



# Gradiane Online Accelerated Learning

Zayd

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Based on Chapter 6 of HMU.

Help

1. Here are the transitions of a deterministic pushdown automaton. The start state is  $q_0$ , and  $f$  is the accepting state.

State-Symbol	a	b	$\epsilon$
$q_0-Z_0$	$(q_1, AAZ_0)$	$(q_2, BZ_0)$	$(f, \epsilon)$
$q_1-A$	$(q_1, AAA)$	$(q_1, \epsilon)$	-
$q_1-Z_0$	-	-	$(q_0, Z_0)$
$q_2-B$	$(q_3, \epsilon)$	$(q_2, BB)$	-
$q_2-Z_0$	-	-	$(q_0, Z_0)$
$q_3-B$	-	-	$(q_2, \epsilon)$
$q_3-Z_0$	-	-	$(q_1, AZ_0)$

Describe informally what this PDA does. Then, identify below the one input string that the PDA accepts.

- a) bababba
- b) abbbab
- c) bbbaabbb
- d) abbbabaab

Answer submitted: **b)**

You have answered the question correctly.

Question Explanation:

This PDA accepts all strings with twice as many  $b$ 's as  $a$ 's. In states  $q_0$  and  $q_1$ , we push two  $A$ 's onto the stack for each input  $a$ , and we pop an  $A$  for every input  $b$ .

push two A's onto the stack for each input  $a$ , and we pop an A for every input  $b$ . You can interpret state  $q_1$  as saying "we've seen more than half as many  $a$ 's as  $b$ 's." In states  $q_0$  and  $q_2$  we push a B for every input  $b$ , and (with the help of  $q_3$ ) we pop two B's for every input  $a$  (using  $q_3$  as an intermediate. You can interpret  $q_2$  as "we have seen more than twice as many  $b$ 's as  $a$ 's."

2. If we convert the context-free grammar G:

$$\begin{aligned} S &\rightarrow AS \mid A \\ A &\rightarrow 0A \mid 1B \mid 1 \\ B &\rightarrow 0B \mid 0 \end{aligned}$$

to a pushdown automaton that accepts  $L(G)$  by empty stack, using the construction of Section 6.3.1, which of the following would be a rule of the PDA?

- a)  $\delta(q, 0, B) = \{(q, B), (q, \epsilon)\}$
- b)  $\delta(q, 0, A) = \{(q, A)\}$
- c)  $\delta(q, \epsilon, S) = \{(q, AS)\}$
- d)  $\delta(q, \epsilon, A) = \{(q, 0A), (q, 1B), (q, 1)\}$

Answer submitted: **d)**

You have answered the question correctly.

Question Explanation:

There is one state,  $q$ . The input symbols are 0 and 1, and the stack symbols are  $\{S, A, B, 0, 1\}$ .  $S$  is the initial stack symbol. The rules are:

$$\begin{aligned} \delta(q, \epsilon, S) &= \{(q, AS), (q, A)\} \\ \delta(q, \epsilon, A) &= \{(q, 0A), (q, 1B), (q, 1)\} \\ \delta(q, \epsilon, B) &= \{(q, 0B), (q, 0)\} \\ \delta(q, 0, 0) &= \{(q, \epsilon)\} \\ \delta(q, 1, 1) &= \{(q, \epsilon)\} \end{aligned}$$

3. Consider the pushdown automaton with the following transition rules:

- 1.  $\delta(q, 0, Z_0) = \{(q, XZ_0)\}$
- 2.  $\delta(q, 0, X) = \{(q, XX)\}$
- 3.  $\delta(q, 1, X) = \{(q, X)\}$
- 4.  $\delta(q, \epsilon, X) = \{(p, \epsilon)\}$
- 5.  $\delta(p, \epsilon, X) = \{(p, \epsilon)\}$
- 6.  $\delta(p, 1, X) = \{(p, XX)\}$
- 7.  $\delta(p, 1, Z_0) = \{(p, \epsilon)\}$

The start state is  $q$ . For which of the following inputs can the PDA first enter state  $p$  with the input empty and the stack containing  $XXZ_0$  [i.e., the ID  $(p, \epsilon, XXZ_0)$ ]?

- a) 011011011
- b) 1001101
- c) 0100110
- d) 011001101

Answer submitted: **a)**

You have answered the question correctly.

Question Explanation:

When in state  $q$ , the PDA adds an  $X$  to the stack whenever it consumes a  $0$ . The PDA may consume a  $1$  with no change to the stack, but only if the stack has top symbol  $X$ . That is, on inputs beginning with  $1$  the PDA has no choice of move and can never enter state  $p$ . Since entering state  $p$  pops an  $X$  from the stack, there must be exactly three  $0$ 's in the consumed inputs, and any number of  $1$ 's. In addition, the first input must be  $0$ .

4. Here are the transitions of a deterministic pushdown automaton. The start state is  $q_0$ , and  $f$  is the accepting state.

State-Symbol	a	b	$\epsilon$
$q_0-Z_0$	$(q_1,AAZ_0)$	$(q_2,BZ_0)$	$(f,\epsilon)$
$q_1-A$	$(q_1,AAA)$	$(q_1,\epsilon)$	-
$q_1-Z_0$	-	-	$(q_0,Z_0)$
$q_2-B$	$(q_3,\epsilon)$	$(q_2,BB)$	-
$q_2-Z_0$	-	-	$(q_0,Z_0)$
$q_3-B$	-	-	$(q_2,\epsilon)$
$q_3-Z_0$	-	-	$(q_1,AZ_0)$

Describe informally what this PDA does. Then, identify below, the one input string that takes the PDA into state  $q_3$  (with any stack).

- a) aabbbbbb
- b) baabba
- c) babbbbaa
- d) bbabbba

Answer submitted: **c)**

Your answer is incorrect.

Hint: try proving that whenever you see twice as many  $b$ 's as  $a$ 's, such as when reading the input babbbba, you go from ID  $(q_0,babbbba,Z_0)$  to ID  $(q_0,\epsilon,Z_0)$ . Pushdown automata are the subject of Section 6.1 (p. 225). See especially the informal description of how these automata move in Section 6.1.1 (p. 225) and the formal definition of their behavior in terms of instantaneous descriptions in Section 6.1.4 (p. 230).

Question Explanation:

This PDA accepts all strings with twice as many  $b$ 's as  $a$ 's. In states  $q_0$  and  $q_1$ , we push two  $A$ 's onto the stack for each input  $a$ , and we pop an  $A$  for every input  $b$ . You can interpret state  $q_1$  as saying "we've seen more than half as many  $a$ 's as  $b$ 's." In states  $q_0$  and  $q_2$  we push a  $B$  for every input  $b$ , and (with the help of  $q_3$ ) we pop two  $B$ 's for every input  $a$ . You can interpret  $q_2$  as "we have seen more than twice as many  $b$ 's as  $a$ 's."

As a result, we enter  $q_3$  when, having previously seen strictly more than twice as many  $b$ 's as  $a$ 's, we see an  $a$  on the input.

The correct choice is: **d)**

5. Suppose one transition rule of some PDA  $P$  is  $\delta(q, 0, X) = \{(p, YZ), (r, XY)\}$ . If we convert PDA  $P$  to an equivalent context-free grammar  $G$  in the manner described in Section 6.3.2 (p. 247), which of the following could be a production of  $G$  derived from this transition rule? You may assume  $s$  and  $t$  are states of  $P$ , as well as  $p$ ,  $q$ , and  $r$ .
- $[qXq] \rightarrow [rXr][rYq]$
  - $[qXq] \rightarrow 0[pYr][sZq]$
  - $[qXq] \rightarrow 0[pYr][rZq]$
  - $[qXq] \rightarrow 0[qXr][rYr]$

Answer submitted: **d)**

Your answer is incorrect.

Remember that reading input symbol 0 results in a state transition out of state  $q$ . See the complete algorithm for construction of the grammar in Section 6.3.2 (p. 247).

Question Explanation:

If  $m$  and  $n$  are any states of  $P$ , then the fact that  $(p, YZ)$  is in  $\delta(q, 0, X)$  says that there will be a production  $[qXm] \rightarrow 0[pYn][nZm]$ . Similarly, the choice  $(r, XY)$  says that  $[qXm] \rightarrow 0[rXn][nYm]$  is a production.

The correct choice is: **c)**

6. Consider the pushdown automaton with the following transition rules:

- $\delta(q, 0, Z_0) = \{(q, XZ_0)\}$
- $\delta(q, 0, X) = \{(q, XX)\}$
- $\delta(q, 1, X) = \{(q, X)\}$
- $\delta(q, \epsilon, X) = \{(p, \epsilon)\}$
- $\delta(p, \epsilon, X) = \{(p, \epsilon)\}$
- $\delta(p, 1, X) = \{(p, XX)\}$
- $\delta(p, 1, Z_0) = \{(p, \epsilon)\}$

From the ID  $(p, 1101, XXZ_0)$ , which of the following ID's can NOT be reached?

- $(p, 101, XZ_0)$

- b)  $(p, 01, Z_0)$
- c)  $(p, 01, XXXXXZ_0)$
- d)  $(p, 1101, Z_0)$

Answer submitted: **c)**

You have answered the question correctly.

Question Explanation:

In state  $p$ , there is no way to consume a 0 from the input, and no way to leave state  $p$ . We can pop X's from the stack spontaneously (on  $\epsilon$  input), and by consuming a 1 we can push an X onto the stack (but only if there was already an X on the top of the stack). Finally, with  $Z_0$  at the top of the stack and 1 as the next input, we can pop the  $Z_0$  and consume the 1. Consequently, the accessible ID's can be categorized as follows. All have state  $p$ .

1. Input = 1101, stack is  $XXZ_0$ ,  $XZ_0$ , or  $Z_0$ .
2. Input = 101, stack is  $XXXZ_0$ ,  $XXZ_0$ ,  $XZ_0$ ,  $Z_0$ , or  $\epsilon$ .
3. Input = 01, stack is  $XXXXZ_0$ ,  $XXXZ_0$ ,  $XXZ_0$ ,  $XZ_0$ ,  $Z_0$ , or  $\epsilon$ .