HW5, due Wed Oct 14 at 9am

In this assignment, you are free to use any language you wish to answer all computational questions. You *do not* need to use all three languages.

1. **5 points** There are three roots to the function $F(x) = e^x - x^4$. Determine all three solutions to $F(x) = 0$ using a bisection algorithm to within a tolerance of 10^{-6}. *Hint:* you will find the roots in the vicinity of -1, 1, and 9. Run your bisection algorithm on three different regions containing those points.

2. **6 points**. Use Newton's method to find any x^* and y^* such that $F(x^*, y^*) = G(x^*, y^*) = 1$, where $F(x, y) = x^2 e^{-x^2} + y^2$ and $G(x, y) = \frac{x^4}{1+x^2y^2}$. Your method may use any existing functions in base matlab, any libraries in python, or should be contained in the STL of C++.

3. **9 points** Exponential decay is common in many physically relevant situations, ranging from chemical reactions to nuclear decay, with $x(t) = Ae^{-t/\tau}$. In cases where two processes are occurring (e.g. a fast and a slow chemical reaction), a bi-exponential decay is a better model, with $x(t) = Ae^{-t/\tau_1} + Be^{-t/\tau_2}$. In this problem, you will attempt to fit the data attached in the class Teams page using both of these models.

   (a) Fit all three datasets to a single exponential model, $x(t) = Ae^{-t/\tau}$ with A and τ fitting parameters, and describe the method you used to find the fit values A and τ (including your initial conditions). Be sure to try a few initial conditions, to see if you find similar best fit parameters.

   (b) Fit all three datasets to a biexponential model, $x(t) = Ae^{-t/\tau_1} + Be^{-t/\tau_2}$ with A, B, τ_1 and τ_2 fitting parameters. Describe the method and initial conditions you used, and be sure to use more than one set of initial conditions.

   (c) Each dataset was drawn from either a single exponential or bi-exponential function (with added noise). Given the results from (a) and (b), which datasets do you think are single-exponential and which do you think are bi-exponential? Justify your conclusions.