
ECE285 Project Progress Report: High Speed Imaging with Compressed Sensing

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Abstract

In this course project, we plan to build a mathematical model for high speed imaging system with compressed sensing. Due to the sparseness of the underlying image, compressed sensing can be applied in the image reconstruction. We'll conduct simulations for a full-functioning imaging system and analyze the results for further improvements.

1 Literature Review

We review a few papers related to compressive imaging systems. In particular, we're interested in the recovery problem formulation, sensing matrix design and algorithms to solve the problem.

1.1 SparseMRI

1.1.1 recovery problem formulation

[2] introduces a method for fast Magnetic Resonance Imaging(MRI). The measurement matrix in MRI system is random partial DFT matrix and the sparsity assumption can be made in either the imaging space or wavelet space. Specifically, we need to solve the inverse problem:

$$\begin{aligned} b &= Ax \\ \text{subject to } \alpha &= \Phi x, \alpha \text{ is sparse} \end{aligned}$$

where A is the random partial DFT matrix and Φ is the domain where the image is sparse(can be either the original image domain or the wavelet matrix).

This inverse problem can be solved with guarantee by L1 minimization since partial DFT matrix satisfy RIP conditions. That is,

$$\begin{aligned} x^* &= \operatorname{argmin} |\Phi x|_1 \\ \text{subject to } |b - Ax|_2 &< \epsilon \end{aligned}$$

One thing that's particular interesting to our project is that, changing the L1 objective function to **Total Variation (TV)** is closed related to sparsity assumption in wavelet space. More generally, even if the sparsity domain is not wavelet space, a TV term can also be added to enforce smoothness. [We hope this is useful in our imaging system.]

$$\begin{aligned} x^* &= \operatorname{argmin} |\Phi x|_1 + \lambda TV(x) \\ \text{subject to } |b - Ax|_2 &< \epsilon \\ \text{where } TV(x) &= |D(x)|, D(x) \text{ is the discrete differential operator.} \end{aligned}$$

1.1.2 discussions about sensing matrix

As the paper discusses, the validity of compressive sensing in MRI largely depends on the nice property of the sensing matrix, which is partial DFT matrix. In their discussion, an intuitive heuristic guideline for evaluation of the sensing matrix is defined. As we desire **incoherent artifacts caused by undersampling**, a natural measure would be the **Point Spread Function (PSF)** as defined in the paper. Actually, this definition is exactly the mutual coherence of a matrix which is discussed in our early lectures. We know it can lead to sufficient conditions for exact recovery.

Thus, we would like to experiment with minimization of mutual coherence of the sensing matrix in our imaging system, although we're given limited freedom of choice (the sensing matrix is related to the optical properties and layout of the materials in the system).

The paper also discusses a lot about the sampling strategy considering smooth sample trajectories due to hardware and physiological limitations. Moreover, their system also has a "calibration" step to correct phase misalignment, which is similar to our system.

1.2 Single-Pixel Imaging

1.2.1 hardware architecture

In [1], the author describes a full-blown imaging system with only a single photon detector. The system is mainly composed of biconvex lenses, a DMD (digital micromirror device) spatial light modulator, a single photon detector and a A/D converter. This whole architecture is essentially a optical computer. Since the DMD is programmable, the system can get M random measurement of the scene by generating different patterns and take the inner product with the image. With sufficiently large number of measurement ($M = O(K \log(N/K))$), we're guaranteed to have exact imaging result.

1.2.2 recovery problem formulation

The formulation is very similar to MRI case, except with a different sensing matrix. Also, this system uses the TV formulation in addition to L1 minimization.

1.2.3 sensing matrix design

The system uses random rows of 0/1 Walsh matrix as the sensing matrix. But the paper also suggests Hadamard or noiselet transform. These are all sampled rows from a deterministic matrix instead of the i.i.d Gaussian case discussed in our lectures.

1.2.4 image classification by smashing filter

The paper demonstrates one interesting experiment of image classification. They argue that the naive approach would first reconstruct the full image and then do classification in the original image space R^n . The smashed filter exploits a recent result that the structure of a smooth k -dimensional manifold in R^n is preserved with high probability under a random projection to the lower dimensional space R^m as long as $m = O(k \log n)$. (note this is quite similar to the guarantee of exact recovery of sparse signal although here it's a subspace). With this guarantee, the smashed filter performs all the operations on the measurements directly without recovery of the full image.

The paper shows that with more measurements, the accuracy of classification by smashed filter increases, which satisfies our intuition.

2 Experiments Results of Our System

2.1 different design of sensing matrix A

Based on the underlying theory of photonics, we simulate two settings of the sensing matrix. (A_1 and A_2) (we leave out the technical details here). Both A_1 and A_2 are generated by randomly placing light sources on our optical material hence has some randomness (the exact distribution is unknown

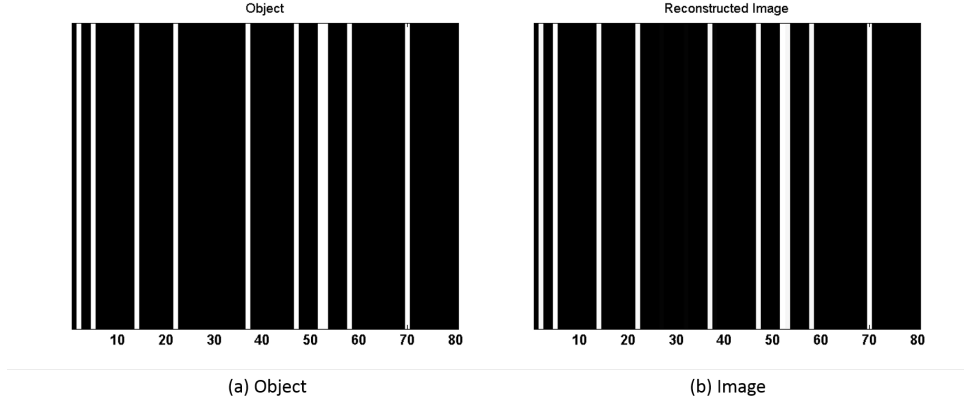


Figure 1: A1: 1D imaging computational experiment

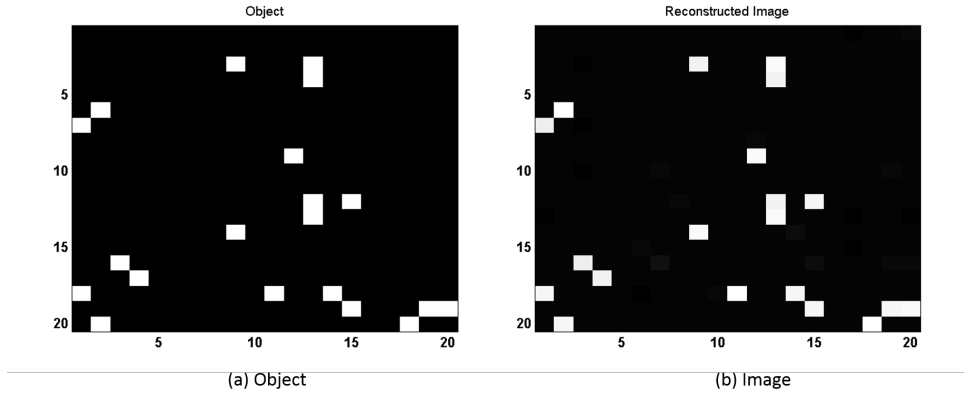


Figure 2: A2: 2D imaging computational experiment

due to complex physical interaction). An example of $A1$ and $A2$ is shown in Figure 1 and Figure 2 respectively.

For $A1$, the sensing structure is relatively simpler and can only sensing one dimension of the underlying image(i.e. determine if *any* pixel in a column is non-zero). figure[??] shows an example of $L1$ recovery result.

From our experiment, with $A1$ of size 30×80 , sparsity 10,image size 80, the system is quite reliable to recover the original image. This is a promising result.

For $A2$, we include more complex layout strategy for the light sources hence resulting sensing matrix that can sense 2-D images. Figure 2 shows an example of the recovery result.

We compare two different algorithms for solving the $L1$ minimization problem. One provided by [4] which is a $l1$ -Regularized Least Squares method. The other is termed as **smoothed $L0$ norm (SL0)**[5] which tries to directly minimize the $L0$ norm with a approximation function (in our experiment SL0 performs better).

From our experiment, with $A2$ of size 160×400 , sparsity 20, image size 20×20 , the system can successfully recover the original image.

3 Future Work

Based on the review of the imaging systems, we would like to experiment with our system in the following aspects:

1. We'll try to add TV norm minimization to our current system. As in the other existed imaging systems, this assumption will lead to smooth images.
2. Also, we'll try to find the optimal layout of the optical material so that the sensing matrix is most "incoherent".
3. To solve sparse recovery problems, a lot of solvers are available online, each with different approaches to relax the original L0 problem. We'll continue to evaluate their performance with respect to imaging quality in our system and hopefully will find the optimal one for our purpose.

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[3] Duarte M F, Davenport M A, &Takhar D, et al. Single-pixel imaging via compressive sampling[J]. *IEEE Signal Processing Magazine*, 2008, 25(2): 83.

[4] Koh K, Kim S J, Boyd S P. An interior-point method for large-scale l1-regularized logistic regression[J]. *Journal of Machine learning research*, 2007, 8(8): 1519-1555.

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