Math 69: Logic	**Winter '23**

Homework assigned February 06, 2023

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Problem 8.

Let $\mathcal{L}$ be a language for first-order logic, including the equality symbol (and any number of other symbols), and $\mathcal{L}^*$ be $\mathcal{L}$ without the equality symbol, but with an additional 2-place predicate symbol E. For any formula α of $\mathcal{L}$, define α^* to be the formula of $\mathcal{L}^*$ obtained fromα by replacing every occurrence of = with E. If Σ is a set of formulas of $\mathcal{L}$, define

$$\Sigma^* = \left\{a^* \mid a \in \Sigma\right\}.$$

(a) This is a short answer problem.

For which logical axiom groups is it true that α is a logical axiom in that group if and only if α^* is a logical axiom in that group?

(b) This is a short answer problem.

Which, if any, of the following are true?

(i) If $\alpha_1, \alpha_2, \ldots, \alpha_n$ is a deduction from Σ in the language $\mathcal{L}$ then $\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*$ is a deduction from Σ^* in the language $\mathcal{L}^*$

(ii) If $\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*$ is a deduction from Σ^* in the language $\mathcal{L}^*$ then $\alpha_1, \alpha_2, \ldots, \alpha_n$ is a deduction from Σ in the language $\mathcal{L}$

(c) Show that if Σ is a consistent set of formulas in $\mathcal{L}$ then Σ^* is a consistent set of formulas in $\mathcal{L}^*$.

You may use your answers in (a) and (b).

Hint: Recall that Σ is inconsistent if there is some α such that $\Sigma \vdash \alpha$ and $\Sigma \vdash \neg\alpha$.

(d) Show by example that it is possible for Σ to be inconsistent but Σ^* to be consistent.

Problem 9.