariables.Max`
b) `mpcobj.ManipulatedVariables.RateMax`
c) `mpcobj.OutputVariables.Max`
d) `mpcobj.Weights.ManipulatedVariablesRate`
e) `mpcobj.Weights.ManipulatedVariables`
f) `mpcobj.Rate.Max`

38. What is the most likely consequence of forgetting to call `xk = mpcstate(mpcobj);` before a simulation loop?
a) The first call to `mpcmove` will initialize states automatically with zeros.
b) MATLAB throws an error because `xk` must be a valid `mpcstate` object.

c) The system starts with a random initial condition.
d) The simulation will run but ignore constraints.
e) The state will be inferred from the plant model history.
f) No issue occurs because `mpcmove` creates the state internally.

39. Consider the command: `mv = mpcmove(mpcobj, xk, ym, ref, v);`. What does the argument `v` represent?

a) A vector of future reference trajectories.
b) Process noise.
c) The control signal from the previous step.
d) The system state at $k-1$.
e) Measured or estimated unmeasured disturbances.
f) An auxiliary tuning gain.

40. A user executes:

```
xk = mpcstate(mpcobj);
for i = 1:N
    u(i) = mpcmove(mpcobj,xk,y(i),r);
end
```

The output response is unexpectedly slow. Which mistake could explain this?

a) The measured output `y(i)` should be the plant output, not a state vector.
b) The reference `r` must be zero-initialized each iteration.
c) `mpcstate` must be redefined inside the loop.
d) The `mpcmove` command must include the disturbance argument.
e) `xk` is being overwritten incorrectly.
f) The control horizon `Hc` is too large.

41. Which of the following statements about `mpcobj.Weights` is true? (more than one is correct)

a) Increasing `mpcobj.Weights.ManipulatedVariablesRate` always speeds up control response.
b) All weights must be positive integers.
c) Setting `mpcobj.Weights.OutputVariables = 0` removes tracking from the cost function.
d) `mpcobj.Weights.ManipulatedVariables` penalizes the rate of change of u.
e) The `ECR` field defines the equilibrium control rate.
f) Weight tuning does not affect control performance.

42. Which statement best describes the relationship between `Hp` and `Hc`?

a) H_u must always equal $H_p - 1$.
b) $H_u > H_p$ yields faster transient response.
c) H_p must be greater than or equal to H_u for feasibility.
d) They are independent and can be chosen arbitrarily.
e) H_p affects only constraints, not predictions.
f) Increasing H_u reduces prediction accuracy.

43. What does the command `sim(mpcobj, N, ref, v)` do?

a) Runs an open-loop simulation of the plant.
b) Initializes the `mpcstate` object for N iterations.
c) Computes only steady-state trajectories.
d) Updates `mpcobj.Weights` adaptively during runtime.
e) Runs an MPC simulation for N steps using reference `ref` and disturbance `v`.
f) Produces the same result as `mpcmove`.

44. Why might the response of an MPC controller remain constant even when the reference changes?

a) The prediction horizon is too small.
b) The observer poles are complex conjugates.
c) The system is unstable in open loop.
d) The sampling time is too large.
e) The weights are all zero.
f) Constraints on u or Δu are active and prevent control movement.

45. Which of the following is the correct logical order in the MPC design flow?

a) Modeling → Initialization → Constraints definition → Tuning → Simulation → Performance evaluation
b) Initialization → Modeling → Simulation → Tuning
c) Tuning → Modeling → Simulation → Constraints
d) Simulation → Performance → Initialization
e) Constraints → Modeling → Tuning
f) Performance evaluation → Simulation → Constraints