MCS 第7次作业

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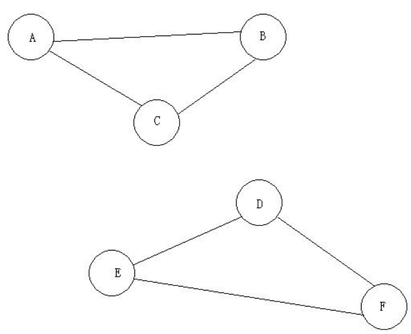
5.1

a.



b. 充分性：
假设二分图存在奇环，设奇环为$v_1 \to v_2 \to v_3 \to \cdots \to v_m \to v_1$
不妨设v_1属于左半集$\Longrightarrow v_n$属于左半集$\Longrightarrow v_1$属于右半集，矛盾

必要性：
假设一个不存在奇环的图不是二分图，则它不可以黑白染色
$\Longrightarrow \exists u_s, u_t$
从u_s沿$u_s \to u_1 \to u_2 \to \cdots \to u_n \to u_t$得$u_t$被染成白色
从u_s沿$u_s \to v_1 \to v_2 \to \cdots \to v_m \to u_t$得$u_t$被染成黑色
则$u_s \to u_1 \to u_2 \to \cdots \to u_n \to u_t \to v_m \to \cdots \to v_1 \to u_s$是奇环，矛盾

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c. 假如最后到每个点的概率收敛，设第n步落在i点的概率为$P(n,i)$
必有$\forall n \sum_{i=1}^{|E|} P(n,i) = 1$
$\Longrightarrow \exists x, \lim\limits_{n\to\infty} P(n,x) > 0$
$\Longrightarrow \exists N, \forall n > N P(n,x) > 0$
由于该图是二分图，$P(n,x)$要么在n为奇数是为零，要么在n为偶数时为零，矛盾

$\square$

5.3
设左边和右边的电势差为U

$$I_1 = \frac{U}{R_1 + R_3}$$

$$I_2 = \frac{U}{R_2}$$

$$I_{eff} = \frac{U}{I_1 + I_2} = \frac{(R1 + R3)R_2}{R1 + R3 + R2}$$

$\square$

5.5

(a) $$\frac{u_c - u_a}{1} + \frac{u_c - u_b}{2} + \frac{u_c - u_d}{1} = 0$$
$$\frac{u_d - u_a}{2} + \frac{u_d - u_b}{1} + \frac{u_d - u_c}{1} = 0 \ u_c = \frac{4}{7} \quad u_d = \frac{3}{7}$$

(b) $$I_{ac} = \frac{u_a - u_c}{1} = \frac{3}{7}$$
$$I_{ad} = \frac{u_a - u_d}{2} = \frac{2}{7}$$
$$I_{cd} = \frac{u_c - u_d}{1} = \frac{1}{7}$$
$$I_{cb} = \frac{u_c - u_d}{2} = \frac{2}{7}$$
$$I_{db} = \frac{u_d - u_b}{1} = \frac{3}{7}$$

(c) $$R_{eff} = \frac{u_a - u_b}{I_{eff}} = \frac{u_a - u_b}{I_{ac} + I_{ad}} = \frac{7}{5}$$

(d) $$p_{ad} = \frac{1}{3} \quad p_{ac} = \frac{2}{3}$$
$$p_{ca} = \frac{2}{5} \quad p_{cb} = \frac{1}{5} \quad p_{cd} = \frac{2}{5}$$

$$p_{da} = \frac{1}{5} \quad p_{db} = \frac{2}{5} \quad p_{dc} = \frac{2}{5}$$
$$p_{bc} = \frac{1}{3} \quad p_{bd} = \frac{2}{3}$$

(e) $u_c = \frac{4}{7} \quad u_d = \frac{3}{7}$

(f) $i_{cd} = \frac{1}{7}$

(g) 由于足够多步之后在$c \leftrightarrow d$打圈的概率足够小

$$p = 1 - p_{escape} = 1 - \frac{1}{r_{eff}c_a} = \frac{11}{21}$$

□

5.9

不妨将u标号为1，v标号为n，令$h(i)$表示i到v的"hitting time"

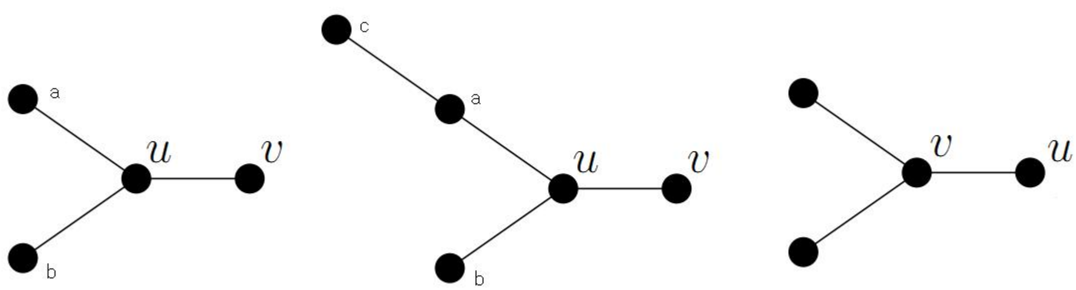
$$\begin{cases} h(1) = 1 + \dfrac{h(2)}{2} \\ h(i) = 1 + \dfrac{h(i-1)+h(i+1)}{2} \quad (2 \le i \le n-1) \\ h(n) = 0 \end{cases}$$

解方程得$h(1) = n - 1$

即$h_{uv} = n - 1$

如果去掉(u, v)，变成一条链，$h_{uv} = (n-1)^2$ □

5.10



首先如图标号

对于第一个图

$$\begin{cases} h_{uv} = 1 + \dfrac{h_{av} + h_{bv}}{3} \\ h_{av} = 1 + h_{uv} \\ h_{bv} = 1 + h_{uv} \end{cases}$$

$h_{uv} = 5$

对于第二个图

$$\begin{cases} h_{uv} = 1 + \dfrac{h_{av} + h_{bv}}{3} \\ h_{av} = 1 + \dfrac{h_{uv} + h_{cv}}{2} \\ h_{cv} = 1 + h_{av} \\ h_{bv} = 1 + h_{uv} \end{cases}$$

$h_{uv} = 7$

对于第三个图(书上的u、v画反了吧。。。)

$h_{uv} = 1$ $\square$