**Math 63: Real Analysis** **Winter 2024**

## PSET 1 — 01/06/2024

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### Problems

#### Problem 1.

Let $A, B, C$ be subsets of a set $S$. Prove that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

To show that the two sets are equal, we will show that each set is a subset of the other. That is, any arbitrary element in the first set must be in the second set, and vice versa.

(i) Let $x \in A \cup (B \cap C)$. Then either $x \in A$ or $x \in (B \cap C)$.

  **1.** If $x \in A$, then $x \in A \cup B$ and $x \in A \cup C$. Therefore, $x \in (A \cup B) \cap (A \cup C)$.

  **2.** If $x \in (B \cap C)$, then $x$ is in **both** $B$ and $C$.
  Therefore, $x \in (A \cup B)$ and $x \in (A \cup C)$.
  Therefore, $x \in (A \cup B) \cap (A \cup C)$.

(ii) Let $x \in (A \cup B) \cap (A \cup C)$.
Then $x \in (A \cup B)$ and $x \in (A \cup C)$. Since $x \in (A \cup B)$, either $x \in A$ or $x \in B$. Similarly, since $x \in (A \cup C)$, either $x \in A$ or $x \in C$. Thus, either $x \in A$ or $x$ is in **both** $B$ and $C$.

  **1.** If $x \in A$, then $x \in A \cup (B \cap C)$.

  **2.** If $x$ is in **both** $B$ and $C$, then $x \in (B \cap C)$, so $x \in A \cup (B \cap C)$.

**Problem 2.**

If $A, B, C$ are sets, show that

$$(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$$

To show that the two sets are equal, we will show that each set is a subset of the other. That is, any arbitrary element in the first set must be in the second set, and vice versa.

(i) Let $x \in (A \cup B) \setminus (A \cap B)$. Then $x \in (A \cup B)$ and $x \notin (A \cap B)$.

   **1.** If $x \in A$, then $x \in (A \setminus B)$.

   **2.** If $x \in B$, then $x \in (B \setminus A)$.

   Therefore, $x \in (A \setminus B) \cup (B \setminus A)$.

(ii) Let $x \in (A \setminus B) \cup (B \setminus A)$. Then either $x \in (A \setminus B)$ or $x \in (B \setminus A)$.

   **1.** If $x \in (A \setminus B)$, then $x \in A$ and $x \notin B$. Therefore, $x \in (A \cup B)$ and $x \notin (A \cap B)$.

   **2.** If $x \in (B \setminus A)$, then $x \in B$ and $x \notin A$. Therefore, $x \in (A \cup B)$ and $x \notin (A \cap B)$.

   Therefore, $x \in (A \cup B) \setminus (A \cap B)$.