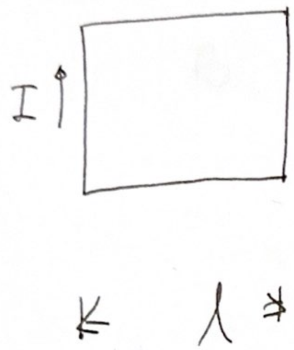
Tarea 9.

22 de mayo de 2024.

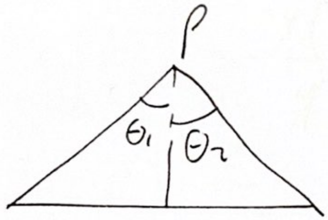
$\boxed{1}$ a) 28.3 μT entrante b) 24.7 μT ~~sent~~ entrante

$I = 10 A$

$l = 0.4\ m$

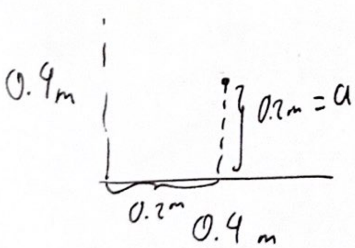


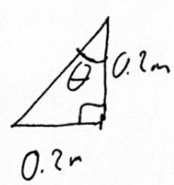
→ Del ejercicio visto en clase el 8 de mayo, tenemos que, para una barra el campo magnético en el centro es.



$$B = \frac{\mu_0 I}{4\pi a}\left(\operatorname{sen}(\theta_1) - \operatorname{sen}(\theta_2)\right)$$

→ Hay que determinar los ángulos





$$\tan\theta = \frac{C.O.}{C.A.} = \frac{0.2m}{0.2m} = 1$$

$$\theta = \arctan(1) = \frac{\pi}{4}$$

$\boxed{I}$

Por simetría, la contribución de cada barra será la misma por lo tanto el campo total será

$$B = \frac{4\mu_0 I}{4\pi a}\left(\text{sen}(\theta_1) - \text{sen}(\theta_2)\right)$$

$$= \frac{4\mu_0 I}{4\pi(0.2m)}\left(\text{sen}(\pi/4) - \text{sen}(-\pi/4)\right)$$

$$= \frac{4\mu_0 I}{4\pi(0.2m)}\left(2\,\text{sen}\left(\frac{\pi}{4}\right)\right)$$

$$= \frac{\cancel{4}(4\pi \times 10^{-7}\,T\cdot m/\cancel{A})(10A)}{\cancel{4}\,\pi\,(0.2m)}\left(2\,\text{sen}\left(\frac{\pi}{4}\right)\right)$$

$$= 2.82 \times 10^{-5}\ T$$

$$= 28.2\ \mu T$$

b) Si es una espira circular

$\cancel{a=0.4m}$ sabemos que

$$B = \frac{\mu_0 I}{2a}$$

→ El ~~diámetro~~ perímetro de la espira debe corresponder con la cuadrada

$$\frac{\mu_0 I}{2a}$$

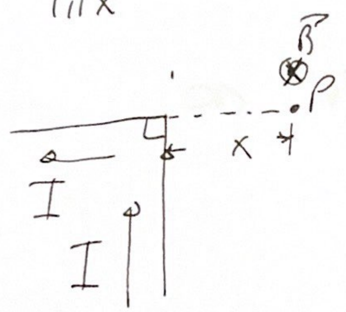
$$4l = 2\pi R \Rightarrow R = \frac{4l}{2\pi} = \frac{2l}{\pi}$$

→ Entonces,

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2\left(\frac{2l}{\pi}\right)} = \frac{\mu_0 \pi I}{4l} = \frac{(4\pi \times 10^{-7}\, T\cdot m/A)(10A)}{4(0.4\, m)}$$

$$= 24.7\, \mu T$$

$\boxed{2}$ $\frac{\mu_0 I}{4\pi x}$



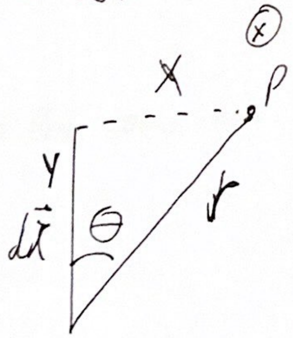
→ El alambre horizontal no contribuye porque

$d\vec{l}$ $\hat{r}$

$$d\vec{l} \times \hat{r} = \vec{0}$$

$$|d\vec{l}||\hat{r}| \operatorname{sen}(0) = 0$$

→ Solo contribuye el otro alambre.



$$d\vec{B} = \cancel{dl}\frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\frac{dl \operatorname{sen}\theta}{r^2}$$

Tenemos

$$\cancel{\tan\theta = \frac{x}{dl} \Rightarrow dl = \frac{x}{\tan\theta}}$$

$$\tan\theta = \frac{x}{y} \Rightarrow y = \frac{x}{\tan\theta}$$

→ Pero, ya sabemos que en el centro de un alambre,

$$B = \frac{\mu_0 I}{4\pi a}\left(\operatorname{sen}(\theta_1) - \operatorname{sen}(\theta_2)\right)$$

→ Como es infinito,

$$\theta_1 \rightarrow +\frac{\pi}{2} \quad ; \quad \theta_2 \rightarrow -\frac{\pi}{2}$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi a}(1+1)$$

$$= \frac{2\mu_0 I}{4\pi a}$$

$$= \frac{\mu_0 I}{2\pi a}$$

→ Pero, como solo contribuye la mitad,

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi a}\right) = \frac{\mu_0 I}{4\pi a} \qquad \text{con } a = x$$

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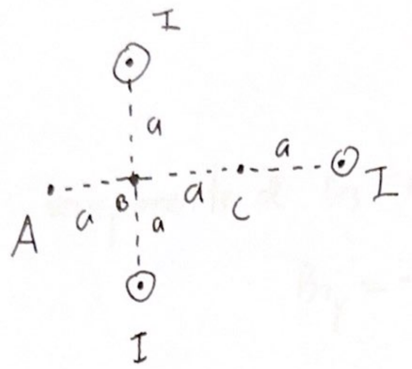
$\boxed{3}$ $I = 2A$; $a = 0.01m$

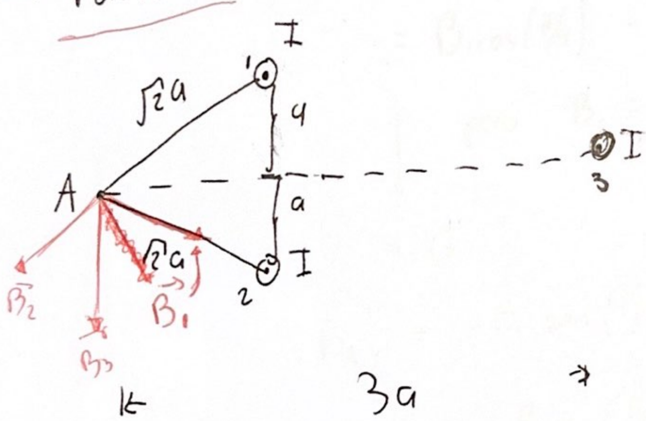


→ El campo magnético de cada alambre es

$$B = \frac{\mu_0 I}{2\pi a}$$

Cambiará dependiendo del punto en el que queramos medir.

→ Punto A



→ Los campos, en magnitud, son

$$B_1 = \frac{\mu_0 I}{2\pi(\sqrt{2}a)}$$

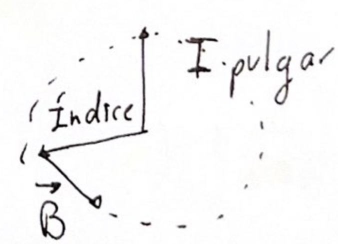
$$B_2 = \frac{\mu_0 I}{2\pi(3a)}$$

$$B_2 = \frac{\mu_0 I}{2\pi\sqrt{2}a}$$

Las componentes de B_1 son

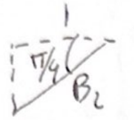
$$B_{1x} = B_1 \cos(\pi/4) \qquad B_{1y} = -B_1 \operatorname{sen}(\pi/4)$$

→ Recordatorio de regla de la mano derecha



→ Las componentes de B_2 son:

$$B_{2x} = -B_2 \cos(\pi/4) \quad ; \quad B_{2y} = -B_2 \operatorname{sen}(\pi/4)$$



→ La componente de B_3 es solo en y,

$$B_{3y} = -B_3$$

→ La contribución total es:

$$B_{Ax} = B_{1x} + B_{2x} + B_{3x}$$

$$= B_1 \cos(\pi/4) - B_2 \cos(\pi/4) + 0$$

pero $B_1 = B_2$

$$= 0$$

$$B_{Ay} = -B_1 \operatorname{sen}(\pi/4) - B_2 \operatorname{sen}(\pi/4) - B_3$$

$$= -2B_1 \operatorname{sen}(\pi/4) - B_3$$

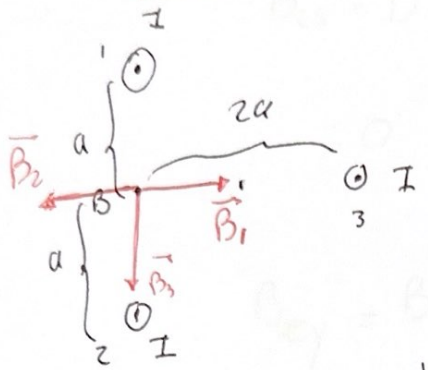
$$= -\frac{\mu_0 I}{\pi\sqrt{2}a} \operatorname{sen}(\pi/4) - \frac{\mu_0 I}{2\pi(3a)}$$

$$= -\frac{\mu_0 I}{\pi a}\left(\frac{\operatorname{sen}(\pi/4)}{\sqrt{2}} + \frac{1}{6}\right)$$

$$= -\frac{(4\pi\times10^{-7}\ T\cdot m/A)(2A)}{\pi\ (0.01m)}\left(\frac{1}{\sqrt{2}}\operatorname{sen}(\pi/4) + \frac{1}{6}\right)$$

$$B_{Ay} = -53.3\ \mu T$$

→ En el punto B tenemos,



$$B_1 = \frac{\mu_0 I}{2\pi a} = B_2$$

$$B_3 = \frac{\mu_0 I}{2\pi (2a)}$$

→ Como B_1 y B_2 tienen la misma magnitud pero con en sentido contrario, se cancelan.

→ Solo tenemos la contribución de B_3,

$$\vec{B}_3 = -\frac{\mu_0 I}{2\pi (2a)} \hat{j}$$

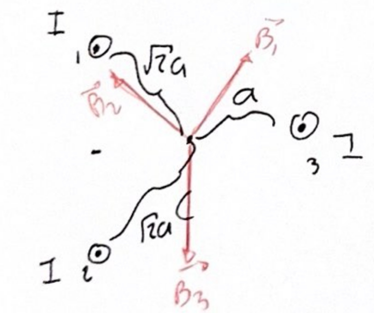
$$= -\frac{(4\pi \times 10^{-7}\ T\cdot m/A)(2A)}{2\pi\ (2(0{,}01\ m))} \hat{j}$$

$$= -20\ \mu T\ \hat{j}$$

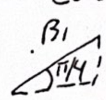
→ En el punto C tenemos.

$$B_1 = \frac{\mu_0 I}{2\pi(\sqrt{2}a)} = B_2$$

$$B_3 = \frac{\mu_0 I}{2\pi a}$$



→ Las componentes de B_1 son



$$B_{1x} = B_1 \cos(\pi/4) \; ; \quad B_{1y} = B_1 \operatorname{sen}(\pi/4)$$

→ Las componentes de B_2 son

$$B_{2x} = -B_2 \cos(\pi/4) \; ; \quad B_{2y} = B_2 \operatorname{sen}(\pi/4)$$

→ La contribución total es:

$$B_{cx} = B_1 \cos(\pi/4) - B_2 \cos(\pi/4)$$

$$= 0 \quad \text{pues } B_1 = B_2$$

$$B_{cy} = B_1 \operatorname{sen}(\pi/4) + B_2 \operatorname{sen}(\pi/4) - B_3$$

$$= 2 B_1 \operatorname{sen}(\pi/4) - B_3$$

$$= \frac{\mu_0 I}{\pi(\sqrt{2}a)} \cdot \frac{1}{\sqrt{2}} - \frac{\mu_0 I}{2\pi a}$$

$$= \frac{\mu_0 I}{2\pi a} - \frac{\mu_0 I}{2\pi a}$$

$$= 0$$

4 $-27\,\mu N\,\hat{\imath}$

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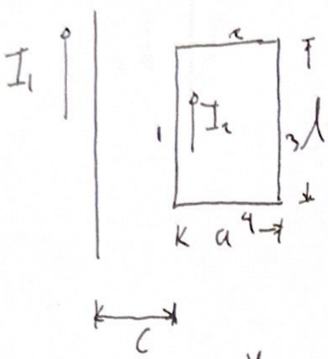
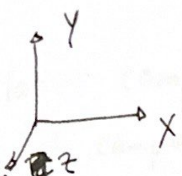
$$I_1 = 5A \quad ; \quad I_2 = 10A$$

$$c = 0.1\,m \qquad a = 0.15\,m \qquad \lambda = 0.45\,m$$



→ Recordemos que la fuerza entre dos alambres paralelos está dada por

$$F = \frac{\mu_0 I_1 I_2 \lambda}{2\pi a}$$



→ Esta expresión viene de:

$$\vec{F} = I_1 \vec{\lambda} \times \vec{B}_2$$

→ Para los alambres paralelos 1 y 3, la magnitud está dada por la expresión

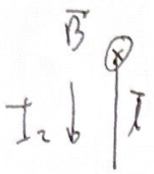
$$F_1 = \frac{\mu_0 I_1 I_2 \lambda}{2\pi c} \quad ; \quad F_3 = \frac{\mu_0 I_1 I_2 \lambda}{2\pi (a + c)}$$

→ Veamos las direcciones, para el alambre 1 tenemos:

donde $B = \frac{\mu_0 I_1}{2\pi c}$ y entra a la página, mientras que $\vec{\lambda}$ tiene dirección $+\hat{\jmath}$. Entonces F_1 tiene dirección $-\hat{\imath}$

$$\vec{F}_1 = \frac{-\mu_0 I_1 I_2 \lambda}{2\pi c}\,\hat{\imath}$$

→ Para $\vec{F_3}$ tenemos algo análogo,



donde $B=\frac{\mu_0 I_1}{2\pi(a+c)}$ y entra a la página, mientras que $\vec{l}$ tiene direción $-\hat{j}$. Entonces: F_2 tiene dirección en $+\hat{z}$.

$$\vec{F_3}=\frac{\mu_0 I_1 I_2 l}{2\pi(a+c)}\hat{z}$$

→ Para las componentes horizontales, debemos hacer la integral, porque el campo depende de la distancia, y esta cambia a lo largo de la espira.

$$\vec{F_2}=\int I_2 d\vec{l}\times\vec{B}=\int I_2 d\vec{x}\times\frac{\mu_0 I}{2\pi x}(\hat{k})$$

$$=-\frac{I_1 I_2 \mu_0}{2\pi}\int d\vec{x}\times\frac{\hat{k}}{x}$$

$$=-\frac{\mu_0 I_1 I_2}{2\pi}\int\frac{dx}{x}(\hat{\imath}\times\hat{k})$$

$$=\frac{-\mu_0 I_1 I_2}{2\pi}\int\frac{dx}{x}(-\hat{j})$$

$$=\frac{\hat{j}\mu_0 I_1 I_2}{2\pi}\int_c^{a+c}\frac{dx}{x}=\frac{\hat{j}\mu_0 I_1 I_2}{2\pi}\ln(x)\Big|_c^{a+c}$$

$$=\hat{j}\frac{\mu_0 I_1 I_2}{2\pi}\left[\ln(a+c)-\ln(c)\right]$$

$$=\hat{j}\frac{\mu_0 I_1 I_2}{2\pi}\ln\left(\frac{a+c}{c}\right)$$

→ Para el lado 4,

$$\vec{F}_4 = \int I_2\, d\vec{l} \times \vec{B} = \int I_2\, d\vec{x} \times \left(\frac{-\mu_0 I_1 \hat{k}}{2\pi x}\right)$$

$$= -\frac{\mu_0 I_1 I_2}{2\pi} \int dx(-\hat{\imath}) \times \frac{\hat{k}}{x}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} \int \frac{dx}{x} (\hat{\imath} \times \hat{k})$$

$$= -\frac{\mu_0 I_1 I_2}{2\pi} \int \frac{dx}{x} \hat{\jmath}$$

$$= -\hat{\jmath} \frac{\mu_0 I_1 I_2}{2\pi} \int_c^{a+c} \frac{dx}{x}$$

$$= -\hat{\jmath} \frac{\mu_0 I_1 I_2}{2\pi} \ln\left(\frac{a+c}{c}\right)$$

→ Entonces, la fuerza total es

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

$$= \frac{-\mu_0 I_1 I_2 l}{2\pi c} \hat{\imath} + \cancel{\frac{\mu_0 I_1 I_2}{2\pi} \ln\left(\frac{a+c}{c}\right) \hat{\jmath}} + \frac{\mu_0 I_1 I_2 l}{2\pi (a+c)} \hat{\imath} - \cancel{\frac{\mu_0 I_1 I_2}{2\pi} \ln\left(\frac{a+c}{c}\right) \hat{\jmath}}$$

$$= \frac{\mu_0 I_1 I_2 l}{2\pi} \left(\frac{-1}{c} + \frac{1}{a+c}\right) \hat{\imath}$$

$$= \frac{\mu_0 I_1 I_2 l}{2\pi} \left(\frac{-a-c+c}{c(a+c)}\right) \hat{\imath}$$

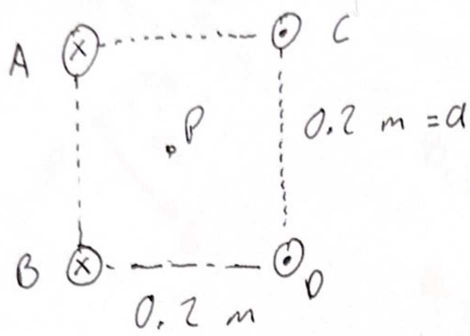
$$= -\frac{\mu_0 I_1 I_2 l}{2\pi} \left(\frac{a}{c(a+c)}\right) \hat{\imath} = \frac{-(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(5\mathrm{A})(10\mathrm{A})(0.45\mathrm{m})(0.15\mathrm{m})\ \hat{\imath}}{2\pi (0.1\mathrm{m})(0.15\mathrm{m} + 0.1\mathrm{m})}$$

$$\vec{F} = -27\,\mu\mathrm{N}\ \hat{\imath}$$

$20\,\mu T \downarrow$

$\boxed{5}$

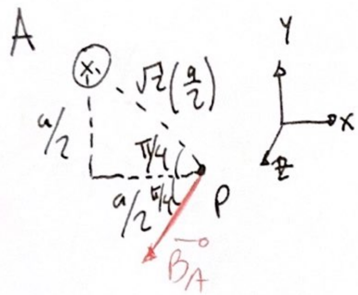
$$I = 5A.$$



→ Ya sabemos que cada alambre produce un campo de magnitud

$$B = \frac{\mu_0 I}{2\pi a}$$

→ Veamos el alambre A,



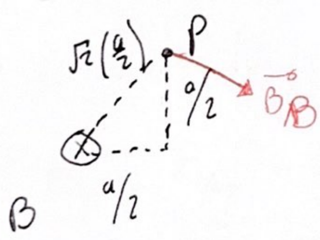
$$B_A = \frac{\mu_0 I}{2\pi\left(\frac{\sqrt{2}}{2} a\right)} = \frac{\mu_0 I}{\sqrt{2}\pi a}$$

$$B_{Ax} = -B_A \cos(\pi/4)$$

$$B_{Ay} = -B_A \operatorname{sen}(\pi/4)$$

→ Alambre B,

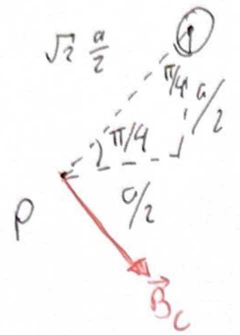
$$B_B = \frac{\mu_0 I}{\sqrt{2}\,\pi a}$$



$$B_{Bx} = B_B \cos(\pi/4)$$

$$B_{By} = -B_B \operatorname{sen}(\pi/4)$$

→ Alambre C.

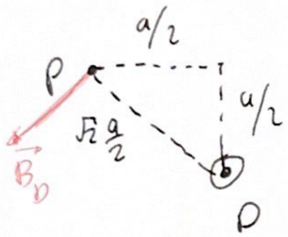


$$B_C = \frac{\mu_0 I}{\sqrt{2}\,\pi a}$$

$$B_{Cx} = \frac{\mu_0 I}{\sqrt{2}\,\pi a}\cos(\pi/4)$$

$$B_{Cy} = -B_C \operatorname{sen}(\pi/4)$$

→ Alambre D.



$$B_D = \frac{\mu_0 I}{\sqrt{2}\,\pi a}$$

$$B_{Dx} = -\frac{\mu_0 I}{\sqrt{2}\,\pi a}\cos(\pi/4)$$

$$B_{Dy} = -B_D \operatorname{sen}(\pi/4)$$

→ Notemos que, como P está a la misma distancia de todos los alambres, las magnitudes de los campos son las mismas. $B_A = B_B = B_C = B_D$

→ Entonces, la contribución total es

$$\begin{aligned}
\underline{B_x} &= B_{Ax} + B_{Bx} + B_{Cx} + B_{Dx} \\
&= -B_A\cos(\pi/4) + B_B\cos(\pi/4) + B_C\cos(\pi/4) - B_D\cos(\pi/4) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\underline{B_y} &= B_{Ay} + B_{By} + B_{Cy} + B_{Dy} \\
&= -B_A\operatorname{sen}(\pi/4) - B_B\operatorname{sen}(\pi/4) - B_C\operatorname{sen}(\pi/4) - B_D\operatorname{sen}(\pi/4) \\
&= -4B_A\operatorname{sen}(\pi/4) = -4\,\frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(5\,\mathrm{A})}{\sqrt{2}\,\pi\,(0.2\,\mathrm{m})}\operatorname{sen}(\pi/4) = -20\,\mu\mathrm{T}
\end{aligned}$$