

**PSET 2 — 01/13/2024**

*Prof. Erchenko*

*Student: Amittai Siavava*

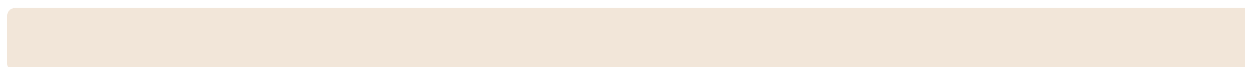
**Credit Statement**

I worked on these problems alone, with reference to class notes and the following books:

- (a) *The Code Book* by **Simon Singh**.
- (b) *Cryptography* by **Simon Rubinsen-Salzedo**

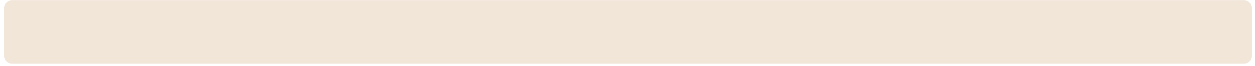
**Problem 1.**

Let  $A$  be a finite set. Show that  $A$  cannot be an ordered field.



**Problem 2.**

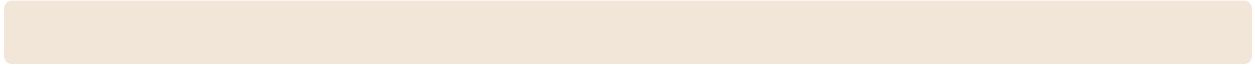
Let  $F = \{0, 1, 2\}$ . Prove that there is exactly one way to define addition and multiplication so that  $F$  is a field if 0 is the additive identity and 1 is the multiplicative identity.



**Problem 3.**

If  $S_1$  and  $S_2$  are nonempty subsets of  $\mathbb{R}$  that are bounded above, prove that

$$\sup \{x + y \mid x \in S_1, y \in S_2\} = \sup S_1 + \sup S_2.$$



**Problem 4.**

Let  $S := \{a_k \mid k \in \mathbb{N}\} \cup \{b_k \mid k \in \mathbb{N}\}$ , ordered such that  $a_k < b_j$  for all  $k$  and  $j$ ,  $a_k < a_m$  whenever  $k < m$ , and  $b_k < b_m$  whenever  $k < m$ .

- (a) Show that  $S$  is an ordered set.
- (b) Show that every subset of  $S$  is bounded above and below.
- (c) Find a bounded subset of  $S$  that has no least upper bound.

**Problem 5.**

Let  $n \in \mathbb{N}$ . Show that  $(\mathbb{R}^n, d_1)$  is a metric space where

$$d_1(p, q) := \sum_1^n |p_i - q_i|$$

for all  $p = (p_1, \dots, p_n)$  and  $q = (q_1, \dots, q_n)$  in  $\mathbb{R}^n$ .



**Problem 6.**

Show that the subset of  $(\mathbb{R}^2, d_E)$  given by

$$S := \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 > x_2\}$$

is open.



**Problem 7.**

Let  $(X, d)$  be a metric space. Let  $A \subset X$ . Show that  $A$  is open if and only if it is equal to the union of a collection of open balls.