CS 83: Computer Vision	Winter 2024

Quiz 5 — 02/15/2024

Prof. Pediredla	*Student: Amittai Siavava*

Credit Statement

I discussed solution ideas with:

1. Ivy (Aiwei) Zhang
2. Angelic McPherson

However, all typed work is my own, with reference to class notes.

Problem 1.

As we discused in class, two cameras are said to form a *rectified pair* if their camera coordinate systems differ only by a translation of their origins (the camera centers) along a direction that is parallel to either the x- or y-axis of their coordinate systems.

(i) Derive an expression for the essential matrix $\mathbf{E}$ of a rectified pair.

(ii) Prove that the epipolar lines of a rectified pair are parallel to the axis of translation.

Problem 2.

Suppose two cameras fixate on a point P (see Figure 1) in space such that their optical axes intersect at that point. Show that if the image coordinates are normalized so that the coordinate system origin $(0, 0)$ coincides with the principal point, the $\mathbf{F}_33$ element of the fundamental matrix $\mathbf{F}$ is zero.

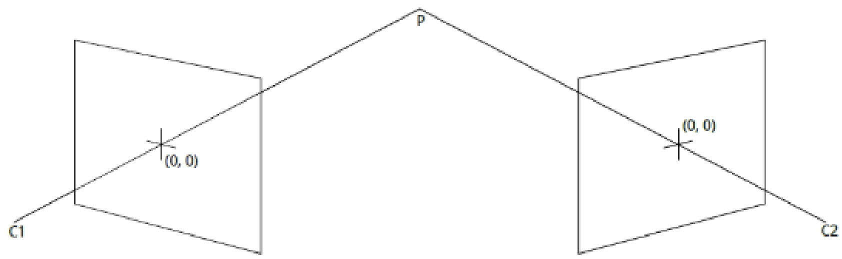

FIGURE 1. C_1 and C_2 are the optical centers. The principal axes intersect at point P.

Problem 3.

Consider three images I_1, I_2 and I_3 that have been captured by a system of three cameras, and suppose the fundamental matrices $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ are known. (Notation: the matrix $\mathbf{F}_{ij}$ satisfies the equation $\mathbf{x}_j^T \mathbf{F}_{ij} \mathbf{x}_i = 0$ for any correspondence $\mathbf{x}_i \leftrightarrow \mathbf{x}_j$ between images I_i and I_j). In general, given a point $\mathbf{x}_1$ in I_1 and a corresponding point $\mathbf{x}_2$ in I_2, the corresponding point in $\mathbf{x}_3$ in I_3 is uniquely determined by the fundamental matrices $\mathbf{F}_{13}$ and $\mathbf{F}_2 3$.

(i) Write an expression for $\mathbf{x}_3$ in terms of $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$.

(ii) Describe a degenerate configuration of three cameras for which the point $\mathbf{x}_3$ cannot be uniquely determined by this expression.

Hint: Consider the epipolar geometry of the situation. Draw a picture!