Comments

I also read through the part on recursions and I thought the recursive definitions of well-formed formulas as n-ary functions taking in n arguments (each of which may be a function applied to its arguments) made a lot of sense.

On the other hand, polish notation without parenthesizing the arguments felt more complicated than the infix notation we have been using thus far. I understand how it reduces ambiguity, but I found it more tasking to process and mentally parse the wffs written in polish notation.

Questions

In the section about 0-ary connectives, we say that $\top$ and $\perp$ can be thought of as the constants T and F having $\overline{v}(\perp) = F$ and $\overline{v}(\top) = T$ for every v. Wouldn't we get a contradiction if we have a function that negates v? For instance, if $w = \neg v$ for some function v, doesn't $\overline{w}(\perp) = \neg\overline{v}(\perp)$ imply that $\overline{w}(\perp) = T$?

Exercises

1. Let G be the following three-place Boolean function.

$$
\begin{aligned}
G(F, F, F) &= T, & \qquad G(T, F, F) &= T, \\
G(F, F, T) &= T, & \qquad G(T, F, T) &= F, \\
G(F, T, F) &= T, & \qquad G(T, T, F) &= F, \\
G(F, T, T) &= F, & \qquad G(T, T, T) &= F.
\end{aligned}
$$

(a) Find a wff using at most the connectives $\wedge$, $\vee$, and $\neg$, that realizes G.

Truth table for G:

α	β	γ	$G(\alpha, \beta, \gamma)$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

$G(\alpha, \beta, \gamma)$ always disagrees with the majority of α, β, and γ.

That is,

$$G(\alpha, \beta, \gamma) = \neg((\alpha \wedge \beta) \vee (\alpha \wedge \gamma) \vee (\beta \wedge \gamma))$$

(b) Then find such a wff in which connective symbols occur at not more than 5 places.

$$G(\alpha, \beta, \gamma) = \neg(\#\alpha\beta\gamma)$$

Using only $\wedge, \vee, \neg$:

$$G(\alpha, \beta, \gamma) = \neg(\ (\alpha \wedge (\beta \vee \gamma))\ \vee\ (\beta \wedge \gamma))$$