Math 69: Logic **Winter '23**

Reading assigned January 11, 2023

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Problem 1.

Is the set of wffs that are not tautologies decidable? Explain briefly.

Yes, the set is decidable.

Since the set of all tautologies is decidable (Taking the set of all tautologies as the tautological implications of the empty set, then Theorem 17C and Corollary 17D tell us that the tautological implications of a finite set are decidable). Let T be the list of all tautologies, then the set of all wffs that are not tautologies is

$$U = \{x : x \text{ is a wff and } x \notin T\}.$$

Since T is decidable:

- We can always evaluate whether a wff is in T, thus determine if it is not in U.
- We can always evaluate whether a wff is not in T, thus determine if it is in U.

Therefore, U is decidable.

$$\forall x\ \forall y\ (([x] = [y]) \vee ([x] \cap [y] = \varnothing))$$

Questions

If we had an infinite set Σ, then it may be semidecidable, then we may not be able to determine if a wff is not in the set. In this case, would it be correct to infer that the set of all wffs that are not tautological consequences of Σ is not decidable?

In general, is it always true to say that the complement of a decidable set is decidable?