CS 83: Computer Vision **Winter '23 Year**

Quiz 3 — 02/02/2023

Prof. Pediredla ***Student: Amittai Siavava***

Credit Statement

I worked on these problems alone, with reference to class notes and the following books:

(i) **Computer Vision: Algorithms and Applications** by **Richard Szeliski**

Problem 1.

Given a set N of points $\mathbf{p}_i = (x_i, y_i), i \in \{1, \ldots, N\}$, in the image plane, we wish to find the best line passing through those points.

(a) One way to solve this problem is to find (a, b) that most closely satisfy the equations $y_i = ax_i + b$, in a least-squares sense. Write these equations in the form of a *heterogeneous* least-squares problem $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x} = (a, b)^T$, and give an expression for the least-squares estimate of $\mathbf{x}$. Give a geometric interpretation of the error being minimized, and use a simple graph to visualize the error. Does this make sense when fitting a line to points in an image?

(b) Another way to solve this problem is to find $\ell = (a, b, c)$, defined up to scale, that most closely satisfies the equations $ax_i + by_i + c = 0$, in a least-squares sense. Write these equations in the form of a *homogeneous* least-squares problem $A\ell = 0$, where $\ell = (a, b, c)^T$ and $\ell \neq 0$. This problem has a trivial solution (zero vector) which is not of much use. Describe some ways of avoiding this trivial solution and corresponding algorithms for solving the resulting optimization problem. Is this approach more or less useful than the previous one? Why?

Hint: Think about how we can express the distance of a point from a line.

Problem 2.

The equation for a conic in the plane using inhomogeneous coordinates (x, y) is

$$ax^2 + bxy + cy^2 + dx + ey + f = 0. \tag{2.1}$$

(a) Suppose you have a given a set of inhomogeneous points $\mathbf{x}_i = (x_i, y_i), i \in \{1, \ldots, N\}$. Derive an expression for the least squares estimate of a conic $\mathbf{c} = (a, b, c, d, e, f)$ passing through those points.

Note: your expression may take the form of a null vector or eigenvector of a matrix. If so, you must provide expressions for the matrix elements.

(b) In general, what is the minimum value of N that allows a unique solution for $\mathbf{c}$?

(c) Homogenize equation 2.1 by making the substitutions $x \leftarrow z_1/x_3$ and $y \leftarrow y_1/y_3$, and show that in terms of homogeneous coordinates ($\mathbf{x} = (x_1, x_2, x_3)$) the conic can be expressed in matrix form

$$\mathbf{x}^T \mathbf{C} \mathbf{x} = 0,$$

where $\mathbf{C}$ is a symmetric matrix.

(d) Suppose we apply a projective transformation to our points $\mathbf{x}_i' = \mathbf{H}\mathbf{x}_i$. The transformed points x_i' will lie on a transformed conic represented by a new symmetric matrix $\mathbf{C}'$. Write an equation that specifies the relationship between $\mathbf{C}'$ and $\mathbf{C}$ in terms of the homography $\mathbf{H}$.

Problem 3.

The affine transform in heterogeneous coordinates is given by $\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$. Affine transformations are combinations of:

(i) Arbitrary linear transformations with 4 degrees of freedom (a, b, c, d, e).

(ii) Translations with 2 degrees of freedom (c, f).

Does affine transformation apply translation first followed by arbitrary linear transformation, or the other way around? Prove your answer mathematically.

Hint: check if

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & c \\ 0 & 1 & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & 0 \\ d & e & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b & 0 \\ d & e & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & c \\ 0 & 1 & f \\ 0 & 0 & 1 \end{bmatrix}$$