Math 29: Computability Theory **Spring 2024**

Mid-Term Exam 05/03/2024

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Problem 1.

Give a register machine which, when given a natural number n in R_0 at ***start***, halts and outputs $x+1$ in R_1 if there is some natural x satisfying $x^2 + n^2 = 2nx$, and returns 0 otherwise.

Problem 2.

Prove that no index set is immune.

Problem 3.

Give an example of a partial computable function $i(x, y)$ such that:

- $i(x, y) \downarrow$ implies $i(x, y) = 0$ or $i(x, y) = 1$.
- If A is computable, then there exists an e such that $i(e, n) = \chi_A(n)$ for all n.
- $I_x \coloneqq \{y \mid i(x, y) \downarrow > 0\}$ is computable for all x.

Why does $i(x, y)$ not contradict Homework 3, Question 2?

Definition of $i(x, y)$

Let $i(x, y)$ be the following partial computable function:

$$i(x, y) = \begin{cases} 1 & \text{if } \varphi_x(y) \downarrow = 1 \\ 0 & \text{if } \varphi_x(y) \downarrow = 0 \end{cases}$$

Problem 4.

Let M be the set

$$\{x \mid \forall\, y < x\, (\varphi_x \neq \varphi_y)\}\,.$$

That is, M is the set of minimal indices of computable functions: the smallest indices which define a given (partial) computable function.

(a) Is M immune? Simple?

(b) Is M productive? Creative?

Problem 5.

Prove that $W_e \leq_1 H$ for all e, then prove that $H \leq_1 K$.

Problem 6.

Suppose C is the set of valid codes for machines based on our coding scheme defined in class. Give a total, computable bijection from ω to C, i.e. a computable function which associates every natural number to a unique machine.

Problem 7.

SHow that there is an e such that

$$W_e = \left\{e + 1, e^2 + 4, e^3 + 9, \ldots\right\}.$$