

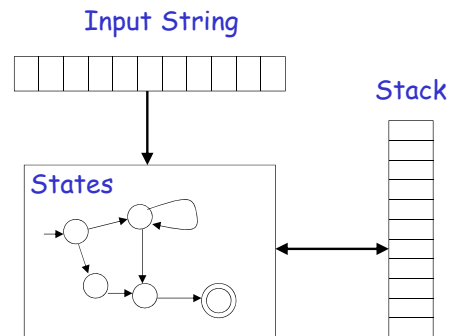
Pushdown Automata

PDAs

class 10

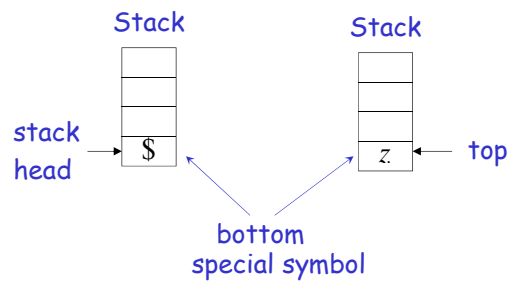
1

Pushdown Automaton -- PDA



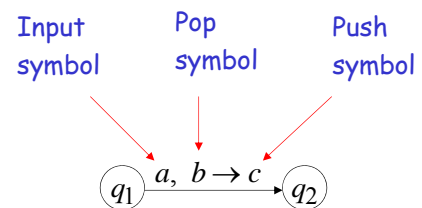
2

Initial Stack Symbol

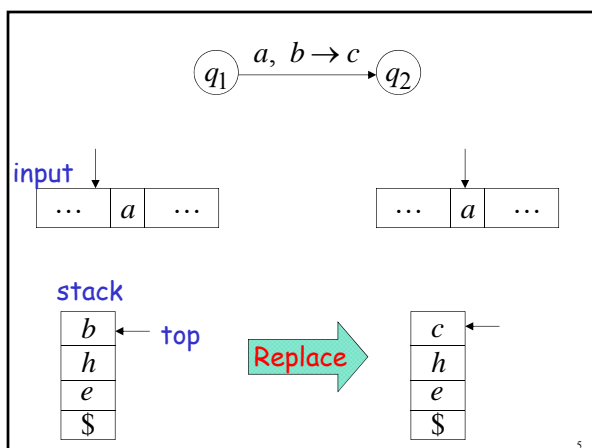


3

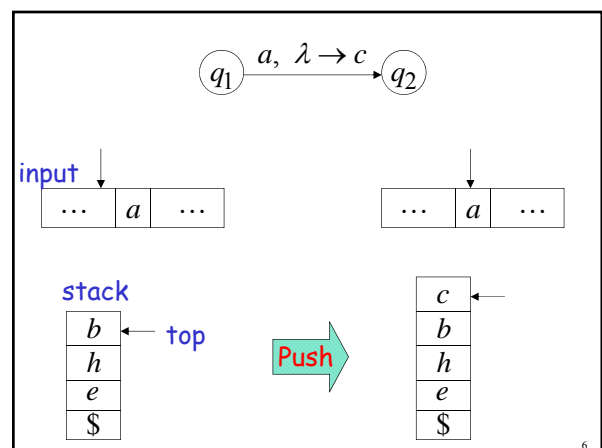
The States



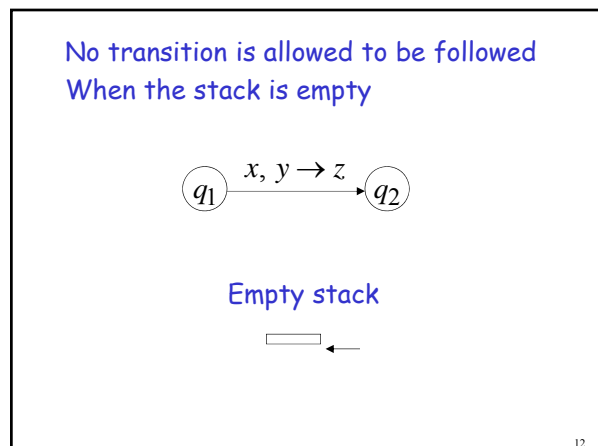
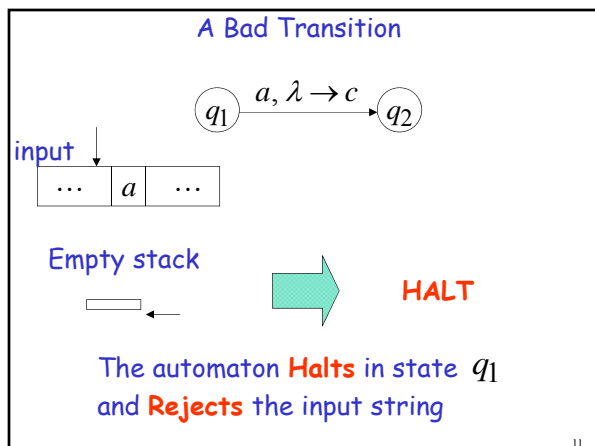
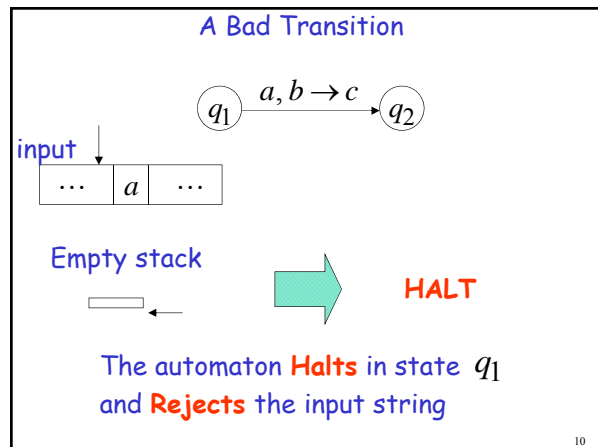
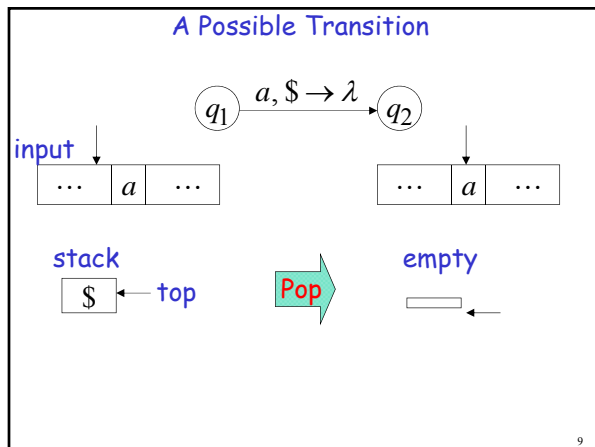
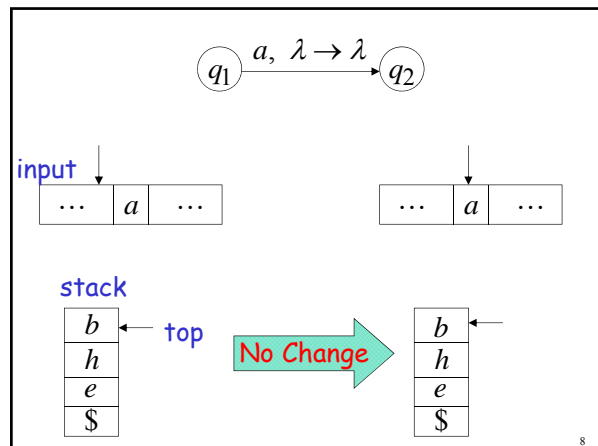
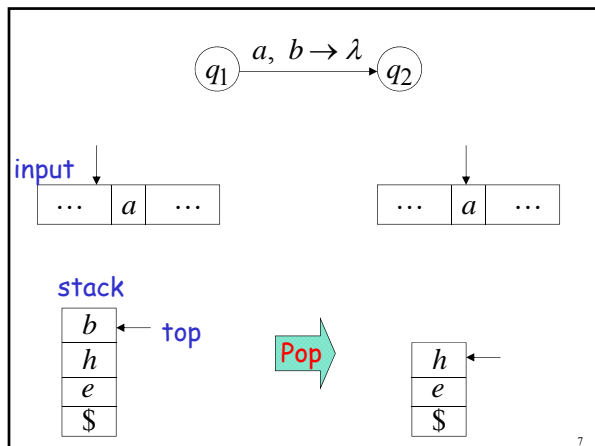
4

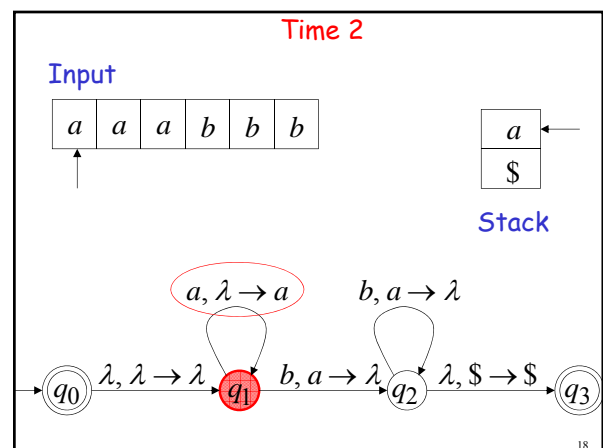
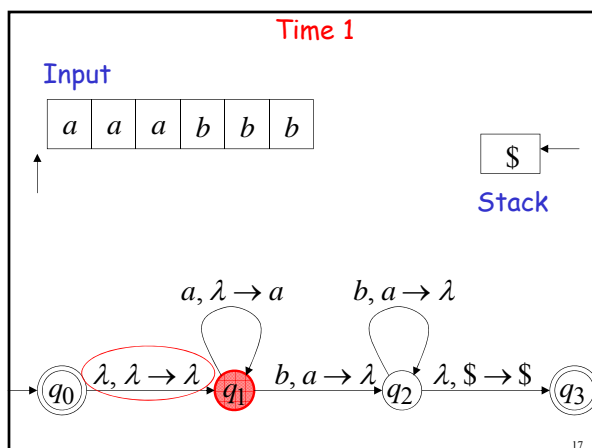
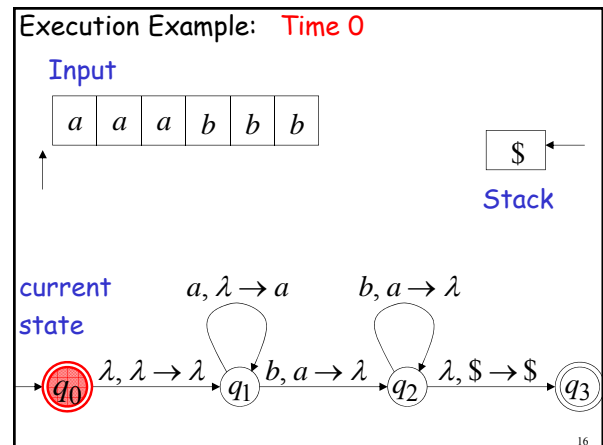
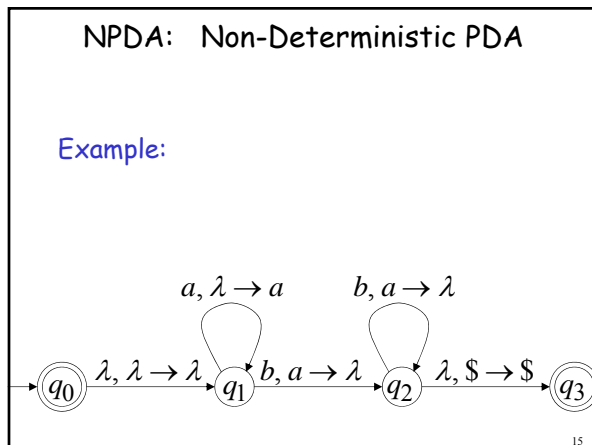
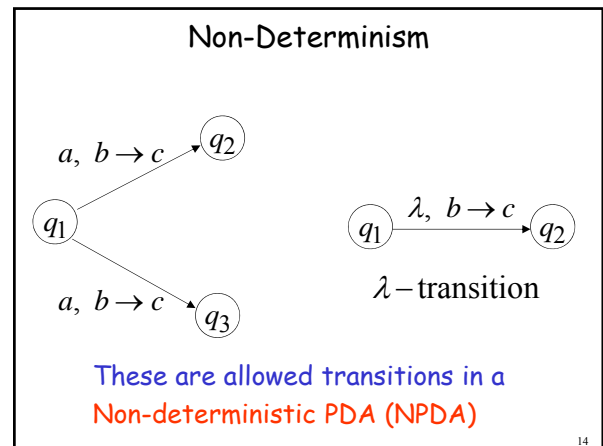
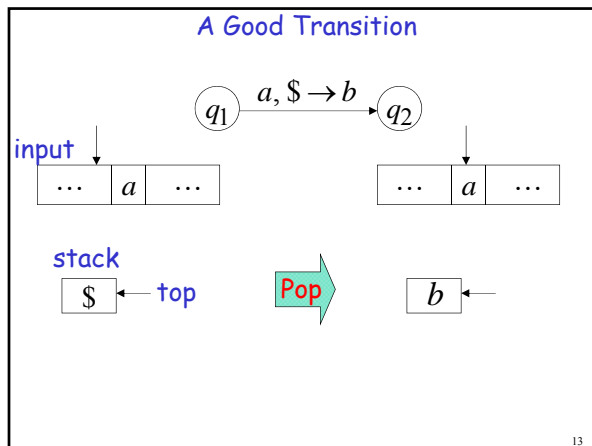


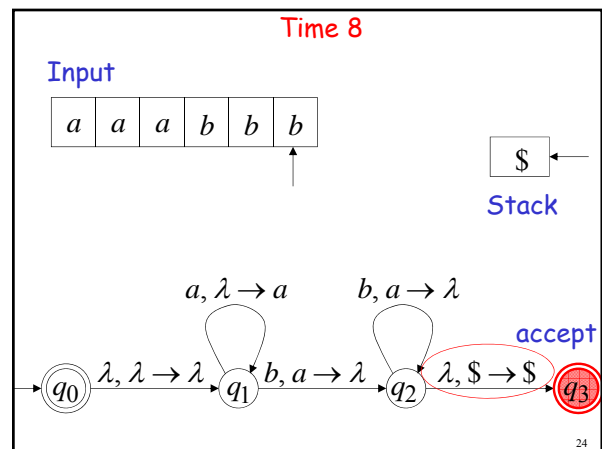
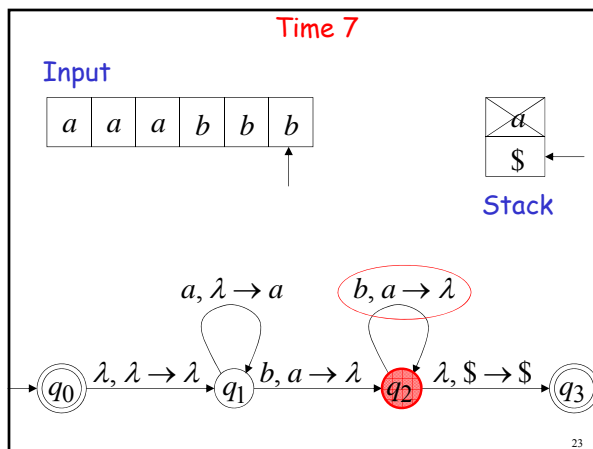
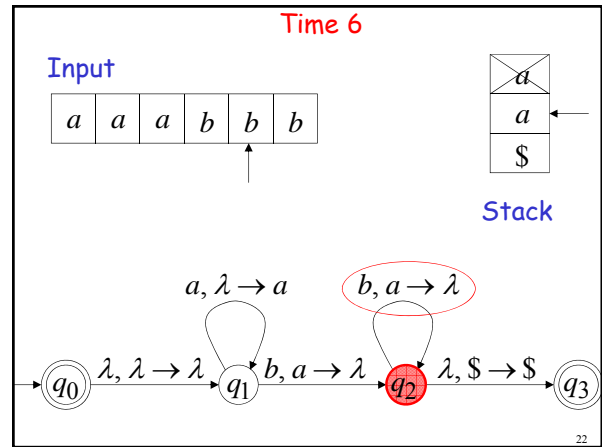
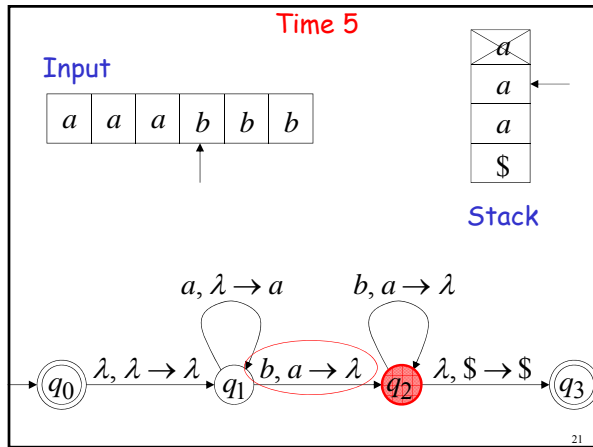
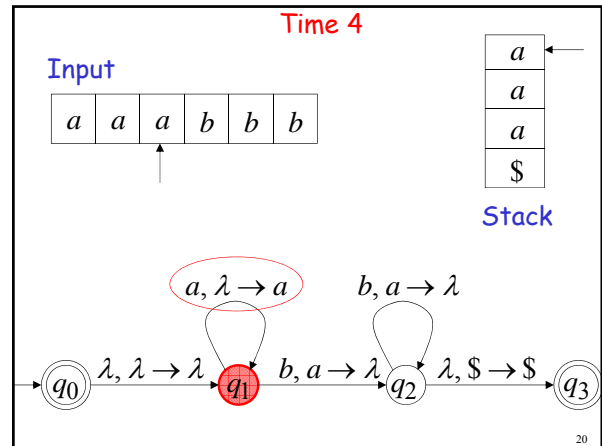
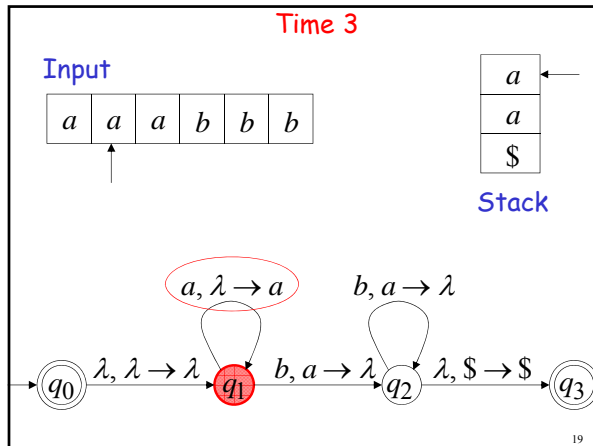
5



6







A string is accepted if there is a computation such that:

All the input is consumed

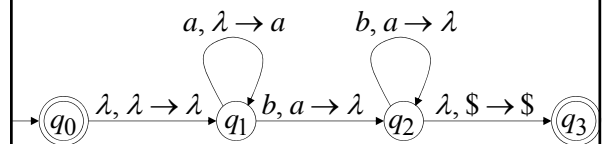
AND

The last state is a final state

At the end of the computation, we do not care about the stack contents

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The input string *aaabbbb* is accepted by the NPDA:

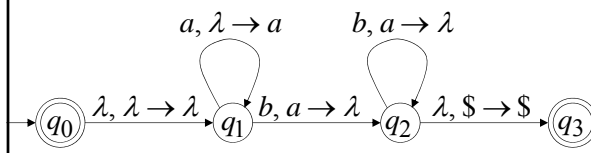


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In general,

$$L = \{a^n b^n : n \geq 0\}$$

is the language accepted by the NPDA:

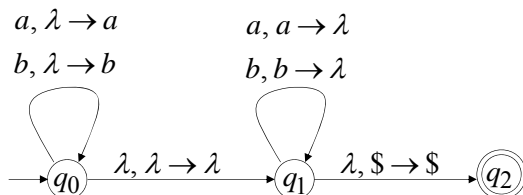


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Another NPDA example

NPDA *M*

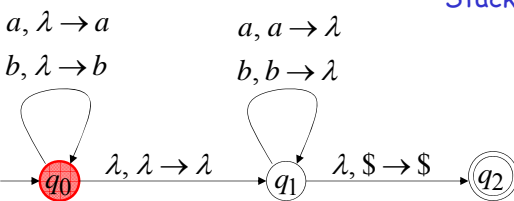
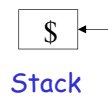
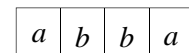
$$L(M) = \{ww^R\}$$



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Execution Example: **Time 0**

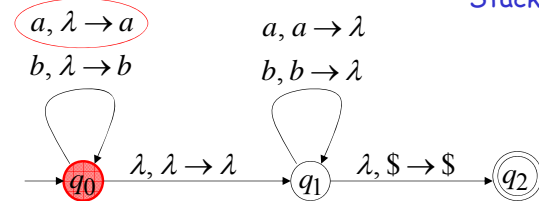
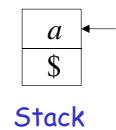
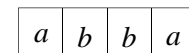
Input



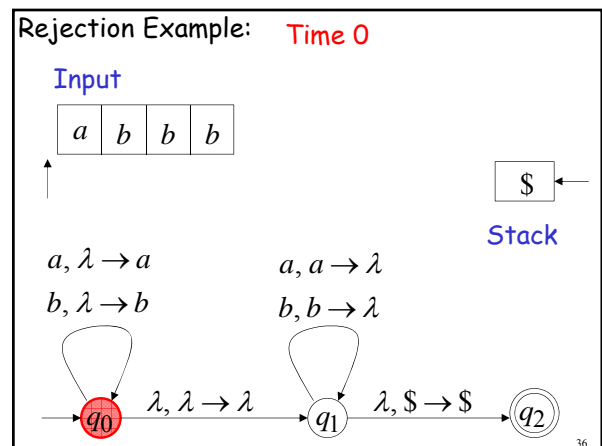
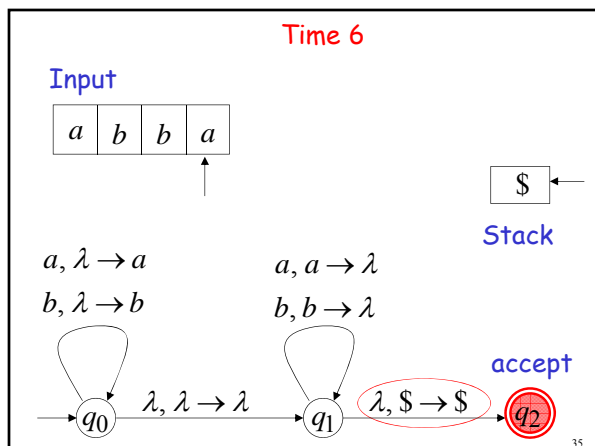
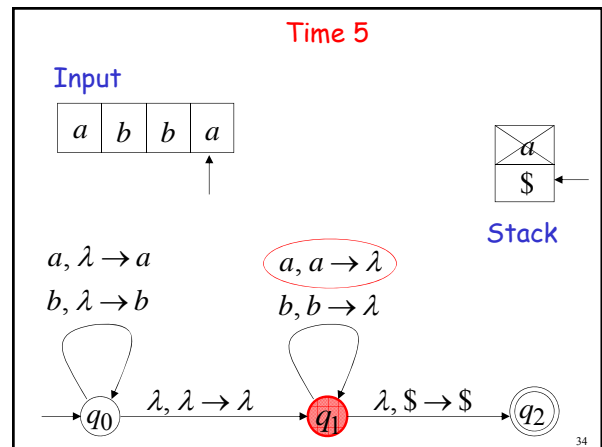
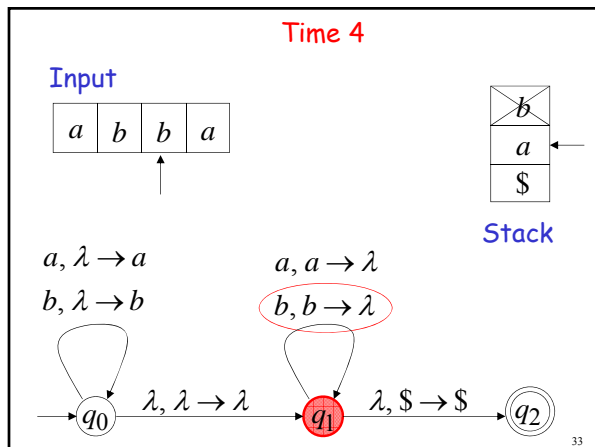
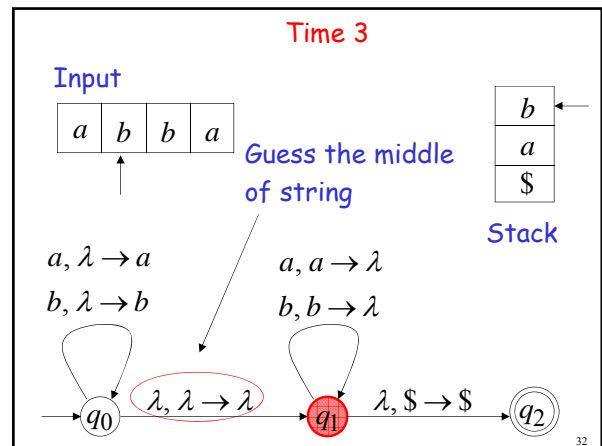
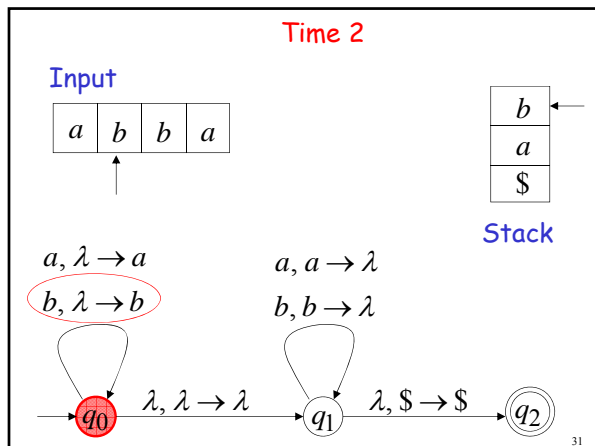
29

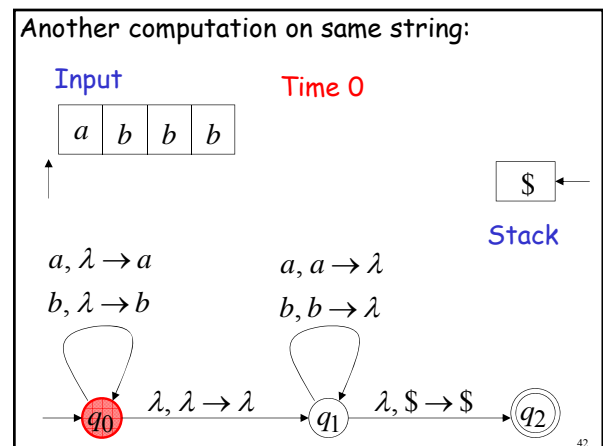
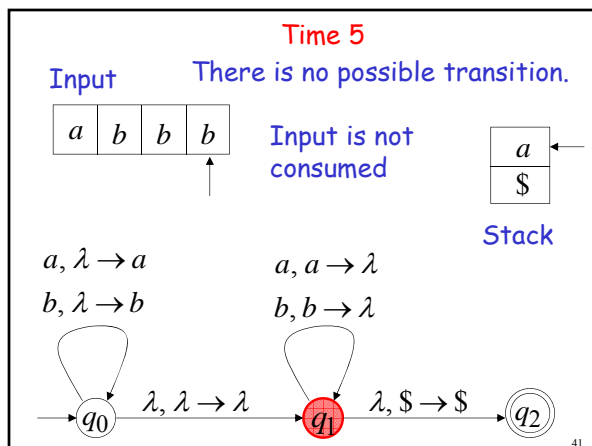
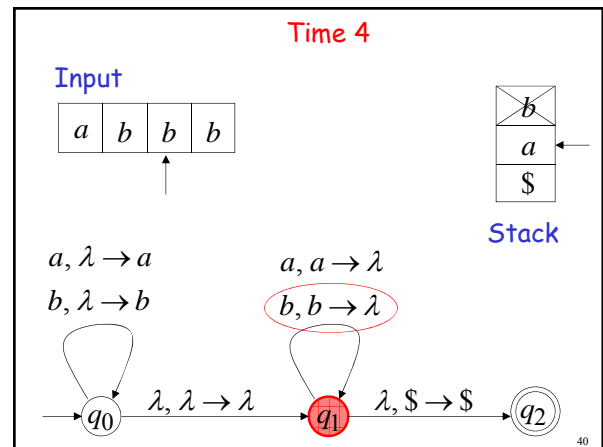
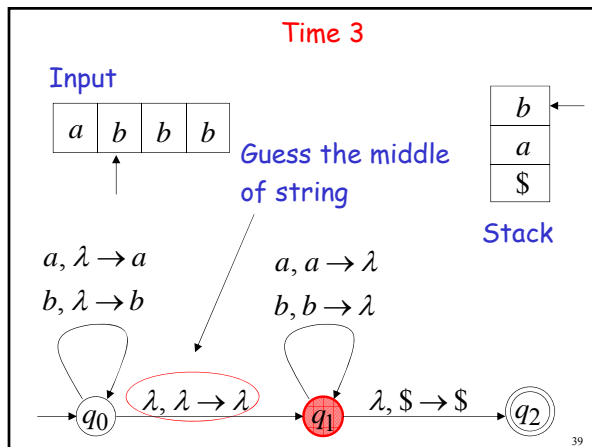
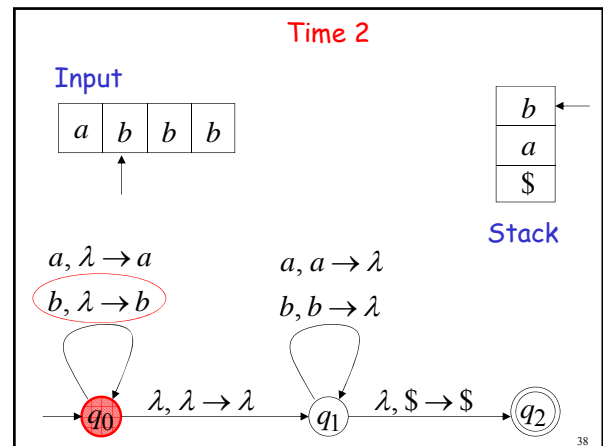
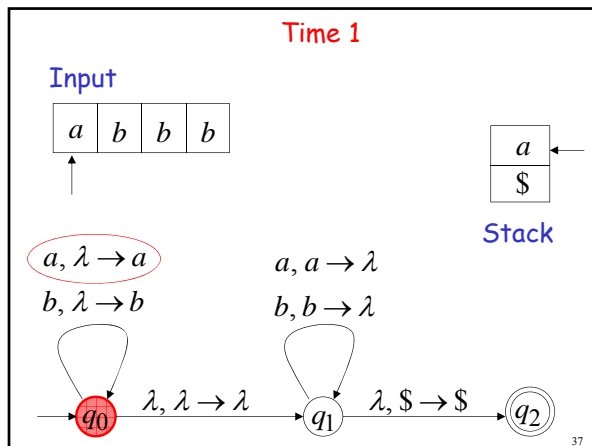
Time 1

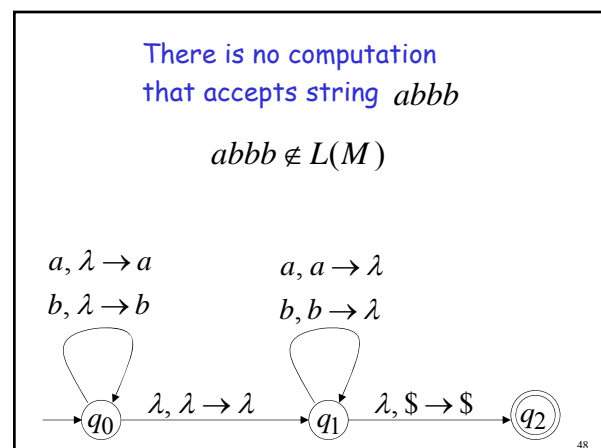
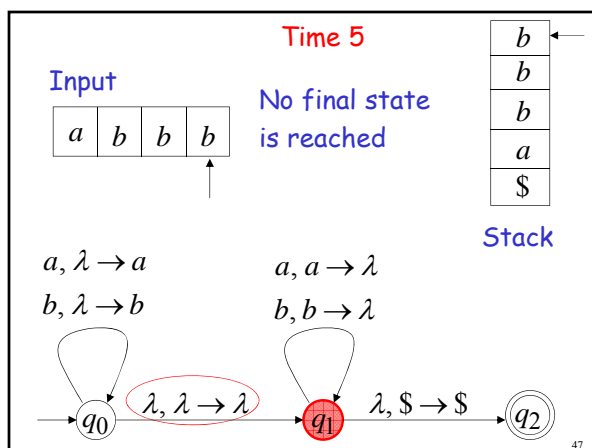
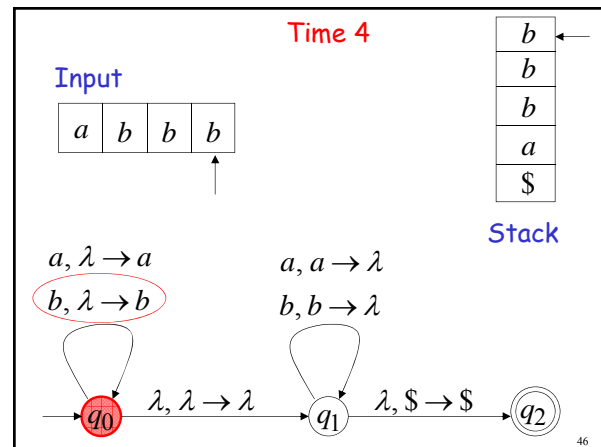
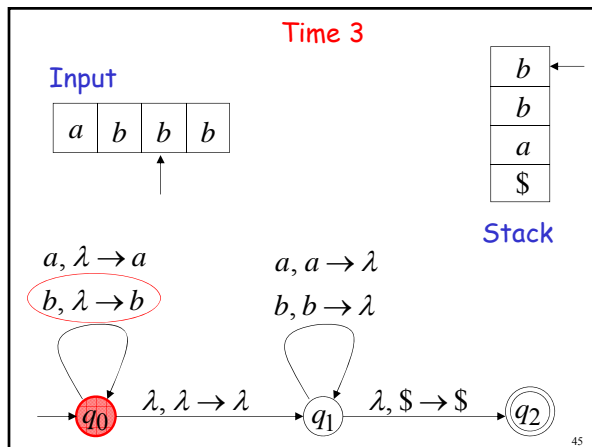
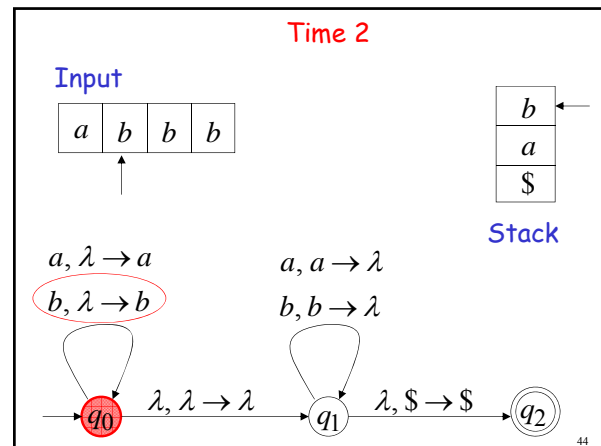
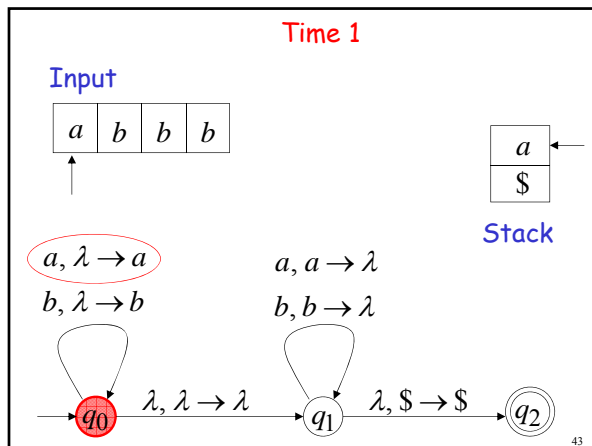
Input



30







A string is rejected if there is no computation such that:

All the input is consumed
AND
 The last state is a final state

At the end of the computation,
 we do not care about the stack contents

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In other words, a string is rejected if in every computation with this string:

The input cannot be consumed
OR

The input is consumed and the last state is not a final state

OR
 The stack head moves below the bottom of the stack

50

Another NPDA example

NPDA M

$$L(M) = \{a^n b^m : n \geq m - 1\}$$

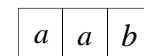
$a, \lambda \rightarrow a$
 $b, a \rightarrow \lambda$



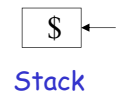
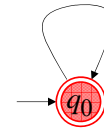
51

Execution Example: Time 0

Input



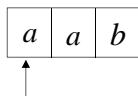
$a, \lambda \rightarrow a$
 $b, a \rightarrow \lambda$
 $b, \$ \rightarrow \lambda$



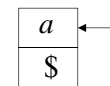
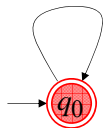
52

Time 1

Input



$a, \lambda \rightarrow a$
 $b, a \rightarrow \lambda$
 $b, \$ \rightarrow \lambda$

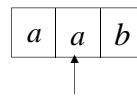


Stack

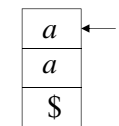
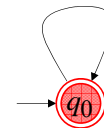
53

Time 2

Input

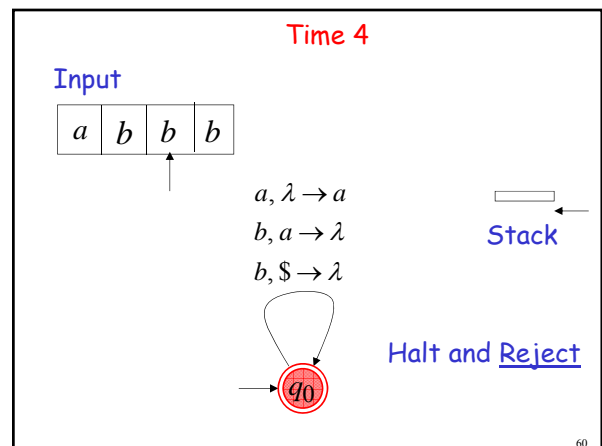
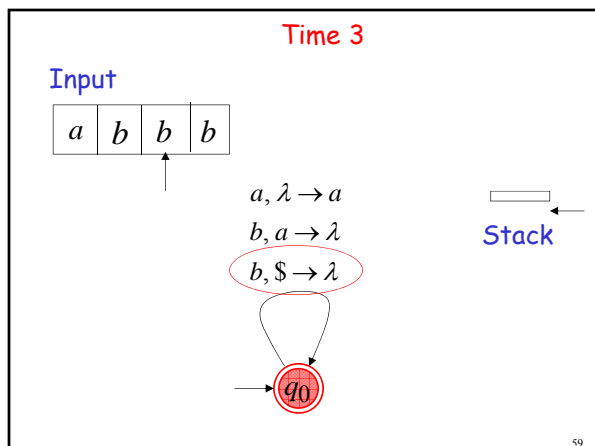
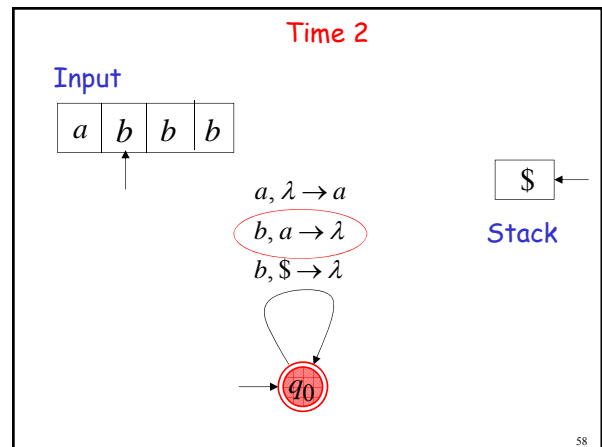
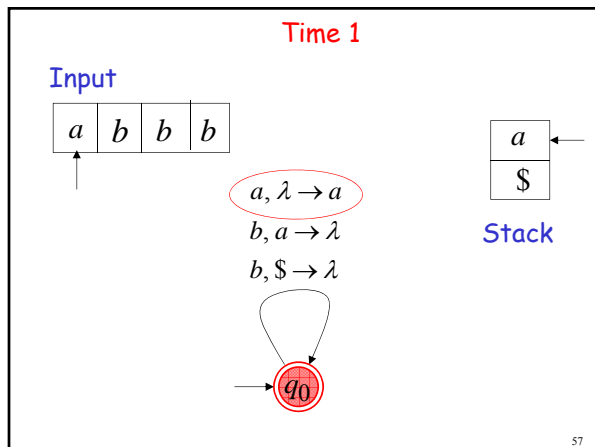
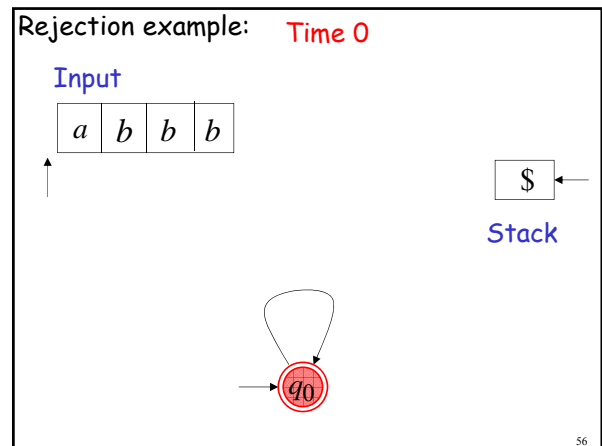
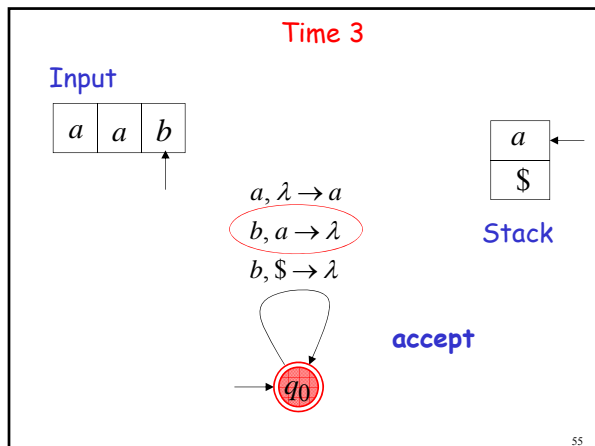


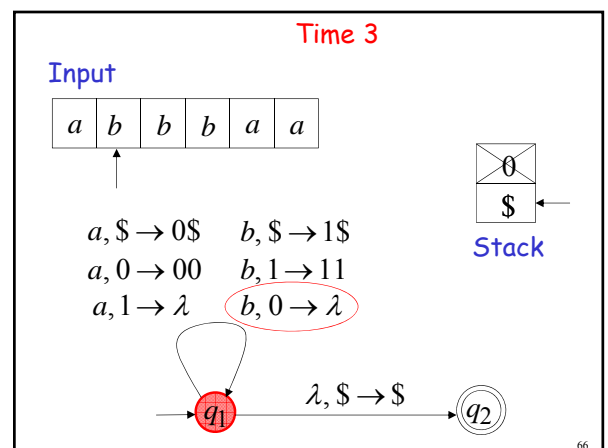
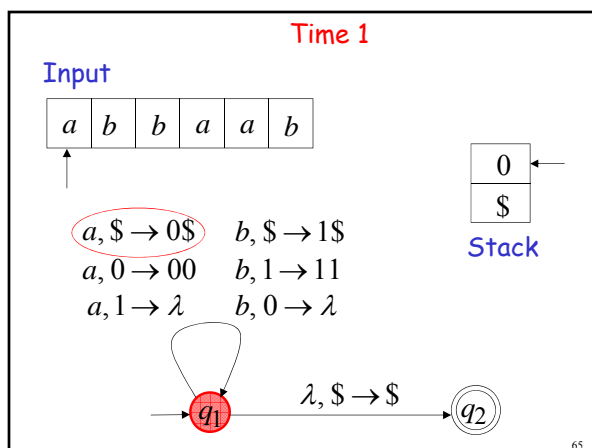
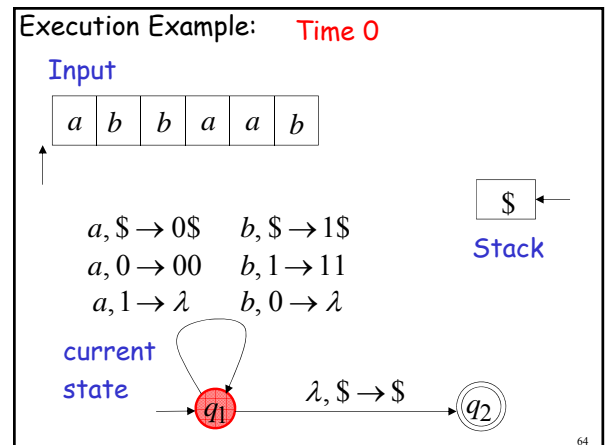
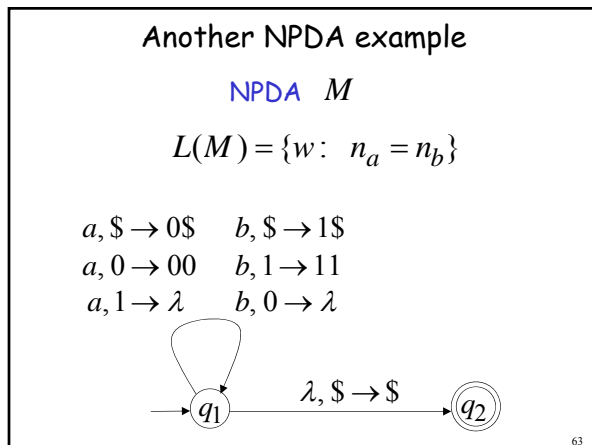
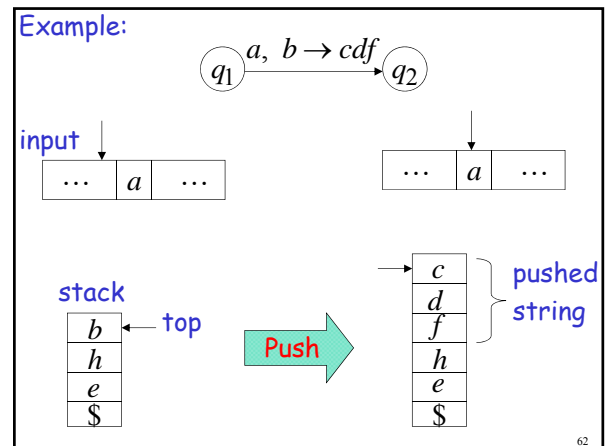
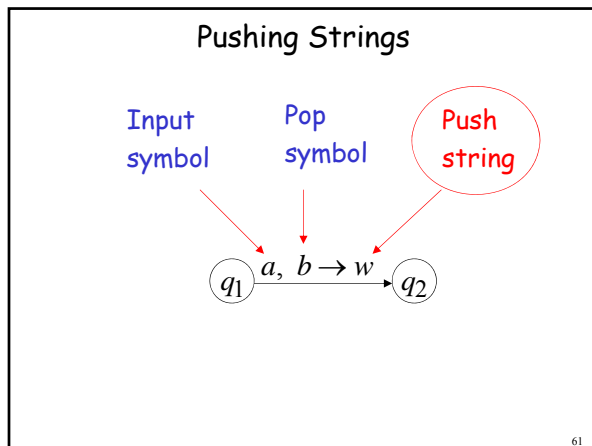
$a, \lambda \rightarrow a$
 $b, a \rightarrow \lambda$
 $b, \$ \rightarrow \lambda$

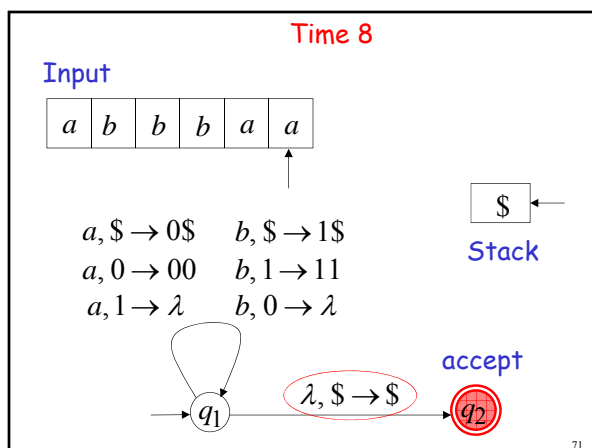
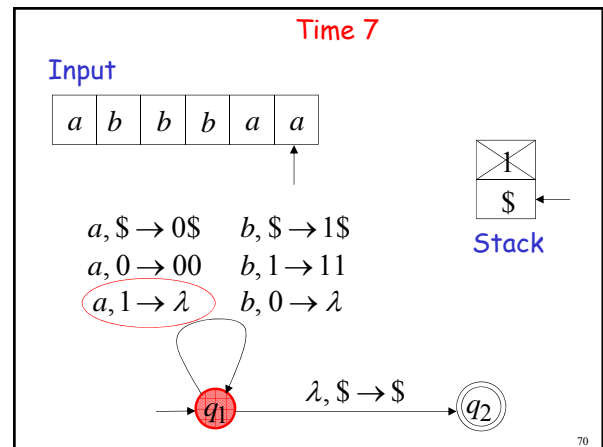
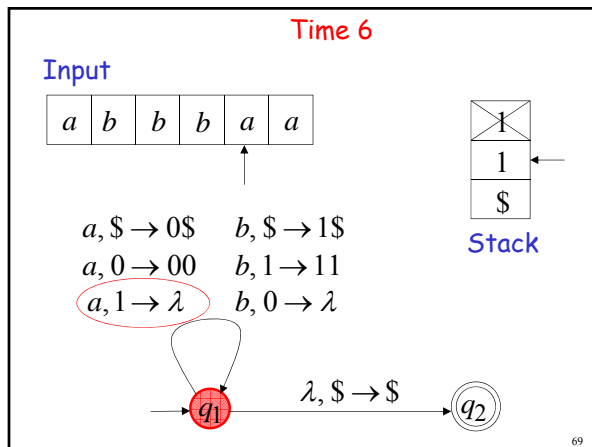
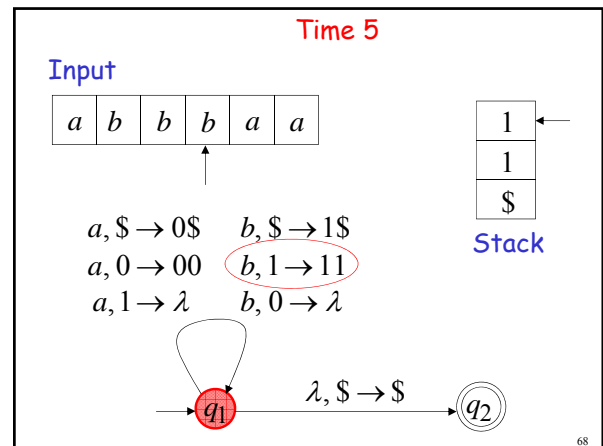
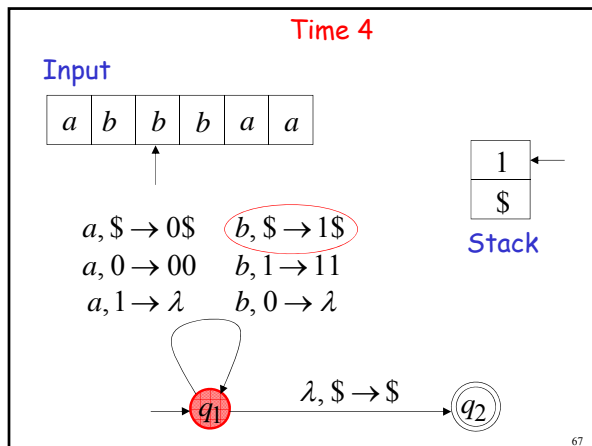


Stack

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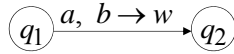






Formalities for NPDAs

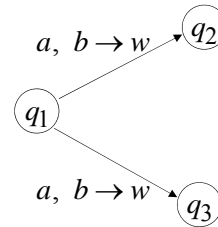
72



Transition function:

$$\delta(q_1, a, b) = \{(q_2, w)\}$$

73



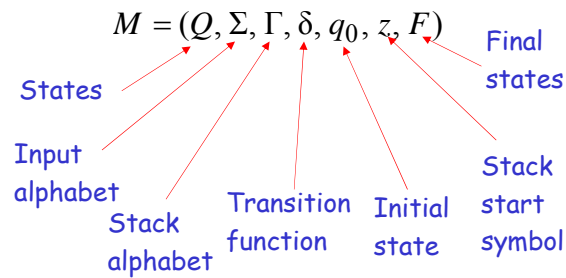
Transition function:

$$\delta(q_1, a, b) = \{(q_2, w), (q_3, w)\}$$

74

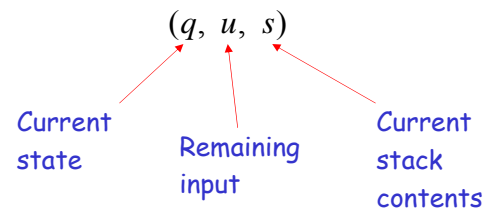
Formal Definition

Non-Deterministic Pushdown Automaton
NPDA



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Instantaneous Description



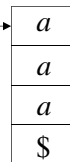
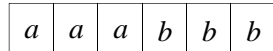
76

Example: **Instantaneous Description**

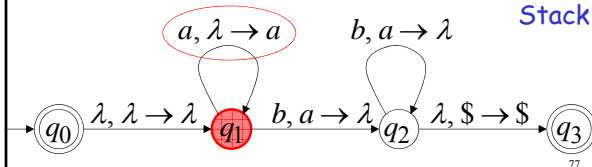
$(q_1, bbb, aaa\$)$

Time 4:

Input



Stack



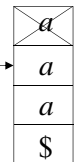
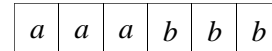
77

Example: **Instantaneous Description**

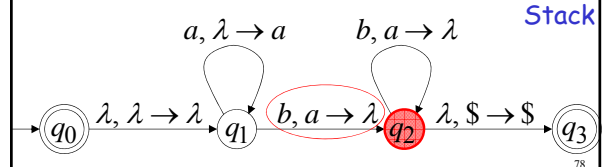
$(q_2, bb, aa\$)$

Time 5:

Input



Stack



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We write:

$$(q_1, bbb, aaa\$) \succ (q_2, bb, aa\$)$$

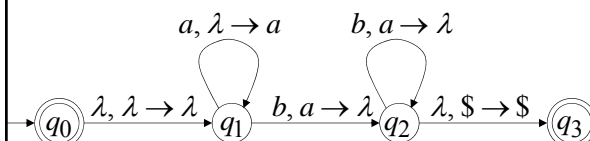
Time 4

Time 5

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A computation:

$$\begin{aligned} &(q_0, aaabbb, \$) \succ (q_1, aaabbb, \$) \succ \\ &(q_1, aabbb, a\$) \succ (q_1, abbb, aa\$) \succ (q_1, bbb, aaa\$) \succ \\ &(q_2, bb, aa\$) \succ (q_2, b, a\$) \succ (q_2, \lambda, \$) \succ (q_3, \lambda, \$) \end{aligned}$$



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$$\begin{aligned} &(q_0, aaabbb, \$) \succ (q_1, aaabbb, \$) \succ \\ &(q_1, aabbb, a\$) \succ (q_1, abbb, aa\$) \succ (q_1, bbb, aaa\$) \succ \\ &(q_2, bb, aa\$) \succ (q_2, b, a\$) \succ (q_2, \lambda, \$) \succ (q_3, \lambda, \$) \end{aligned}$$

For convenience we write:

$$(q_0, aaabbb, \$) \overset{*}{\succ} (q_3, \lambda, \$)$$

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Formal Definition

Language $L(M)$ of NPDA M :

$$L(M) = \{w : (q_0, w, s) \overset{*}{\succ} (q_f, \lambda, s')\}$$

Initial state

Final state

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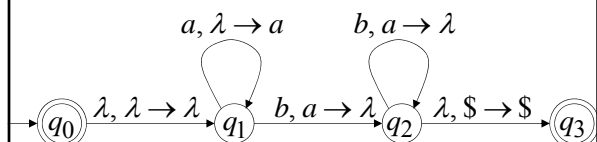
Example:

$$(q_0, aaabbb, \$) \overset{*}{\succ} (q_3, \lambda, \$)$$



$$aaabbb \in L(M)$$

NPDA M :



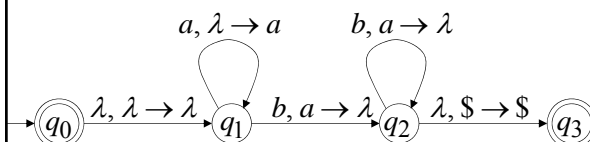
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$$(q_0, a^n b^n, \$) \overset{*}{\succ} (q_3, \lambda, \$)$$



$$a^n b^n \in L(M)$$

NPDA M :



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Therefore: $L(M) = \{a^n b^n : n \geq 0\}$

NPDA M :

