Turing's Thesis

class 16

1

Turing's thesis:

Any computation carried out
by mechanical means
can be performed by a Turing Machine

(1930)

2

Computer Science Law:

A computation is mechanical
if and only if
it can be performed by a Turing Machine

There is no known model of computation
more powerful than Turing Machines

3

Definition of Algorithm:

An algorithm for function $f(w)$
is a
Turing Machine which computes $f(w)$

4

Algorithms are Turing Machines

When we say:

There exists an algorithm

We mean:

There exists a Turing Machine
that executes the algorithm

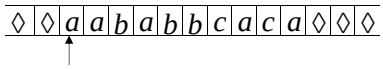
5

Variations of the Turing Machine

6

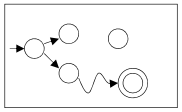
The Standard Model

Infinite Tape



Read-Write Head (Left or Right)

Control Unit



Deterministic

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Variations of the Standard Model

Turing machines with:

- Stay-Option
- Semi-Infinite Tape
- Off-Line
- Multitape
- Multidimensional
- Nondeterministic

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The variations form different Turing Machine **Classes**

We want to prove:

Each **Class** has the same power with the Standard Model

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Same Power of two classes means:

Both classes of Turing machines accept the same languages

10

Same Power of two classes means:

For any machine M_1 of first class

there is a machine M_2 of second class

such that: $L(M_1) = L(M_2)$

And vice-versa

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Simulation: a technique to prove same power

Simulate the machine of one class with a machine of the other class

First Class
Original Machine

M_1

Second Class
Simulation Machine

M_2

M_1

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Configurations in the Original Machine correspond to configurations in the Simulation Machine

Original Machine: $d_0 \succ d_1 \succ \cdots \succ d_n$

Simulation Machine: $d'_0 \overset{*}{\succ} d'_1 \overset{*}{\succ} \cdots \overset{*}{\succ} d'_n$

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Final Configuration

Original Machine: d_f

Simulation Machine: d'_f

The Simulation Machine and the Original Machine accept the same language

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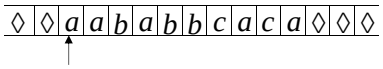
Turing Machines with Stay-Option

The head can stay in the same position



Left, Right, Stay

L,R,S: moves

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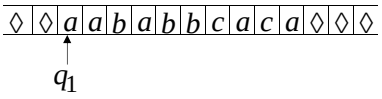
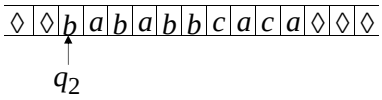
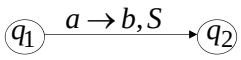
Example:

Time 1



Time 2





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Theorem: Stay-Option Machines have the same power with Standard Turing machines

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Proof:

Part 1: Stay-Option Machines are at least as powerful as Standard machines

Proof: a Standard machine is also a Stay-Option machine (that never uses the S move)

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Proof:

Part 2: Standard Machines are at least as powerful as Stay-Option machines

Proof: a standard machine can simulate a Stay-Option machine

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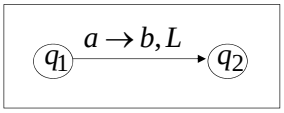
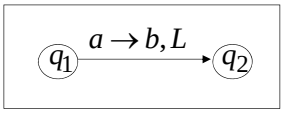
Stay-Option Machine



Simulation in Standard Machine



Similar for Right moves

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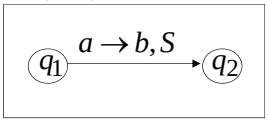
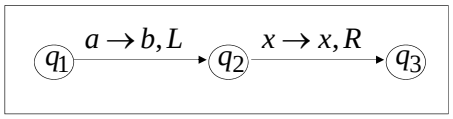
Stay-Option Machine



Simulation in Standard Machine

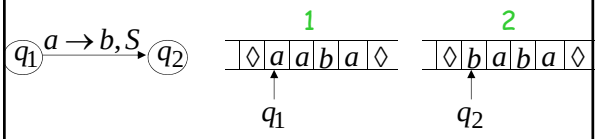


For every symbol x

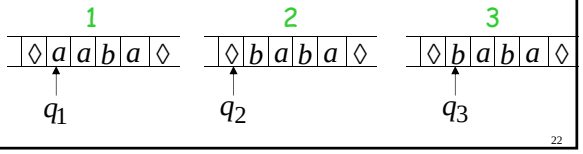
21

Example

Stay-Option Machine:



Simulation in Standard Machine:



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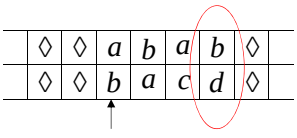
Standard Machine--Multiple Track Tape

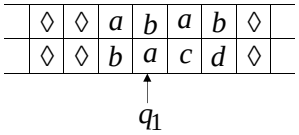


track 1

track 2

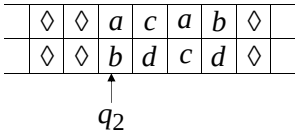
one symbol

23



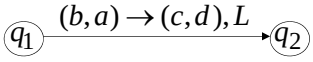
track 1

track 2



track 1

track 2



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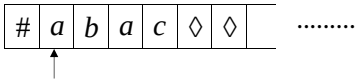
Semi-Infinite Tape



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Standard Turing machines simulate Semi-infinite tape machines:

Trivial

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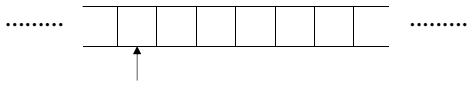
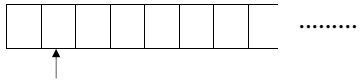
Semi-infinite tape machines simulate Standard Turing machines:

Standard machine



Semi-infinite tape machine



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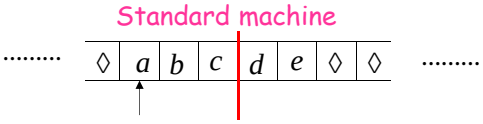
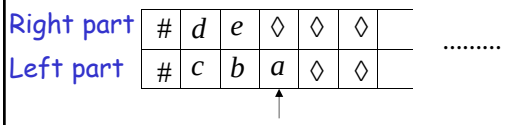
Standard machine



Semi-infinite tape machine with two tracks



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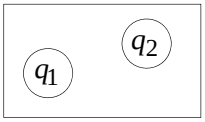
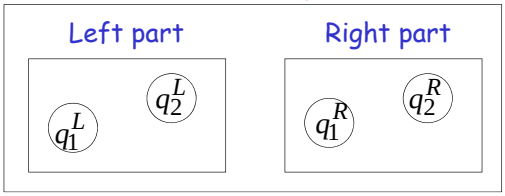
Standard machine



Semi-infinite tape machine



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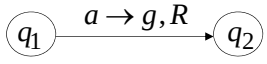
Standard machine



Semi-infinite tape machine

Right part q_1^R $(a,x) \to (g,x), R$ q_2^R

Left part q_1^L $(x,a) \to (x,g), L$ q_2^L

For all symbols x

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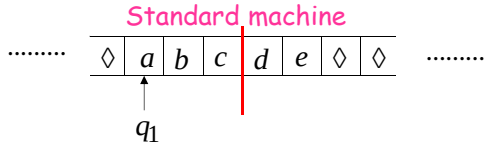
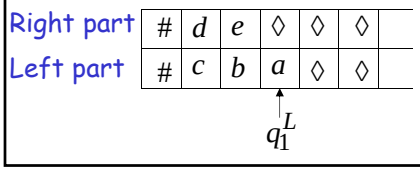
Time 1

Standard machine



Semi-infinite tape machine



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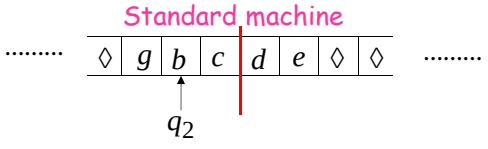
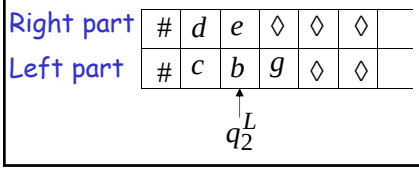
Time 2

Standard machine



Semi-infinite tape machine



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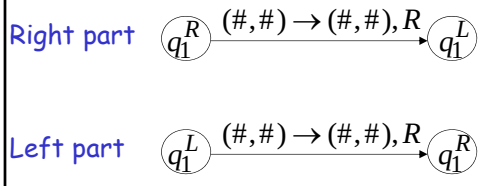
At the border:

Semi-infinite tape machine



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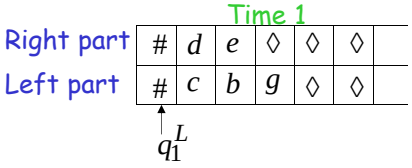
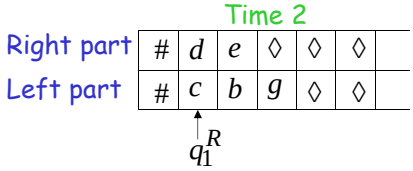
Semi-infinite tape machine

Time 1



Time 2



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Theorem: Semi-infinite tape machines have the same power with Standard Turing machines

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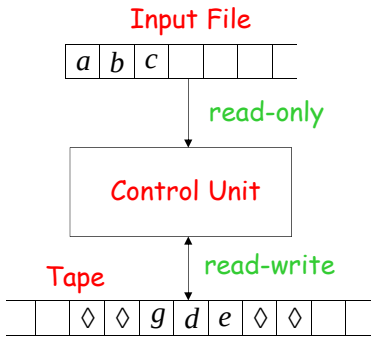
The Off-Line Machine



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Off-line machines simulate Standard Turing Machines:

Off-line machine:

1. Copy input file to tape

2. Continue computation as in Standard Turing machine

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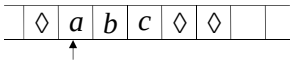
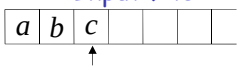
Standard machine

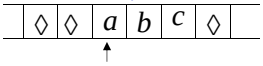


Off-line machine

Input File



Tape



1. Copy input file to tape

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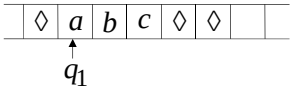
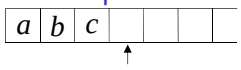
Standard machine

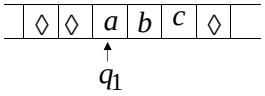


Off-line machine

Input File



Tape



2. Do computations as in Turing machine

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Standard Turing machines simulate Off-line machines:

Use a Standard machine with four track tape to keep track of the Off-line input file and tape contents

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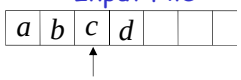
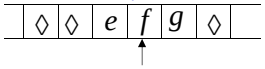
Off-line Machine

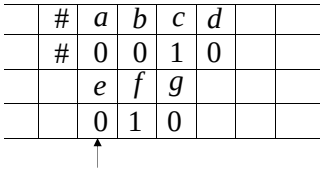
Input File



Tape



Four track tape -- Standard Machine



Input File head position

Tape head position

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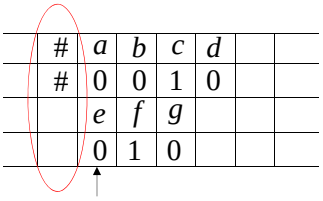
Reference point



Input File head position

Tape head position

Repeat for each state transition:

- Return to reference point
- Find current input file symbol
- Find current tape symbol
- Make transition

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Theorem: Off-line machines have the same power with Stansard machines

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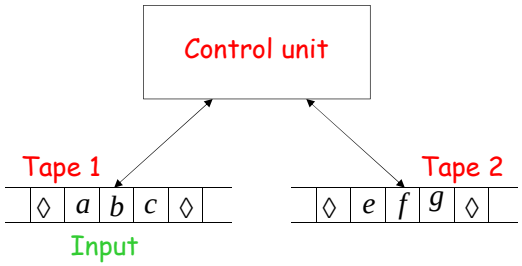
Multitape Turing Machines



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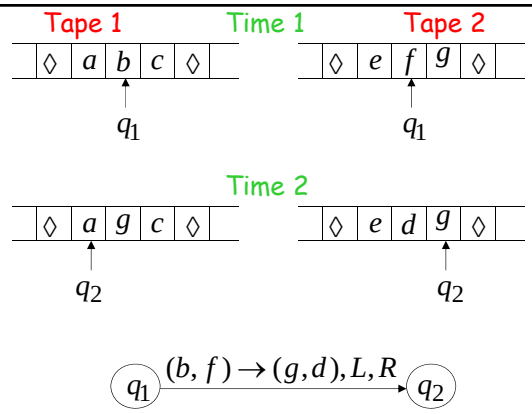

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Multitape machines simulate Standard Machines:

Use just one tape

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Standard machines simulate Multitape machines:

Standard machine:

- Use a multi-track tape
- A tape of the Multiple tape machine corresponds to a pair of tracks

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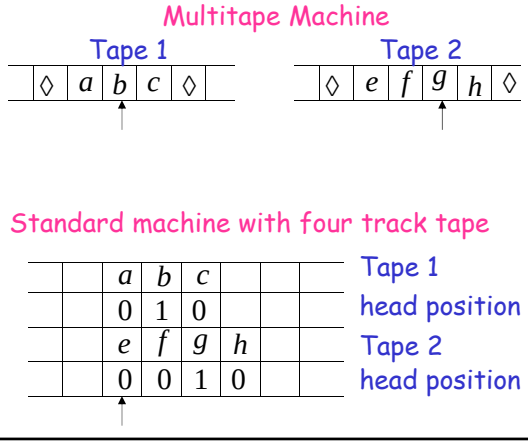

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Reference point

#	a	b	c			
#	0	1	0			
#	e	f	g	h		
#	0	0	1	0		

Tape 1 head position

Tape 2 head position

Repeat for each state transition:

- Return to reference point
- Find current symbol in Tape 1
- Find current symbol in Tape 2
- Make transition

Theorem: Multi-tape machines have the same power with Standard Turing Machines

Same power doesn't imply same speed:

Language $L = \{a^n b^n\}$

	Acceptance Time
Standard machine	n^2
Two-tape machine	n

$L = \{a^n b^n\}$

Standard machine:

Go back and forth n^2 times

Two-tape machine:

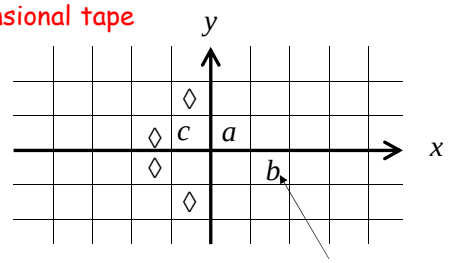
Copy b^n to tape 2 (n steps)

Leave a^n on tape 1 (n steps)

Compare tape 1 and tape 2 (n steps)

MultiDimensional Turing Machines

Two-dimensional tape



MOVES: L,R,U,D

U: up D: down

HEAD

Position: +2, -1

Multidimensional machines simulate Standard machines:

Use one dimension

Standard machines simulate Multidimensional machines:

Standard machine:

- Use a two track tape
- Store symbols in track 1
- Store coordinates in track 2

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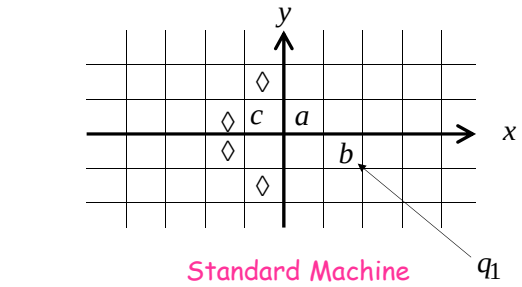
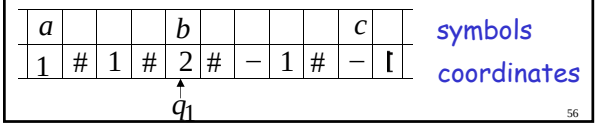
Two-dimensional machine



Standard Machine



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Standard machine:

Repeat for each transition

- Update current symbol
- Compute coordinates of next position
- Go to new position

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Theorem: MultiDimensional Machines have the same power with Standard Turing Machines

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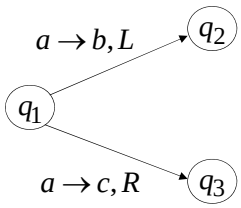
NonDeterministic Turing Machines



Non Deterministic Choice

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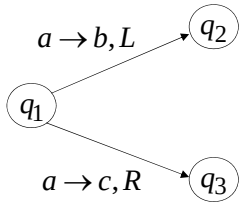
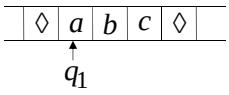
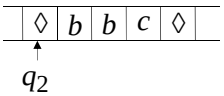


Time 0

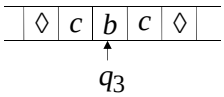


Time 1

Choice 1



Choice 2



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Input string w is accepted if this a possible computation

$$q_0 w \overset{*}{\succ} x\, q_f\, y$$

Initial configuration

Final Configuration

Final state

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NonDeterministic Machines simulate Standard (deterministic) Machines:

Every deterministic machine is also a nondeterministic machine

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Deterministic machines simulate NonDeterministic machines:

Deterministic machine:

Keeps track of all possible computations

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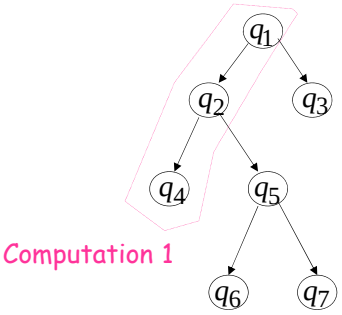
Non-Deterministic Choices



Computation 1

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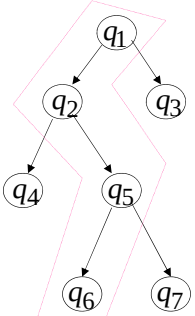
Non-Deterministic Choices



Computation 2

65

Simulation

Deterministic machine:

- Keeps track of all possible computations
- Stores computations in a two-dimensional tape

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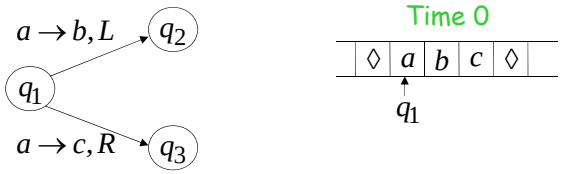
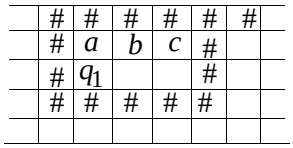
NonDeterministic machine



Time 0

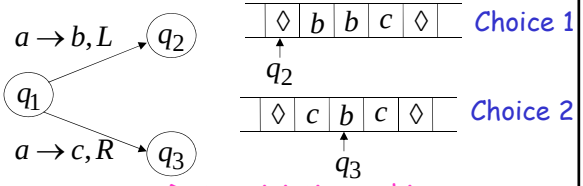
Deterministic machine



Computation 1

NonDeterministic machine

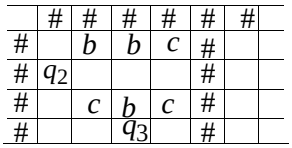
Time 1



Choice 1

Choice 2

Deterministic machine



Computation 1

Computation 2

Repeat

- Execute a step in each computation:
- If there are two or more choices in current computation:
  1. Replicate configuration
  2. Change the state in the replica

Theorem: NonDeterministic Machines have the same power with Deterministic machines

Remark:

The simulation in the Deterministic machine takes time exponential time compared to the NonDeterministic machine