Gradiance Online Accelerated Learning

Zayd

- Home Page
- Assignments Due
- Progress Report
- Handouts
- Tutorials
- Homeworks
- Lab Projects
- Log Out

Help

| | |
|---|---|
| **Submission number:** | 73532 |
| **Submission certificate:** | DE336935 |
| **Submission time:** | 2014-03-30 18:54:59 PST (GMT - 8:00) |

| | |
|---|---|
| **Number of questions:** | 6 |
| **Positive points per question:** | 3.0 |
| **Negative points per question:** | 1.0 |
| **Your score:** | 14 |

Based on Sections 7.2, 7.3, and 7.4 of HMU.

**1.** $G_1$ is a context-free grammar with start symbol $S_1$, and no other nonterminals whose name begins with "S." Similarly, $G_2$ is a context-free grammar with start symbol $S_2$, and no other nonterminals whose name begins with "S." $S_1$ and $S_2$ appear on the right side of no productions. Also, no nonterminal appears in both $G_1$ and $G_2$.

We wish to combine the symbols and productions of $G_1$ and $G_2$ to form a new grammar G, whose language is the concatenation of the languages of $G_1$ and $G_2$. The start symbol of G will be S. All productions and symbols of $G_1$ and $G_2$ will be symbols and productions of G. Which of the following sets of productions, added to those of G, is guaranteed to make L(G) be $L(G_1)L(G_2)$?

a) $S \rightarrow S_1S_3$, $S_3 \rightarrow S_1S_2$

b) $S \rightarrow S_1S \mid S_2S \mid \varepsilon$

c) $S \rightarrow S_1S_3$, $S_3 \rightarrow S_2$

d) $S \rightarrow S_1 \mid S_2$

Answer submitted: **c)**

You have answered the question correctly.

Question Explanation:

Each of the choices involves only S, $S_1$, $S_2$, and in some cases other nonterminals whose names begin with "S" and that therefore are known not to appear in $G_1$ or $G_2$. As a result, we need only to look at the strings involving $S_1$ and $S_2$ only, that are derivable from S, using the new productions. In order for L(G) to be $L(G_1)L(G_2)$, it is necessary and sufficient that the strings involving only symbols $S_1$ and $S_2$ that are derived from S using only the additional productions be exactly the one string

$\{S_1S_2\}$.

For example, adding $S \rightarrow S_1S_2$ obviously has this property. So does the set of productions

$S \rightarrow S_1S_3S_2$
$S_3 \rightarrow e$

2. The language L = {ss | s is a string of a's and b's} is not a context-free language. In order to prove that L is not context-free we need to show that for every integer n, there is some string z in L, of length at least n, such that no matter how we break z up as $z = uvwxy$, subject to the constraints |vwx| ≤ n and |vx| > 0, there is some $i \geq 0$ such that $uv^iwx^iy$ is not in L.

   Let us focus on a particular z = aabaaaba and n = 7. It turns out that this is the wrong choice of z for n = 7, since there are some ways to break z up for which we can find the desired $i$, and for others, we cannot. Identify from the list below the choice of u,v,w,x,y for which there is an $i$ that makes $uv^iwx^iy$ not be in L. We show the breakup of aabaaaba by placing four |'s among the a's and b's. The resulting five pieces (some of which may be empty), are the five strings. For instance, aa|b||aaaba| means u=aa, v=b, w=ε, x=aaaba, and y=ε.

   a) |a|abaa|a|ba
   b) a|ab|aa|ab|a
   c) aa|ba|a|ab|a
   d) a|a|baaab|a|

   Answer submitted: **c)**

   Your answer is incorrect.

   If we pump v and x $i$ times, we get the string aa(ba)$^i$a(ab)$^i$a, which we can rewrite as aa(ba)$^i$aa(ba)$^i$ by regrouping the last $a$. All these strings are aa(ba)$^i$ repeated. You should examine the statement of the Pumping Lemma in Section 7.2.2 (p. 280) and the examples in Section 7.2.3.

   Question Explanation:

   In all cases for this problem, $i = 0$ works if anything works. For example one correct choice is aab|a|a|a|ba, where if we remove the second and 4th pieces, we get aababa, which is not of the form ss. That is, the first half, aab, is not the same as the second half, and therefore aababa is not in L.

   For each of the incorrect choices, the choice explanation gives an argument as to why no "pumping" of v and x will lead to a string whose first half differs from the second half.

   The correct choice is: **d)**

3. The intersection of two CFL's need not be a CFL. Identify in the list below a pair of CFL's such that their intersection is not a CFL.

a) $L_1=\{aba^nb^nc^iba \mid n>0, i>0\}$
$L_2=\{aba^nb^ic^iba \mid n>0, i>0\}$

b) $L_1=\{aba^nb^nc^nba \mid n>0, i>0\}$
$L_2=\{aba^nb^ic^jba \mid n>0, i>0, j>0\}$

c) $L_1=\{a^nb^jc^ic \mid n>0, i>0, j>0\}$
$L_2=\{a^ja^nb^ic^i \mid n>0, i>0, j>0\}$

d) $L_1=\{aca^nb^jc^ic \mid n>0, i>0, j>0\}$
$L_2=\{aca^ja^nb^ic^i \mid n>0, i>0, j>0\}$

Answer submitted: **a)**

You have answered the question correctly.

Question Explanation:

The incorrect choices fall in two categories: either one of the two given languages is not a CFL, or one is a CFL and the other is a regular language, in which case we know that their intersection is a CFL (see Section 7.3.4 on p. 291).

For the four correct choices the intersections of the two given languages are:

1. $\{a^nb^nc^n \mid n> 0\}$.
2. $\{aba^nb^nc^n \mid n> 0\}$.
3. $\{aba^nb^nc^nba \mid n> 0\}$.
4. $\{a^nb^nc^n \mid n> 0\}$.

In all cases we can prove the language not to be context-free by using the pumping lemma on a word $a^nb^nc^n$ for sufficiently large n. If we pump any pair of strings (of length much smaller than n) then the balance on the number of a's, b's and c's will be ruined.

**4.** $G_1$ is a context-free grammar with start symbol $S_1$, and no other nonterminals whose name begins with "S." Similarly, $G_2$ is a context-free grammar with start symbol $S_2$, and no other nonterminals whose name begins with "S." $S_1$ and $S_2$ appear on the right side of no productions. Also, no nonterminal appears in both $G_1$ and $G_2$.

We wish to combine the symbols and productions of $G_1$ and $G_2$ to form a new grammar G, whose language is the union of the languages of $G_1$ and $G_2$. The start symbol of G will be S. All productions and symbols of $G_1$ and $G_2$ will be symbols and productions of G. Which of the following sets of productions, added to those of G, is guaranteed to make L(G) be $L(G_1)$ [union] $L(G_2)$?

a) $S \rightarrow S_1S_2$

b) $S \rightarrow S_1S_3$, $S_3 \rightarrow S_2$

c) $S \rightarrow S_3S_4$, $S_3 \rightarrow S_1 \mid \varepsilon$, $S_4 \rightarrow S_2 \mid \varepsilon$

d) $S \rightarrow S_1$, $S_1 \rightarrow S_2$

Answer submitted: **d)**

You have answered the question correctly.

Question Explanation:

Each of the choices involves only S, $S_1$, $S_2$, and in some cases other nonterminals

whose names begin with "S" and that therefore are known not to appear in $G_1$ or $G_2$. As a result, we need only to look at the strings involving $S_1$ and $S_2$ only, that are derivable from S, using the new productions. In order for L(G) to be L($G_1$) [union] L($G_2$), it is necessary and sufficient that the strings involving only symbola $S_1$ and $S_2$ that are derived from S using only the additional productions be exactly the two strings $\{S_1, S_2\}$.

For example, adding $S \rightarrow S_1 \mid S_2$ obviously has this property. So does the set of productions

$S \rightarrow S_1$
$S_1 \rightarrow S_2$

Note that if $S_1$ could appear on the right side of productions of $G_1$, then this choice would not work --- derivations of $G_2$ could suddenly appear in the middle of derivations that should be in $G_1$ only.

$S \rightarrow S_1 \mid S_3$
$S_3 \rightarrow S_2$

**5.** Apply the CYK algorithm to the input ababaa and the grammar:

```
S → AB | BC
A → BA | a
B → CC | b
C → AB | a
```

Compute the table of entries $X_{ij}$ = the set of nonterminals that derive positions *i* through *j*, inclusive, of the string ababaa. Then, identify a true assertion about one of the $X_{ij}$'s in the list below.

a) $X_{15} = \{S,A,C\}$

b) $X_{15} = \{B\}$

c) $X_{23} = \{S\}$

d) $X_{16} = \{S,A,C\}$

Answer submitted: **a)**

You have answered the question correctly.

Question Explanation:

Here is the table:

| | | | | | |
|---|---|---|---|---|---|
| B | | | | | |
| SAC | SA | | | | |
| B | B | SA | | | |
| B | SC | B | - | | |
| SC | SA | SC | SA | B | |
| AC | B | AC | B | AC | AC |
| **a** | **b** | **a** | **b** | **a** | **a** |

**6.** If $h$ is the homomorphism defined by h(a) = 0 and h(b) = ε, which of the following strings is in $h^{-1}(000)$?

a) bababab

b) bbb

c) abbbabaab

d) babab

Answer submitted: **a)**

You have answered the question correctly.

Question Explanation:

Since there are three 0's in h(w), *w* must have exactly three *a*'s. It can have any number of *b*'s, since h(b) = ε. Of the choices, only bababab has exactly three *a*'s.