Math 29: Computability Theory **Spring 2024**

PSET 2 (Corrections) — 04/18/2024

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Problem 2.

Show that the set of powers of 2 is computable by building a Turing machine.

Idea

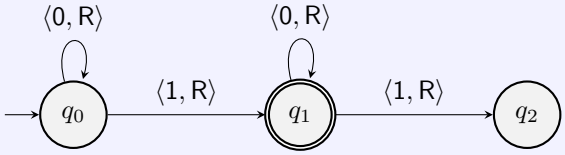
A number (in binary) is a power of 2 if and only if it has exactly one 1 bit. To show that the set is computable, we can construct a Turing machine that computes the characteristic function $\chi_{\text{pow}_2}(x)$, which is 1 if x is a power of 2 and 0 otherwise:

To show that the

1. ***start*** in state q_0.
2. While in q_0, read the tape at the current position.
   (a) If a 0 is read, move right and remain in q_0.
   (b) If a 1 is read, move right and transition to q_1.
   (c) If tape is blank at current position, move left and transition to q_3.
3. While in q_1, read the tape at the current position.
   (a) If a 0 is read, it move right and remain in q_1.
   (b) If a 1 is read, input cannot be a power of 2. HALT and output 0.
   (c) If tape is blank at current position, input is a power of 2. HALT and output 1.

Turing Machine

1. $\langle q_0, 0, \text{R}, q_0 \rangle$
2. $\langle q_0, 1, \text{R}, q_1 \rangle$
3. $\langle q_1, 0, \text{R}, q_1 \rangle$
4. $\langle q_1, 1, \text{R}, q_2 \rangle$



Problem 3.

Show that the set of multiples of 4 is computable by building a Turing machine.

Idea

A number (in binary) is a multiple of 4 if either the number is 0 or the last two bits are 0. We check this by reading the input from left to right, until we reach the end of the input, then checking the last two bits on the tape.

1. The Turing machine starts in q_0. It reads the tape at the current position.
   (a) If it reads a 0, it moves right and remains in q_0.
   (b) If it reads a 1, it moves right and remains in q_0.
   (c) If it reads a blank ($*$), it moves left on the tape and transitions to q_1.
2. While in q_1, it reads the tape at the current position.
   (a) If it reads a 0, it moves left and transitions to q_2.
   (b) If it reads a 1, or a blank symbol ($*$), it halts in q_1 and does not accept.
3. While in q_2, it reads the tape at the current position.
   (a) If it reads a 0 or a blank ($*$), it moves right and transitions to q_3.
   NOTE: we include the empty symbol $*$ here to account for the number 0.
   (b) If it reads a 1, it halts in q_2 and does not accept.
4. q_3 is an accepting state. If the Turing machine reaches q_3, whatever it reads next halts the machine and accepts the input as a multiple of 4.

Turing Machine

1. $\langle q_0, 0, \mathsf{R}, q_0 \rangle$
2. $\langle q_0, 1, \mathsf{R}, q_0 \rangle$
3. $\langle q_0, *, \mathsf{L}, q_1 \rangle$
4. $\langle q_1, 0, \mathsf{L}, q_2 \rangle$
5. $\langle q_2, 0, \mathsf{L}, q_3 \rangle$
6. $\langle q_2, *, \mathsf{L}, q_3 \rangle$