Math 29: Computability Theory **Spring 2024**

PSET 1 — 04/05/2024

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Problem 1.

The rational numbers, $\mathbb{Q}$, are $\left\{\frac{p}{q} \mid p, q \in \mathbb{Z}, q > 0\right\}$. The Gaussian rationals are complex nunbers of the form $\{r + si \mid r, s \in \mathbb{Q}\}$. Provide a bijection between the Gaussian rationals and ω.

Prove that it is a bijection.

This... is a bijection.

Problem 2.

Give a register machine which converges if there is a 2 in R_0 and diverges otherwise.

Problem 3.

Prove that the squaring function is computable by providing a register machine which takes in n in R_0 and outputs n^2 in R_1. You may use the multiplication function $x(n, m)$ — which starts with n in R_0 and m in R_1 and outputs $n \cdot m$ in R_2 — as a black box function labeled M.

Problem 4.

Prove that the set of ordered pairs of natural numbers (x, y) such that $x \leq y$ is computable by providing a register machine which takes n and m as imputs in R_0 and R_1 respectively and outputs 1 if $n \leq m$ and 0 in R_2 otherwise.

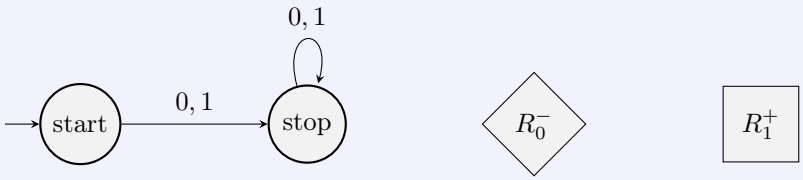

Figure 1: Only accept the empty string.