Math 69: Logic Winter '23

Reading assigned January 9, 2023

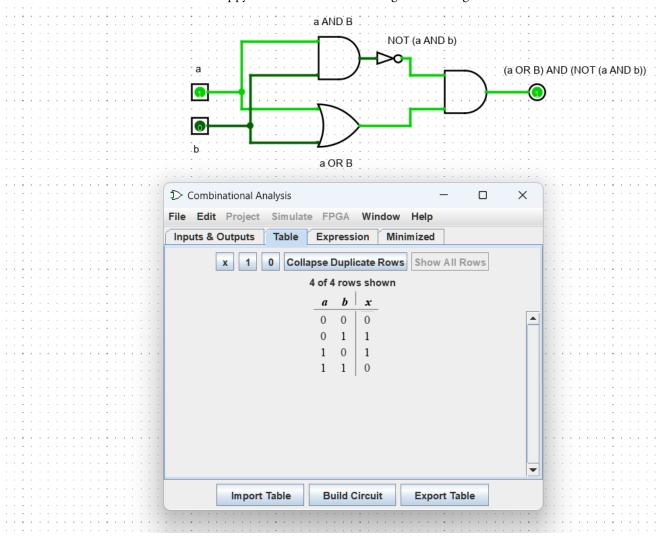
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Comments

I did not have any questions or confusions about the reading, but I also found the section on switching circuits to be very interesting since I've seen them show up in Computer Architecture. Will we be reviewing that section later, or is not relevant to our study of logic?

Just for fun I built a simple circuit implementing exclusive-or from basic AND and OR gates, but the modeling program also supports exclusive-or as a built-in gate (and much more complicated logic and computing circuits) If classmates are interested I would be happy to demo how to model logic as with logisim.



Exercises

1. Give an example of a set S of wffs such that every two-element subset of S is satisfiable but S is not satisfiable. (It is possible to do this with S having exactly 3 elements, but it is okay if your set is bigger.)

Let S be a set of 3 wffs such that two wffs are independent of each other but the third wff places a condition on the first two wffs, such as:

$$S = \{\alpha, \beta, \neg(\alpha \land \beta)\}.$$

with the two-element subsets

$$S_1 = \{\alpha, \beta\}$$

$$S_2 = \{\alpha, \neg(\alpha \land \beta)\}$$

$$S_3 = \{\beta, \neg(\alpha \land \beta)\}\$$

Then, S is not satisfiable (since setting $\alpha = T$ and $\beta = T$ violates the third wff).

However, each of the subsets S_1, S_2, S_3 is satisfiable.

Another example:

$$S = \{ \neg \alpha, \neg \beta, (\alpha \lor \beta) \}$$

$$S_1 = \{ \neg \alpha, \neg \beta \}$$

$$S_2 = \{ \neg \alpha, (\alpha \lor \beta) \}$$

$$S_3 = \{ \neg \beta, (\alpha \lor \beta) \}$$