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CXX 01: Class Title **Term Year**

PSET 1 — April 8, 2022

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Credit Statement

I worked on these problems alone, with reference to class notes and the following books:

(a) ***The Code Book*** by **Simon Singh**.

(b) ***Cryptography*** by **Simon Rubinsen-Salzedo**

Problems

Problem 1.

What is the message embedded in the following?

> Dear George,
> Greetings to all at Oxford. Many thanks for your
> letter and for the Summer examination package.
> All Entry Forms and Fees Forms should be ready
> for final despatch to the syndicate by Friday
> 20th or at the very latest, I'm told, by the 21st.
> Admin has improved here, though there's room
> for improvement still; just give us all two or three
> more years and we'll really show you! Please
> don't let these wretched 16+ proposals destroy
> your basic O and A pattern. Certainly this
> sort of change, if implemented immediately,
> would bring chaos.

I thought it was unlikely that the text was actually encrypted (a strategy that did not work that well on problem 2). On this problem, I tried to look at the words in different positions in the text — every first word in a sentence,

every last word, every nth word, in sentence n, etc.

Using this strategy, I deciphered this message (highlighted in red above):

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Your package ready Friday 21st, room three. Please destroy this immediately.
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Problem 2.

Encrypt the message 001100001010 using two rounds of SDES and (9 bit) key 111000101, as explained in lecture. Show all your steps! *[Hint: After one round, the output is* 001010010011*.]*

First, let's define our permutation function:

permute (123456) = 12434356

And tables:

s_1	1	2	s_2	1	2
000	101	001		100	101
001	010	100		000	011
010	001	110		110	000
011	110	010		101	111
100	011	000		111	110
101	100	111		001	010
110	111	101		011	001
111	000	011		010	100

Round 1

L_0 = 001100

R_0 = 001010

K_0 = 11100010

permute (R_0) = 00010110

00010110 **xor** K_0 = 11110100

$S_1(1111)$ = 011

$S_2(0100)$ = 111

011111 **xor** L_0 = 010011

$L_1 \leftarrow R_0$

$R_1 \leftarrow$ 010011

Excryption after 1 round: 001010010011

Round 2

L_1 = 001010

R_1 = 010011

K_1 = 11000101

permute (R_1) = 01000011

01000011 **xor** K_1 = 10000110

$S_1(1000)$ = 001

$S_2(0110)$ = 011

001011 **xor** L_1 = 000001

$L_2 \leftarrow R_1$

$R_2 \leftarrow$ 000001

Excryption after 2 rounds: 010011000001

Problem 3.

In the Rijndael field $F = \mathbb{F}_2[X]/(X^8 + X^4 + X^3 + X + 1)$, where bytes are associated to polynomials modulo $X^8 + X^4 + X^3 + X + 1$, compute the product $01010010 \cdot 10010010 \in F$.

We can represent polynomials in F as binary numbers, where the state of each bit (whether 0 or 1) represents whether the corresponding power in the polynomial has a factor of 0 or 1.

Then:

$$X^8 + X^4 + X^3 + X + 1 = 100011011$$

Then, we can perform the multiplication modulo 2:

$$\begin{array}{r} 1\,0\,0\,1\,0\,0\,1\,0 \\ 1\,0\,1\,0\,0\,1\,0 \\ \times\, 1\,0\,0\,1\,0\,0\,1\,0\,\cdot \\ \hline 1\,0\,0\,1\,0\,0\,1\,0\,\cdot\,\cdot\,\cdot\,\cdot \\ 1\,0\,0\,1\,0\,0\,1\,0\,\cdot\,\cdot\,\cdot\,\cdot\,\cdot\,\cdot \\ \hline 1\,0\,1\,1\,0\,2\,1\,0\,2\,0\,0\,1\,0\,0 \end{array}$$

Shifting back to base 2, we get: 10110010000100

We then need to find this number $\bmod 100011011$

$$\begin{array}{r|l} & \bmod 10110010000100, 100011011 \\ 100011011 & 10110010000100 \\ 100000 & 100011011\,\cdot\,\cdot\,\cdot\,\cdot\,\cdot \\ & 111111100\,\cdot\,\cdot\,\cdot \\ 1000 & 100011011\,\cdot\,\cdot\,\cdot \\ & 111001111\,\cdot\,\cdot \\ 100 & 100011011\,\cdot\,\cdot \\ & 110101000\,\cdot \\ 10 & 100011011\,\cdot \\ & 101100110 \\ 1 & 100011011 \\ \hline 101111 & 1111101 \end{array}$$

Thus, the product in F is 1111101