

Reading assigned January 25, 2023*Prof. Marcia Groszek**Student: Amittai Siavava***Problem 1.**

Suppose our language contains the equality symbol, constant symbol 0, and no other parameters, and our structure \mathfrak{A} is defined by setting $|\mathfrak{A}| = \mathbb{N}$, and $c^{\mathfrak{A}} = 0$. Which sets of natural numbers are definable in this structure?

By definition; $|\mathfrak{A}| = \mathbb{N}$ and $c^{\mathfrak{A}} = 0$.

First, $0 \in |\mathfrak{A}|$ (since \mathfrak{A} assigns $c^{\mathfrak{A}}$ to a member of $|\mathfrak{A}|$).

We also have a two-place predicate symbol $=$, but we do not have a “forall” quantifier so we cannot enforce the predicate on every member of $|\mathfrak{A}|$. Therefore, all sets $S \subseteq \mathbb{N}$ such that $0 \in S$ are definable in the structure.

Questions

The textbook gives an example that, on the structure $(\mathbb{R}, <)$, a function h is an automorphism if it is strictly increasing (i.e. $a < b \implies h(a) < h(b)$), e.g. $h(a) = a^3$, but that \mathbb{N} is not definable in this structure since h maps elements out of \mathbb{N} into \mathbb{N} . I don't fully understand what this means.