Problem n.1 (15 points)

Given the discrete-time system S(A,B,C,D) described below:

$$\begin{cases} x_1(k+1) = -x_1(k) + x_2(k) \\ x_2(k+1) = x_1(k) + u(k) \end{cases} \qquad y(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Where:

$$r(k) = \begin{bmatrix} 1(k) \\ 1(k) \end{bmatrix} \qquad 1(k) = 1 \quad \forall k \geq 0$$

And the cost function is:

$$V(k) = \sum_{i=H_W}^{H_P} [r(k+i) - x(k+i)]^T \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} [r(k+i) - x(k+i)]$$

$$H_P = 2 \qquad H_w = 1 \qquad H_c = 1$$

Questions:

A. Compute the cost function $V(k)$.
B. Compute $u(k)$
C. Compute $u(k+1)$
D. Considering the computed u(k), what can be said about the stability of the closed-loop system?
E. By considering the cost function defined below, compute $V_1(k), u(k), u(k+1)$

$$V_1(k) = \sum_{i=H_W}^{H_P} [r(k+i) - x(k+i)]^T \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} [r(k+i) - x(k+i)] + \sum_{i=0}^{H_C-1} 2[u(k+i)]^2$$

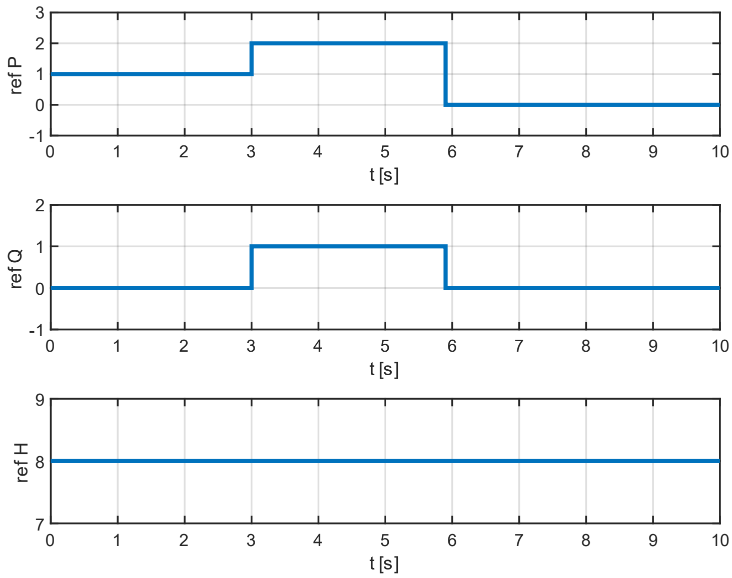
Table n.1 (see Problem n.2)

Name	Description	Value [unit]
P	Pressure in the tank	[Pa]
Q	Flow rate into the tank	[m^3 /s]
H	Water level in the tank	[m]
u_P	Pressure at the inflow to the tank	[m^5/s^2]
u_Q	Flow rate at the inflow to the tank	[1/s]
α	Model parameter	0.1 [1/h]
β	Model parameter	0.5 [Pa/(hr*m^3]
γ	Model parameter	0.2 [1/hr]
x_0	Reference	[0; 0; 5]

Problem n.2

(15 points)

Consider a model of a water distribution system, where the objective is to control the pressure (P), flow rate (Q), and water level (H) in a tank. The state of the system can be described by three variables: the pressure in the tank (P), the flow rate into the tank (Q), and the water level in the tank (H).



The equations of motion for this system can be derived using basic principles of fluid mechanics and are as follows:

$$\dot{\boldsymbol{x}} = [\dot{P} \quad \dot{Q} \quad \dot{H}]^T$$

$$\boldsymbol{y} = \boldsymbol{x} \qquad \boldsymbol{u} = [u_P \quad u_Q]^{\boldsymbol{T}}$$

$$\begin{cases} \dot{P} = \alpha(u_P - P) + \beta Q \\ \dot{Q} = \gamma(u_Q - Q) \\ \dot{H} = \beta Q^2 \end{cases}$$

Table n.1 lists the system parameters.

Questions:

A. Calculate and specify an analytical Jacobian for the output function and the state function.
B. Consider $T_S = 0.1s, H_P = 10, H_C = 2$. Initialize the MPC controller with the proper parameters. Validate the MPC Object. Run a simulation lasting $10s$ and comment the results. Consider the static reference = [1; 0; 7] and x0 as described in table 1.
C. The pressure at the inflow to the tank is bounded between [-1000:1000]. Constrains the controller accordingly. Run a simulation lasting $10s$ and comment the results. Did the constraints come into effect?
D. Consider the reference trajectory in the figure above. We want to reduce the transient, reduce overshoots. Tune the controller to meet the requirements. Explain all the design choices and comment on the results obtained.
E. Analyze the computational time required for each move update. Can you control the system in real time?
F. Plot the evolution of the cost over time and comment it.

IMPORTANT: To get a full score it is necessary to comment on the design choices.