HW 7

1\. The dimensionless NS

$$\nabla \cdot u = 0$$

$$\dot{u} + (\vec{u} \cdot \nabla)\vec{u} = -\nabla p + Re^{-1} \nabla^2 \vec{u}$$

with

$$\vec{u} = (u_x, u_y, 0)$$

BC $\quad \vec{u}(y=1) = \vec{u}(y=-1) = 0$

Periodic BCs on $u$ in $x$ direction

@ $\quad P = P_0 - \alpha x$

$$\nabla p = -\alpha$$

$$u_x = \alpha Re (1-y^2)/2$$

$$u_y = 0$$

$$\vec{u} \cdot \nabla = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0$$

$\dot{u} = 0 \quad$ [From the BCs]

$$\nabla^2 u = -\alpha Re$$

RHS : $\quad -(-\alpha) + Re^{-1}(-\alpha Re) = \alpha - \alpha = 0$

LHS : $\quad 0 + 0 = 0$

Also, $\nabla \cdot u = u \cdot \nabla = 0$

Hence, the flow profile given satisfies the N-S equations.

b) $\quad \dot{\omega} + (\vec{u} \cdot \nabla)\omega = Re^{-1} \nabla^2 \omega$

Let $\quad u = (u_x, u_y, 0)$

$$\omega = \nabla \times u = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y}\right) \times (i u_x + j u_y)$$

$$\omega = k\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) \qquad \nabla^2 \omega = \frac{\partial^2 \omega}{\partial x^2} = k\frac{\partial^2}{\partial x^2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)$$

$$u \cdot \nabla = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}$$

$$(u \cdot \nabla)\omega = k\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right)\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)$$

$$= k\left[\frac{\partial u_x}{\partial x}\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial x}\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial y}\frac{\partial u_y}{\partial x} - \frac{\partial u_y}{\partial y}\frac{\partial u_x}{\partial y}\right]$$

c) The 2nd term includes a non-linear coefficient. Hence, we cannot use the typical linear algebra methods.