Math 63: Real Analysis **Winter 2024**

PSET 3 — 01/24/2024

Prof. Erchenko ***Student: Amittai Siavava***

Credit Statement

I worked on these problems alone, with reference to class notes and the following books:

(a) ***Introduction to Analysis*** by Maxwell Rosenlicht

Problem 1.

Let $a_i, b_i \in \mathbb{R}$ for $i = 1, 2, \ldots, n$. Show that $(a_1, b_1) \times (a_2, b_2) \times \cdots \times (a_n, b_n)$ is open in $(\mathbb{R}^n, d_E)$ and $[a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_n, b_n]$ is closed in $(\mathbb{R}^n, d_E)$.

Problem 2.

Prove that any bounded open subset of $\mathbb{R}$ is the union of disjoint open intervals.

Problem 3.

Prove that if the points of a convergent sequence of points in a metric space are reordered, the new sequence converges to the same limit.

Let $\{p_i\}_1^\infty$ be a convergent sequence in a metric space (X, d).

Problem 4.

Prove that if $\lim_{n\to\infty} p_n = p$ in a given metrix space then the set of points $\{p, p_1, p_2, \dots\}$ is closed.

Problem 5.

Let $a_n = \frac{n}{n+1}$ for $n \in \mathbb{N}$. Show, using the definition of a limit, that $\lim_{n \to \infty} a_n = 1$.

Problem 6.

Consider the sequence $\{a_n\}_{n=1}^{\infty}$ such that $a_1 \geq a_2 \geq a_3 \geq \dots$ (i.e., it is a monotonically decreasing sequence). Assume that there exists $m > 0$ such that $a_n > m$ for all n. Show that $\{a_n\}_{n=1}^{\infty}$ converges in $\mathbb{R}$.