Math 69: Logic **Winter '23**

Homework assigned February 08, 2023

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Problem 2.

Prove the equivalence of parts (a) and (b) of the completeness theorem.

Suggestion: $\Gamma \vDash \varphi$ iff $\Gamma \cup \{\neg\varphi\}$ is unsatisfiable. And Δ is satisfiable iff $\Delta \nvdash \bot$, where $\bot$ is some unsatisfiable, refutable formula like $\neg\forall x x = x$. (Similarly, the soundness theorem is equivalent to the statement that every satisfiable set of formulas is consistent.)

Problem 8.

Assume the language (with equality) has just the parameters $\forall$ and P, where P is a two-place predicate symbol. Let $\mathfrak{A}$ be the structure with $|\mathfrak{A}| = \mathbb{Z}$, the set of the integers (positive, negative, zero) and with $\langle a, b \rangle \in P^{\mathfrak{A}}$ iff $|a - b| = 1$.

Thus, $\mathfrak{A}$ looks like an infinite graph:

$$\cdots \longleftrightarrow \bullet \longleftrightarrow \bullet \longleftrightarrow \bullet \longleftrightarrow \cdots$$

Show that there is an elementarily equivalent structure $\mathfrak{B}$ that is not connected (being *connected* means that for every two members of $|\mathfrak{B}|$, there is a path between them. A path — of length n — is a sequence $\langle p_0, p_1, \ldots, p_n \rangle$ of elements of $|\mathfrak{B}|$ such that $a = p_0$ and $b = p_n$ and $p_i, \vec{p}_{i+1} \in P^{\mathfrak{B}}$ for each i).

Write down sentences saying c and d are far apart, applying compactness.