Math 69: Logic **Winter '23**

Homework assigned January 20, 2023

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Credit Statement

I worked on these problems alone, with reference to class notes and the following books:

(a) **A Mathematical Introduction to Logic** by **Herbert Enderton**.

Problem 7.

Write down 4 sentences for a language $\mathfrak{L}$ such that any structure $\mathfrak{U} = \langle X, \leq \rangle$ is a linear ordering if and only if it satisfies those four sentences.

$\forall\, x P x x$	(reflexive)
$\forall\, x \forall\, y\, ((Pxy \wedge Pyx) \rightarrow (x = y))$	(antisymmetric)
$\forall\, x \forall\, y \forall\, z\, ((Pxy \wedge Pyz) \rightarrow Pxz)$	(transitive)
$\forall\, x \forall\, y\, (Pxy \vee Pyx)$	(total)

Problem 9.

Suppose that X is a set and $\le$ is a preordering of X. Define a new binary relation on X by

$$x \equiv y \iff (x \le y \wedge y \le x).$$

Show that $\equiv$ is an equivalence relation on X, that $\le$ induces a well-defined relation on equivalence classes, and that this induced relation is a partial ordering of $X/\equiv$.

Problem 11.

Define the notion of isomorphism between two equivalence relations

$$\mathfrak{A} = \langle X, \equiv_X \rangle \text{ and } \mathfrak{B} = \langle Y, \equiv_Y \rangle .$$