Math 69: Logic **Winter '23**

Homework assigned January 18, 2023

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Credit Statement

I worked on these problems alone, with reference to class notes and the following books:

(a) **A Mathematical Introduction to Logic** by **Herbert Enderton**.

Problem 2.

Assume that we have a language with the following parameters:

(i) $\forall$, intended to mean, "for all things";

(ii) N, intended to mean "is a number";

(iii) I, intended to mean "is interesting";

(iv) $<$, intended to mean "is less than"; and

(v) 0, a constant symbol intended to denote zero.

Translate back into good English the wff

$$\forall\, x(Nx \to Ix \to \neg \forall\, y(Ny \to Iy \to \neg x < y))$$

Everything that is a number and is interesting is NOT greater than or equal to every interesting number.

Equivalently, every interesting number is less than some interesting number.

Equivalently, for every interesting number, there exists some interesting number that is less than it.

Problem 5.

Translate each English sentence into the first-order language specified. *(You may want to carry out the translation in several steps, as in some of the examples.) Make full use of the notational conventions and abbreviations to make the end result as readable as possible.*

Symbols:

(i) $\forall$, for all things;
(ii) P, is a person;
(iii) T, is a time;
(iv) Fxy, you can fool x at y.

One or more of the above may be ambiguous, in which case you will need more than one translation.

(a) You can fool some of the people all of the time.

There exists a person you can fool all the time. Equivalently, it is not the case that for all people, you cannot fool them all the time.

$$\neg\forall\, x(\neg\forall\, t((Px \wedge Ty) \rightarrow Fxy))$$

(b) You can fool all of the people some of the time.

There exists a time when you can fool all of the people. Equivalently, it is not the case that for all times, there exists a person that you cannot fool.

$$\neg\forall\, t(\neg\forall\, x((Px \wedge Ty) \rightarrow Fxy))$$

(c) You can't fool all of the people all of the time.

There exists a time when you cannot fool all of the people. Equivalently, it is not the case that for all times, you can fool all of the people.

$$\neg\forall\, t(\forall\, x((Px \wedge Ty) \rightarrow Fxy))$$