Math 63: Real Analysis **Winter 2024**

PSET 2 — 01/14/2024

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Credit Statement

I worked on these problems alone, with reference to class notes and the following books:

(a) ***The Code Book*** by **Simon Singh**.

(b) ***Cryptography*** by **Simon Rubinsen-Salzedo**

Problem 1.

Let A be a finite set. Show that A cannot be an ordered field.

Suppose A is an ordered field, with 0 as the additive identity element, and 1 as the multiplicative identity element such that $0 < 1$.

First, note that for any $a \in A$, $a + 1 \in A$, and $a \leq a + 1$ since $0 < 1$.

Let $a \in A$ be arbitrary. Since A is an ordered field, $a + 1 \in A$. Since A is an ordered field, $a + 1 > a$.

Problem 2.

Let $F = \{0, 1, 2\}$. Prove that there is exactly one way to define addition and multiplication so that F is a field if 0 is the additive identity and 1 is the multiplicative identity.

Problem 3.

If S_1 and S_2 are nonempty subsets of $\mathbb{R}$ that are bounded above, prove that

$$\sup\{x + y \mid x \in S_1, y \in S_2\} = \sup S1 + \sup S_2.$$

Problem 4.

Let $S := \{a_k \mid k \in \mathbb{N}\} \cup \{b_k \mid k \in \mathbb{N}\}$, ordered such that $a_k < b_j$ for all k and j, $a_k < a_m$ whenever $k < m$, and $b_k < b_m$ whenever $k < m$.

(a) Show that S is an ordered set.

(b) Show that every subset of S is bounded above and below.

(c) Find a bounded subset of S that has no least upper bound.

Problem 5.

Let $n \in \mathbb{N}$. Sshow that $(\mathbb{R}^n, d_1)$ is a metric space where

$$d_1(p, q) := \sum_{1}^{n} |p_i - q_i|$$

for all $p = (p_1, \ldots, p_n)$ and $q = (q_1, \ldots, q_n)$ in $\mathbb{R}^n$.

Problem 6.

Show that the subset of (R^2, d_E) given by

$$S := \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 > x_2\}$$

is open.

Problem 7.

Let (X, d) be a metric space. Let $A \subset X$. Show that A is open if and only if it is equal to the union of a collection of open balls.