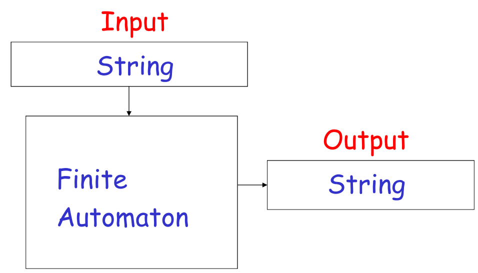
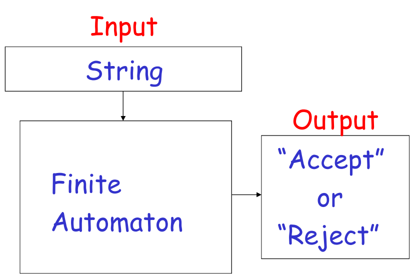
Finite Automata

class 2

Finite Automaton

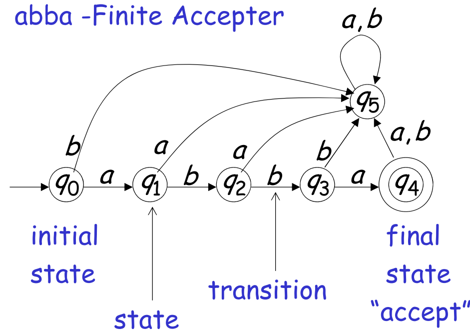


Finite Accepter



Transition Graph

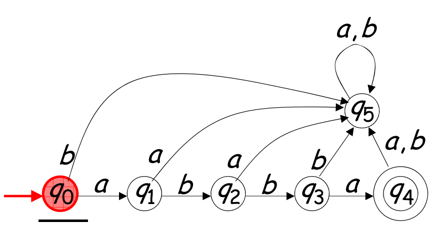
abba -Finite Accepter



Initial Configuration

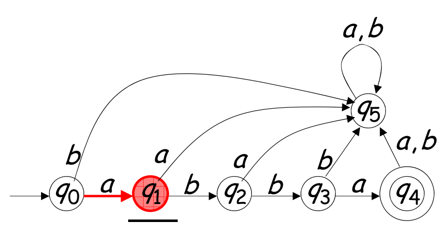
Input String

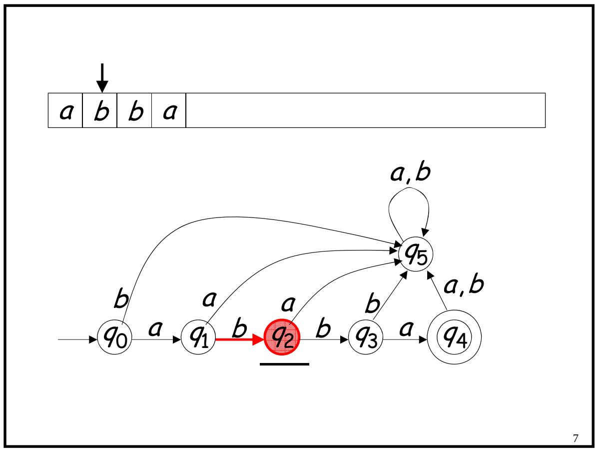
a	b	b	a	

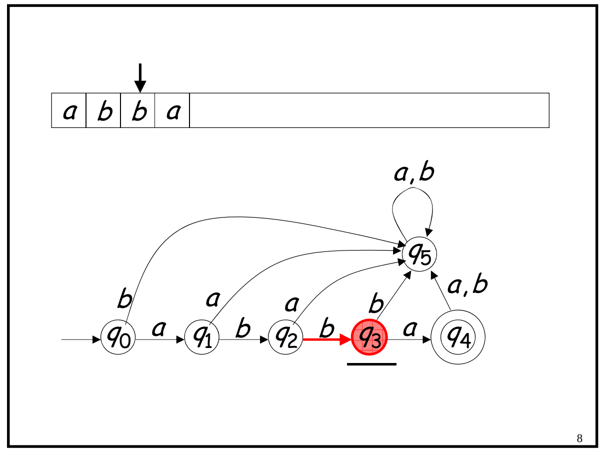


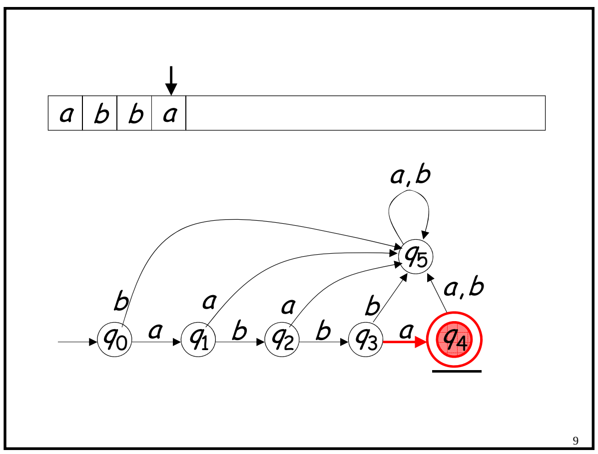
Reading the Input

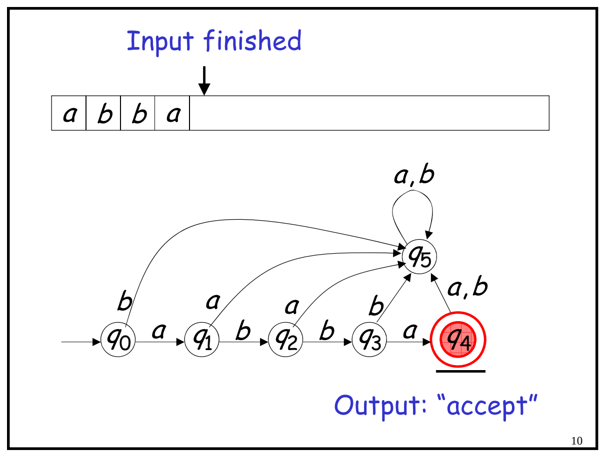
a	b	b	a	

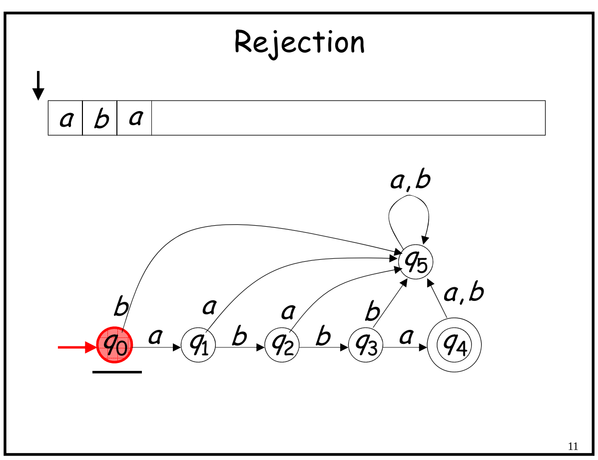


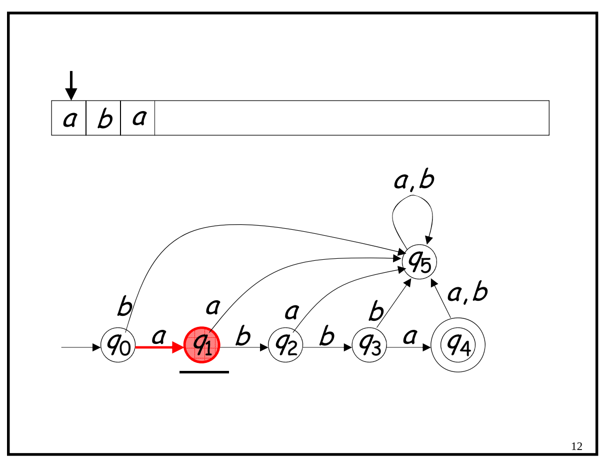

a b b a
a,b
q5
b a a b a,b
q0 a q1 b q2 b q3 a q4
7

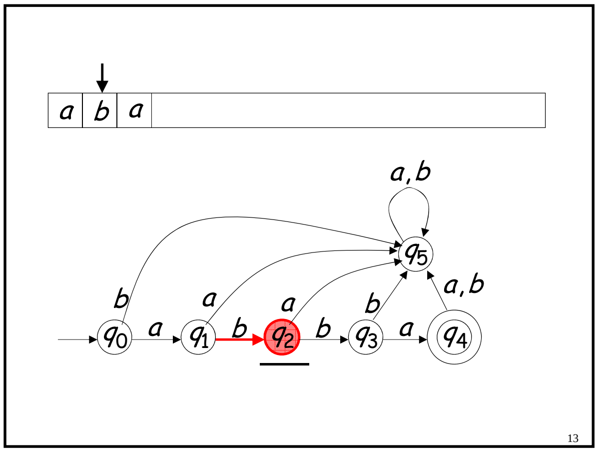

a b b a
a,b
q5
b a a b a,b
q0 a q1 b q2 b q3 a q4
8

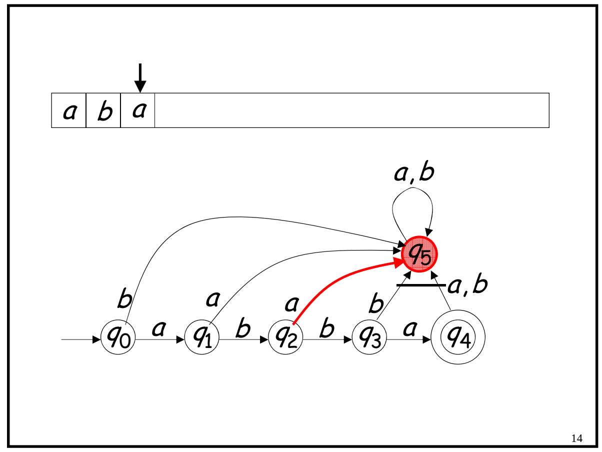

a b b a
a,b
q5
b a a b a,b
q0 a q1 b q2 b q3 a q4
9

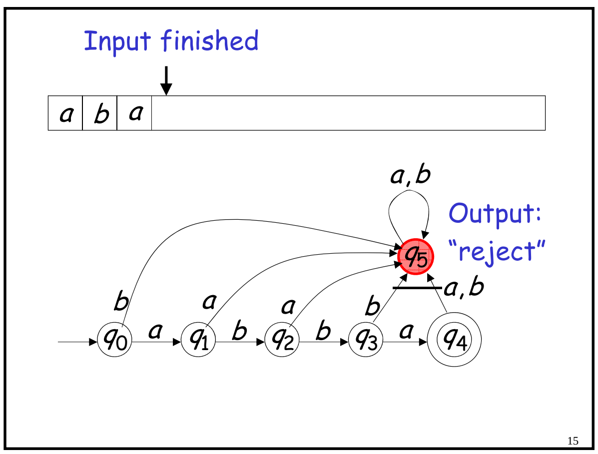

Input finished
a b b a
a,b
q5
b a a b a,b
q0 a q1 b q2 b q3 a q4
Output: "accept"
10

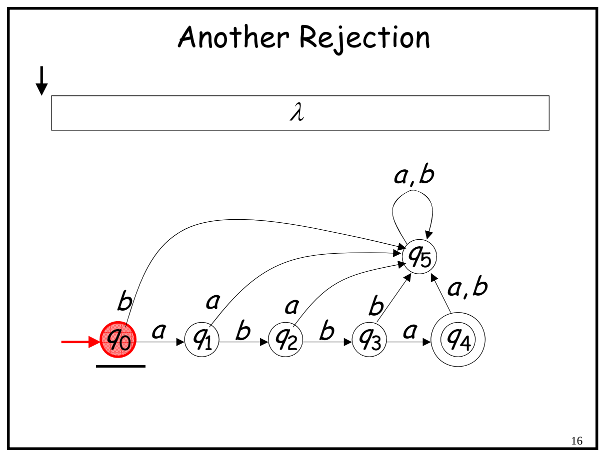

Rejection
a b a
a,b
q5
b a a b a,b
q0 a q1 b q2 b q3 a q4
11

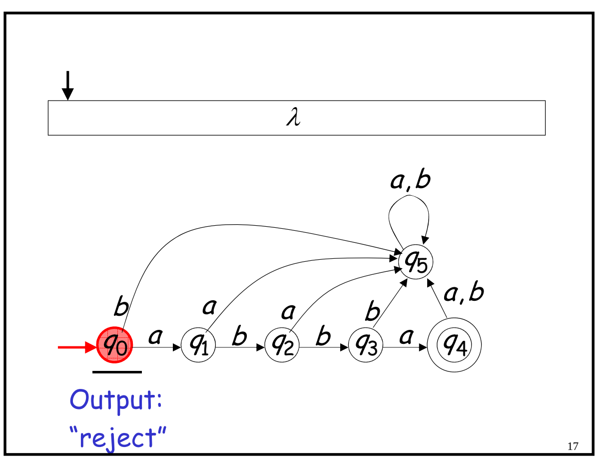

a b a
a,b
q5
b a a b a,b
q0 a q1 b q2 b q3 a q4
12

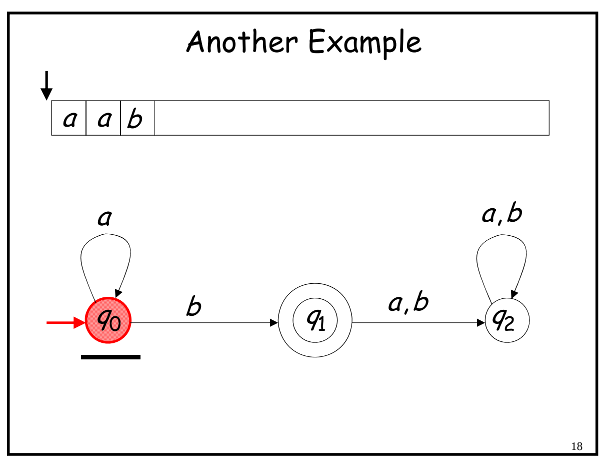

a b a
a,b
b a b a b a
a,b
13

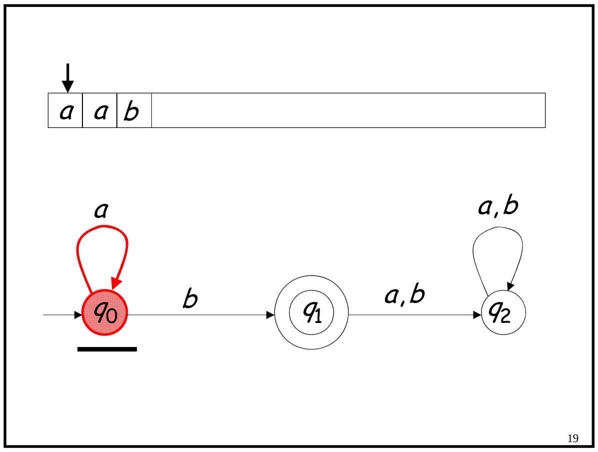

a b a
a,b
b a b a b a
a,b
14

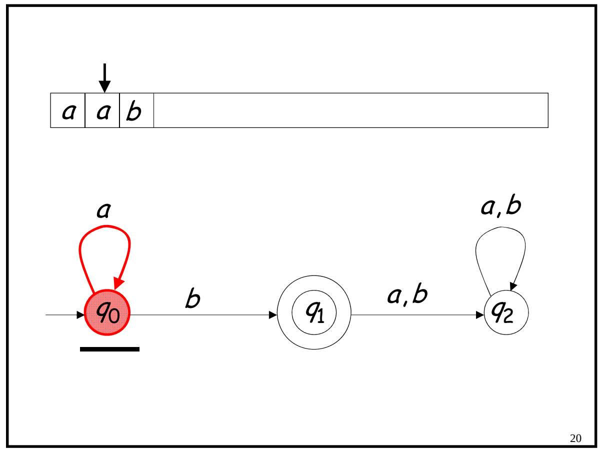

Input finished
a b a
a,b
Output:
"reject"
a,b
15

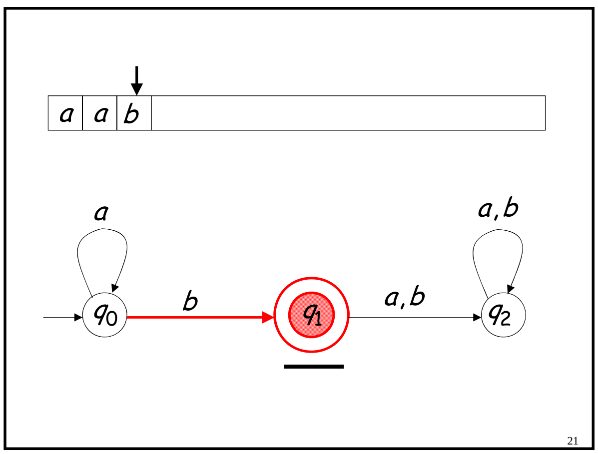

Another Rejection
λ
a,b
a,b
16

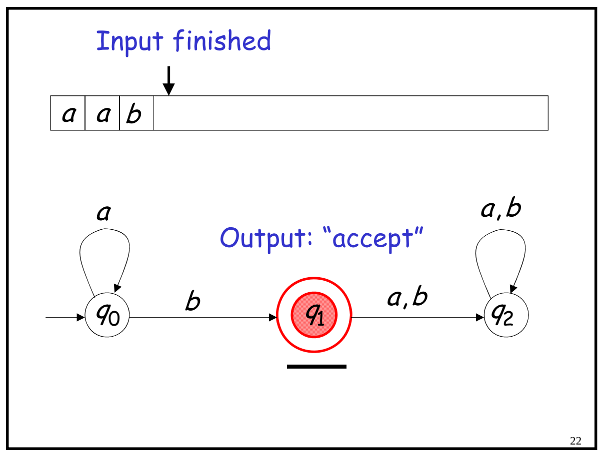

λ
a,b
a,b
Output:
"reject"
17

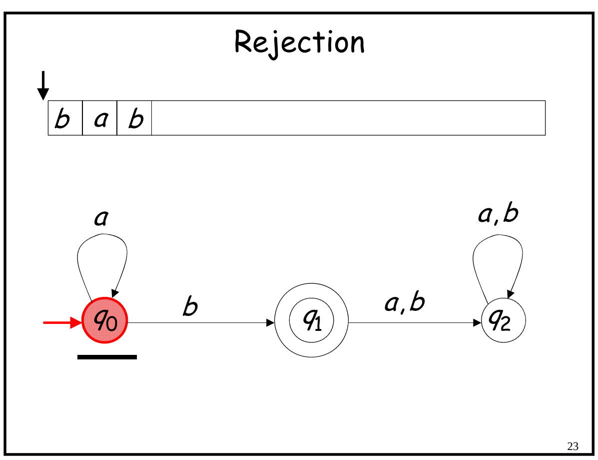

Another Example
a a b
a
b
a,b
a,b
18

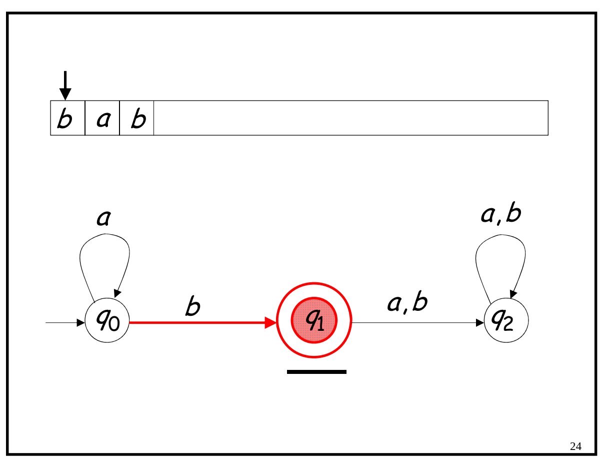

a a b
a
a,b
b
a,b
q_0 q_1 q_2
19

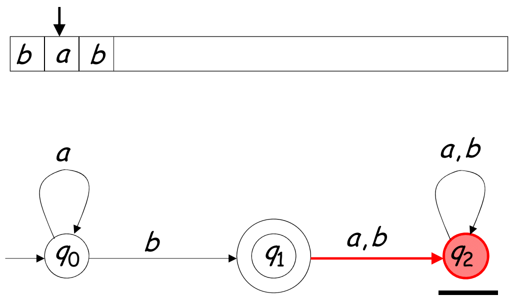

a a b
a
a,b
b
a,b
q_0 q_1 q_2
20


a a b
a
a,b
b
a,b
q_0 q_1 q_2
21


Input finished
a a b
a
Output: "accept"
a,b
b
a,b
q_0 q_1 q_2
22


Rejection
b a b
a
a,b
b
a,b
q_0 q_1 q_2
23


b a b
a
a,b
b
a,b
q_0 q_1 q_2
24



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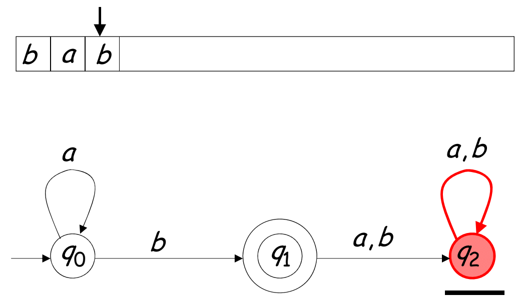

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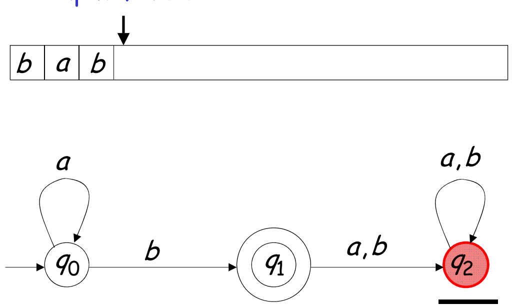
Input finished



Output: "reject"

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Formalities

Deterministic Finite Accepter (DFA)

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : set of states

Σ : input alphabet

δ : transition function

q_0 : initial state

F : set of final states

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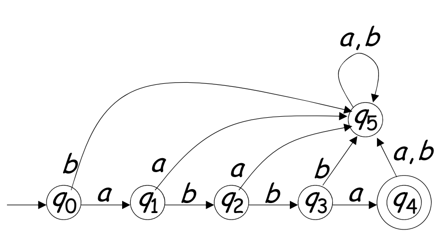
Input Alphabet Σ

$\Sigma = \{a, b\}$



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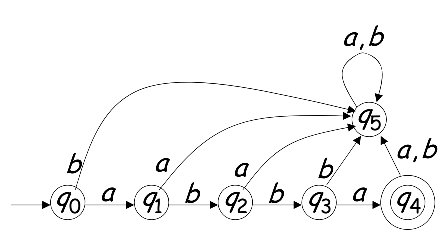
Set of States Q

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$



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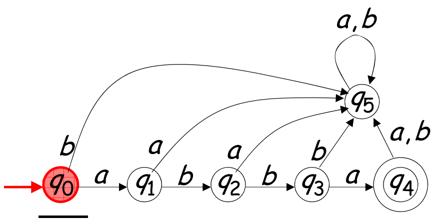
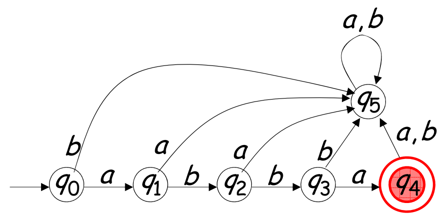
Initial State q_0



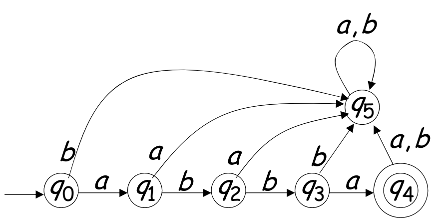
Set of Final States F

$F = \{q_4\}$

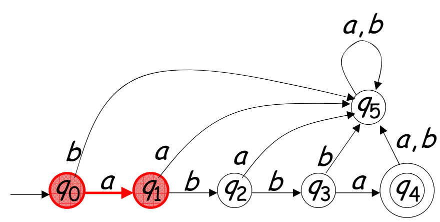


Transition Function δ

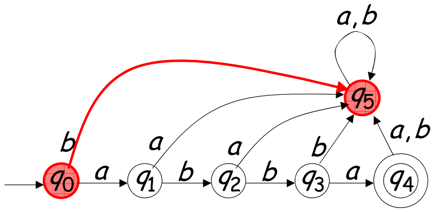
$\delta : Q \times \Sigma \rightarrow Q$



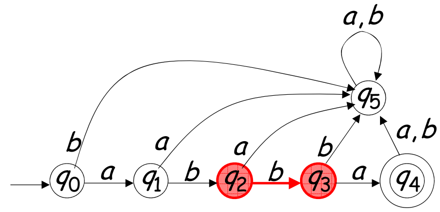
$\delta(q_0, a) = q_1$



$\delta(q_0, b) = q_5$

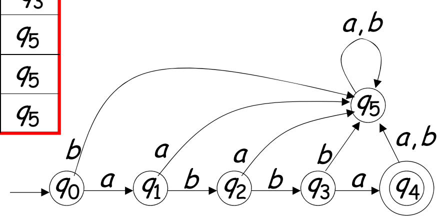


$\delta(q_2, b) = q_3$



Transition Function δ

δ	a	b
q_0	q_1	q_5
q_1	q_5	q_2
q_2	q_5	q_3
q_3	q_4	q_5
q_4	q_5	q_5
q_5	q_5	q_5



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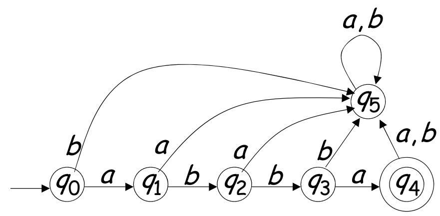
Extended Transition Function $\delta *$

$$\delta * : Q \times \Sigma * \rightarrow Q$$



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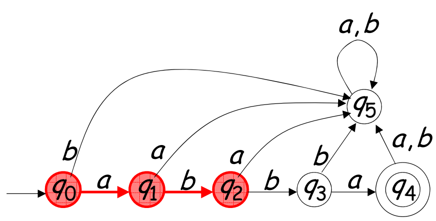
$$\delta * (q_0, ab) = q_2$$



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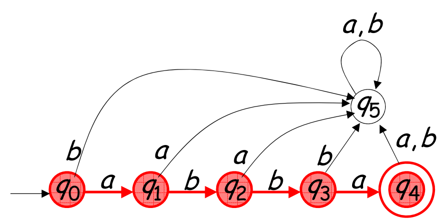
$$\delta * (q_0, abba) = q_4$$



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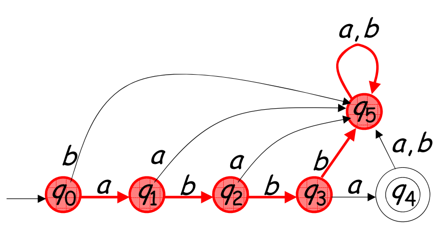
$$\delta * (q_0, abbbaa) = q_5$$



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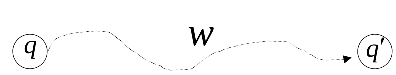
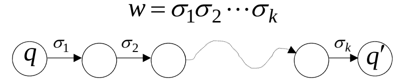
Observation: There is a walk from q to q' with label w

$$\delta * (q, w) = q'$$



$$w = \sigma_1 \sigma_2 \cdots \sigma_k$$



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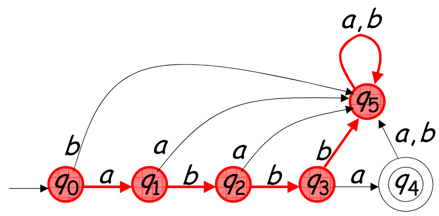
Example: There is a walk from q_0 to q_5 with label $abbbaa$

$$\delta^*(q_0, abbbaa) = q_5$$



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Recursive Definition

$$\delta^*(q, \lambda) = q$$

$$\delta^*(q, w\sigma) = \delta(\delta^*(q, w), \sigma)$$

$\left.\begin{array}{l}\delta^*(q, w\sigma) = q' \\ \delta(q_1, \sigma) = q'\end{array}\right\} \Rightarrow \delta^*(q, w\sigma) = \delta(q_1, \sigma)$

$\left.\begin{array}{l}\delta^*(q, w\sigma) = \delta(q_1, \sigma) \\ \delta^*(q, w) = q_1\end{array}\right\} \Rightarrow \delta^*(q, w\sigma) = \delta(\delta^*(q, w), \sigma)$

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$\delta^*(q_0, ab) =$

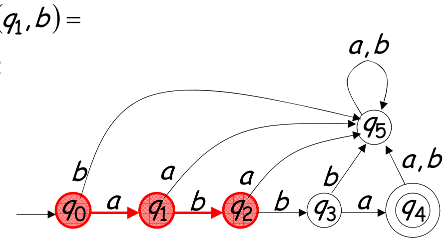
$\delta(\delta^*(q_0, a), b) =$

$\delta(\delta(\delta^*(q_0, \lambda), a), b) =$

$\delta(\delta(q_0, a), b) =$

$\delta(q_1, b) =$

q_2



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Languages Accepted by DFAs

Take DFA M

Definition:

The language $L(M)$ contains all input strings accepted by M

$L(M)$ = { strings that drive M to a final state}

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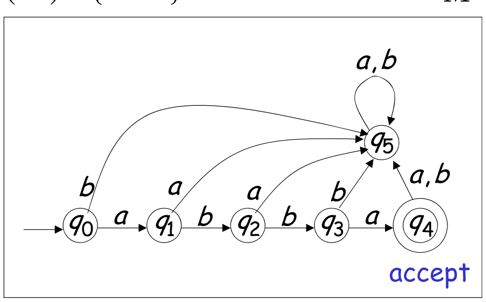
Example

$L(M) = \{abba\}$ $\qquad M$



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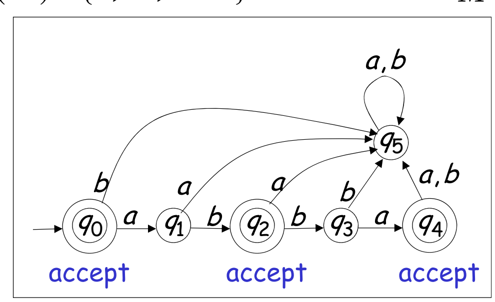
Another Example

$L(M) = \{\lambda, ab, abba\}$ $\qquad M$



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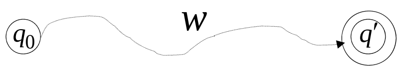
Formally

For a DFA $M = (Q, \Sigma, \delta, q_0, F)$

Language accepted by M :

$$L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F\}$$



$q' \in F$

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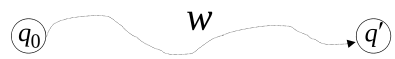
Observation

Language rejected by M :

$$\overline{L(M)} = \{w \in \Sigma^* : \delta^*(q_0, w) \notin F\}$$



$q' \notin F$

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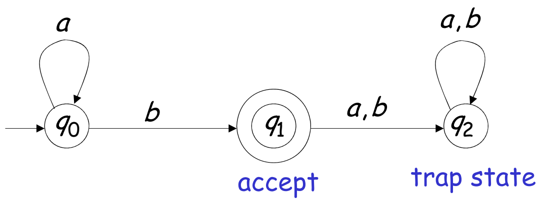
More Examples

$$L(M) = \{a^n b : n \geq 0\}$$



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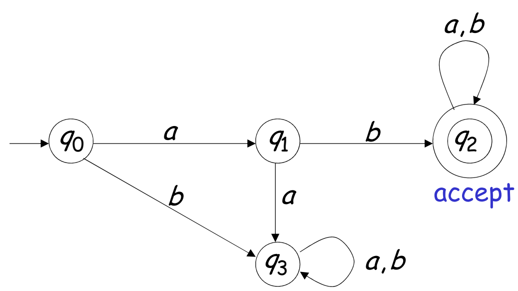
$L(M)$ = { all strings with prefix ab }



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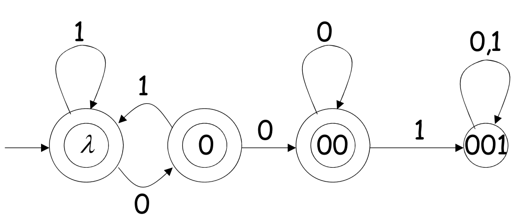
$L(M)$ = { all strings without substring 001 }



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Regular Languages

A language L is regular if there is a DFA M such that $L = L(M)$

All regular languages form a language family

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Examples of regular languages:

$\{abba\}$ $\{\lambda, ab, abba\}$ $\{a^n b : n \geq 0\}$

{ all strings with prefix ab }

{ all strings with prefix ab }

{ all strings without substring 001 }

There exist automata that accept these Languages (see previous slides).

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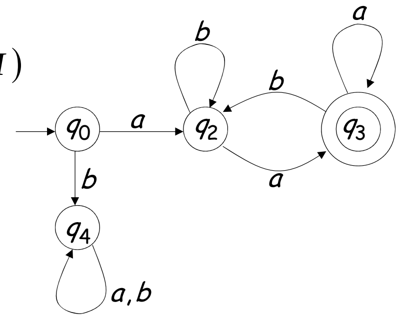
Another Example

The language $L = \{awa : w \in \{a,b\}^*\}$ is regular:

$L = L(M)$



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There exist languages which are not Regular:

Example: $L = \{a^n b^n : n \geq 0\}$

There is no DFA that accepts such a language

(we will prove this later in the class)

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