Gradiance Online Accelerated Learning

Zayd

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| **Your score:** | 34 |

These questions, based on Section 5.1 of HMU, are not in either of the other two homeworks on CFG's, but are available for use.

**1.** Consider the grammar G1: $S \to \epsilon$, $S \to aS$, $S \to aSbS$ and the language L that contains exactly those strings of a's and b's such that every prefix has at least as many a's as b's. We want to prove the claim: G1 generates all strings in L.

We take the following inductive hypothesis to prove the claim:

For n < k, G1 generates every string of length n in L.

To prove the inductive step we argue as follows:

"For each string w in L either ____________(a1) or ______________(a2) holds. In both cases we use the inductive hypothesis and one of the rules to show that string w can be generated by the grammar. In the first case we use rule $S \to aS$ and in the second case we use rule $S \to aSbS$."

Which phrases can replace the ____________ so that this argument is correct?

a) a1: each prefix has more a's than b's.
a2: w can be written as w=aw'bw'' where for both w' and w'' it holds that each prefix has as many a's as b's.

b) a1: w can be written as w=aw'bw'' where for both w' and w'' it holds that each prefix has as many a's as b's.
a2: each prefix has more a's than b's.

c) a1: each prefix has equal number of b's and a's.

a2: w can be written as w=aw'bw'' where for both w' and w'' it holds that each prefix has as many a's as b's.

d) a1: each prefix has more a's than b's.
a2: there is a unique b in string w such that for the part of the string until the b (b also included) each prefix has as many a's as b's and for the part after b each prefix has as many a's as b's.

Answer submitted: **a)**

You have answered the question correctly.

**2.** Consider the grammar G with start symbol S:

$S \to bS \mid aA \mid b$
$A \to bA \mid aB$
$B \to bB \mid aS \mid a$

Which of the following is a word in L(G)?

a) ababbbbbbbb

b) abababbbb

c) aababbabbbabba

d) ababbb

Answer submitted: **c)**

You have answered the question correctly.

**3.** Let L be the language of all strings of a's and b's such that no prefix (proper or not) has more b's than a's. Let G be the grammar with productions

```
S → aS | aSbS | ε
```

To prove that L = L(G), we need to show two things:

1. If S =>* w, then w is in L.
2. If w is in L, then S =>* w.

We shall consider only the proof of (1) here. The proof is an induction on n, the number of steps in the derivation S =>* w. Here is an outline of the proof, with reasons omitted. You need to supply the reasons.

Basis:

1) If n=1, then w is ε because _______.

2) w is in L because _______.

Induction:

3) Either (a) S => aS =>$^{n-1}$ w or (b) S => aSbS =>$^{n-1}$ w because _______.

4a ) In case (a), w = ax, and S =>$^{n-1}$ x because _______.

5a) In case (a), x is in L because _______.

6a) In case (a), w is in L because _______.

4b) In case (b), w can be written w = aybz, where S =>$^p$ y and S =>$^q$ z for some p and q less than n because _______.

5b) In case (b), y is in L because _______.

6b) In case (b), z is in L because _______.

7b) In case (b), w is in L because _______.

For which of the steps above the appropriate reason is contained in the following argument:
"The following two statements are true
(i) if string x has no prefix with more b's than a's, then neither does string ax,
(ii) if strings y and z are such that no prefix has more b's than a's, then neither does string aybz."

a) 3

b) 7b

c) 4b

d) 6b

Answer submitted: **b)**

You have answered the question correctly.

**4.** Consider the grammar G and the language L:

G: S → AB | a | abC, A → b, C → abC | c

L: {w | w a string of a's, b's, and c's with an equal number of a's and b's}.
Grammar G does not define language L. To prove, we use a string that either is produced by G and not contained in L or is contained in L but is not produced by G. Which string can be used to prove it?

a) cacaba
b) cacacab
c) cccabab
d) abacccc

Answer submitted: **c)**

You have answered the question correctly.

**5.** Which of the following grammars derives a subset of the language:

{x | x contains a and c in proportion 4:3 and there are no two consecutive c's}?

a) S → ε S → aacacac S → SaScSaScSaScSa
b) S → ε S → SaScSaScSaSaS
c) S → acacaca S → SaScSaScSaScSaS S → SaSaSaScSaScSaS
d) S → acacacacacaaca S → SaScSaScSaScSaS

Answer submitted: **b)**

Your answer is incorrect.

acacaaa is a word in this language and the number of a's and c's is as 5:2 Derivations and the terminal strings they derive are introduced in Section 5.1.3 (p. 175).

**6.** Let L be the language of all strings of a's and b's such that no prefix (proper or not) has more b's than a's. Let G be the grammar with productions

```
S → aS | aSbS | ε
```

To prove that L = L(G), we need to show two things:

1. If S =>* w, then w is in L.
2. If w is in L, then S =>* w.

We shall consider only the proof of (1) here. The proof is an induction on n, the number of steps in the derivation S =>* w. Here is an outline of the proof, with reasons omitted. You need to supply the reasons.

Basis:
1) If n=1, then w is ε because _______.
2) w is in L because _______.
Induction:
3) Either (a) S => aS =>$^{n-1}$ w or (b) S => aSbS =>$^{n-1}$ w because _______.
4a ) In case (a), w = ax, and S =>$^{n-1}$ x because _______.
5a) In case (a), x is in L because _______.
6a) In case (a), w is in L because _______.
4b) In case (b), w can be written w = aybz, where S =>$^{p}$ y and S =>$^{q}$ z for some p and q less than n because _______.
5b) In case (b), y is in L because _______.
6b)

In case (b), z is in L because _______.

7b)

In case (b), w is in L because _______.

For which of the steps above the appropriate reason is contained in the following argument:
"All n-step derivations of w produce either ε (for n=1) or use one of the productions with at least one nonterminal in the body (for n > 1). In case the production S → aS is used, then w=ax with x being produced by a (n-1)-step derivation. In case the production S → aSbS is used then w=aybz with y and z being produced by derivations with number of steps less than n."

a) 6a

b) 6b

c) 5b

d) 4b

Answer submitted: **d)**

You have answered the question correctly.

7. Consider the grammars:

$G_1$: S → AB, A → aAA|ε , B → abBB|ε
$G_2$:S → CB, C → aCC|aC|a, B → abBB|abB|ab

$G_3$:S → CB|C|B| ε , C → aCC|aC|a, B → abBB|abB|ab
$G_4$:S → ASB|ε, A → aA|ε, B → abB|ε
$G_5$:S → ASB|AB, A → aA|a, B → abB|ab
$G_6$:S → ASB|aab, A → aA|a, B → abB|ab

Describe the language of each of these grammars. Then, identify from the list below a pair of grammars that define the same language?

a) $G_2$ and $G_6$

b) $G_4$ and $G_6$

c) $G_4$ and $G_5$

d) $G_1$ and $G_4$

Answer submitted: **d)**

You have answered the question correctly.

8. Identify in the list below a sentence of length 6 that is generated by the grammar S → (S)S | ε

a) ) ) ( ( ) (

b) ( ) ( ) ) (

c) ( ) ( ) ( )

d) ) ( ) ) ) (

Answer submitted: **c)**

You have answered the question correctly.

9. Here are eight simple grammars, each of which generates an infinite language of strings. These strings tend to look like alternating *a*'s and *b*'s, although there are some exceptions, and not all grammars generate all such strings.

1. S → abS | ab
2. S → SS | ab
3. S → aB; B → bS | a
4. S → aB; B → bS | b
5. S → aB; B → bS | ab
6. S → aB | b; B → bS

7. $S \rightarrow aB \mid a; B \rightarrow bS$
8. $S \rightarrow aB \mid ab; B \rightarrow bS$

The initial symbol is S in all cases. Determine the language of each of these grammars. Then, find, in the list below, the pair of grammars that define the same language.

a) G1: $S \rightarrow aB, B \rightarrow bS, B \rightarrow b$
G2: $S \rightarrow aB, B \rightarrow bS, S \rightarrow b$

b) G1: $S \rightarrow abS, S \rightarrow ab$
G2: $S \rightarrow aB, B \rightarrow bS, B \rightarrow b$

c) G1: $S \rightarrow aB, B \rightarrow bS, B \rightarrow ab$
G2: $S \rightarrow SS, S \rightarrow ab$

d) G1: $S \rightarrow aB, B \rightarrow bS, B \rightarrow ab$
G2: $S \rightarrow aB, B \rightarrow bS, S \rightarrow ab$

Answer submitted: **b)**

You have answered the question correctly.

**10.** Consider the following languages and grammars. $G_1$: $S \rightarrow aA|aS, A \rightarrow ab$
$G_2$: $S \rightarrow abS|aA, A \rightarrow a$
$G_3$: $S \rightarrow Sa|AB, A \rightarrow aA|a, B \rightarrow b$
$G_4$: $S \rightarrow aS|b$
$L_1$: $\{a^ib \mid i=1,2,...\}$
$L_2$: $\{(ab)^iaa \mid i=0,1,...\}$
$L_3$: $\{a^ib \mid i=2,3,...\}$
$L_4$: $\{a^iba^j \mid i=1,2,..., j=0,1,...\}$
$L_5$: $\{a^ib \mid i=0,1,...\}$

Match each grammar with the language it defines. Then, identify a correct match from the list below.

a) $G_4$ defines $L_1$.
b) $G_3$ defines $L_2$.
c) $G_4$ defines $L_5$.
d) $G_1$ defines $L_2$.

Answer submitted: **c)**

You have answered the question correctly.

**11.** Which of the following grammars derives a subset $L_s$ of the language: L= {x | (i) x contains a and c in proportion 4:3, (ii) x does not begin with c and (iii) there are no two consecutive c's} such that $L_s$ is missing at most a finite number of strings from L.

a) $S \rightarrow \varepsilon$ , $S \rightarrow SaScSaScSa$
b) $S \rightarrow \varepsilon$ , $S \rightarrow SaScSaScaSaSaS$
c) $S \rightarrow \varepsilon$, $S \rightarrow SaScSaScSaScSaS$, $S \rightarrow A$, $A \rightarrow acaca$
d) $S \rightarrow acacaca$, $S \rightarrow SaScSaScSaScSaS$, $S \rightarrow SaSaSaScSaScSa$

Answer submitted: **c)**

You have answered the question correctly.

**12.** Programming languages are often described using an extended form of context-free grammar, where curly brackets are used to denote a construct that can repeat 0, 1, 2, or any number of times. For example, $A \rightarrow B\{C\}D$ says that an *A* can be replaced by a *B* and a *D*, with any number of *C*'s (including 0) between them. This notation does not allow us to describe anything but context-free languages, since an extended production can always be replaced by several conventional productions.

Suppose a grammar has the extended production:

```
A → B{C}D
```

Convert this extended production to conventional productions. Identify, from the list below, the conventional productions that are equivalent to the extended production above.

a) $A \rightarrow BA_1D$
$A_1 \rightarrow CA_1 \mid C$

b) $A \rightarrow BA_1D$
$A_1 \rightarrow A_1C \mid \varepsilon$

c) $A \rightarrow BA_1CD$
$A_1 \rightarrow A_1C \mid \varepsilon$

d) $A \rightarrow BCA_1D$
$A_1 \rightarrow CA_1 \mid \varepsilon$

Answer submitted: **b)**

You have answered the question correctly.

**13.** Consider the grammar G1:

$S \rightarrow \varepsilon \mid aS \mid aSbS$

Which of the following is correct (for a choice to be correct, all propositions must be correct)?

a) The string aaba is not generated by the grammar.

b) The string aaabbbababaabba is not generated by the grammar.

c) For any word w with every prefix having at least as many a's as b's, either there is a unique b in w such that w can be written as aw''bw''' with w'' and w''' being such that every prefix has as many a's as b's--hence can be generated from shorter words of the grammar using the rule $S \rightarrow aSbS$-- or w can be written as w=aw' with w' having every prefix with as many a's as b's -- hence can be generated by the rule $S \rightarrow aS$.

d) a) G1 generates all and only the strings of a's and b's such that every prefix has at least as many a's as b's. b) The following inductive hypothesis will prove it: For n < k, it holds that: Any word in G1 of length n, is such that all its prefixes contain at least as many a's as b's.

Answer submitted: **d)**

Your answer is incorrect.

This inductive hypothesis is incomplete. It shows that the grammar generates only strings such that every prefix has at least as many a's as b's. It does not show however that ALL strings such that every prefix has at least as many a's as b's are generated by the grammar. See the discussion of inductive proofs in Section 1.4 (p. 19).

**14.** Which of the following pairs of grammars define the same language?

a) $G_1$: $S \rightarrow SaScSa|aca|\varepsilon$
$G_2$: $S \rightarrow SaSAaS|aca$, $A \rightarrow cS|\varepsilon$

b) $G_1$: $S \rightarrow SS|a$
$G_2$: $S \rightarrow Sa|a$

c) $G_1$: $S \rightarrow SaScSaS|aca|\varepsilon$
$G_2$: $S \rightarrow SaBaS|aca$, $B \rightarrow cS|\varepsilon$

d) $G_1$: $S \rightarrow AB|a$, $A \rightarrow b$
$G_2$: $S \rightarrow a|b$

Answer submitted: **b)**

You have answered the question correctly.