



## Gradiance Online Accelerated Learning

Zayd

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Help

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1. Let  $L$  be the language of all strings of a's and b's such that no prefix (proper or not) has more b's than a's. Let  $G$  be the grammar with productions

$$S \rightarrow aS \mid aSbS \mid \epsilon$$

To prove that  $L = L(G)$ , we need to show two things:

1. If  $S \Rightarrow^* w$ , then  $w$  is in  $L$ .
2. If  $w$  is in  $L$ , then  $S \Rightarrow^* w$ .

We shall consider only the proof of (1) here. The proof is an induction on  $n$ , the number of steps in the derivation  $S \Rightarrow^* w$ . Here is an outline of the proof, with reasons omitted. You need to supply the reasons.

Basis:

- 1) If  $n=1$ , then  $w$  is  $\epsilon$  because \_\_\_\_\_.

- 2)  $w$  is in  $L$  because \_\_\_\_\_.

Induction:

- 3) Either (a)  $S \Rightarrow aS \Rightarrow^{n-1} w$  or (b)  $S \Rightarrow aSbS \Rightarrow^{n-1} w$  because \_\_\_\_\_.

- 4a) In case (a),  $w = ax$ , and  $S \Rightarrow^{n-1} x$  because \_\_\_\_\_.

- 5a) In case (a),  $x$  is in  $L$  because \_\_\_\_\_.

- 6a) In case (a),  $w$  is in  $L$  because \_\_\_\_\_.

- 4b) In case (b),  $w$  can be written  $w = aybz$ , where  $S \Rightarrow^p y$  and  $S \Rightarrow^q z$  for some  $p$  and  $q$  less than  $n$  because \_\_\_\_\_.

- 5b) In case (b),  $y$  is in  $L$  because \_\_\_\_\_.

- 6b) In case (b),  $z$  is in  $L$  because \_\_\_\_\_.

- 7b) In case (b),  $w$  is in  $L$  because \_\_\_\_\_.

Some of the steps above have one of the following reasons:

I) "The following two statements are true:

- (i) if string  $x$  has no prefix with more b's than a's, then neither does string  $ax$ ,
- (ii) if strings  $y$  and  $z$  are such that no prefix has more b's than a's, then neither does string  $aybz$ ."

II) "All  $n$ -step derivations of  $w$  produce either  $\epsilon$  (for  $n=1$ ) or use one of the productions with at least one nonterminal in the body (for  $n > 1$ ). In case the production  $S \rightarrow aS$  is used, then  $w=ax$  with  $x$  being produced by a  $(n-1)$ -step derivation. In case the production  $S \rightarrow aSbS$  is used then  $w=aybz$  with  $y$  and  $z$  being produced by derivations with number of steps less than  $n$ ."

III) "by the inductive hypothesis"

Choose as correct a (STEP, REASON) pair. (I.e., a correct pair means that step STEP is true because of reason REASON.)

- a) (5b,II)
- b) (5b,III)
- c) (4a,III)
- d) (7b,II)

You did not answer this question.

2. Here is the transition table of a DFA that we shall call  $M$ :

	0	1
→A	B	G
B	C	H
*C	D	G
*D	A	H
E	F	C
F	G	I
*G	H	C
*H	A	D
I	E	I

Find the minimum-state DFA equivalent to the above. States in the minimum-state DFA are each the merger of some of the states of  $M$ . Find in the list below a set of states of  $M$  that forms one state of the minimum-state DFA.

- a) {G,H}
- b) {B,E}
- c) {D,G}
- d) {C,G}

You did not answer this question.

3. The grammar  $G$ :

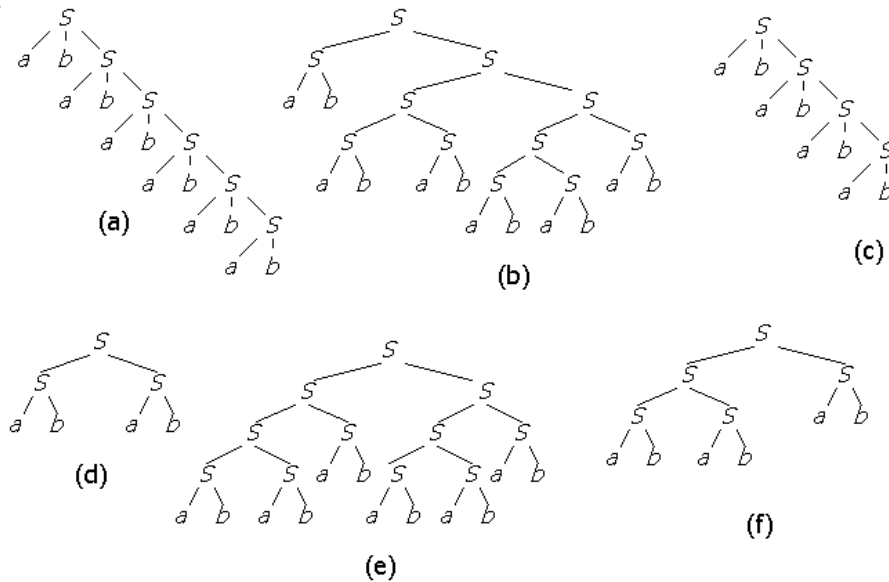
$$S \rightarrow SS \mid a \mid b$$

is ambiguous. That means at least some of the strings in its language have more than one leftmost derivation. However, it may be that some strings in the language have only one derivation. Identify from the list below a string that has exactly TWO leftmost derivations in  $G$ .

- a) a
- b) bab
- c) ba
- d) ab

You did not answer this question.

4. Which of the following is a parse tree for the grammar  $S \rightarrow abS$ ,  $S \rightarrow ab$ ?



- a) (d)
- b) (a)
- c) (b)
- d) (f)

You did not answer this question.

5. Consider the grammar G1:

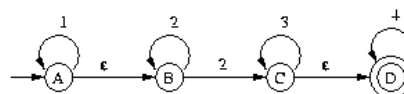
$$S \rightarrow \varepsilon \mid aS \mid aSbS$$

Which of the following is correct (for a choice to be correct, all propositions must be correct)?

- a) There are strings that have as many a's as b's and are not generated by the grammar G1.
- b) a) G1 generates all and only the strings of a's and b's such that every string has at least as many a's as b's. b) The inductive hypothesis to prove it is: For  $n < k$ , it holds: Any word in G1 of length  $n$ , is such that all its prefixes contain more a's than b's or as many a's as b's.
- c) a) G1 generates all and only the strings of a's and b's such that every prefix has at least as many a's as b's. b) The inductive hypothesis to prove it is: For  $n < k$ , it holds that: For any word in G1, any prefix of length  $n$ , is such that all its prefixes contain at least as many a's as b's.
- d) The string aaba is not generated by the grammar.

You did not answer this question.

6. Here is a nondeterministic finite automaton with epsilon-transitions:



Suppose we use the extended subset construction from Section 2.5.5 (p. 77) to convert this epsilon-NFA to a deterministic finite automaton with a dead state, with all transitions defined, and with no state that is inaccessible from the start state. Which of the following would be a transition of the DFA?

Note: we use  $S \cdot x \rightarrow T$  to say that the DFA has a transition on input  $x$  from state  $S$  to state  $T$ .

- a)  $\{D\} \cdot 3 \rightarrow \{\}$

- b)  $\{A,B,C\} \cdot 3 \rightarrow \{C,D\}$
- c)  $\{C,D\} \cdot 4 \rightarrow \{C,D\}$
- d)  $\{A,B\} \cdot \varepsilon \rightarrow \{B\}$

You did not answer this question.

7. Here are the transitions of a deterministic pushdown automaton. The start state is  $q_0$ , and  $f$  is the accepting state.

State-Symbol	a	b	$\varepsilon$
$q_0-Z_0$	$(q_1,AAZ_0)$	$(q_2,BZ_0)$	$(f,\varepsilon)$
$q_1-A$	$(q_1,AAA)$	$(q_1,\varepsilon)$	-
$q_1-Z_0$	-	-	$(q_0,Z_0)$
$q_2-B$	$(q_3,\varepsilon)$	$(q_2,BB)$	-
$q_2-Z_0$	-	-	$(q_0,Z_0)$
$q_3-B$	-	-	$(q_2,\varepsilon)$
$q_3-Z_0$	-	-	$(q_1,AZ_0)$

Describe informally what this PDA does. Then, identify below the one input string that the PDA accepts.

- a) bbbaababb
- b) bababbaa
- c) bbbab
- d) bababba

You did not answer this question.

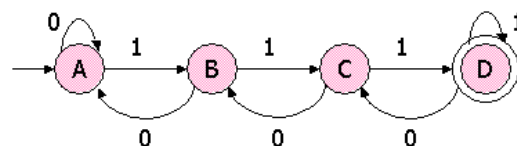
8.  $G_1$  is a context-free grammar with start symbol  $S_1$ , and no other nonterminals whose name begins with "S." Similarly,  $G_2$  is a context-free grammar with start symbol  $S_2$ , and no other nonterminals whose name begins with "S."  $S_1$  and  $S_2$  appear on the right side of no productions. Also, no nonterminal appears in both  $G_1$  and  $G_2$ .

We wish to combine the symbols and productions of  $G_1$  and  $G_2$  to form a new grammar  $G$ , whose language is the concatenation of the languages of  $G_1$  and  $G_2$ . The start symbol of  $G$  will be  $S$ . All productions and symbols of  $G_1$  and  $G_2$  will be symbols and productions of  $G$ . Which of the following sets of productions, added to those of  $G$ , is guaranteed to make  $L(G)$  be  $L(G_1)L(G_2)$ ?

- a)  $S \rightarrow S_2S_1$
- b)  $S \rightarrow S_3S_2, S_3 \rightarrow S_4S_1, S_4 \rightarrow \varepsilon$
- c)  $S \rightarrow S_1S_3, S_3 \rightarrow S_1S_2$
- d)  $S \rightarrow S_1S_1, S_1 \rightarrow S_2$

You did not answer this question.

9. Converting a DFA such as the following:



to a regular expression requires us to develop regular expressions for limited sets of paths --- those that take the automaton from one particular state to another particular state, without passing through some set of states. For the automaton above, determine the languages for the following limitations:

- $L_{AA}$  = the set of path labels that go from A to A without passing through C or D.
- $L_{AB}$  = the set of path labels that go from A to B without passing through C or D.

3.  $L_{BA}$  = the set of path labels that go from B to A without passing through C or D.
4.  $L_{BB}$  = the set of path labels that go from B to B without passing through C or D.

Then, identify a correct regular expression from the list below. Note: there are several different regular expressions possible for each of these languages. However, each of the correct answers can be thought of as built from more limited components. For example, the regular expression **1** is the set of path labels that go from A to B without passing through any of the four states.

- a)  $L_{BB} = 0(0^*1)^*$
- b)  $L_{BB} = (00^*1)^*$
- c)  $L_{AA} = (0+1(10)^*0)^*$
- d)  $L_{AB} = 0^*1(01+10)^*$

You did not answer this question.

10. Find, in the list below, a regular expression whose language is the reversal of the language of this regular expression:  $012^*$ . Recall that the reversal of a language is formed by reversing all its strings, and the reversal of a string  $a_1a_2\dots a_n$  is  $a_n\dots a_2a_1$ .
  - a)  $12^*0$
  - b)  $2^*10$
  - c)  $2^*01$
  - d)  $102^*$

You did not answer this question.

11. In this question,  $L_1, L_2, L_3, L_4$  refer to languages and  $M, M_1, M_2$  refer to Turing machines. Let
 
$$L_1 = \{(M_1, M_2) \mid L(M_1) \text{ is a subset of } L(M_2)\},$$

$$L_2 = \{M \mid \text{There exists an input on which TM } M \text{ halts within 100 steps}\},$$

$$L_3 = \{M \mid \text{There exists an input } w \text{ of size less than 100, such that } M \text{ accepts } w\},$$

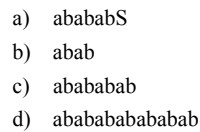
$$L_4 = \{M \mid L(M) \text{ contains at least 2 strings}\}.$$

Decide whether each of  $L_1, L_2, L_3$  and  $L_4$  are recursive, RE or neither. Then identify the true statement below.

- a) The complement of  $L_3$  is recursively enumerable.
- b)  $L_2$  is not recursive.
- c) The complement of  $L_4$  is not recursive.
- d)  $L_4$  is not recursive but cannot be proved so by Rice's Theorem.

You did not answer this question.

12. Consider the grammar  $G: S \rightarrow abS, S \rightarrow ab$ . Which of the following strings is a word of  $L(G)$  AND is the yield of one of the parse trees for grammar  $G$  in the figure below?

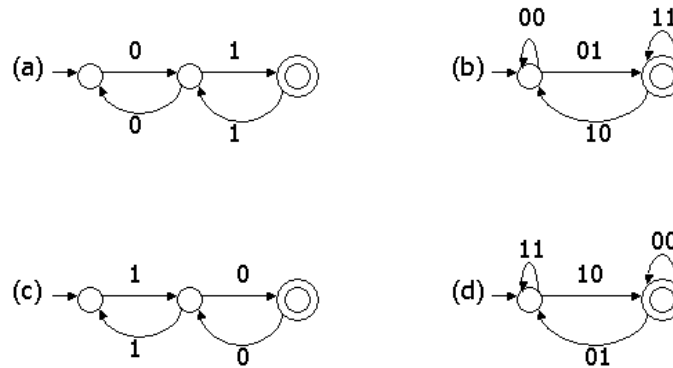


**13.** Use the construction from Theorem 10.15 (p. 457) to convert the following clauses:

- to products of 3 literals per clause. In each case, the new clauses must be satisfiable if and only if the original clause is satisfiable. For the first clause, introduce variables  $x_1, x_2, \dots$  in that order from the left; for the second introduce  $y_1, y_2, \dots$  in that order from the left, and for the third introduce  $z_1, z_2, \dots$  in that order from the left. Use -w as shorthand for NOT w. Then identify, in the list below, the one clause that would appear among the clauses generated by the construction.

- Remember that the last of the generated clauses is a special case; it is able to handle two of the original literals. Check case 4 of the construction. The reduction of CSAT to 3SAT is in Section 10.3.4 (p. 456).

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Note: (b) and (d) use transitions on strings. You may assume that there are nonaccepting intermediate states, not shown, that are in the middle of these transitions, or just accept the extension to the conventional finite automaton that allows strings on transitions and, like the conventional FA accepts strings that are the concatenation of labels along any path from the start state to an accepting state.

- a) b and d
- b) a and d
- c) a and c
- d) a and b

You did not answer this question.

15. Consider the following languages and grammars.  $G_1: S \rightarrow aA|aS, A \rightarrow ab$   
 $G_2: S \rightarrow abS|aA, A \rightarrow a$   
 $G_3: S \rightarrow Sa|AB, A \rightarrow aA|a, B \rightarrow b$   
 $G_4: S \rightarrow aS|b$   
 $L_1: \{a^i b \mid i=1,2,\dots\}$   
 $L_2: \{(ab)^i aa \mid i=0,1,\dots\}$   
 $L_3: \{a^i b \mid i=2,3,\dots\}$   
 $L_4: \{a^i ba^j \mid i=1,2,\dots, j=0,1,\dots\}$   
 $L_5: \{a^i b \mid i=0,1,\dots\}$

Match each grammar with the language it defines. Then, identify a correct match from the list below.

- a)  $G_3$  defines  $L_1$ .
- b)  $G_2$  defines  $L_3$ .
- c)  $G_1$  defines  $L_3$ .
- d)  $G_3$  defines  $L_3$ .

You did not answer this question.

16. A Turing machine  $M$  with start state  $q_0$  and accepting state  $q_f$  has the following transition function:

$\delta(q,a)$	0	1	B
$q_0$	$(q_0, 1, R)$	$(q_1, 1, R)$	$(q_f, B, R)$
$q_1$	$(q_2, 0, L)$	$(q_2, 1, L)$	$(q_2, B, L)$
$q_2$	-	$(q_0, 0, R)$	-
$q_f$	-	-	-

Deduce what  $M$  does on any input of 0's and 1's. Hint: consider what happens when  $M$  is started in state  $q_0$  at the left end of a sequence of any number of 0's (including zero of them) and a 1. Demonstrate your understanding by identifying the true transition of  $M$  from the list below.

- a)  $q_0 0011 \vdash^* 1100 q_f$
- b)  $q_0 1010 \vdash^* 0101 B q_f$
- c)  $q_0 0101 \vdash^* 0100 B q_f$

d)  $q_01010 \mid \neg^* 0101q_f$

You did not answer this question.

17. Consider the languages.

- (a)  $\{0^{2n}1^n \mid n > 0\}$
- (b)  $\{0^{5n}1^n \mid n > 0\}$
- (c)  $\{w \mid w \text{ a string of 0's and 1's such that when interpreted in reverse as a binary integer it is a multiple of 5}\}$
- (d)  $\{0^n1^n \mid n > 0\}$
- (e)  $\{w \mid w \text{ a string of 0's and 1's such that its length is a perfect square}\}$
- (f)  $\{w \mid w \text{ string of 0's and 1's such that when interpreted as a binary integer it is not a multiple of 5}\}$
- (g)  $\{w \mid w \text{ a string of 0's and 1's such that its length is not a perfect cube}\}$
- (h)  $\{w \mid w \text{ a string of 0's and 1's such that the number of 0's is not equal to twice the number of 1's}\}$

Which is a regular language?

- a) (h)
- b) (b)
- c) (g)
- d) (c)

You did not answer this question.

18. Suppose we execute the Chomsky-normal-form conversion algorithm of Section 7.1.5 (p. 272). Let  $A \rightarrow BCDE$  be one of the productions of the given grammar, which has already been freed of  $\epsilon$ -productions and unit productions. Suppose that in our construction, we introduce new variable  $X_a$  to derive a terminal  $a$ , and when we need to split the right side of a production, we use new variables  $Y_1, Y_2, \dots$

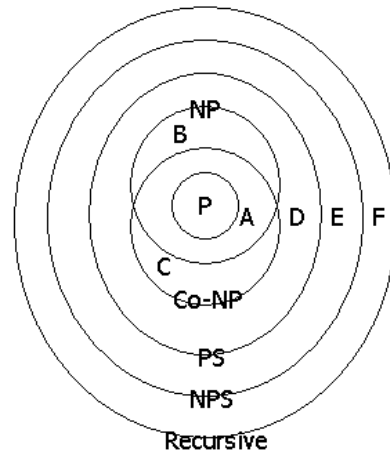
What productions would replace  $A \rightarrow BCDE$ ? Identify one of these replacing productions from the list below.

- a)  $Y_1 \rightarrow Y_2C$
- b)  $Y_3 \rightarrow DE$
- c)  $Y_3 \rightarrow DY_4$
- d)  $Y_2 \rightarrow CY_3$

You did not answer this question.

19. In the diagram below we see certain complexity classes (represented as circles or ovals) and certain regions labeled A through F that represent the differences of some of these complexity classes.





The state of our knowledge regarding the existence of problems in the regions A-F is imperfect. In some cases, we know that a region is nonempty, and in other cases we know that it is empty. Moreover, if  $P=NP$ , then we would know more about the emptiness or nonemptiness of some of these regions, but still would not know everything.

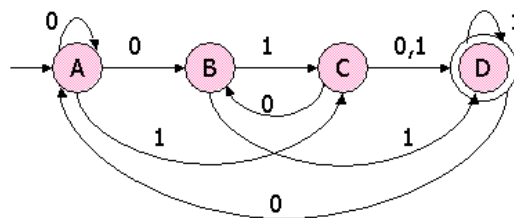
Decide what we know about the regions A-F currently, and also what we would know if  $P=NP$ . Then, identify the true statement from the list below.

- Region C is definitely empty.
- If  $P=NP$ , it would still not be known whether region B is empty.
- Region B is definitely empty.
- Region E is definitely empty.

Answer submitted: **d)**

You have answered the question correctly.

20. Here is a nondeterministic finite automaton:



Some input strings lead to more than one state. Find, in the list below, a string that leads from the start state A to three different states (possibly including A).

- 110110
- 00100
- 1010
- 0110

You did not answer this question.

21. A *unit pair*  $(X,Y)$  for a context-free grammar is a pair where:

- X and Y are variables (nonterminals) of the grammar.

2. There is a derivation  $X \Rightarrow^* Y$  that uses only unit productions (productions with a body that consists of exactly one occurrence of some variable, and nothing else).

For the following grammar:

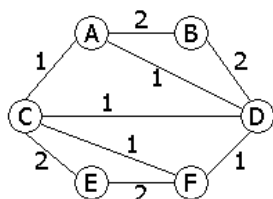
$S \rightarrow A \mid B \mid 2$   
 $A \rightarrow C0 \mid D$   
 $B \rightarrow C1 \mid E$   
 $C \rightarrow D \mid E \mid 3$   
 $D \rightarrow E0 \mid S$   
 $E \rightarrow D1 \mid S$

Identify all the unit pairs. Then, select from the list below the pair that is NOT a unit pair.

- a) (D,D)
- b) (D,E)
- c) (D,C)
- d) (C,E)

You did not answer this question.

22. Find all the minimum-weight Hamilton circuits in the graph below:



Then, identify in the list below the edge that is NOT on any minimum-weight Hamilton circuit.

- a) (E,F)
- b) (C,E)
- c) (A,D)
- d) (A,C)

You did not answer this question.

23. Let  $L$  be the language of all strings of a's and b's such that no prefix (proper or not) has more b's than a's. Let  $G$  be the grammar with productions

$S \rightarrow aS \mid aSbS \mid \epsilon$

To prove that  $L = L(G)$ , we need to show two things:

- 1. If  $S \Rightarrow^* w$ , then  $w$  is in  $L$ .
- 2. If  $w$  is in  $L$ , then  $S \Rightarrow^* w$ .

We shall consider only the proof of (1) here. The proof is an induction on  $n$ , the number of steps in the derivation  $S \Rightarrow^* w$ . Here is an outline of the proof, with reasons omitted. You need to supply the reasons.

Basis:

- 1) If  $n=1$ , then  $w$  is  $\epsilon$  because \_\_\_\_\_.

- 2)  $w$  is in  $L$  because \_\_\_\_\_.

Induction:

- 3) Either (a)  $S \Rightarrow aS \Rightarrow^{n-1} w$  or (b)  $S \Rightarrow aSbS \Rightarrow^{n-1} w$  because \_\_\_\_\_.

- 4a) In case (a),  $w = ax$ , and  $S \Rightarrow^{n-1} x$  because \_\_\_\_\_.

- 5a)

- In case (a),  $x$  is in  $L$  because \_\_\_\_\_.
- 6a) In case (a),  $w$  is in  $L$  because \_\_\_\_\_.
- 4b) In case (b),  $w$  can be written  $w = aybz$ , where  $S \Rightarrow^p y$  and  $S \Rightarrow^q z$  for some  $p$  and  $q$  less than  $n$  because \_\_\_\_\_.
- 5b) In case (b),  $y$  is in  $L$  because \_\_\_\_\_.
- 6b) In case (b),  $z$  is in  $L$  because \_\_\_\_\_.
- 7b) In case (b),  $w$  is in  $L$  because \_\_\_\_\_.

For which of the steps above the appropriate reason is contained in the following argument:

"The following two statements are true

- (i) if string  $x$  has no prefix with more b's than a's, then neither does string  $ax$ ,  
 (ii) if strings  $y$  and  $z$  are such that no prefix has more b's than a's, then neither does string  $aybz$ ."

- a) 3  
 b) 5b  
 c) 7b  
 d) 4a

[You did not answer this question.](#)

24. The operation  $DM(L)$  is defined as follows:

1. Throw away every even-length string from  $L$ .
2. For each odd-length string, remove the middle character.

For example, if  $L = \{001, 1100, 10101\}$ , then  $DM(L) = \{01, 1001\}$ . That is, even-length string 1100 is deleted, the middle character of 001 is removed to make 01, and the middle character of 10101 is removed to make 1001.

It turns out that if  $L$  is a regular language,  $DM(L)$  may or may not be regular. For each of the following languages  $L$ , determine what  $DM(L)$  is, and tell whether or not it is regular.

- $L_1$ : the language of regular expression  $(01)^*0$ .
- $L_2$ : the language of regular expression  $(0+1)^*1(0+1)^*$ .
- $L_3$ : the language of regular expression  $(101)^*$ .
- $L_4$ : the language of regular expression  $00^*11^*$ .

Now, identify the true statement below.

- a)  $DM(L_2)$  is not regular; it consists of all strings of the form  $(0+1)^n00(0+1)^n$ .  
 b)  $DM(L_1)$  is not regular; it consists of all strings of the form  $(01)^n(10)^n$  for  $n \geq 1$ .  
 c)  $DM(L_3)$  is not regular; it consists of all strings of the form  $(101)^n11(101)^n$ .  
 d)  $DM(L_1)$  is not regular; it consists of all strings of the form  $(01)^n(00)(10)^n$ .

[You did not answer this question.](#)

25. Here is an instance of the Modified Post's Correspondence Problem:

	List A	List B
1	01	010
2	11	110
3	0	01

If we apply the reduction of MPCP to PCP described in Section 9.4.2 (p. 404), which of the following would be a pair in the resulting PCP instance.

- a)  $(\$, \$\$)$   
 b)  $(*0^*, *0^*1)$   
 c)  $(\$, \$^*)$   
 d)  $(*\$, \$)$

Answer submitted: **a)**

You have answered the question correctly.