Math 69: Logic **Winter '23**

Homework assigned January 9, 2023

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Credit Statement

I worked on these problems alone, with reference to class notes and the following books:

(a) **A Mathematical Introduction to Logic** by **Herbert Enderton**.

Problem 8.

Prove Theorem 17F: *A set of expressions is decidable iff both it and its complement (relative to the set of all expressions) are effectively enumerable.*

Remark: Two semidecision procedures make a whole.

Let Σ be a set of expressions, with Γ as its complement.

We first prove that if Σ is decidable then both Σ and Γ are effectively enumerable:
Suppose Σ is decidable, then we can always determine whether a wff α is in Σ or is not in Σ. Therefore, we can implement an enumeration algorithm as follows:

(i) Pick an arbitrary wff, β, that has not yet been listed as a member of Σ or β.
(ii) If $\beta \in \Sigma$, then list it as an element of Σ.
(iii) However, if $\beta \notin \Sigma$, then list it as an element of Γ.
(iv) Repeat from step 1.

We then prove that if both Σ and its complement, Γ, are effectively enumerable then Σ is decidable.
Assume that both Σ and Γ are effectively enumerable. Then, by definition of effectively enumeration, we may list members of Σ and non-members of Σ (i.e. members of Γ), and every member or non-member will eventually be listed in the appropriate category even if the enumeration might never end in the case of an infinite Σ or Γ. Consequently, by checking the listed wffs we may always determine whether a wff β is in Σ or not in Σ, implying that Σ is decidable.