Automata Theory, Languages, and Computation

Name: ______________________ Date: ____________

Note: The purpose of the following questions is:

• *Enhance learning*	• *Summarized points*	• *Analyze abstract ideas*

Class 13: Pumping Lemma for Context-Free Languages

The family of context-free languages occupies a central position in a hierarchy of formal languages. On the one hand, context-free languages include important but restricted language families such as regular and deterministic context-free languages. On the other hand, there are broader language families of which context-free languages are special case. To study the relationship between language families and to exhibit their similarities and differences, we investigate characteristic properties of the various families. As in class 4, we look at closure under a variety of operations, algorithms for determining properties of members of the family, and structural results such as pumping lemmas. These all provides us with a means of understanding relations between different families as well as for classifying specific languages in an appropriate category.

Two Pumping Lemmas

The pumping lemma given in class 6 is an effective tool for showing that certain languages are not regular. Similar pumping lemmas are known for other language families.

1. Take an infinite context-free language

$$S \rightarrow AB$$
$$A \rightarrow aBb$$
$$B \rightarrow Sb$$
$$B \rightarrow b$$

Exercise:

i. *Show the leftmost derivation and the derivation tree for the string abbabbbb, what are your observations regarding repeated nodes.*

ii. *Show the leftmost derivation and the derivation tree for* $B \overset{*}{\Rightarrow} aBbbb$ *, what are your observations regarding repeated nodes.*

iii. *Show the leftmost derivation and the derivation tree for* $S \overset{*}{\Rightarrow} abbBb$ *, what are your observations regarding repeated nodes.*

A Pumping Lemma for Context-Free Languages

2. Prove the **Theorem**:

> For infinite context-free language L
>
> there exists an integer m such that
>
> for any string $w \in L, \quad |w| \geq m$
>
> we can write $w = uvxyz$
>
> with lengths $|vxy| \leq m$ and $|vy| \geq 1$
>
> and it must be:
>
> $$uv^i xy^i z \in L, \quad \text{for all } i \geq 0$$
>
> 42

This pumping lemma is useful in showing that a language does not belong to the family of context-free languages. Its application is typical of pumping lemmas in general; they are used negatively to show that a given language doesn't belong to some family. As in the pumping lemma for regular languages - class 6, the correct argument can be visualized as a game against an intelligent opponent. But now the rules make it a little more difficult for us. For regular languages, the substring xy whose length is bounded by m starts at the left end of w. Therefore, the substring y that can be pumped in within m symbols of the beginning of w. For context-Free languages, we only have a bound on $|vxy|$. The substring u that precedes vxy can be arbitrarily long. This gives additional freedom to the adversary, making arguments involving The Pumping Lemma for Context-Free languages a little more complicated.

Applications of the Pumping Lemma

3. Show that the language

$$L = \{a^n b^n c^n : n \geq 0\}$$

is not context-free