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Gradiance Online Accelerated Learning

Zayd

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Help

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These questions, based on Section 5.1 of HMU, are not in either of the other two homeworks on CFG's, but are available for use.

1. Let L be the language of all strings of a's and b's such that no prefix (proper or not) has more b's than a's. Let G be the grammar with productions

```
S → aS | aSbS | ε
```

To prove that L = L(G), we need to show two things:

1. If S =>* w, then w is in L.
2. If w is in L, then S =>* w.

We shall consider only the proof of (1) here. The proof is an induction on n, the number of steps in the derivation S =>* w. Here is an outline of the proof, with reasons omitted. You need to supply the reasons.

Basis:

1) If n=1, then w is ε because _______.

2) w is in L because _______.

Induction:

3) Either (a) S => aS =>$^{n-1}$ w or (b) S => aSbS =>$^{n-1}$ w because _______.

4a) In case (a), w = ax, and S =>$^{n-1}$ x because _______.

5a) In case (a), x is in L because _______.

6a) In case (a), w is in L because _______.

4b) In case (b), w can be written w = aybz, where S =>^{p} y and S =>^{q} z for some p and q less than n because _______.

5b) In case (b), y is in L because _______.

6b) In case (b), z is in L because _______.

7b) In case (b), w is in L because _______.

This question is giving two ways to reach the final conclusion. As such, it has to be either 6a or 7b.

For which of the steps above the appropriate reason is contained in the following argument:
"The following two statements are true
(i) if string x has no prefix with more b's than a's, then neither does string ax,
(ii) if strings y and z are such that no prefix has more b's than a's, then neither does string aybz."

a) 2

b) 6b

c) 7b

c) 7b

d) 4a

Answer submitted: **c)**

You have answered the question correctly.

Keys to look for in wrong answers:
1. String provided has a way to make two c's next to each other (choice a).
2. Major productions do not have 4 a's and 3 c's in order.

2. Which of the following grammars derives a subset of the language:

{x | x contains a and c in proportion 4:3 and there are no two consecutive c's}?

a) S → ε S → aScScaSaScSaS

b) S → acacaca S → SaScSaScSaScSaS S → SaSaSaScSaScSa

c) S → acacaca S → SacSaScSaScSaS

d) S → ε S → aacacac S → SaScSaScSaScSa

Answer submitted: **c)**

You have answered the question correctly.

3. Consider the grammar G with start symbol S:

S → bS | aA | b
A → bA | aB
B → bB | aS | a

To be accepted by this grammar, the string must have the number of a's be a multiple of three.

Which of the following is a word in L(G)?

a) bababbabaababaa

b) ababbbbb

c) babbbbaaaab

d) bbbaababaaa

Answer submitted: **d)**

You have answered the question correctly.

4. Consider the grammar G1: S → ε, S → aS, S → aSbS and the language L that contains exactly those strings of a's and b's such that every prefix has at least as many a's as b's. We want to prove the claim: G1 generates all strings in L.

We take the following inductive hypothesis to prove the claim:

For n < k, G1 generates every string of length n in L.

To prove the inductive step we argue as follows:

"For each string w in L either ____________(a1) or ______________(a2) holds. In both cases we use the inductive hypothesis and one of the rules to show that string w can be generated by the grammar. In the first case we use rule S → aS and in the second case we use rule S → aSbS."

Be careful to read the second half of the question after the blanks. In choice a you use the production S->aS so must have at least as many a's as b's. For the second one, they could be equal or there could be more a's than b's.

Which phrases can replace the ____________ so that this argument is correct?

a) a1: each prefix has equal number of a's and b's.
a2: there is a b in string w such that the part of the string until the b belongs in L by inductive hypothesis and the part after this b belongs in L by inductive hypothesis.

b) a1: there is a b in string w such that for the part of the string until the b (b also included) each prefix has as many a's as b's and for the part after b each prefix has as many a's as b's.
a2: each prefix has more a's than b's.

c) a1: w can be written as w=aw' where each prefix of w' has as many a's as b's.
a2: w can be written as w=aw'bw'' where for both w' and w'' it holds that each prefix has as many a's as b's.

d) a1: each prefix has equal number of b's and a's.
a2: w can be written as w=aw'bw" where for both w' and w" it holds that each prefix has as many a's as b's.

Answer submitted: **c)**

You have answered the question correctly.

Choose the right production that can lead to any number of b's. Selection b is invalid as there must be a finite number of productions.

5. Programming languages are often described using an extended form of context-free grammar, where curly brackets are used to denote a construct that can repeat 0, 1, 2, or any number of times. For example, A → B{C}D says that an *A* can be replaced by a *B* and a *D*, with any number of *C*'s (including 0) between them. This notation does not allow us to describe anything but context-free languages, since an extended production can always be replaced by several conventional productions.

Suppose a grammar has the extended production:

```
A → a{b}B
```

Convert this extended production to conventional productions. Identify, from the list below, the conventional productions that are equivalent to the extended production above.

a) $A \to aA_1B$
$A_1 \to bA_1 \mid b$

b) $A \to aB \mid abB \mid abbB \mid abbbB \mid \ldots$

c) $A \to aA_1B$
$A_1 \to b \mid \varepsilon$

d) $A \to aA_1B$
$A_1 \to bA_1 \mid \varepsilon$

Answer submitted: **d)**

You have answered the question correctly.

G1 defines L3.
G2 defines L2.
G3 defines L4.
G4 defines L5.

6. Consider the following languages and grammars. G_1: S → aA|aS, A → ab
G_2: S → abS|aA, A → a
G_3: S → Sa|AB, A → aA|a, B → b
G_4: S → aS|b
L_1: $\{a^ib \mid i=1,2,...\}$
L_2: $\{(ab)^iaa \mid i=0,1,...\}$
L_3: $\{a^ib \mid i=2,3,...\}$
L_4: $\{a^iba^j \mid i=1,2,..., j=0,1,...\}$
L_5: $\{a^ib \mid i=0,1,...\}$

Match each grammar with the language it defines. Then, identify a correct match from the list below.

a) G_4 defines L_5.

b) G_1 defines L_2.

c) G_4 defines L_2.

d) G_4 defines L_1.

Answer submitted: **a)**

You have answered the question correctly.

7. Here are eight simple grammars, each of which generates an infinite language of strings. These strings tend to look like alternating *a*'s and *b*'s, although there are some exceptions, and not all grammars generate all such strings.

G1, G2, G4, and G8 produce same language

1. S → abS | ab
2. S → SS | ab
3. S → aB; B → bS | a
4. S → aB; B → bS | b
5. S → aB; B → bS | ab
6. S → aB | b; B → bS
7. S → aB | a; B → bS
8. S → aB | ab; B → bS

The initial symbol is S in all cases. Determine the language of each of these grammars. Then, find, in the list below, the pair of grammars that define the same language.

a) G1: S → aB, B → bS, B → ab
   G2: S → aB, B → bS, S → ab

b) G1: S → aB, B → bS, B → b
   G2: S → aB, B → bS, S → b

c) G1: S → abS, S → ab
   G2: S → aB, B → bS, B → ab

d) G1: S → SS, S → ab
   G2: S → aB, B → bS, S → ab

Answer submitted: **d)**

You have answered the question correctly.

8. Which of the following pairs of grammars define the same language?

a) G_1: S → AB|a, A → b
   G_2: S → a

b) G_1: S → AB, A → aAA|ε , B → baBB|ε
   G_2: S → CB|B|ε , C → aCC|aC|a, B → baBB|baB|ba

c) G_1: S → SaScSaS|aca|ε
   G_2: S → SaBaS|aca, B → cS|ε

d) G_1: S → AB, A → aAA|ε , B → baB|ε
   G_2: S → CB|C|B, C → aCC|aC|a, B → baBB|baB|ba

For choice a, S->AB is useless as no production for B.
For choice c, the issue arises when B->epsilon so no middle c.
For choice d, the issue is the first grammar can produce the empty string while the second cannot.

Answer submitted: **a)**

You have answered the question correctly.

Keys to look for in wrong answers:
1. String provided has a way to make two c's next to each other (choice a).
2. Major productions do not have 4 a's and 3 c's in order.

9. Which of the following grammars derives a subset L_s of the language: L= {x | (i) x contains a and c in proportion 4:3, (ii) x does not begin with c and (iii) there are no two consecutive c's} such that L_s is missing at most a finite number of strings from L.

a) S → ε , S → SaScSaScSa
b) S → ε , S → SaScSaScSaSaS
c) S → ε, S → acacaScSaS
d) S → ε, S → SaScSaScSaScSaS

Answer submitted: **d)**

You have answered the question correctly.

10. Let L be the language of all strings of a's and b's such that no prefix (proper or not) has more b's than a's. Let G be the grammar with productions

```
S → aS | aSbS | ε
```

To prove that L = L(G), we need to show two things:

1. If S =>* w, then w is in L.
2. If w is in L, then S =>* w.

We shall consider only the proof of (1) here. The proof is an induction on n, the number of steps in the derivation S =>* w. Here is an outline of the proof, with reasons omitted. You need to supply the reasons.

Basis:

1) If n=1, then w is ε because _______.

2) w is in L because _______.

Induction:

3) Either (a) S => aS $=>^{n-1}$ w or (b) S => aSbS $=>^{n-1}$ w because _______.

4a) In case (a), w = ax, and S $=>^{n-1}$ x because _______.

5a) In case (a), x is in L because _______.

6a) In case (a), w is in L because _______.

4b) In case (b), w can be written w = aybz, where S $=>^{p}$ y and S $=>^{q}$ z for some p and q less than n because _______.

5b) In case (b), y is in L because _______.

6b) In case (b), z is in L because _______.

7b) In case (b), w is in L because _______.

This question is asking for the step that divides the problem into two separate cases. It has to be step #3, #4a, or #4b as those either define the "a" and "b" cases below or are the "a" and "b" cases.

For which of the steps above the appropriate reason is contained in the following argument:
"All n-step derivations of w produce either ε (for n=1) or use one of the productions with at least one nonterminal in the body (for n > 1). In case the production S → aS is used, then w=ax with x being produced by a (n-1)-step derivation. In case the production S → aSbS is used then w=aybz with y and z being produced by derivations with number of steps less than n."

a) 2

b) 5a

c) 7b

d) 3

Answer submitted: **d)**

You have answered the question correctly.

11. Identify in the list below a sentence of length 6 that is generated by the grammar S → (S)S | ε

All that is required is to find a valid parenthesis string of six characters.

a)) ( ) (( )

b) (( ))) (

c) ( )) (( )

d) (( )) ( )

Answer submitted: **d)**

You have answered the question correctly.

12. Consider the grammar G and the language L:

G: S → AB | a | abC, A → b, C → abC | c

L: {w | w a string of a's, b's, and c's with an equal number of a's and b's}.

In the grammar, a is always before b but that is not required by the language.

Grammar G does not define language L. To prove, we use a string that either is produced by G and not contained in L or is contained in L but is not produced by G. Which string can be used to prove it?

a) abababc
b) cacabbb
c) bababa
d) ababc

Answer submitted: **c)**

You have answered the question correctly.

13. Consider the grammar G1:

S → ε | aS | aSbS

Which of the following is correct (for a choice to be correct, all propositions must be correct)?

In choice d, the substring aaabbbabb has more b's than a's.

a) The string aaba is not generated by the grammar.

b) a) G1 generates all and only the strings of a's and b's such that every string has at least as many a's as b's. b) The inductive hypothesis to prove it is: For n < k, it holds: Any word in G1 of length n, is such that all its prefixes contain more a's than b's or as many a's as b's.

c) a) G1 generates all and only the strings of a's and b's such that every prefix has at least as many a's as b's. b) The following inductive hypothesis will prove it: For n < k, it holds that: Any word in G1 of length n, is such that all its prefixes contain at least as many a's as b's.

d) The string aaabbbabbaabbaaabb is not generated by the grammar.

Answer submitted: **d)**

You have answered the question correctly.

14. Consider the grammars:

G_1: S → AB, A → aAA|ε , B → abBB|ε
G_2:S → CB, C → aCC|aC|a, B → abBB|abB|ab
G_3:S → CB|C|B| ε , C → aCC|aC|a, B → abBB|abB|ab
G_4:S → ASB|ε, A → aA|ε, B → abB|ε
G_5:S → ASB|AB, A → aA|a, B → abB|ab
G_6:S → ASB|aab, A → aA|a, B → abB|ab

G1, G3, and G4 are equal.
G2 and G5 are equal.

Describe the language of each of these grammars. Then, identify from the list below a pair of grammars that define the same language?

a) G_1 and G_6
b) G_1 and G_3
c) G_5 and G_6
d) G_3 and G_2

Answer submitted: **b)**

You have answered the question correctly.

You have answered the question correctly.