MCS 第一次作业

李青林[*]

May 21, 2012

2.11

对$\forall d \in \mathbf{N}$

若$d = 2k(k \in \mathbf{N}) \qquad \Gamma(d) = (k-1)!$

若$d = 2k+1(k \in \mathbf{N}) \quad \Gamma(d) = (k-1)!\sqrt{\pi}$

则对于$\forall d = 2k, V(d) = \dfrac{\pi^k}{k*(k-1)!} = \dfrac{\pi^k}{k!}$

对于$\forall d = 2k+1, V(d) = \dfrac{\pi^{k+1/2}}{\pi^{1/2}*(k+1/2)*\prod_{i=1}^{k}(i-\frac{1}{2})} = \dfrac{\pi^k}{\prod_{i=1}^{k+1}(i-\frac{1}{2})}$

$\therefore \dfrac{V(2k+2)}{V(2k)} = \dfrac{\pi}{k+1} \qquad \dfrac{V(2k+3)}{V(2k+1)} = \dfrac{\pi}{k+3/2}$

$\therefore k = 3$时,$V(2k)$最大 $\quad k = 2$时$V(2k+1)$最大

$\therefore V_{max} =$Max$(V(5), V(6)) = \dfrac{8\pi^2}{15}$ $\square$

2.12

$V(r,d) = r^d \dfrac{\pi^{d/2}}{\frac{d}{2}\Gamma(\frac{d}{2})}$

当$r = 2$时,体积先增加，再减小，最后收敛于0

当$r \neq 2$但为常数时,同上

若$V(r,d) \equiv v, r = \left(\dfrac{vd\ \Gamma(\frac{d}{2})}{2\pi^{d/2}}\right)^{\frac{1}{d}}$ $\square$

[*]jack951753@gmail.com

2.17

令$f(x)$表示圆柱体积,设高度为h

底面积为$(1-h^2)^{\frac{d-1}{2}}V(d-1)$

$f(x)=h(1-h^2)^{\frac{d-1}{2}}V(d-1)$

当$f(x)'=(1-h^2)^{\frac{d-3}{2}}(1-dh^2)V(d-1)=0$时,$f(x)$取极大值

得$h=\frac{1}{\sqrt{d}}$或$h=1$

$\because h=1, f(1)=0$,不可能为极大值

$\therefore h=\frac{1}{\sqrt{d}}$时,f(x)取极大值 $\square$

2.20

$\forall\epsilon>0$，令$S=\{x|x$到赤道的距离不超过$\epsilon\}$

当$d\to\infty$,$S\to$整个球

因而，当维数足够大时，任意一个赤道周围都聚集绝大部分的点，可以任选北极 $\square$