



Zayd

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Submission number: 67764
Submission certificate: HE532744
Submission time: 2014-03-08 22:33:29 PST (GMT - 8:00)

Number of questions: 5
Positive points per question: 3.0
Negative points per question: 1.0
Your score: 3

Help

These are some questions in which students are asked to understand proofs about context-free grammars. The material is based on Chapter 5 of HM

1. Let G be the grammar:

$$S \rightarrow SS \mid (S) \mid \varepsilon$$

$L(G)$ is the language BP of all strings of balanced parentheses, that is, those strings that could appear in a well-formed arithmetic expression. We want to prove that $L(G) = BP$, which requires two inductive proofs:

1. If w is in $L(G)$, then w is in BP.
2. If w is in BP, then w is in $L(G)$.

We shall here prove only the first. You will see below a sequence of steps in the proof, each with a reason left out. These reasons belong to one of three classes:

- A) Use of the inductive hypothesis.
- B) Reasoning about properties of grammars, e.g., that every derivation has at least one step.
- C) Reasoning about properties of strings, e.g., that every string is longer than any of its proper substrings.

The proof is an induction on the number of steps in the derivation of w . You should decide on the reason for each step in the proof below, and then identify from the available choices a correct pair consisting of a step and a kind of reason (A, B, or C).

Basis: One step.

- (1) The only 1-step derivation of a terminal string is $S \Rightarrow \varepsilon$ because _____
- (2) ε is in BP because _____
Induction: An n -step derivation for some $n > 1$.
- (3) The derivation $S \Rightarrow^n w$ is either of the form
(a) $S \Rightarrow SS \Rightarrow^{n-1} w$ or of the form
(b) $S \Rightarrow (S) \Rightarrow^{n-1} w$
because _____
Case (a):
- (4) $w = xy$, for some strings x and y such that $S \Rightarrow^p x$ and $S \Rightarrow^q y$, where $p < n$ and $q < n$ because _____
- (5) x is in BP because _____
- (6) y is in BP because _____
- (7) w is in BP because _____
Case (b):
- (8) $w = (z)$ for some string z such that $S \Rightarrow^{n-1} z$ because _____
- (9) z is in BP because _____
- (10) w is in BP because _____

- a) (10) for reason C
- b) (7) for reason A
- c) (6) for reason B
- d) (10) for reason B

Answer submitted: **d)**

Your answer is incorrect.

Can the fact that w has balanced parentheses be determined by talking only about general properties of grammars like G ? A good model of the proof is Section 5.2.6 (p. 191), where there is an induction on the length of a derivation. The proof in Section 5.1.5 (p. 179) may also be instructive.

2. Consider the grammars:

$G_1: S \rightarrow AB \mid a \mid abC, A \rightarrow b, C \rightarrow abC \mid c$

$G_2: S \rightarrow a \mid b \mid cC, C \rightarrow cC \mid c$

These grammars do not define the same language. To prove, we use a string that is generated by one but not by the other grammar. Which of the following strings can be used for this proof?

- a) ababababcc
- b) cacaba
- c) ccc
- d) abababcc

Answer submitted: **c)**

You have answered the question correctly.

3. Let L be the language of all strings of a's and b's such that no prefix (proper or not) has more b's than a's. Let G be the grammar with productions

$S \rightarrow aS \mid aSbS \mid \epsilon$

To prove that $L = L(G)$, we need to show two things:

1. If $S \Rightarrow^* w$, then w is in L .
2. If w is in L , then $S \Rightarrow^* w$.

We shall consider only the proof of (2) here. The proof is an induction on n , the length of w . Here is an outline of the proof, with reasons omitted. You need to supply the reasons.

Basis:

- 1) If $n=0$, then w is ϵ because _____.

- 2) $S \Rightarrow^* w$ because _____.

Induction:

- 3) Either (a) w can be written as $w=aw'$ where for w' each prefix has as many a's as b's or (b) w can be written as $w=aw'bw''$ where for both w' and w'' hold that each prefix has as many a's as b's because _____.

- 4a) In case (a), w' is in the language because _____.

- 5a) In case (a), $S \Rightarrow^* w'$ because _____.

- 6a) In case (a), $S \Rightarrow^* w$ because _____.

- 4b) In case (b), both w' and w'' are in the language because _____.

- 5b) In case (b), $S \Rightarrow^* w'$ because _____.

- 6b) In case (b), $S \Rightarrow^* w''$ because _____.

- 7b) In case (b), $S \Rightarrow^* w$ because _____.

For which of the steps above is the appropriate reason "by the inductive hypothesis"?

- a) 5a
- b) 1
- c) 2
- d) 4b

Answer submitted: **d)**

Your answer is incorrect.

The reason for 4b is:

Both w' and w'' have property that no prefix (proper or not) has more b's than a's.

The "If" part of Theorem 5.7 (p. 179) is a useful example. Also, see Section 1.4.2 (p. 22) on the general form of inductions on integers (which includes an induction on the lengths of strings).

4. Let G be the grammar:

$$S \rightarrow SS \mid (S) \mid \varepsilon$$

$L(G)$ is the language BP of all strings of balanced parentheses, that is, those strings that could appear in a well-formed arithmetic expression. We want to prove that $L(G) = BP$, which requires two inductive proofs:

1. If w is in $L(G)$, then w is in BP.
2. If w is in BP, then w is in $L(G)$.

We shall here prove only the second. You will see below a sequence of steps in the proof, each with a reason left out. These reasons belong to one of three classes:

- A) Use of the inductive hypothesis.
- B) Reasoning about properties of grammars, e.g., that every derivation has at least one step.
- C) Reasoning about properties of strings, e.g., that every string is longer than any of its proper substrings.

The proof is an induction on the length of w . You should decide on the reason for each step in the proof below, and then identify from the available choices a correct pair consisting of a step and a kind of reason (A, B, or C).

Basis: Length = 0.

- (1) The only string of length 0 in BP is ε because _____
- (2) ε is in $L(G)$ because _____
- Induction: $|w| = n > 0$.
- (3) w is of the form $(x)y$, where (x) is the shortest proper prefix of w that is in BP, and y is the remainder of w because _____
- (4) x is in BP because _____
- (5) y is in BP because _____
- (6) $|x| < n$ because _____
- (7) $|y| < n$ because _____
- (8) x is in $L(G)$ because _____
- (9) y is in $L(G)$ because _____
- (10) (x) is in $L(G)$ because _____
- (11) w is in $L(G)$ because _____

- a) (6) for reason A
- b) (3) for reason A
- c) (11) for reason B
- d) (4) for reason A

Answer submitted: **d)**

Your answer is incorrect.

Remember: the inductive hypothesis is used to conclude that a certain string s is in $L(G)$, and the only way to apply the inductive hypothesis is to be sure that the length of s is less than that of the length (typically n) being considered in the proof of the inductive step. The "If" part of Theorem 5.7 (p. 179) is a useful example. Also, see Section 1.4.2 (p. 22) on the general form of inductions on integers (which includes an induction on lengths of strings).

5. Let L be the language of all strings of a 's and b 's such that no prefix (proper or not) has more b 's than a 's. Let G be the grammar with productions

$$S \rightarrow aS \mid aSbS \mid \varepsilon$$

To prove that $L = L(G)$, we need to show two things:

1. If $S \Rightarrow^* w$, then w is in L .
2. If w is in L , then $S \Rightarrow^* w$.

We shall consider only the proof of (1) here. The proof is an induction on n , the number of steps in the derivation $S \Rightarrow^* w$. Here is an outline of the proof, with reasons omitted. You need to supply the reasons.

Basis:

- 1) If $n=1$, then w is ε because _____.

- 2) w is in L because _____.

Induction:

- 3) Either (a) $S \Rightarrow aS \Rightarrow^{n-1} w$ or (b) $S \Rightarrow aSbS \Rightarrow^{n-1} w$ because _____.

- 4a) In case (a), $w = ax$, and $S \Rightarrow^{n-1} x$ because _____.

- 5a) In case (a), x is in L because _____.

- 6a) In case (a), w is in L because _____.

- 4b) In case (b), w can be written $w = aybz$, where $S \Rightarrow^p y$ and $S \Rightarrow^q z$ for some p and q less than n because _____.

- 5b) In case (b), y is in L because _____.

- 6b) In case (b), z is in L because _____.

- 7b) In case (b), w is in L because _____.

Some of the steps above have one of the following reasons:

I) "The following two statements are true:

- (i) if string x has no prefix with more b 's than a 's, then neither does string ax ,
- (ii) if strings y and z are such that no prefix has more b 's than a 's, then neither does string $aybz$."

II) "All n -step derivations of w produce either ε (for $n=1$) or use one of the productions with at least one nonterminal in the body (for $n > 1$). In case the production $S \rightarrow aS$ is used, then $w=ax$ with x being produced by a $(n-1)$ -step derivation. In case the production $S \rightarrow aSbS$ is used then $w=aybz$ with y and z being produced by derivations with number of steps less than n ."

III) "by the inductive hypothesis"

Choose as correct a (STEP, REASON) pair. (I.e., a correct pair means that step STEP is true because of reason REASON.)

- a) (1,II)
- b) (6a,II)
- c) (5a,II)
- d) (4a,III)

Answer submitted: **a)**

You have answered the question correctly.

