



## Gradiance Online Accelerated Learning

Zayd

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### Help

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1. We can represent questions about context-free languages and regular languages by choosing a standard encoding for context-free grammars (CFG's) and another for regular expressions (RE's), and phrasing the question as recognition of the codes for grammars and/or regular expressions such that their languages have certain properties. Some sets of codes are decidable, while others are not.

In what follows, you may assume that  $G$  and  $H$  are context-free grammars with terminal alphabet  $\{0,1\}$ , and  $R$  is a regular expression using symbols 0 and 1 only. You may assume that the problem "Is  $L(G) = (0+1)^*$ ?", that is, the problem of recognizing all and only the codes for CFG's  $G$  whose language is all strings of 0's and 1's, is undecidable.

There are certain other problems about CFG's and RE's that are decidable, using well-known algorithms. For example, we can test if  $L(G)$  is empty by finding the pumping-lemma constant  $n$  for  $G$ , and checking whether or not there is a string of length  $n$  or less in  $L(G)$ . It is not possible that the shortest string in  $L(G)$  is longer than  $n$ , because the pumping lemma lets us remove at least one symbol from a string that long and find a shorter string in  $L(G)$ .

You should try to determine which of the following problems are decidable, and which are undecidable:

- Is  $\text{Comp}(L(G))$  equal to  $(0+1)^*$ ? [ $\text{Comp}(L)$  is the complement of language  $L$  with respect to the alphabet  $\{0,1\}$ .]
- Is  $\text{Comp}(L(G))$  empty?
- Is  $L(G) \cap L(H)$  equal to  $(0+1)^*$ ?
- Is  $L(G) \cup L(H)$  equal to  $(0+1)^*$ ?
- Is  $L(G)$  finite?
- Is  $L(G)$  contained in  $L(H)$ ?
- Is  $L(G) = L(H)$ ?
- Is  $L(G) = L(R)$ ?
- Is  $L(G)$  contained in  $L(R)$ ?
- Is  $L(R)$  contained in  $L(G)$ ?

Then, identify the true statement from the list below:

- a) "Is  $L(G)$  finite?" is decidable.
- b) "Is  $L(G)$  infinite?" is undecidable.
- c) "Is  $L(R)$  contained in  $L(G)$ ?" is decidable.
- d) "Is  $L(G) = L(H)$ ?" is decidable.

Answer submitted: a)

You have answered the question correctly.

2. Which of the following grammars derives a subset of the language:

$\{x \mid x \text{ contains a and c in proportion 4:3 and there are no two consecutive c's}\}$ ?

- a)  $S \rightarrow \epsilon \ S \rightarrow aScSaScSaS$
- b)  $S \rightarrow acacaca \ S \rightarrow SacSaScSaScSaS$
- c)  $S \rightarrow acacaca \ S \rightarrow SaSaSaScSaScSaS$
- d)  $S \rightarrow \epsilon \ S \rightarrow SaScSaScaSaSaSaS$

You did not answer this question.

3. Consider the pushdown automaton with the following transition rules:

- 1.  $\delta(q, 0, Z_0) = \{(q, XZ_0)\}$
- 2.  $\delta(q, 0, X) = \{(q, XX)\}$
- 3.  $\delta(q, 1, X) = \{(q, X)\}$
- 4.  $\delta(q, \epsilon, X) = \{(p, \epsilon)\}$
- 5.  $\delta(p, \epsilon, X) = \{(p, \epsilon)\}$
- 6.  $\delta(p, 1, X) = \{(p, XX)\}$
- 7.  $\delta(p, 1, Z_0) = \{(p, \epsilon)\}$

The start state is  $q$ . For which of the following inputs can the PDA first enter state  $p$  with the input empty and the stack containing  $XXZ_0$  [i.e., the ID  $(p, \epsilon, XXZ_0)$ ]?

- a) 111001
- b) 1100101
- c) 010101
- d) 0111011

You did not answer this question.

4. Suppose there are three languages (i.e., problems), of which we know the following:

- 1.  $L_1$  is in P.
- 2.  $L_2$  is NP-complete.
- 3.  $L_3$  is not in NP.

Suppose also that we do not know anything about the resolution of the "P vs. NP" question; for example, we do not know definitely whether  $P=NP$ . Classify each of the following languages as (a) Definitely in P, (b) Definitely in NP (but perhaps not in P and perhaps not NP-complete) (c) Definitely NP-complete (d) Definitely not in NP:

- $L_1 \cup L_2$ .
- $L_1 \cap L_2$ .
- $L_2 c L_3$ , where  $c$  is a symbol not in the alphabet of  $L_2$  or  $L_3$  (i.e., the *marked concatenation* of  $L_2$  and  $L_3$ , where there is a unique marker symbol between the strings from  $L_2$  and  $L_3$ ).
- The complement of  $L_3$ .

Based on your analysis, pick the correct, definitely true statement from the list below.

- a)  $L1 \cap L2$  is definitely in P.
- b)  $L2 \subseteq L3$  is definitely not in NP.
- c)  $L2 \subseteq L3$  is definitely NP-complete.
- d)  $L1 \cup L2$  is definitely not in NP.

Answer submitted: **b)**

You have answered the question correctly.

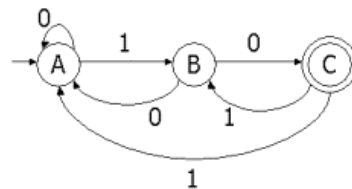
5. In the following expressions, - represents negation of a variable. For example, -x stands for "NOT x", + represents logical OR, and juxtaposition represents logical AND (e.g.,  $(x+y)(y+z)$  represents  $(x \text{ OR } y) \text{ AND } (y \text{ OR } z)$ ).

Identify the expression that is satisfiable, from the list below.

- a)  $(-z+-x)(z)(-x-y)(y+-z)$
- b)  $(z)(-z+x)(-z+-x)(x+y)$
- c)  $(z+-x)(z+x)(-z+-x)(-z+x)$
- d)  $(z+x)(-z)(z+-x)(x+y)$

You did not answer this question.

6. When we convert an automaton to a regular expression, we need to build expressions for the labels along paths from one state to another state that do not go through certain other states. Below is a nondeterministic finite automaton with three states. For each of the six orders of the three states, find regular expressions that give the set of labels along all paths from the first state to the second state that never go through the third state.



Then identify one of these expressions from the list of choices below.

- a)  $(01)^+$  represents the paths from B to C that do not go through A.
- b)  $(01)^*0$  represents the paths from B to C that do not go through A.
- c)  $1(0+1010)^*$  represents the paths from C to A that do not go through B.
- d)  $0(10)^*$  represents the paths from B to A that do not go through C.

You did not answer this question.

7. Design the minimum-state DFA that accepts all and only the strings of 0's and 1's that end in 010. To verify that you have designed the correct automaton, we will ask you to identify the true statement in a list of choices. These choices will involve:

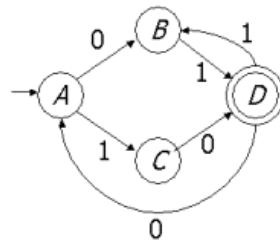
- 1. The number of *loops* (transitions from a state to itself).
- 2. The number of transitions into a state (including loops) on input 1.
- 3. The number of transitions into a state (including loops) on input 0.

Count the number of transitions into each of your states ("in-transitions") on input 1 and also on input 0. Count the number of loops on input 1 and on input 0. Then, find the true statement in the following list.

- There are two states that have no in-transitions on input 0.
- There are two states that have one in-transition on input 0.
- There is one state that has one in-transition on input 1.
- There are three states that have one in-transition on input 1.

You did not answer this question.

8. Here is a finite automaton:



Which of the following regular expressions defines the same language as the finite automaton? Hint: each of the correct choices uses component expressions. Some of these components are:

- The ways to get from A to D without going through D.
- The ways to get from D to itself, without going through D.
- The ways to get from A to itself, without going through A.

It helps to write down these expressions first, and then look for an expression that defines all the paths from A to D.

- $((01+10)0+11)^*(01+10)(11)^*$
- $(01+10)(0(01+10))^*(11)^*$
- $(01+10)(11^*+0(01+10))^*$
- $((01+10)(11)^*0)^*(01+10)(11)^*$

You did not answer this question.

9. The homomorphism  $h$  is defined by  $h(a) = 01$  and  $h(b) = 10$ . What is  $h(abbb)$ ?

- 01010101
- 011010
- 01101010
- abbb

You did not answer this question.

10. For the grammar:

$S \rightarrow AB \mid CD$   
 $A \rightarrow BC \mid a$   
 $B \rightarrow AC \mid C$

$C \rightarrow AB \mid CD$   
 $D \rightarrow AC \mid d$

1. Find the generating symbols. Recall, a grammar symbol is *generating* if there is a derivation of at least one terminal string, starting with that symbol.
2. Eliminate all *useless productions* --- those that contain at least one symbol that is not a generating symbol.
3. In the resulting grammar, eliminate all symbols that are not *reachable* --- they appear in no string derived from S.

In the list below, you will find several statements about which symbols are generating, which are reachable, and which productions are useless. Select the one that is FALSE.

- a)  $A \rightarrow BC$  is not useless.
- b)  $B \rightarrow C$  is useless.
- c) S is not generating.
- d) A is not reachable.

You did not answer this question.

11. A *k-clique* in a graph G is a set of *k* nodes of G such that there is an edge between every pair of nodes in the set. The problem *k-CLIQUE* is: Given a graph G and a positive integer *k*, does G have a *k-clique*?

We can prove *k-CLIQUE* to be NP-complete by reducing the 3SAT problem to it. Consider the 3-CNF expression:

$$E = (x_1' + x_2 + x_3)(x_1' + x_2' + x_3')(x_3 + x_4 + x_5')(x_1 + x_2 + x_4)$$

[Note:  $a'$  denotes the negation NOT( $a$ ) of variable  $a$ .] Let G be the graph constructed from this expression as in Theorem 10.18 (p. 460). Then, let H be the *complement* of G, that is, the graph with the same nodes as G and an edge between two nodes if and only if G DOES NOT have an edge between those two nodes. Let us denote the vertices of H by using the corresponding (clause, literal) pair. The node ( $i,j$ ) corresponds to the  $j^{\text{th}}$  literal of the  $i^{\text{th}}$  clause. Which of the following nodes form a maximum-sized clique in H?

- a)  $\{(1,2), (2,2), (3,1), (4,3)\}$
- b)  $\{(1,2), (2,1), (3,2), (4,3)\}$
- c)  $\{(1,2), (2,1), (4,3), (3,2), (4,2)\}$
- d)  $\{(1,1), (2,1), (2,3), (3,2), (4,3)\}$

You did not answer this question.

12. A nondeterministic Turing machine *M* with start state  $q_0$  and accepting state  $q_f$  has the following transition function:

$\delta(q,a)$	0	1	B
$q_0$	$\{(q_1, 0, R)\}$	$\{(q_1, 0, R)\}$	$\{(q_1, 0, R)\}$
$q_1$	$\{(q_1, 1, R), (q_2, 0, L)\}$	$\{(q_1, 1, R), (q_2, 1, L)\}$	$\{(q_1, 1, R), (q_2, B, L)\}$
$q_2$	$\{(q_f, 0, R)\}$	$\{(q_2, 1, L)\}$	$\{\}$
$q_f$	$\{\}$	$\{\}$	$\{\}$

Deduce what  $M$  does on any input of 0's and 1's. Demonstrate your understanding by identifying, from the list below, the ID that CANNOT be reached on some number of moves from the initial ID  $q_010100101$ .

- a)  $0q_f1110101$
- b)  $0q_f1111111111$
- c)  $0111q_20101$
- d)  $q_f011111111$

You did not answer this question.

13. A *linear grammar* is a context-free grammar in which no production body has more than one occurrence of one variable. For example,  $A \rightarrow 0B1$  or  $A \rightarrow 001$  could be productions of a linear grammar, but  $A \rightarrow BB$  or  $A \rightarrow A0B$  could not. A *linear language* is a language that has at least one linear grammar.

The following statement is false: "The concatenation of two linear languages is a linear language." To prove it we use a counterexample: We give two linear languages  $L_1$  and  $L_2$  and show that their concatenation is not a linear language. Which of the following can serve as a counterexample?

- a)  $L_1 = \{w | w = (aaa)^n(ab)^n, \text{ where } n \text{ is a positive integer}\}$   
 $L_2 = \{w | w = (ab)^n, \text{ where } n \text{ is a positive integer}\}$
- b)  $L_1 = \{w | w = (aa)^n a^n b^n, \text{ where } n \text{ is a positive integer}\}$   
 $L_2 = \{w | w = c(aaa)^n(ab)^n, \text{ where } n \text{ is a positive integer}\}$
- c)  $L_1 = \{w | w = aaac^{n-1}(ab), \text{ where } n \text{ is a positive integer}\}$   
 $L_2 = \{w | w = c(aaa)^n, \text{ where } n \text{ is a positive integer}\}$
- d)  $L_1 = \{w | w = (aaa)^n(ab)^n, \text{ where } n \text{ is a positive integer}\}$   
 $L_2 = \{w | w = (ab)^n b^n a^n, \text{ where } n \text{ is a positive integer}\}$

You did not answer this question.

14. Consider the grammar  $G$  and the language  $L$ :

$G: S \rightarrow AB \mid a \mid abC, A \rightarrow b, C \rightarrow abC \mid c$

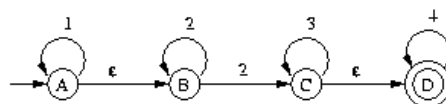
$L: \{w \mid w \text{ a string of a's, b's, and c's with an equal number of a's and b's}\}.$

Grammar  $G$  does not define language  $L$ . To prove, we use a string that either is produced by  $G$  and not contained in  $L$  or is contained in  $L$  but is not produced by  $G$ . Which string can be used to prove it?

- a) cabaca
- b) abacccc
- c) cccabab
- d) caca

You did not answer this question.

15. Here is a nondeterministic finite automaton with epsilon-transitions:



Suppose we use the extended subset construction from Section 2.5.5 (p. 77) to convert this epsilon-NFA to a deterministic finite automaton with a dead state, with all transitions defined, and with no state that is inaccessible from the start state. Which of the following would be a transition of the DFA?

Note: we use  $S \cdot x \rightarrow T$  to say that the DFA has a transition on input  $x$  from state  $S$  to state  $T$ .

- a)  $\{A, B\} \cdot 3 \rightarrow \{C, D\}$
- b)  $\{C, D\} \cdot 4 \rightarrow \{C, D\}$
- c)  $\{A, B\} \cdot 2 \rightarrow \{A, B, C, D\}$
- d)  $\{B, C, D\} \cdot 2 \rightarrow \{B, C, D\}$

You did not answer this question.

16. Consider the language  $L = \{a\}$ . Which grammar defines  $L$ ?

- a)  $G_1: S \rightarrow AC|C, A \rightarrow b$
- b)  $G_1: S \rightarrow AC|a, A \rightarrow c|\epsilon$
- c)  $G_1: S \rightarrow AB|a|b, A \rightarrow b$
- d)  $G_1: S \rightarrow AC|a|ab, A \rightarrow c|\epsilon$

You did not answer this question.

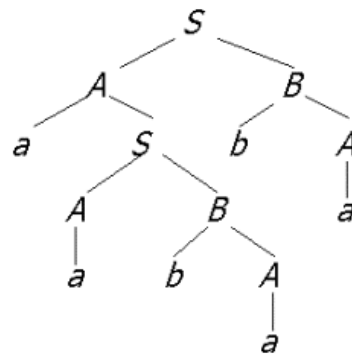
17. The language  $L = \{ss \mid s \text{ is a string of a's and b's}\}$  is not a context-free language. In order to prove that  $L$  is not context-free we need to show that for every integer  $n$ , there is some string  $z$  in  $L$ , of length at least  $n$ , such that no matter how we break  $z$  up as  $z = uvwxy$ , subject to the constraints  $|vwx| \leq n$  and  $|vx| > 0$ , there is some  $i \geq 0$  such that  $uv^iwx^iy$  is not in  $L$ .

Let us focus on a particular  $z = aabaaaba$  and  $n = 7$ . It turns out that this is the wrong choice of  $z$  for  $n = 7$ , since there are some ways to break  $z$  up for which we can find the desired  $i$ , and for others, we cannot. Identify from the list below the choice of  $u, v, w, x, y$  for which there is an  $i$  that makes  $uv^iwx^iy$  not be in  $L$ . We show the breakup of  $aabaaaba$  by placing four  $|$ 's among the a's and b's. The resulting five pieces (some of which may be empty), are the five strings. For instance,  $aa|b|aaaba|$  means  $u=aa, v=b, w=\epsilon, x=aaaba$ , and  $y=\epsilon$ .

- a)  $a|ab|aaa|b|a$
- b)  $|a|ab|a|aaba$
- c)  $aa|ba|a|ab|a$
- d)  $aab|a|aab|a|$

You did not answer this question.

18. Here is a parse tree that uses some unknown grammar  $G$ .

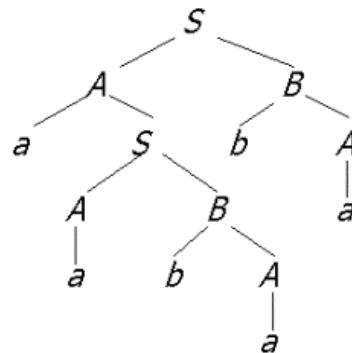


Which of the following productions is surely one of those for grammar G?

- a)  $S \rightarrow AB$
- b)  $B \rightarrow A$
- c)  $A \rightarrow S$
- d)  $A \rightarrow SB$

[You did not answer this question.](#)

19. The parse tree below represents a leftmost derivation according to the grammar  $S \rightarrow AB$ ,  $A \rightarrow aS|a$ ,  $B \rightarrow bA$ .



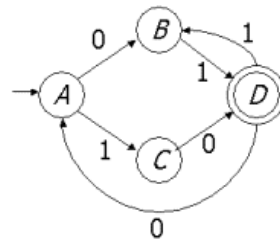
Which of the following is a left-sentential form in this derivation?

- a) aaABBB
- b) aabaB
- c) aABbA
- d) aSbA

[You did not answer this question.](#)



20. Examine the following DFA:



Identify in the list below the string that this automaton accepts.

- a) 1011
- b) 010011
- c) 100011
- d) 1100111

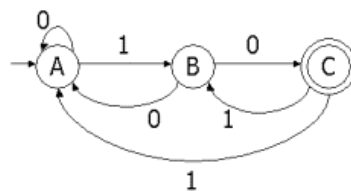
You did not answer this question.

21. Identify in the list below a sentence of length 6 that is generated by the grammar  $S \rightarrow (S)S \mid \varepsilon$

- a) ))( ( (
- b) )( ( (
- c) ( ( ( (
- d) ( ( ( ) (

You did not answer this question.

22. Convert the following nondeterministic finite automaton:



to a DFA, including the dead state, if necessary. Which of the following sets of NFA states is **not** a state of the DFA that is accessible from the start state of the DFA?

- a) {A,B,C}
- b) {A}
- c) {}
- d) {A,C}

You did not answer this question.

23. Let G be the grammar:

$$S \rightarrow SS \mid (S) \mid \varepsilon$$

$L(G)$  is the language BP of all strings of balanced parentheses, that is, those strings that could appear in a well-formed arithmetic expression. We want to prove that  $L(G) = BP$ , which requires two inductive proofs:

1. If  $w$  is in  $L(G)$ , then  $w$  is in  $BP$ .
2. If  $w$  is in  $BP$ , then  $w$  is in  $L(G)$ .

We shall here prove only the first. You will see below a sequence of steps in the proof, each with a reason left out. These reasons belong to one of three classes:

- A) Use of the inductive hypothesis.
- B) Reasoning about properties of grammars, e.g., that every derivation has at least one step.
- C) Reasoning about properties of strings, e.g., that every string is longer than any of its proper substrings.

The proof is an induction on the number of steps in the derivation of  $w$ . You should decide on the reason for each step in the proof below, and then identify from the available choices a correct pair consisting of a step and a kind of reason (A, B, or C).

Basis: One step.

- (1) The only 1-step derivation of a terminal string is  $S \Rightarrow \epsilon$  because \_\_\_\_\_

- (2)  $\epsilon$  is in  $BP$  because \_\_\_\_\_  
Induction: An  $n$ -step derivation for some  $n > 1$ .

- (3) The derivation  $S \Rightarrow^n w$  is either of the form  
(a)  $S \Rightarrow SS \Rightarrow^{n-1} w$  or of the form  
(b)  $S \Rightarrow (S) \Rightarrow^{n-1} w$   
because \_\_\_\_\_  
Case (a):

- (4)  $w = xy$ , for some strings  $x$  and  $y$  such that  $S \Rightarrow^p x$  and  $S \Rightarrow^q y$ , where  $p < n$  and  $q < n$  because \_\_\_\_\_

- (5)  $x$  is in  $BP$  because \_\_\_\_\_

- (6)  $y$  is in  $BP$  because \_\_\_\_\_

- (7)  $w$  is in  $BP$  because \_\_\_\_\_  
Case (b):

- (8)  $w = (z)$  for some string  $z$  such that  $S \Rightarrow^{n-1} z$  because \_\_\_\_\_

- (9)  $z$  is in  $BP$  because \_\_\_\_\_

- (10)  $w$  is in  $BP$  because \_\_\_\_\_

- a) (1) for reason B
- b) (3) for reason A
- c) (3) for reason C
- d) (1) for reason C

You did not answer this question.

24. The proof that the Independent-Set problem is NP-complete depends on a construction given in Theorem 10.18 (p. 460), which reduces 3SAT to Independent Sets. Apply this construction to the 3SAT instance:

$$(u+v+w)(-v+-w+x)(-u+-x+y)(x+-y+z)(u+-w+-z)$$

Note that - denotes negation, e.g., -v stands for the literal NOT v. Also, remember that the construction involves the creation of nodes denoted  $[i,j]$ . The node  $[i,j]$  corresponds to the  $j$ th literal of the  $i$ th clause. For example,  $[1,2]$  corresponds to the occurrence of  $v$ .

After performing the construction, identify from the list below the one pair of nodes that does NOT have an edge between them.

- a)  $[2,3]$  and  $[3,2]$
- b)  $[1,3]$  and  $[2,2]$
- c)  $[2,2]$  and  $[5,2]$
- d)  $[3,1]$  and  $[5,1]$

[You did not answer this question.](#)

25. Which of the following strings is NOT in the Kleene closure of the language  $\{011, 10, 110\}$ ?
- a) 110011110
  - b) 10111011
  - c) 1010110
  - d) 01110011

[You did not answer this question.](#)