Math 29: Computability Theory **Spring 2024**

PSET 7 — 05/24/2024

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Problem 1.

Are there sets A and B such that $A' \leq_T B'$ but $A \not\leq_T B$? Justify your answer.

Problem 2.

(a) Prove that if there is $g \leq_T X$ such that $\varphi_{g(x)} \neq \varphi_x$, then there is $h \leq_T X$ such that $h(e) \neq \varphi_e(e)$ for all e.

(b) Given $h \leq_T X$ such that $h(e) \neq \varphi_e(e)$ for all e, show that there is $f \leq_T X$ such that $W_{f(e)} \neq W_e$ for all e.
Hint: make $\left|W_{f(e)}\right| = 1$.

Problem 3.

Verify that the f constructed in the High-Low lecture notes dominates every total computable function, but does not compute K.

Problem 4.

Prove that not ML-random set has an infinite c.e. subset. (*Hint: use a lemma from the class notes*).