Math 69: Logic **Winter '23**

Homework assigned January 20, 2023

Prof. Marcia Groszek ***Student: Amittai Siavava***

Credit Statement

I worked on these problems alone, with reference to class notes and the following books:

(a) **A Mathematical Introduction to Logic** by **Herbert Enderton**.

Problem 7.

Write down 4 sentences for a language $\mathfrak{L}$ such that any structure $\mathfrak{U} = \langle X, \leq \rangle$ is a linear ordering if and only if it satisfies those four sentences.

$$\forall\, x\; Pxx \qquad \text{(reflexive)}$$

$$\forall\, x\, \forall\, y\; ((Pxy \wedge Pyx) \rightarrow (x = y)) \qquad \text{(antisymmetric)}$$

$$\forall\, x\, \forall\, y\, \forall\, z\, ((Pxy \wedge Pyz) \rightarrow Pxz) \qquad \text{(transitive)}$$

$$\forall\, x\, \forall\, y\; (Pxy \vee Pyx) \qquad \text{(total)}$$

Problem 9.

Suppose that X is a set and $\leq$ is a preordering of X. Define a new binary relation on X by

$$x \equiv y \iff (x \leq y \land y \leq x).$$

Show that $\equiv$ is an equivalence relation on X, that $\leq$ induces a well-defined relation on equivalence classes, and that this induced relation is a partial ordering of $X/\equiv$.

Problem 11.

Define the notion of isomorphism between two equivalence relations

$$\mathfrak{A} = \langle X, \equiv_X \rangle \text{ and } \mathfrak{B} = \langle Y, \equiv_Y \rangle .$$