**Math 69: Logic** **Winter '23**

# Reading assigned January 9, 2023

***Prof. Marcia Groszek*** ***Student: Amittai Siavava***

### Comments

I also read through the part on recursions and I thought the recursive definitions of well-formed formulas as $n$-ary functions taking in $n$ arguments (each of which may be a function applied to its arguments) made a lot of sense. On the other hand, polish notation without parenthesizing the arguments felt more complicated than the infix notation we have been using thus far. I understand how it reduces ambiguity, but I found it more tasking to process and mentally parse the wffs written in polish notation.

### Questions

In the section about 0-ary connectives, we say that $\top$ and $\perp$ can be thought of as the constants $T$ and $F$ having $\overline{v}(\perp) = F$ and $\overline{v}(\top) = T$ for every $v$. Wouldn't we get a contradiction if we have a function that negates $v$? For instance, if $w = \neg v$ for some function $v$, doesn't $\overline{w}(\perp) = \neg\overline{v}(\perp)$ imply that $\overline{w}(\perp) = T$?

### Exercises

**1.** Let G be the following three-place Boolean function.

$$G(F, F, F) = T, \qquad G(T, F, F) = T,$$
$$G(F, F, T) = T, \qquad G(T, F, T) = F,$$
$$G(F, T, F) = T, \qquad G(T, T, F) = F,$$
$$G(F, T, T) = F, \qquad G(T, T, T) = F.$$

(a) Find a wff using at most the connectives $\wedge$, $\vee$, and $\neg$, that realizes $G$.

Truth table for $G$:

| $\alpha$ | $\beta$ | $\gamma$ | $G(\alpha, \beta, \gamma)$ |
|---|---|---|---|
| $T$ | $T$ | $T$ | $F$ |
| $T$ | $T$ | $F$ | $F$ |
| $T$ | $F$ | $T$ | $F$ |
| $T$ | $F$ | $F$ | $T$ |
| $F$ | $T$ | $T$ | $F$ |
| $F$ | $T$ | $F$ | $T$ |
| $F$ | $F$ | $T$ | $T$ |
| $F$ | $F$ | $F$ | $T$ |

$G(\alpha, \beta, \gamma)$ always disagrees with the majority of $\alpha$, $\beta$, and $\gamma$.

That is,

$$G(\alpha, \beta, \gamma) = \neg((\alpha \wedge \beta) \vee (\alpha \wedge \gamma) \vee (\beta \wedge \gamma))$$

(b) Then find such a wff in which connective symbols occur at not more than 5 places.

$$G(\alpha, \beta, \gamma) = \neg(\#\alpha\beta\gamma)$$

Using only $\wedge, \vee, \neg$:

$$G(\alpha, \beta, \gamma) = \neg(\ (\alpha \wedge (\beta \vee \gamma))\ \vee\ (\beta \wedge \gamma))$$