CS 39: Theory of Computation **Winter '23**

PSET 4 — 02/06/2023

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Credit Statement

I discussed ideas for this homework assignment with Paul Shin.

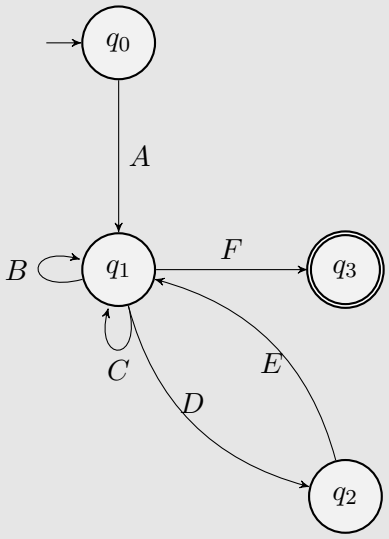
I also referred to the following books:

(a) **Introduction to the Theory of Computation** by **Michael Sipser**.

(b) **A Mathematical Introduction to Logic** by **Herbert Enderton**.

Problem 1.

Draw a PDA that recognizes the language $L = \left\{x \in \{0,1\}^* : N_1(x) \geq 2N_0(x)\right\}$. Give a high-level proof that your that your PDA works correctly.



KEY:

Item	Meaning
A	$\varepsilon, \varepsilon \to \$$
B	$1, p \to pp, 1, m \to \varepsilon, 1, \$ \to p\$$
C	$0, m \to mmm, 0, \$ \to mm\$$
D	$0, p \to \varepsilon$
E	$\varepsilon, p \to \varepsilon, \varepsilon, \$ \to m\$$

Figure 1. DFA for L

Problem 2.

In class, we wrote a formal construction of a PDA that proves that context-free languages are closed under union. Give similar constructions for PDAs to prove closure under:

(a) concatenation.

(b) Kleene star.

Problem 3.

Give an alternate proof, using CFGs alone (no PDAs), to prove that context-free grammars are closed under:

(a) union.

(b) concatenation.

(c) Kleene star.

Problem 4.

A string $x \in \Sigma^*$ is called a *square* if $x = w^2$ for some $w \in \Sigma^*$. Let $L_{sq} = \left\{w^2 : w \in \{0,1\}^*\right\}$. Consider its complement:

$$\overline{L}_{sq} = \left\{x \in \{0,1\}^* : x \text{ is not of the form } w^2 \text{ for any } w \in \{0,1\}^* \right\}.$$

(a) Prove that every even-length string is in $\overline{L}_{sq}$ can be decomposed as $x = uv$ where the middle symbol of u differs from the middle symbol of v.

(b) Using this property, design a context-free grammar that generates $\overline{L}_{sq}$.

Problem 5.

Let Σ be an alphabet, $L \subseteq \Sigma^*$, and $\# \notin \Sigma$. Define the language

$$\textsc{Intersperse}(\#, L) := \{a_1 \# a_2 \# \dots \# a_n\}, \text{ each } a_i \in \Sigma \text{ and } a_1 a_2 \dots a_n \in L.$$

Let $M_1 = (Q, \Sigma, \Gamma, \delta, q_0, F)$ be a PDA that recognizes L. Formally describe a PDA that recognizes $\textsc{Intersperse}(\#, L)$. Also give a high-level proof that your PDA works correctly.

Problem 6.

Consider the following CFG:

$$S \to 1S00 \mid 00S1 \mid SS \mid 0S1S0 \mid \varepsilon$$

(a) Give a simple description of the language it generates using set-builder notation.

$$L = \left\{x \in \{0,1\}^* : N_1(x) = 2N_0(x)\right\}$$

(b) Now for the hard and fun part: prove the correctness of your answer.