



## Gradiance Online Accelerated Learning

Zayd

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1. Here are the transitions of a deterministic pushdown automaton. The start state is  $q_0$ , and  $f$  is the accepting state.

State-Symbol	a	b	$\epsilon$
$q_0-Z_0$	$(q_1, AAZ_0)$	$(q_2, BZ_0)$	$(f, \epsilon)$
$q_1-A$	$(q_1, AAA)$	$(q_1, \epsilon)$	-
$q_1-Z_0$	-	-	$(q_0, Z_0)$
$q_2-B$	$(q_3, \epsilon)$	$(q_2, BB)$	-
$q_2-Z_0$	-	-	$(q_0, Z_0)$
$q_3-B$	-	-	$(q_2, \epsilon)$
$q_3-Z_0$	-	-	$(q_1, \Lambda Z_0)$

Describe informally what this PDA does. Then, identify below, the one input string that takes the PDA into state  $q_3$  (with any stack).

- a) babbabaa
- b) baabbbba
- c) babbbba
- d) bababba

You did not answer this question.

2. The Turing machine  $M$  has:

- States  $q$  and  $p$ ;  $q$  is the start state.
- Tape symbols 0, 1, and  $B$ ; 0 and 1 are input symbols, and  $B$  is the blank.
- The following next-move function:

State	Tape	Move
	Symbol	
$q$	0	$(q, 0, R)$
$q$	1	$(p, 0, R)$
$q$	$B$	$(q, B, R)$
$p$	0	$(q, 0, L)$
$p$	1	none (halt)
$p$	$B$	$(q, 0, L)$

Simulate  $M$  on the input 1010110, and identify one of the ID's (instantaneous descriptions) of  $M$  from the list below.

- a) 10101p10

- b) 000000p0
- c) 000001q0
- d) 000q0110

You did not answer this question.

3. Here is a context-free grammar:

```

S → AB | CD
A → BG | 0
B → AD | ε
C → CD | 1
D → BB | E
E → AF | B1
F → EG | 0C
G → AG | BD

```

Find all the nullable symbols, and then use the construction from Section 7.1.3 (p. 265) to modify the grammar's productions so there are no  $\epsilon$ -productions. The language of the grammar should change only in that  $\epsilon$  will no longer be in the language.

- a)  $G \rightarrow AG | BD | A | B | D$
- b)  $S \rightarrow AB | CD | A | B | C | \epsilon$
- c)  $D \rightarrow BB | E | B | B$
- d)  $E \rightarrow AF | B1 | F | 1$

You did not answer this question.

4. Let  $L$  be the language of all strings of a's and b's such that no prefix (proper or not) has more b's than a's. Let  $G$  be the grammar with productions

```

S → aS | aSbS | ε

```

To prove that  $L = L(G)$ , we need to show two things:

- 1. If  $S \Rightarrow^* w$ , then  $w$  is in  $L$ .
- 2. If  $w$  is in  $L$ , then  $S \Rightarrow^* w$ .

We shall consider only the proof of (1) here. The proof is an induction on  $n$ , the number of steps in the derivation  $S \Rightarrow^* w$ . Here is an outline of the proof, with reasons omitted. You need to supply the reasons.

Basis:

- 1) If  $n=1$ , then  $w$  is  $\epsilon$  because \_\_\_\_\_.

- 2)  $w$  is in  $L$  because \_\_\_\_\_.

Induction:

- 3) Either (a)  $S \Rightarrow aS \Rightarrow^{n-1} w$  or (b)  $S \Rightarrow aSbS \Rightarrow^{n-1} w$  because \_\_\_\_\_.

- 4a) In case (a),  $w = ax$ , and  $S \Rightarrow^{n-1} x$  because \_\_\_\_\_.

- 5a) In case (a),  $x$  is in  $L$  because \_\_\_\_\_.

- 6a) In case (a),  $w$  is in  $L$  because \_\_\_\_\_.

- 4b) In case (b),  $w$  can be written  $w = aybz$ , where  $S \Rightarrow^p y$  and  $S \Rightarrow^q z$  for some  $p$  and  $q$  less than  $n$  because \_\_\_\_\_.

- 5b) In case (b),  $y$  is in  $L$  because \_\_\_\_\_.

- 6b) In case (b),  $z$  is in  $L$  because \_\_\_\_\_.

- 7b) In case (b),  $w$  is in  $L$  because \_\_\_\_\_.

For which of the steps above the appropriate reason is contained in the following argument:  
 "All  $n$ -step derivations of  $w$  produce either  $\epsilon$  (for  $n=1$ ) or use one of the productions with at least one

nonterminal in the body (for  $n > 1$ ). In case the production  $S \rightarrow aS$  is used, then  $w=ax$  with  $x$  being produced by a  $(n-1)$ -step derivation. In case the production  $S \rightarrow aSbS$  is used then  $w=aybz$  with  $y$  and  $z$  being produced by derivations with number of steps less than  $n$ ."

- a) 2
- b) 6a
- c) 4b
- d) 6b

You did not answer this question.

5. Here is the transition table of a DFA:

	0	1
→A	E	D
*B	A	C
C	G	B
D	E	A
*E	H	C
F	C	B
G	F	E
H	B	H

Find the minimum-state DFA equivalent to the above. Then, identify in the list below the pair of equivalent states (states that get merged in the minimization process).

- a) A and B
- b) A and C
- c) B and C
- d) F and G

You did not answer this question.

6. Suppose one transition rule of some PDA  $P$  is  $\delta(q, 0, X) = \{(p, YZ), (r, XY)\}$ . If we convert PDA  $P$  to an equivalent context-free grammar  $G$  in the manner described in Section 6.3.2 (p. 247), which of the following could be a production of  $G$  derived from this transition rule? You may assume  $s$  and  $t$  are states of  $P$ , as well as  $p$ ,  $q$ , and  $r$ .

- a)  $[qXq] \rightarrow 0[rXr][sYq]$
- b)  $[qXq] \rightarrow [pYr][rZq]$
- c)  $[qXq] \rightarrow 0[pYr][rZq]$
- d)  $[qXq] \rightarrow 0[qYr][rZp]$

You did not answer this question.

7. The language of regular expression  $(0+10)^*$  is the set of all strings of 0's and 1's such that every 1 is immediately followed by a 0. Describe the complement of this language (with respect to the alphabet  $\{0,1\}$ ) and identify in the list below the regular expression whose language is the complement of  $L((0+10)^*)$ .

- a)  $(1+01)^*$
- b)  $(0+10)^*(1+11(0+1)^*)$
- c)  $(0+1)^*11(0+10)^* + (0+10)^*1$
- d)  $(0+10)^*11(0+1)^*$

You did not answer this question.

8. There is a Turing transducer  $T$  that transforms problem  $P_1$  into problem  $P_2$ .  $T$  has one read-only input tape, on which an input of length  $n$  is placed.  $T$  has a read-write scratch tape on which it uses  $O(S(n))$

cells.  $T$  has a write-only output tape, with a head that moves only right, on which it writes an output of length  $O(U(n))$ . With input of length  $n$ ,  $T$  runs for  $O(T(n))$  time before halting. You may assume that each of the upper bounds on space and time used are as tight as possible.

A given combination of  $S(n)$ ,  $U(n)$ , and  $T(n)$  may:

1. Imply that  $T$  is a polynomial-time reduction of P1 to P2.
2. Imply that  $T$  is NOT a polynomial-time reduction of P1 to P2.
3. Be impossible; i.e., there is no Turing machine that has that combination of tight bounds on the space used, output size, and running time.

What are all the constraints on  $S(n)$ ,  $U(n)$ , and  $T(n)$  if  $T$  is a polynomial-time reducer? What are the constraints on feasibility, even if the reduction is not polynomial-time? After working out these constraints, identify the true statement from the list below.

- a)  $S(n) = \log_2 n$ ;  $U(n) = n^2$ ;  $T(n) = n^4$  is a polynomial-time reduction
- b)  $S(n) = n^2$ ;  $U(n) = n^2$ ;  $T(n) = n!$  is not physically possible.
- c)  $S(n) = n$ ;  $U(n) = n^2$ ;  $T(n) = n \log_2 n$  is a polynomial-time reduction
- d)  $S(n) = n^3$ ;  $U(n) = n$ ;  $T(n) = (1.01)^n$  is possible, but not a polynomial-time reduction.

Answer submitted: **d)**

You have answered the question correctly.

9. Which of the following problems about a Turing Machine  $M$  does Rice's Theorem imply is undecidable?
- a) Is there some input that causes  $M$  to enter more than 100 states?
  - b) Is there some input that causes  $M$  to halt after no more than 500 moves?
  - c) Is the language of  $M$  a set of strings?
  - d) Does the language of  $M$  contain at least 10 strings?

Answer submitted: **d)**

You have answered the question correctly.

10. Consider the following identities for regular expressions; some are false and some are true. You are asked to decide which and in case it is false to provide the correct counterexample.

- (a)  $R(S+T) = RS + RT$
  - (b)  $(R^*)^* = R^*$
  - (c)  $(R^*S^*)^* = (R+S)^*$
  - (d)  $(R+S)^* = R^* + S^*$
  - (e)  $S(RS+S)^*R = RR^*S(RR^*S)^*$
  - (f)  $(RS+R)^*R = R(SR+R)^*$
- a) (d) is false and a counterexample is:  
 $R = \{a\}$ ,  $T = \{a\}$ ,  $S = \{b\}$
  - b) (d) is false and a counterexample is:  
 $R = \{a, \epsilon\}$ ,  $T = \{b\}$ ,  $S = \{a, \epsilon\}$
  - c) (a) is false and a counterexample is:  
 $R = \{ab\}$ ,  $T = \{a\}$ ,  $S = \{b\}$
  - d) (e) is false and a counterexample is:  
 $R = \{a, \epsilon\}$ ,  $T = \{b\}$ ,  $S = \{a, \epsilon\}$

You did not answer this question.

11. The polynomial-time reduction from SAT to CSAT, as described in Section 10.3.3 (p. 452), needs to introduce new variables. The reason is that the obvious manipulation of a boolean expression into an equivalent CNF expression could exponentiate the size of the expression, and therefore could not be polynomial time.

Suppose we apply this construction to the expression  $(u+(vw))^+x$ , with the parse implied by the parentheses. Suppose also that when we introduce new variables, we use  $y_1, y_2, \dots$

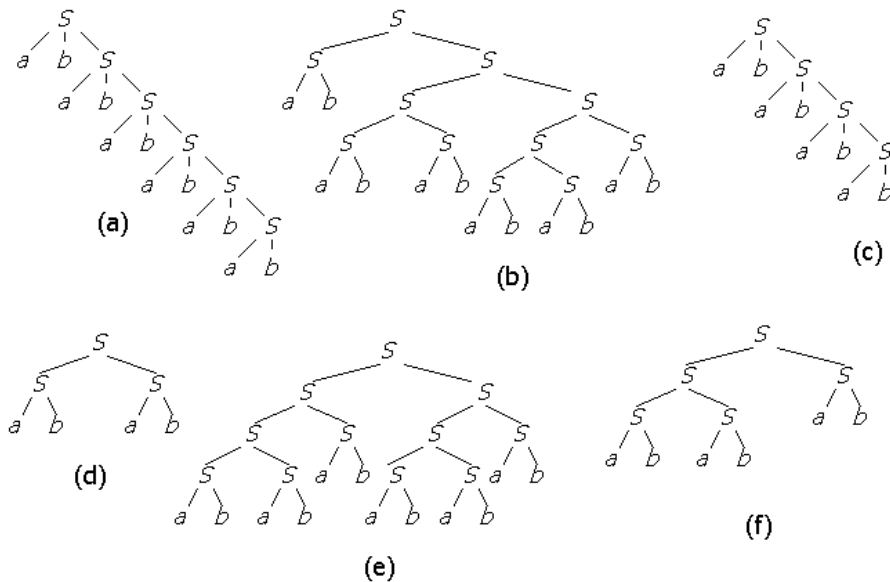
After constructing the corresponding CNF expression, identify one of its clauses from the list below.  
Note: logical OR is represented by +, logical AND by juxtaposition, and logical NOT by -.

- a)  $(\neg y_2 + x)$
- b)  $(y_3 + y_2 + u)$
- c)  $(\neg y_1 + w)$
- d)  $(y_3 + \neg y_2 + \neg y_1 + w)$

Answer submitted: a)

You have answered the question correctly.

12. Consider the grammar  $G: S \rightarrow SS, S \rightarrow ab$ . Which of the following strings is a word of  $L(G)$  AND is the yield of one of the parse trees for grammar  $G$  in the figure below?



- a) abababababababab
- b) abba
- c) abab
- d) abababab

You did not answer this question.

13. Let  $h$  be the homomorphism defined by  $h(a) = 01$ ,  $h(b) = 10$ ,  $h(c) = 0$ , and  $h(d) = 1$ . If we take any string  $w$  in  $(0+1)^*$ ,  $h^{-1}(w)$  contains some number of strings,  $N(w)$ . For example,  $h^{-1}(1100) = \{ddcc, dbc\}$ , i.e.,  $N(1100) = 2$ . We can calculate the number of strings in  $h^{-1}(w)$  by a recursion on the length of  $w$ . For example, if  $w = 00x$  for some string  $x$ , then  $N(w) = N(0x)$ , since the first 0 in  $w$  can only be produced from  $c$ , not from  $a$ .

Complete the reasoning necessary to compute  $N(w)$  for any string  $w$  in  $(0+1)^*$ . Then, choose the correct value of  $N(1011010)$ .

- a) 21
- b) 64
- c) 9
- d) 15

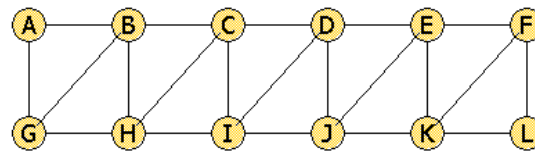
You did not answer this question.

14. Suppose a problem  $P_1$  reduces to a problem  $P_2$ . Which of the following statements can we conclude to be TRUE based on the above?
- If  $P_1$  is non-RE, then it must be that  $P_2$  is non-RE.
  - If  $P_2$  is non-RE, then it must be that  $P_1$  is non-RE.
  - If  $P_2$  is undecidable, then it must be that  $P_1$  is decidable.
  - If  $P_1$  is undecidable, then it must be that  $P_2$  is decidable.

Answer submitted: a)

You have answered the question correctly.

15. What is the size of a minimal node cover for the graph below?

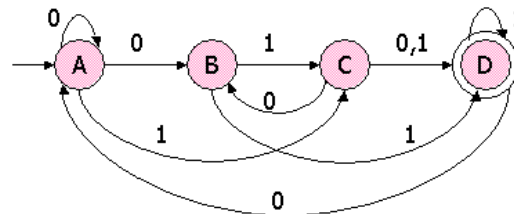


Identify one of the minimal node covers below.

- $\{A, C, E, G, H, I, J, K, L\}$
- $\{B, D, F, G, H, I, K\}$
- $\{B, C, D, F, G, I, K\}$
- $\{A, C, D, F, G, H, J, K\}$

You did not answer this question.

16. Here is a nondeterministic finite automaton:



Convert this NFA to a DFA, using the "lazy" version of the subset construction described in Section 2.3.5 (p. 60), so only the accessible states are constructed. Which of the following sets of NFA states becomes a state of the DFA constructed in this manner?

- $\{A, B, C, D\}$
- $\{A, C, D\}$
- $\{C, D\}$
- $\{B\}$

You did not answer this question.

17. Let  $G$  be the grammar:

$$S \rightarrow SS \mid (S) \mid \varepsilon$$

$L(G)$  is the language BP of all strings of balanced parentheses, that is, those strings that could appear in a well-formed arithmetic expression. We want to prove that  $L(G) = BP$ , which requires two inductive proofs:

1. If  $w$  is in  $L(G)$ , then  $w$  is in BP.
2. If  $w$  is in BP, then  $w$  is in  $L(G)$ .

We shall here prove only the second. You will see below a sequence of steps in the proof, each with a reason left out. These reasons belong to one of three classes:

- A) Use of the inductive hypothesis.
- B) Reasoning about properties of grammars, e.g., that every derivation has at least one step.
- C) Reasoning about properties of strings, e.g., that every string is longer than any of its proper substrings.

The proof is an induction on the length of  $w$ . You should decide on the reason for each step in the proof below, and then identify from the available choices a correct pair consisting of a step and a kind of reason (A, B, or C).

Basis: Length = 0.

- (1) The only string of length 0 in BP is  $\epsilon$  because \_\_\_\_\_
  - (2)  $\epsilon$  is in  $L(G)$  because \_\_\_\_\_  
Induction:  $|w| = n > 0$ .
  - (3)  $w$  is of the form  $(x)y$ , where  $(x)$  is the shortest proper prefix of  $w$  that is in BP, and  $y$  is the remainder of  $w$  because \_\_\_\_\_
  - (4)  $x$  is in BP because \_\_\_\_\_
  - (5)  $y$  is in BP because \_\_\_\_\_
  - (6)  $|x| < n$  because \_\_\_\_\_
  - (7)  $|y| < n$  because \_\_\_\_\_
  - (8)  $x$  is in  $L(G)$  because \_\_\_\_\_
  - (9)  $y$  is in  $L(G)$  because \_\_\_\_\_
  - (10)  $(x)$  is in  $L(G)$  because \_\_\_\_\_
  - (11)  $w$  is in  $L(G)$  because \_\_\_\_\_
- a) (11) for reason C
  - b) (3) for reason A
  - c) (2) for reason A
  - d) (9) for reason A

[You did not answer this question.](#)

18. Consider the grammars:

$G_1: S \rightarrow AB, A \rightarrow aAA|\epsilon, B \rightarrow abBB|\epsilon$   
 $G_2: S \rightarrow CB, C \rightarrow aCC|aC|a, B \rightarrow abBB|abB|ab$   
 $G_3: S \rightarrow CB|C|B|\epsilon, C \rightarrow aCC|aC|a, B \rightarrow abBB|abB|ab$   
 $G_4: S \rightarrow ASB|\epsilon, A \rightarrow aA|\epsilon, B \rightarrow abB|\epsilon$   
 $G_5: S \rightarrow ASB|AB, A \rightarrow aA|a, B \rightarrow abB|ab$   
 $G_6: S \rightarrow ASB|aab, A \rightarrow aA|a, B \rightarrow abB|ab$

Describe the language of each of these grammars. Then, identify from the list below a pair of grammars that define the same language?

- a)  $G_3$  and  $G_4$
- b)  $G_1$  and  $G_2$
- c)  $G_2$  and  $G_6$
- d)  $G_4$  and  $G_6$

[You did not answer this question.](#)

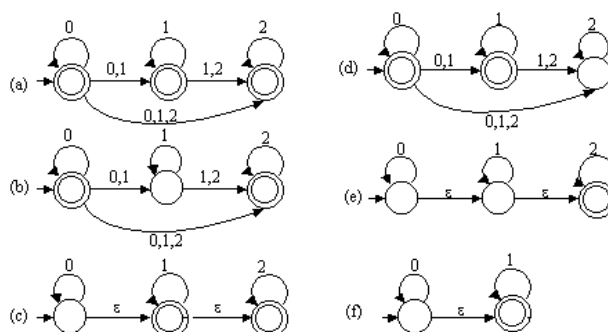
19.  $G_1$  is a context-free grammar with start symbol  $S_1$ , and no other nonterminals whose name begins with "S." Similarly,  $G_2$  is a context-free grammar with start symbol  $S_2$ , and no other nonterminals whose name begins with "S."  $S_1$  and  $S_2$  appear on the right side of no productions. Also, no nonterminal appears in both  $G_1$  and  $G_2$ .

We wish to combine the symbols and productions of  $G_1$  and  $G_2$  to form a new grammar  $G$ , whose language is the union of the languages of  $G_1$  and  $G_2$ . The start symbol of  $G$  will be  $S$ . All productions and symbols of  $G_1$  and  $G_2$  will be symbols and productions of  $G$ . Which of the following sets of productions, added to those of  $G$ , is guaranteed to make  $L(G)$  be  $L(G_1)$  [union]  $L(G_2)$ ?

- $S \rightarrow S_1 S_3 \mid S_2 S_3, S_3 \rightarrow \epsilon$
- $S \rightarrow S_3 S_2, S_3 \rightarrow S_1 \mid \epsilon$
- $S \rightarrow S_1 S_3, S_3 \rightarrow S_2$
- $S \rightarrow S_3 S_2, S_3 \rightarrow S_1$

You did not answer this question.

20. Identify which automata define the same language and provide the correct counterexample if they don't. Choose the correct statement from the list below.



- (c) and (b) do not define the same language and the following counterexample shows it. String 0012 is accepted by one and not by the other.
- (e) and (b) do not define the same language and the following counterexample shows it. String 0111 is accepted by one and not by the other.
- (a) and (d) do not define the same language and the following counterexample shows it. String 0012 is accepted by one and not by the other.
- (a) and (f) define the same language.

You did not answer this question.

21. Which of the following grammars derives a subset  $L_s$  of the language:  $L = \{x \mid \text{(i) } x \text{ contains a and c in proportion 4:3, (ii) } x \text{ does not begin with c and (iii) there are no two consecutive c's such that } L_s \text{ is missing at most a finite number of strings from } L\}$

- $S \rightarrow \epsilon, S \rightarrow SaScSaScSa$
- $S \rightarrow \epsilon, S \rightarrow SaScSaScSaScSaS, S \rightarrow A, A \rightarrow acaca$
- $S \rightarrow \epsilon, S \rightarrow SaScSaScSaSaSaS$
- $S \rightarrow acacaca, S \rightarrow SaSaSaScSaScSaS$

You did not answer this question.

22. Programming languages are often described using an extended form of context-free grammar, where square brackets are used to denote an optional construct. For example,  $A \rightarrow B[C]D$  says that an  $A$  can be replaced by a  $B$  and a  $D$ , with an optional  $C$  between them. This notation does not allow us to



describe anything but context-free languages, since an extended production can always be replaced by several conventional productions.

Suppose a grammar has the extended productions:

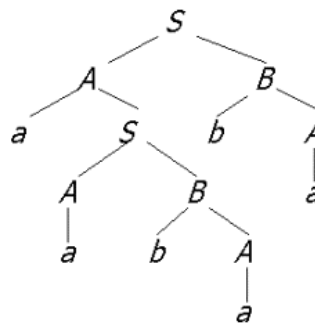
$$A \rightarrow B[CD]EF \mid BC[DE]F$$

Convert this pair of extended productions to conventional productions. Identify, from the list below, the conventional productions that are equivalent to the extended productions above.

- a)  $A \rightarrow BA_1EF \mid BCA_2F$   
 $A_1 \rightarrow CD \mid \varepsilon$   
 $A_2 \rightarrow DE \mid \varepsilon$
- b)  $A \rightarrow BA_1F$   
 $A_1 \rightarrow CD \mid DE$
- c)  $A \rightarrow BCDEF \mid BEF \mid BCF \mid BF$
- d)  $A \rightarrow BCDEF \mid BF$

You did not answer this question.

23. The following is a parse tree in some unknown grammar G:



Which of the following productions is **definitely not** a production of G?

- a)  $S \rightarrow aC$
- b)  $B \rightarrow CD$
- c) None of the other choices.
- d)  $S \rightarrow AB$

You did not answer this question.

24. Here is the transition function of a simple, deterministic automaton with start state A and accepting state B:

	0	1
A	A	B
B	B	A

We want to show that this automaton accepts exactly those strings with an odd number of 1's, or more formally:

$$\delta(A, w) = B \text{ if and only if } w \text{ has an odd number of 1's.}$$

Here,  $\delta$  is the extended transition function of the automaton; that is,  $\delta(A, w)$  is the state that the automaton is in after processing input string  $w$ . The proof of the statement above is an induction on the

length of  $w$ . Below, we give the proof with reasons missing. You must give a reason for each step, and then demonstrate your understanding of the proof by classifying your reasons into the following three categories:

- A) Use of the inductive hypothesis.
- B) Reasoning about properties of deterministic finite automata, e.g., that if string  $s = yz$ , then  $\delta(q,s) = \delta(\delta(q,y),z)$ .
- C) Reasoning about properties of binary strings (strings of 0's and 1's), e.g., that every string is longer than any of its proper substrings.

Basis ( $|w| = 0$ ):

- (1)  $w = \epsilon$  because \_\_\_\_\_
- (2)  $\delta(A,\epsilon) = A$  because \_\_\_\_\_
- (3)  $\epsilon$  has an even number of 0's because \_\_\_\_\_

Induction ( $|w| = n > 0$ )

- (4) There are two cases: (a) when  $w = x1$  and (b) when  $w = x0$  because \_\_\_\_\_  
Case (a):
- (5) In case (a),  $w$  has an odd number of 1's if and only if  $x$  has an even number of 1's because \_\_\_\_\_
- (6) In case (a),  $\delta(A,x) = A$  if and only if  $w$  has an odd number of 1's because \_\_\_\_\_
- (7) In case (a),  $\delta(A,w) = B$  if and only if  $w$  has an odd number of 1's because \_\_\_\_\_  
Case (b):
- (8) In case (b),  $w$  has an odd number of 1's if and only if  $x$  has an odd number of 1's because \_\_\_\_\_
- (9) In case (b),  $\delta(A,x) = B$  if and only if  $w$  has an odd number of 1's because \_\_\_\_\_
- (10) In case (b),  $\delta(A,w) = B$  if and only if  $w$  has an odd number of 1's because \_\_\_\_\_
- a) (4) for reason C.
- b) (2) for reason C.
- c) (5) for reason B.
- d) (1) for reason A.

[You did not answer this question.](#)

25. Which among the following languages is not regular (cannot be defined by a regular expression or finite automaton)?

- a)  $L = \{x \mid x = (ab^4c)^n, n \text{ a positive integer}\}$
- b)  $L = \{x \mid x = (a^m b^n c^m)^n, n, m \text{ positive integers}\}$
- c)  $L = \{x \mid x = a^m (bc^6)^n, n, m \text{ positive integers}\}$
- d)  $L = \{x \mid x = a^m (bc)^n, n, m \text{ positive integers}\}$

[You did not answer this question.](#)