

## PSET 3 — 01/24/2024

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I worked on these problems alone, with reference to class notes and the following books:

- (a) *Introduction to Analysis* by Maxwell Rosenlicht

**Problem 1.**

Let  $a_i, b_i \in \mathbb{R}$  for  $i = 1, 2, \dots, n$ . Show that  $(a_1, b_1) \times (a_2, b_2) \times \cdots \times (a_n, b_n)$  is open in  $(\mathbb{R}^n, d_E)$  and  $[a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_n, b_n]$  is closed in  $(\mathbb{R}^n, d_E)$ .

**Problem 2.**

Prove that any bounded open subset of  $\mathbb{R}$  is the union of disjoint open intervals.



**Problem 3.**

Prove that if the points of a convergent sequence of points in a metric space are reordered, the new sequence converges to the same limit.

Let  $\{p_i\}_1^\infty$  be a convergent sequence in a metric space  $(X, d)$ .

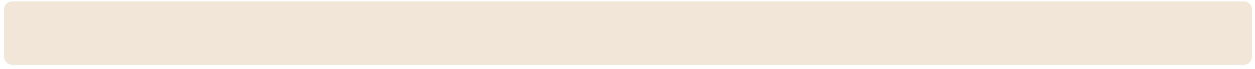
**Problem 4.**

Prove that if  $\lim_{n \rightarrow \infty} p_n = p$  in a given metric space then the set of points  $\{p, p_1, p_2, \dots\}$  is closed.



**Problem 5.**

Let  $a_n = \frac{n}{n+1}$  for  $n \in \mathbb{N}$ . Show, using the definition of a limit, that  $\lim_{n \rightarrow \infty} a_n = 1$ .



**Problem 6.**

Consider the sequence  $\{a_n\}_{n=1}^{\infty}$  such that  $a_1 \geq a_2 \geq a_3 \geq \dots$  (i.e., it is a monotonically decreasing sequence). Assume that there exists  $m > 0$  such that  $a_n > m$  for all  $n$ . Show that  $\{a_n\}_{n=1}^{\infty}$  converges in  $\mathbb{R}$ .

