

PSET 1 — 2022-11-14

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Credit Statement

I worked on these problems alone, with reference to class notes and the following books:

- (a) *Abstract Algebra* by David S. Dummit & Richard M. Foote.
- (b) *Algebra* by Jacob K. Goldhaber & Gertrude Ehrlich

Problems

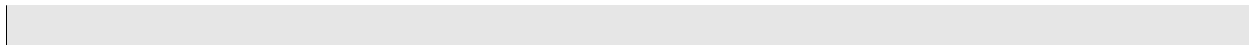
1. What is the message embedded in the following?

Dear George,
Greetings to all at Oxford. Many thanks for **your**
letter and for the Summer examination **package**.
All Entry Forms and Fees Forms should be **ready**
for final despatch to the syndicate by **Friday**
20th or at the very latest, I'm told, by the **21st**.
Admin has improved here, though there's **room**
for improvement still; just give us all two or **three**
more years and we'll really show you! **Please**
don't let these wretched 16+ proposals **destroy**
your basic O and A pattern. Certainly **this**
sort of change, if implemented **immediately**,
would bring chaos.

I thought it was unlikely that the text was actually encrypted (a strategy that did not work that well on problem 2). On this problem, I tried to look at the words in different positions in the text — every first word in a sentence, every last word, every n th word, in sentence n , etc.

Using this strategy, I deciphered this message (highlighted in red above):

Your package ready Friday 21st, room three. Please destroy this immediately.



2. Encrypt the message 001100001010 using two rounds of SDES and (9 bit) key 111000101, as explained in lecture. Show all your steps! *[Hint: After one round, the output is 001010010011.]*

First, let's define our permutation function:

permute (123456) = 12434356

And tables:

s_1	1	2	s_2	1	2
000	101	001		100	101
001	010	100		000	011
010	001	110		110	000
011	110	010		101	111
100	011	000		111	110
101	100	111		001	010
110	111	101		011	001
111	000	011		010	100

ROUND 1	ROUND 2
$L_0 = 001100$	$L_1 = 001010$
$R_0 = 001010$	$R_1 = 010011$
$K_0 = 11100010$	$K_1 = 11000101$
permute (R_0) = 00010110	permute (R_1) = 01000011
00010110 xor K_0 = 11110100	01000011 xor K_1 = 10000110
$S_1(1111) = 011$	$S_1(1000) = 001$
$S_2(0100) = 111$	$S_2(0110) = 011$
011111 xor L_0 = 010011	001011 xor L_1 = 000001
$L_1 \leftarrow R_0$	$L_2 \leftarrow R_1$
$R_1 \leftarrow 010011$	$R_2 \leftarrow 000001$
Exryption after 1 round: 001010010011	Exryption after 2 rounds: 010011000001

3. In the Rijndael field $F = \mathbb{F}_2[X]/(X^8 + X^4 + X^3 + X + 1)$, where bytes are associated to polynomials modulo $X^8 + X^4 + X^3 + X + 1$, compute the product $01010010 \cdot 10010010 \in F$.

We can represent polynomials in F as binary numbers, where the state of each bit (whether 0 or 1) represents whether the corresponding power in the polynomial has a factor of 0 or 1.

Then:

$$X^8 + X^4 + X^3 + X + 1 = 100011011$$

Then, we can perform the multiplication modulo 2:

$$\begin{array}{r} 10010010 \\ 1010010 \\ \times 0010010 \\ \hline 10010010 \dots \\ 10010010 \dots \\ \hline 10110210200100 \end{array}$$

Shifting back to base 2, we get: 10110010000100

We then need to find this number mod 100011011

$$\begin{array}{r|l} \text{mod } 10110010000100, 100011011 & \\ 100011011 & 10110010000100 \\ 100000 & 100011011 \dots \\ & 111111100 \dots \\ 1000 & 100011011 \dots \\ & 111001111 \dots \\ 100 & 100011011 \dots \\ & 110101000 \dots \\ 10 & 100011011 \dots \\ & 101100110 \\ 1 & 100011011 \\ \hline 101111 & 1111101 \end{array}$$

Thus, the product in F is 1111101