CS 83: Computer Vision	**Winter 2024**

Quiz 4 — 02/07/2024

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Credit Statement

I discussed solution ideas with:

1. Ivy (Aiwei) Zhang
2. Angelic McPherson

However, all typed work is my own, with reference to class notes especially on homographies and transformations. I also referred to some of my earlier notes on linear algebra (from **MATH 22**) on the interpretations of matrices and their null-spaces, column-spaces, and row-spaces.

Problem 1.

(i) Prove that there exists a homography $\mathbf{H}$ that satisfies

$$\mathbf{x}_1 \equiv \mathbf{H}\mathbf{x}_2 \tag{1.1}$$

between the 2D points (in homogeneous coordinates) $\mathbf{x}_1$ and $\mathbf{x}_2$ in the images of a *plane* Π captured by two 3×4 camera projection matrices $\mathbf{P}_1$ and $\mathbf{P}_2$ respectively. The $\equiv$ symbol is equality up to scale. *Note: A degenerate case happens when the plane* Π *contains both cameras' centers, in which case there are infinite choices of* $\mathbf{H}$ *satisfying the above equation. You can ignore this special case in your answer.*

(ii) Prove that there exists a homography $\mathbf{H}$ that satisfies equaton 1.1 given two cameras separated by a pure rotation. That is, for camera 1, $\mathbf{x}_1 = \mathbf{K}_1 \begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix} \mathbf{X}$, and for camera 2, $\mathbf{x}_2 = \mathbf{K}_2 \begin{bmatrix}\mathbf{R} & \mathbf{0}\end{bmatrix} \mathbf{X}$. Note that $\mathbf{K}_1$ and $\mathbf{K}_2$ are the 3×3 intrinsic matrices of the two cameras and are different. $\mathbf{I}$ is the 3×3 identity matrix, $\mathbf{0}$ is the 3×1 zero vector, and $\mathbf{X}$ is a point in 3D space. $\mathbf{R}$ is the 3×3 rotation matrix of the camera.

(iii) Suppose that a camera is rotating about its center $\mathbf{C}$, keeping the intrinsic parameters $\mathbf{K}$ constant. Let $\mathbf{H}$ be the homography that maps the view from one camera orientation to the view at a second orientation. Let θ be the angle of rotation between the two orientations. Show that $\mathbf{H}^2$ is the homography corresponding to a rotation of 2θ.

Problem 2.

In class, we say that a camera matrix satisfies the equation $\mathbf{x}_i = \mathbf{P}\mathbf{X}_i$, and that six 3D-2D matches $\mathbf{x} \leftrightarrow \mathbf{X}$ are sufficient to recover $\mathbf{P}$ using a linear (non-iterative) algorithm.

Find a linear algorithm for computing the camera matrix $\mathbf{P}$ in the special case when the camera location (but not orientation) is known. Ignoring degenerate configurations, how many 2D-3D matches are required for there to be a unique solution? Justify your answer.