Math 69: Logic **Winter '23**

Reading assigned January 19, 2023

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Problem 7.

Write down 4 sentences for a language $\mathfrak{L}$ such that any structure $\mathfrak{U} = \langle X, \leq \rangle$ is a linear ordering if and only if it satisfies those four sentences.

$$\forall x Pxx \qquad \text{(reflexive)}$$

$$\forall x \forall y \left((Pxy \wedge Pyx) \rightarrow (x = y) \right) \qquad \text{(antisymmetric)}$$

$$\forall x \forall y \forall z \left((Pxy \wedge Pyz) \rightarrow Pxz \right) \qquad \text{(transitive)}$$

$$\forall x \forall y \left(Pxy \vee Pyx \right) \qquad \text{(total)}$$

Questions

I have never encountered the idea of a preordering (i.e. having $x \leq y$ and $y \leq x$ not imply $x = y$) before. Is there any example of such a structure that maybe occurs frequently in Mathematics?