Problem n.1 (15 points)

Given the discrete-time system S(A,B,C,D) described below:

$$\begin{cases} x_1(k+1) = -x_1(k) + x_2(k) \\ x_2(k+1) = x_1(k) + u(k) \end{cases} \qquad y(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Where:

$$r(k) = \begin{bmatrix} 1(k) \\ 1(k) \end{bmatrix} \qquad 1(k) = 1 \quad \forall k \geq 0$$

Cost function:

$$V(k) = \sum_{i=H_W}^{H_P} [r(k+i) - x(k+i)]^T \begin{bmatrix} 4 & 0 \\ 0 & -3 \end{bmatrix} [r(k+i) - y(k+i)]$$

$$H_P = 2 \qquad H_w = 1 \qquad H_c = 2$$

Questions:

A. Compute the cost function V(k)
B. Compute u(k)
C. Compute u(k+1)
D. Considering the computed u(k), what can be said about the stability of the closed-loop system?
E. By considering the cost function defined below, compute $V_1(k), u(k), u(k+1)$

$$V_1(k) = V(k) + \sum_{i=1}^{H_C-1} 2[u(k+i)]^2$$

Table n.1 (see Problem n.2)		
Name	**Description**	**Value [unit]**
x_1	Car position on the x axis (x)	[m]
x_2	Car position on the y axis (y)	[m]
x_3	Orientation of the car with respect to the x axis. (θ)	[rad]
x_4	The angle of the front wheel with respect to the longitudinal symmetry axis of the car.(φ)	[rad]
u_1	Longitudinal car speed (v)	[m/s]
u_2	The angle of the front wheel with respect to the longitudinal symmetry axis of the car.(φ)	[m/s]
y_1	Car position on the x axis (x)	[m]
y_2	Car position on the y axis (y)	[m]

# Problem n.2   (15 points)

The follow image depicts the kinematic car in the horizontal plane. Let us suppos that the Ackermann steering assumptions hold true, hence all wheels turn arount the same point (denoted by P) which lies on the line of the rear axle. It follows that the kinematics of the car can be fully described by the kinematics of a bicycle fitted in the middle of the car.

All lengths involved in the kinematic calculations, and in particular l, equal to one.

$$\begin{cases} x_1(k+1) = x_1(k) + u_1(k) * \cos(x_3(k)) \\ x_2(k+1) = x_2(k) + u_1(k) * \sin(x_3(k)) \\ x_3(k+1) = x_3(k) + u_1(k) * \tan(x_4(k)) \\ x_4(k+1) = u_2(k) \end{cases} \qquad y(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$
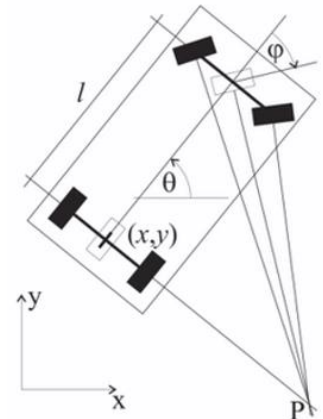
The vehicle parameters are written in Table n.1 in page 1.

Questions:

A. Consider x0=[0 0 0 0]', yref=[1 1]'. The settling time required is 1s. Calculate Ts,Hp,Hc.
B. The vehicle considered is a golf cart. It can reach maximum 3.6km / h. In reverse it can reach a maximum of 1.8km / h. The steering can orient the wheels in a range of + -60 °. Forward from 0 to maximum speed takes 5 seconds. In reverse the vehicle has the same acceleration. Deceleration has the same magnitude.
   1. Initialize the MPC controller with the found parameters.
   2. Constrain the system accordingly.
   3. Run a simulation lasting 5s.
   4. Observe and comment the simulation results.
C. Specify an analytical Jacobian for the output function and the state function.
D. Run a simulation considering x0=[0 0 0 0]', yref=[5 1]' and Set the RateMin of u1 = -0.025.
   As you can see, the car does not reach its goal smoothly.
   What parameters can be tuned to improve the simulation?
E. Tune the identified parameters and perform a simulation in Matlab.

SUGGESTION: Analyze the data using 4 figures:

-   fig. 1 with the input subplots
-   fig. 2 with state subplots
-   fig. 3 with the subplots of the outputs
-   fig. 4 representation on the xy plane of the path traveled by the car.

Add titles to charts and anything else that makes them easier to understand.

IMPORTANT: remember that your system is not continuous but it is discrete!

IMPORTANT: To get a full score it is necessary to comment on the design choices.