Turing Categories and Computability

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Introduction

A Turing category is a category C equipped with:

- cartesian products — to pair (the codes of) data and programs,
- a notion of partiality — to represent programs (morphisms) which do not necessarily halt,
- and a ***Turing object*** A — to represent the "codes" of all programs. A Turing object is an object A such that for any $X, Y \in C$, there is a universal application morphism $\tau_{X,Y} : A \times X \to Y$ that represents the application of a program (in A) to data (in X) to produce a result (in Y). [2]

Turing categories provide an abstract framework for computability: a "category with partiality" equipped with a "universal computer", whose programs and codes thereof constitute the objects of interest. [2]

I am interested in doing an expository study on Turing categories with the goal of explaining the main ideas it builds on and demonstrating some of the main results it achieves.

My main references will be:

1. ***Basic category theory*** [4], a textbook by Tom Leinster accessible on the internet here.
2. Introductory paper on ***Turing Categories*** [2], accessible on the internet here.
3. Introductory notes on ***Effective Applicative Structures*** [1], accessible on the internet here.
4. Enderton's textbook on mathematical logic [3], accessible on the internet here and discussed here.

The final deliverable will be a paper (LaTeX) that explains the main ideas of Turing categories and demonstrates how it can be used to derive some of the main results in computability.

I am thinking of two possible directions for the presentation:

1. A video presentation of the paper using `manim`, a Python library for making mathematical animations (developed by Grant Sanderson of `3Blue1Brown`).. I've experimented a little with the library and I think it could be an exciting challenge. More information on `manim` can be found here.

2. If the video doesn't work out, my alternative will be presenting the paper in class.

References

[1] Andrea Asperti and Agata Ciabattoni, *Effective applicative structures*, Category theory and computer science, 1995, pp. 81–95.

[2] J.R.B. Cockett and P.J.W. Hofstra, *Introduction to turing categories*, Annals of Pure and Applied Logic **156** (2008), no. 2, 183–209.

[3] Herbert Enderton and Herbert B Enderton, *A mathematical introduction to logic*, Elsevier, 2001.

[4] Tom Leinster, *Basic category theory*, 2016.