

Homework assigned January 19, 2023*Prof. Marcia Groszek**Student: Amittai Siavava***Credit Statement**

I worked on these problems alone, with reference to class notes and the following books:

- (a) **A Mathematical Introduction to Logic** by **Herbert Enderton**.

Problem 1.

Show that tautological equivalence is an equivalence relation on the set of wffs of sentential logic; that is, if we define

$$\alpha \equiv \beta \iff \alpha \models \beta,$$

then *equiv* is an equivalence relation on the set of wffs.

Problem 3.

Let X be the set of all wffs of sentential logic and \equiv be tautological equivalence. Define a binary (2-place) function on equivalence classes, which we could call conjunction, by

$$[\alpha] \wedge [\beta] = [\alpha \wedge \beta]$$

Prove that this function is well-defined.

As you do this, at some point you are going to have to prove that two wffs are tautologically equivalent.

For this exercise, please do this by showing explicitly that any truth assignment that satisfies one of the formulas also satisfies the other, and conversely.

You may think it's obvious that these wffs are tautologically equivalent. I agree, and after this proof, you can get away with saying so, or giving a more informal explanation, in similar circumstances.

Problem 5.

Let X be the set of all wffs of sentential logic and \equiv be tautological equivalence. Define a binary (2-place) relation on equivalence classes by

$$[\alpha] \models [\beta] \iff \alpha \models \beta.$$

Determine whether this relation is well-defined and prove your answer is correct.