Math 69: Logic **Winter '23**

Homework assigned February 03, 2023

Prof. Marcia Groszek ***Student: Amittai Siavava***

Problem 6.

Show that the formula θ is valid iff $\forall x\theta$ is valid.

$\Longrightarrow$

Suppose θ is valid, we show that $\forall x\theta$ is also valid.

Let $\mathfrak{A}$ be a structure and $s : V \to |\mathfrak{A}|$ be an assignment function. Since θ is valid, $\vDash_{\mathfrak{A}} \theta[s]$.

Then for all $a \in |\mathfrak{A}|$, we have that $\vDash_{\mathfrak{A}} \forall x\theta[s(x|a)]$.

Since s and $\mathfrak{A}$ are arbitrary, it must be that $\vDash \forall x\theta$, meaning $\forall x\theta$ is valid.

$\Longleftarrow$

Suppose $\forall x\theta$ is valid, we show that θ is also valid.

Let $\mathfrak{A}$ be a structure and $s : V \to |\mathfrak{A}|$ be an assignment function. Since $\forall x\theta$ is valid, we have that $\vDash_{\mathfrak{A}} \forall x\theta[s]$.

Then $\vDash_{\mathfrak{A}} \theta[s(x|s(x))]$.

But $s(x|s(x)) = s$, so $\vDash_{\mathfrak{A}} \theta[s]$, meaning θ is valid.

Problem 10.

Show that

$$\vDash_{\mathfrak{A}} \forall v_2\ Qv_1v_2[\![c^{\mathfrak{A}}]\!] \quad \text{iff} \quad \vDash_{\mathfrak{A}} \forall v_2 Qcv_2.$$

Here, Q is a two-place predicate symbol and c is a constant symbol.

Let $\mathfrak{A}$ be a structure and $s : V \to |\mathfrak{A}|$ be an assignment function such that $\vDash_{\mathfrak{A}} \forall v_2\ Qv_1v_2[\![c^{\mathfrak{A}}]\!]$.

$$\vDash_{\mathfrak{A}} \forall v_2\ Qv_1v_2[\![c^{\mathfrak{A}}]\!] \iff \vDash_{\mathfrak{A}} Qv_1v_2[s(v_1|c^{\mathfrak{A}}, v_2|a)] \text{ for all } a \in \mathfrak{A} \tag{10.1}$$

$$\iff \text{A deduction } \left\langle c^{\mathfrak{A}}, a \right\rangle \text{ exists in } \mathfrak{A} \text{ for all } a \in |\mathfrak{A}| \tag{10.2}$$

$$\iff \vDash_{\mathfrak{A}} Qcv_2[s(v_2|a)] \tag{10.3}$$

$$\iff \vDash_{\mathfrak{A}} \forall v_2 Qcv_2 \quad \text{(since } \mathfrak{A} \text{ and } s \text{ are arbitrary)} \tag{10.4}$$