CS 39: Theory of Computation **Winter '23**

Mid-Term Exam 1 — 02/13/2023

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Credit Statement

All work on the mid-term is my own. I referred to class notes and the following books:

(i) **Introduction to the Theory of Computation** by **Michael Sipser**.

Problem 1.

Let $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the alphabet of all decimal digits. A string $x \in D^*$ is said to be stable if, for each pair of adjacent digits in x, those two digits have a numerical difference of at most 1.

For example:

$$\text{Stable:} \quad 433321001012,\ 556677654,\ 7,\ 0000,\ \varepsilon.$$

$$\text{Unstable:} \quad 6554667,\ 1213141516,\ 7890123.$$

Give a formal description of a DFA that recognizes $L_1 = \{x \in D^* : x \text{ is stable}\}$. Provide a high-level explanation of your design idea (no formal completeness and soundness proofs are required).

Problem 2.

For each CFG G_i;

(i) Describe $\mathcal{L}(G_i)$ using set notation, as simply as possible.

(ii) Either *draw* an NFA that recognizes $\mathcal{L}(G_i)$ or *prove* that $\mathcal{L}(G_i)$ is not regular.

(a) G_1

$$S \Rightarrow 0T0 \mid 1T1$$

$$T \Rightarrow 0T0 \mid 1T1 \mid X$$

$$X \Rightarrow AX \mid A$$

$$A \Rightarrow 0 \mid 1$$

(i) $\mathcal{L}(G_1) = \{0^n 1^n : n \geq 0\}$

(ii) *Draw* an NFA that recognizes $\mathcal{L}(G_1)$.

(b) G_2

$$S \Rightarrow 0X \mid 1Y$$

$$X \Rightarrow AXA \mid 0$$

$$Y \Rightarrow AYA \mid 1$$

$$A \Rightarrow 0 \mid 1$$

(i) $\mathcal{L}(G_2) = \{0^n 1^n : n \geq 0\}$

(ii) *Prove* that $\mathcal{L}(G_2)$ is not regular.

(c) G_3

$$S \Rightarrow AAT \mid BBT$$

$$T \Rightarrow AAT \mid BBT \mid A \mid B$$

$$A \Rightarrow 0$$

$$B \Rightarrow 1$$

(i) $\mathcal{L}(G_3) = \{0^n 1^n : n \geq 0\}$

(ii) *Draw* an NFA that recognizes $\mathcal{L}(G_3)$.

Problem 3.

Give a simple CFG that generates the language $L_3 = \left\{x \in \{0,1\}^* : x \neq x^R\right\}$.

Formally prove that your CFG is sound and complete.

Problem 4.

For each $x \in \{0, 1\}^*$, define grow(x) to be the string obtained by replacing every occurrence of '0' in x with '00'. For example:

$$\text{grow}(10110) = 1001100, \quad \text{grow}(000) = 000000, \quad \text{grow}(\varepsilon) = \varepsilon, \quad \text{grow}(11) = 11.$$

Let $P = (Q, \{0, 1\}, \Gamma, \delta, q_0, F0)$ be a PDA. Formally describe a PDA that recognizes $\{\text{grow}(x) : x \in \mathcal{L}(P)\}$. Do not assume anything about the design of P.

Give a high-level explanation of your construction (no formal completeness and soundness proofs are required).

Problem 5.

For a language L, define $\textsc{unique}(L) := \{x \in L : \nexists y \in L \text{ such that } |y| = |x|\}$.

In other words, $\textsc{unique}(L)$ is the set of all strings $x \in L$ such that x is the only string in L that has length $|x|$.

Is the class of regular languages closed under the operation $\textsc{unique}$? Prove your answer.

Problem 6.

Every language falls into into one of the following three categories:

(i) regular

(ii) context-free but not regular

(iii) not context-free

Which of these categories is the following language in?

$$L_6 = \left\{x \in \{0,1\}^* : \exists y, z \in \{0,1\}^* \text{ such that } x = yz, |y| = |z|, \text{ and } \beta(y) \equiv 0 \pmod 3\right\}.$$

Reminder: For a string $x \in \{0,1\}^*$, $\beta(x)$ is the string x interpreted as a binary number. For instance, $\beta(11001) = 25$ and $\beta(0011) = 3$. We also define $\beta(\varepsilon) = 0$.