Math 29: Computability Theory **Spring 2024**

PSET 5 — 05/12/2024

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Problem 1.

Show that if C is a computable set, then $C \leq_m X$ for any set which is empty and has nonempty complement.

Problem 2.

Let B be an infinite c.e. set. Is there an immune set I such that $B \leq_1 I$? Justify your answer.

Problem 3.

Are there uncountably many Turing degrees? Justify your answer.

Problem 4.

$A \oplus B$, "A join B", is defined as

$$A \oplus B = \{2x \mid x \in A\} \cup \{2x + 1 \mid x \in B\}.$$

Prove that thfe Turing degree of $A \oplus B$ is a least upper bound of the Turing degrees of A and B. In other words, show that it computes both A and B, and that any C which computes both A and B also computes $A \oplus B$.

Problem 5.

Given a countable sequence of sets $\{A_i\}_{i\in\omega}$, define the *infinite join* $A = \oplus_{i\in\omega} A_i$ by

$$A = \{\langle i, n\rangle \mid n \in A_i\}\,.$$

Prove that there are sequences $\{A_i\}_{i\in\omega}$ and $\{B_i\}_{i\in\omega}$ such that $A_i \equiv_T B_i$ for all $i<$ but $A \not\equiv_T B$. In other words, this operation is defined on sets, but not on degrees (unlike the finite joins).

Hint: make A computable but B not computable.