Intersection of Two Lines

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1 INTERSECTION OF TWO LINES

Consider the following two lines given in point-slope form:

$$y - y_1 = m_1 (x - x_1)$$

$$y - y_2 = m_2 (x - x_2)$$

If these two lines intersect at $(x_{\text{int}}, y_{\text{int}})$, then

$$y_{\text{int}} - y_1 = m_1 (x_{\text{int}} - x_1) \tag{1}$$

$$y_{\text{int}} - y_2 = m_2 (x_{\text{int}} - x_2) \tag{2}$$

Solving Eqs. (1) and (2) for y_{int},

$$y_{\text{int}} = y_1 + m_1 (x_{\text{int}} - x_1) \tag{3}$$

$$y_{\text{int}} = y_2 + m_2 (x_{\text{int}} - x_2) \tag{4}$$

Equating Eqs. (3) and (4),

$$y_1 + m_1 (x_{\text{int}} - x_1) = y_2 + m_2 (x_{\text{int}} - x_2)$$

Solving for x_{int},

$$\boxed{x_{\text{int}} = \frac{(m_1 x_1 - m_2 x_2) - (y_1 - y_2)}{m_1 - m_2}} \tag{5}$$

To obtain y_{int}, we can use either line. We choose to use line 1.

$$\boxed{y_{\text{int}} = y_1 + m_1 (x_{\text{int}} - x_1)} \tag{6}$$