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Math 71: Algebra	Fall 2022

PSET 1 — 2022-11-14

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Credit Statement

I worked on these problems alone, with reference to class notes and the following books:

(a) ***Abstract Algebra*** by **David S. Dummit & Richard M. Foote**.

(b) ***Algebra*** by **Jacob K. Goldhaber & Gertrude Ehrlich**

Problems

1. What is the message embedded in the following?

> Dear George,
> Greetings to all at Oxford. Many thanks for your
> letter and for the Summer examination package.
> All Entry Forms and Fees Forms should be ready
> for final despatch to the syndicate by Friday
> 20th or at the very latest, I'm told, by the 21st.
> Admin has improved here, though there's room
> for improvement still; just give us all two or three
> more years and we'll really show you! Please
> don't let these wretched 16+ proposals destroy
> your basic O and A pattern. Certainly this
> sort of change, if implemented immediately,
> would bring chaos.

I thought it was unlikely that the text was actually encrypted (a strategy that did not work that well on problem 2). On this problem, I tried to look at the words in different positions in the text — every first word in a sentence, every last word, every nth word, in sentence n, etc.

Using this strategy, I deciphered this message (highlighted in red above):

`Your package ready Friday 21st, room three. Please destroy this immediately.`

2. Encrypt the message 001100001010 using two rounds of SDES and (9 bit) key 111000101, as explained in lecture. Show all your steps! *[Hint: After one round, the output is* 001010010011.*]*

First, let's define our permutation function:

$\textbf{permute}\ (123456) = 12434356$

And tables:

s_1	1	2	s_2	1	2
000	101	001		100	101
001	010	100		000	011
010	001	110		110	000
011	110	010		101	111
100	011	000		111	110
101	100	111		001	010
110	111	101		011	001
111	000	011		010	100

Round 1

$$L_0 = 001100$$

$$R_0 = 001010$$

$$K_0 = 11100010$$

$$\textbf{permute}\ (R_0) = 00010110$$

$$00010110\ \textbf{xor}\ K_0 = 11110100$$

$$S_1(1111) = 011$$

$$S_2(0100) = 111$$

$$011111\ \textbf{xor}\ L_0 = 010011$$

$$L_1 \leftarrow R_0$$

$$R_1 \leftarrow 010011$$

Excryption after 1 round: 001010010011

Round 2

$$L_1 = 001010$$

$$R_1 = 010011$$

$$K_1 = 11000101$$

$$\textbf{permute}\ (R_1) = 01000011$$

$$01000011\ \textbf{xor}\ K_1 = 10000110$$

$$S_1(1000) = 001$$

$$S_2(0110) = 011$$

$$001011\ \textbf{xor}\ L_1 = 000001$$

$$L_2 \leftarrow R_1$$

$$R_2 \leftarrow 000001$$

Excryption after 2 rounds: 010011000001

3. In the Rijndael field $F = \mathbb{F}_2[X]/(X^8+X^4+X^3+X+1)$, where bytes are associated to polynomials modulo $X^8+X^4+X^3+X+1$, compute the product $01010010 \cdot 10010010 \in F$.

We can represent polynomials in F as binary numbers, where the state of each bit (whether 0 or 1) represents whether the corresponding power in the polynomial has a factor of 0 or 1.

Then:

$$X^8+X^4+X^3+X+1 = 100011011$$

Then, we can perform the multiplication modulo 2:

$$\begin{array}{r}
1\;0\;0\;1\;0\;0\;1\;0 \\
1\;0\;1\;0\;0\;1\;0 \\
\cancel{1}\;0\;0\;1\;0\;0\;1\;0\;\cdot \\
\hline
1\;0\;0\;1\;0\;0\;1\;0\;\cdot\;\cdot\;\cdot\;\cdot \\
1\;0\;0\;1\;0\;0\;1\;0\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot \\
\hline
1\;0\;1\;1\;0\;2\;1\;0\;2\;0\;0\;1\;0\;0
\end{array}$$

Shifting back to base 2, we get: 10110010000100

We then need to find this number $\mod 100011011$

$$\mod 10110010000100, 100011011$$

$$\begin{array}{r|r}
100011011 & 10110010000100 \\
100000 & 100011011\;.\;.\;.\;.\;. \\
 & 111111100\;.\;.\;. \\
1000 & 100011011\;.\;.\;. \\
 & 111001111\;.\;. \\
100 & 100011011\;.\;. \\
 & 110101000\;. \\
10 & 100011011\;. \\
 & 101100110 \\
1 & 100011011 \\
\hline
101111 & 1111101
\end{array}$$

Thus, the product in F is 1111101