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Zayd

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Help

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Questions about languages classes NP and above, based on Sections 10.1, 11.1, 11.2, and 11.3 of HMU.

1. The classes of languages P and NP are closed under certain operations, and not closed under others, just like classes such as the regular languages or context-free languages have closure properties. Decide whether P and NP are closed under each of the following operations.

1. Union.
2. Intersection.
3. Intersection with a regular language.
4. Concatenation.
5. Kleene closure (star).
6. Homomorphism.
7. Inverse homomorphism.

Then, select from the list below the true statement.

- a) NP is not closed under homomorphism.
- b) P is not closed under concatenation.
- c) P is not closed under union.
- d) P is not closed under intersection with a regular language.

Answer submitted: **a)**

You have answered the question correctly.

Question Explanation:

Both P and NP are closed under each of these operations, except for homomorphism. To see why neither class is closed under homomorphism, start with a very hard language L , say one that requires time 2^{2^n} . Make it easy by appending to each word of length n exactly 2^{2^n} c 's, where c is a new symbol. That is, let $L' = Lc^{2^{2^n}}$. Then L' is surely in P and NP. But L is $h(L')$, if h is the homomorphism that sends c to ϵ and is the identity on all symbols of L . If P or NP were closed under homomorphism, then L would be in NP, which it is not.

Here are the constructions that show P and NP closed under the other six operations:

1. $L_1 \cup L_2$: Apply the tests for membership in L_1 and then for membership in L_2 . Accept if either accepts. If L_1 and L_2 are in P, then both tests are polynomial, so the entire process is polynomial. If L_1 and L_2 are in NP, then there is a nondeterministic polynomial algorithm for the entire process.
2. $L_1 \cap L_2$: The argument is the same, but accept only if both tests accept.
3. Intersection with a regular set: The argument is the same as (2), since a regular language is surely in P and NP.
4. $L_1 L_2$: If L_1 and L_2 are in NP, just guess the point on the input where the string in L_2 begins, and apply the nondeterministic polynomial tests to the two parts of the input. If L_1 and L_2 are in P, we have to try systematically all possible breakpoints between the prefix of the input that is

in L_1 and the suffix that is in L_2 . If the tests for L_1 and L_2 are polynomial, say $p(n)$ and $q(n)$ running time, then we must apply each at most n times on input of length n . Since $n(p(n)+q(n))$ is a polynomial, we can recognize L_1L_2 in polynomial time.

5. L^* : If L is in NP, guess all the places on the input where strings in L end. Run the nondeterministic polynomial-time test on each segment. Since there are at most n segments, the whole process is nondeterministic polynomial. If L is in P, we again must be systematic. For each pair of input positions i and j , run the polynomial-time test to see whether positions i through j of the input is in L . If L has a $p(n)$ -time algorithm, then this process takes $O(n^2p(n))$ time, a polynomial. Then, use a dynamic programming algorithm, taking $O(n^3)$ time, to determine whether positions i through j can be composed of $1, 2, \dots, n$ strings in L . At the end, we only care about whether positions 1 through n can be composed of some number of strings in L .
6. $h^{-1}(L)$: Any homomorphism h can only expand the length of the string to which it is applied by a constant factor. To recognize $h^{-1}(L)$, apply h to the input w , and see whether $h(w)$ is in L . If there is a polynomial-time, or nondeterministic polynomial-time test for membership in L , this test will, in the same order of magnitude time complexity tell us whether w is in $h^{-1}(L)$.

2. There is a Turing transducer T that transforms problem P_1 into problem P_2 . T has one read-only input tape, on which an input of length n is placed. T has a read-write scratch tape on which it uses $O(S(n))$ cells. T has a write-only output tape, with a head that moves only right, on which it writes an output of length $O(U(n))$. With input of length n , T runs for $O(T(n))$ time before halting. You may assume that each of the upper bounds on space and time used are as tight as possible.

A given combination of $S(n)$, $U(n)$, and $T(n)$ may:

1. Imply that T is a polynomial-time reduction of P_1 to P_2 .
2. Imply that T is NOT a polynomial-time reduction of P_1 to P_2 .
3. Be impossible; i.e., there is no Turing machine that has that combination of tight bounds on the space used, output size, and running time.

What are all the constraints on $S(n)$, $U(n)$, and $T(n)$ if T is a polynomial-time reducer? What are the constraints on feasibility, even if the reduction is not polynomial-time? After working out these constraints, identify the true statement from the list below.

- a) $S(n) = n^2$; $U(n) = n^2$; $T(n) = n!$ is possible, but not a polynomial-time reduction.
- b) $S(n) = \log n$; $U(n) = n$; $T(n) = n^2$ is not physically possible.
- c) $S(n) = n^3$; $U(n) = n$; $T(n) = (1.01)^n$ is not physically possible.
- d) $S(n) = n^2$; $U(n) = n^2$; $T(n) = n!$ is not physically possible.

Answer submitted: **d)**

Your answer is incorrect.

There are two reasons a combination might be impossible. Perhaps the running time is so large that the Turing machine cannot run that long without entering a loop because it repeats an ID (combination of state, head positions on the input and scratch tapes, and scratch-tape contents). Or the running time is so small that the Turing machine cannot possibly use that many cells of the scratch or output tapes. The conditions for a reduction to be polynomial-time are in Section 10.1.5 (p. 433).

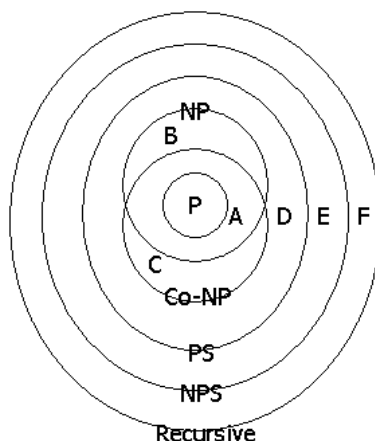
Question Explanation:

There is one constraint on T being a polynomial-time reduction: $T(n)$ must be a polynomial. However, there are also some constraints on feasibility. First, in time $T(n)$, T cannot write more than $T(n)$ cells on either its scratch or output tape. Thus, $S(n)$ and $U(n)$ must both be $O(T(n))$. Second, if $T(n)$ is larger than $O(n \log S(n) c^{S(n)})$ for any constant c , then T must repeat an "ID" consisting of the state, scratch-tape contents, and head positions on the input and scratch tapes.

The correct choice is: **a)**

3. In the diagram below we see certain complexity classes (represented as circles or ovals) and certain regions labeled A through F that represent the differences of some of these complexity classes.

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The state of our knowledge regarding the existence of problems in the regions A-F is imperfect. In some cases, we know that a region is nonempty, and in other cases we know that it is empty. Moreover, if $P=NP$, then we would know more about the emptiness or nonemptiness of some of these regions, but still would not know everything.

Decide what we know about the regions A-F currently, and also what we would know if $P=NP$. Then, identify the true statement from the list below.

- Region C is definitely empty.
- It is not known whether region B is empty.
- Region D is definitely empty.
- Region A is definitely empty.

Answer submitted: **c)**

Your answer is incorrect.

If D is empty, then $PS = NP \cup co-NP$. What do we know about this question? See Section 11.2.2 (p. 488).

Question Explanation:

Since we do not know whether $P=NP$, or whether $NP=co-NP$ (i.e., whether NP is closed under complementation), we do not know whether any of A, B, or C is empty. If we know $P=NP$, then surely A and B are empty. But if $P=NP$, then $co-NP=NP=P$, since P is closed under complementation. Thus, C would be empty as well.

Savitch's theorem tells us that $PS=NPS$; i.e., region E is empty. We also know that there are arbitrarily complex recursive languages, so region F is definitely *not* empty. Finally, we do not know about region D, since it is open whether $PS=NP$ or even $PS=P$. And even if we knew $P=NP$, we would still not be sure whether or not $PS=NP=P$.

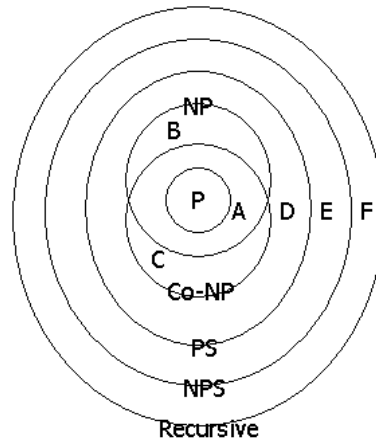
The correct choice is: **b)**

4. Consider the following problems:

- SP (Shortest Paths): given a weighted, undirected graph with nonnegative integer edge weights, given two nodes in that graph, and given an integer limit k , determine whether the length of the shortest path between the nodes is k or less.
- WHP (Weighted Hamilton Paths): given a weighted, undirected graph with nonnegative integer edge weights, and given an integer limit k , determine whether the length of the shortest Hamilton path in the graph is k or less.
- TAUT (Tautologies): given a propositional boolean formula, determine whether it is true for all possible truth assignments to its variables.

4. QBF (Quantified Boolean Formulas): given a boolean formula with quantifiers for-all and there-exists, such that there are no free variables, determine whether the formula is true.

In the diagram below are seven regions, P and A through F.



Place each of the four problems in its correct region, on the assumption that NP is equal to neither P nor co-NP nor PS.

- Problem TAUT is in region B.
- Problem QBF is in region D.
- Problem WHP is in region P.
- Problem TAUT is in region P.

Answer submitted: **b)**

You have answered the question correctly.

Question Explanation:

SP is in P. You can use Dijkstra's algorithm to solve the problem in quadratic time.

WHP is NP-complete. It is a generalization of Hamilton-path, but the weights don't affect the nondeterministic-polynomial-time "guessing" algorithm to solve it. On the assumption that NP is neither P nor co-NP, WHP must be in region B.

TAUT is essentially the complement of SAT. Since SAT is NP-complete, TAUT must be in region C, unless NP is equal to one of P or co-NP (which we assume not to be the case).

QBF is complete for PS. Thus, assuming PS is not NP, QBF is in D.

5. Let us denote a problem X as NP-Easy if it is polynomial-time reducible to some problem Y that is in NP. Let us denote as NP-Equivalent, the class of problems that are both NP-Easy and NP-Hard. Let A,B,C,D and E be problems such that A is NP-Hard, B is NP-Complete, C is NP-Equivalent, D is NP-Easy and E is in NP. Which of the following statements is TRUE?

- If C is in P then $P=NP$.
- If C is in P then so is A.
- If $P=NP$ then A is in P.
- If $P = NP$ then B is not in P

Answer submitted: **a)**

You have answered the question correctly.

Question Explanation:

The question is based on definitions of NP, NP-hardness, NP-completeness, and Theorems 10.4 and 10.5 in Section 10.1.6, p. 434--435. Some key facts we are using include:

1. If a problem A reduces to a problem B, then A is at least as hard as B.
2. Since A is NP-Hard, for every language L in NP, there is a polynomial-time reduction from L to A.
3. Since C is NP-Easy, it implies that C is reducible in polynomial time to another problem F in NP. F is reducible in polynomial time to A. So C is polynomial-time reducible to A. A similar reasoning holds for D.
4. Since A is NP-Hard, every problem in NP reduces to A in polynomial time. If A is in P, all such problems R can be solved in polynomial time using the algorithm for reducing R to A combined with the algorithm for A. This would imply that $P = NP$.
5. We denote a problem X as NP-Easy if it is polynomial-time reducible to some problem Y that is in NP. Then the fact that Y is in NP, combined with the reduction, implies that X is also in NP. Moreover, if X is NP-equivalent, then X is also NP-Hard. Hence the notions of NP-Equivalence and NP-Completeness are the same.

6. Suppose there are three languages (i.e., problems), of which we know the following:

1. L_1 is in P.
2. L_2 is NP-complete.
3. L_3 is not in NP.

Suppose also that we do not know anything about the resolution of the "P vs. NP" question; for example, we do not know definitely whether $P=NP$. Classify each of the following languages as (a) Definitely in P, (b) Definitely in NP (but perhaps not in P and perhaps not NP-complete) (c) Definitely NP-complete (d) Definitely not in NP:

- $L_1 \cup L_2$.
- $L_1 \cap L_2$.
- $L_2 c L_3$, where c is a symbol not in the alphabet of L_2 or L_3 (i.e., the *marked concatenation* of L_2 and L_3 , where there is a unique marker symbol between the strings from L_2 and L_3).
- The complement of L_3 .

Based on your analysis, pick the correct, definitely true statement from the list below.

- a) The complement of L_3 is definitely not NP-complete.
- b) $L_1 \cap L_2$ is definitely in NP.
- c) $L_1 \cap L_2$ is definitely in P.
- d) $L_1 \cap L_2$ is definitely NP-complete.

Answer submitted: **b)**

You have answered the question correctly.

Question Explanation:

Let $L = L_1 \cup L_2$. It is possible that L_1 is empty; that is certainly one language in P. If so, then $L = L_2$, and L is NP-complete. On the other hand, suppose L_1 is all strings over the alphabet of L_2 --- another language in P. Then $L = L_1$, a language in P. Since we don't know whether $P = NP$, we cannot conclude L is definitely in P and we cannot conclude that L is definitely NP-complete.

On the other hand, we can conclude definitely that L is in NP. A nondeterministic, polynomial-time algorithm for L starts by applying a polynomial-time algorithm for L_1 to its input, and if the result is negative applies the NP-recognizer for L_2 to the same input. It accepts the input if either recognizer accepts.

The argument for $L = L_1 \cap L_2$ is essentially the same, although now L is in P if L_1 is empty and L is NP-complete if L_1 is all strings over the alphabet of L_2 . Also, the nondeterministic polynomial-time algorithm for L works by trying the polynomial-time recognizer for L_1 first, and only accepting if that recognizer accepts and the NP-recognizer for L_2 also accepts.

Now, consider $L = L_2 c L_3$. Suppose L had an NP-recognizer. Let x be some string in L_2 (since L_2 is NP-complete, and we do not know that $P = NP$, we can be sure L_2 is not empty, so x exists). Then there is a nondeterministic polynomial-time algorithm for L_3 that works as follows. Take input w , and test it for membership in L_3 by feeding xcw to the NP-recognizer for L . Respond exactly as this recognizer responds. Since we know x is in L_2 , the recognizer for L accepts xcw if and only if w is in L_3 . We now have an NP-recognizer for L_3 , but we were told that L_3 is *not* in NP. Thus, our assumption that L is in NP must be false.

Last, let L be the complement of L_3 . If L is in P, then the complement of L , which is L_3 , is also in P.

But we know L_3 is not even in NP. We do not even know that L is in NP; for example, L_3 could be an undecidable problem. On the other hand, L could be NP-complete. For example, the complement of the problem SAT is not known to be in NP. But it is possible that L_3 is the complement of SAT and therefore $L = \text{SAT}$, a known NP-complete problem.
