



## Gradiane Online Accelerated Learning

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- [Home Page](#)
- [Assignments Due](#)
- [Progress Report](#)
- [Handouts](#)
- [Tutorials](#)
- [Homeworks](#)
- [Lab Projects](#)
- [Log Out](#)

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Based on Sections 7.2, 7.3, and 7.4 of HMU.

Help

1. The intersection of two CFL's need not be a CFL. Identify in the list below a pair of CFL's such that their intersection is not a CFL.

- a)  $L_1 = \{a^n b^n c^i \mid n > 0, i > 0\}$   
 $L_2 = \{a^j a^n b^i c^j \mid n > 0, i > 0, j > 0\}$
- b)  $L_1 = \{a c a^n b^j c^i \mid n > 0, i > 0, j > 0\}$   
 $L_2 = \{a c a^j a^n b^i c^j \mid n > 0, i > 0, j > 0\}$
- c)  $L_1 = \{a b a^n b^n c^n b a \mid n > 0, i > 0\}$   
 $L_2 = \{a b a^n b^i c^j b a \mid n > 0, i > 0, j > 0\}$
- d)  $L_1 = \{a^n b^j c^i \mid n > 0, i > 0, j > 0\}$   
 $L_2 = \{a^j a^n b^i c^j \mid n > 0, i > 0, j > 0\}$

Answer submitted: **c)**

Your answer is incorrect.

$L_1$  is not a context free language. You can use the pumping lemma (Section 7.2, p. 279) to prove this fact.

Question Explanation:

The incorrect choices fall in two categories: either one of the two given languages is not a CFL, or one is a CFL and the other is a regular language, in which case we know that their intersection is a CFL (see Section 7.3.4 on p. 291).

For the four correct choices the intersections of the two given languages are:

1.  $\{a^n b^n c^n \mid n > 0\}$ .
2.  $\{a b a^n b^n c^n \mid n > 0\}$ .
3.  $\{a b a^n b^n c^n b a \mid n > 0\}$ .

$$4. \{a^n b^n c^n \mid n > 0\}.$$

In all cases we can prove the language not to be context-free by using the pumping lemma on a word  $a^n b^n c^n$  for sufficiently large  $n$ . If we pump any pair of strings (of length much smaller than  $n$ ) then the balance on the number of a's, b's and c's will be ruined.

The correct choice is: **a)**

2. The language  $L = \{ss \mid s \text{ is a string of a's and b's}\}$  is not a context-free language. In order to prove that  $L$  is not context-free we need to show that for every integer  $n$ , there is some string  $z$  in  $L$ , of length at least  $n$ , such that no matter how we break  $z$  up as  $z = uvwxy$ , subject to the constraints  $|vwx| \leq n$  and  $|vx| > 0$ , there is some  $i \geq 0$  such that  $uv^iwx^iy$  is not in  $L$ .

Let us focus on a particular  $z = aabaaaba$  and  $n = 7$ . It turns out that this is the wrong choice of  $z$  for  $n = 7$ , since there are some ways to break  $z$  up for which we can find the desired  $i$ , and for others, we cannot. Identify from the list below the choice of  $u, v, w, x, y$  for which there is an  $i$  that makes  $uv^iwx^iy$  not be in  $L$ . We show the breakup of  $aabaaaba$  by placing four |'s among the a's and b's. The resulting five pieces (some of which may be empty), are the five strings. For instance,  $aa|b|aaaba|$  means  $u=aa$ ,  $v=b$ ,  $w=\epsilon$ ,  $x=aaaba$ , and  $y=\epsilon$ .

- a)  $aab|a|aab|a|$
- b)  $a|ab|aa|ab|a$
- c)  $|aa|b|aa|aba$
- d)  $a|ab|aaa|b|a$

Answer submitted: **d)**

You have answered the question correctly.

#### Question Explanation:

In all cases for this problem,  $i = 0$  works if anything works. For example one correct choice is  $aab|a|a|a|ba$ , where if we remove the second and 4th pieces, we get  $aababa$ , which is not of the form  $ss$ . That is, the first half,  $aab$ , is not the same as the second half, and therefore  $aababa$  is not in  $L$ .

For each of the incorrect choices, the choice explanation gives an argument as to why no "pumping" of  $v$  and  $x$  will lead to a string whose first half differs from the second half.

3.  $G_1$  is a context-free grammar with start symbol  $S_1$ , and no other nonterminals whose name begins with "S." Similarly,  $G_2$  is a context-free grammar with start symbol  $S_2$ , and no other nonterminals whose name begins with "S."  $S_1$  and  $S_2$  appear on the right side of no productions. Also, no nonterminal appears in both  $G_1$  and  $G_2$ .

We wish to combine the symbols and productions of  $G_1$  and  $G_2$  to form a new grammar  $G$ , whose language is the concatenation of the languages of  $G_1$  and  $G_2$ .

The start symbol of  $G$  will be  $S$ . All productions and symbols of  $G_1$  and  $G_2$  will be symbols and productions of  $G$ . Which of the following sets of productions, added to those of  $G$ , is guaranteed to make  $L(G)$  be  $L(G_1)L(G_2)$ ?

- a)  $S \rightarrow S_3S_2, S_3 \rightarrow S_4S_1, S_4 \rightarrow \varepsilon$
- b)  $S \rightarrow S_1S_3, S_3 \rightarrow S_1S_2$
- c)  $S \rightarrow S_1S_2 \mid \varepsilon$
- d)  $S \rightarrow S_1S_3, S_3 \rightarrow S_2 \mid \varepsilon$

Answer submitted: **a)**

You have answered the question correctly.

#### Question Explanation:

Each of the choices involves only  $S, S_1, S_2$ , and in some cases other nonterminals whose names begin with "S" and that therefore are known not to appear in  $G_1$  or  $G_2$ . As a result, we need only to look at the strings involving  $S_1$  and  $S_2$  only, that are derivable from  $S$ , using the new productions. In order for  $L(G)$  to be  $L(G_1)L(G_2)$ , it is necessary and sufficient that the strings involving only symbols  $S_1$  and  $S_2$  that are derived from  $S$  using only the additional productions be exactly the one string  $\{S_1S_2\}$ .

For example, adding  $S \rightarrow S_1S_2$  obviously has this property. So does the set of productions

$$\begin{aligned} S &\rightarrow S_1S_3S_2 \\ S_3 &\rightarrow \varepsilon \end{aligned}$$

4.  $G_1$  is a context-free grammar with start symbol  $S_1$ , and no other nonterminals whose name begins with "S." Similarly,  $G_2$  is a context-free grammar with start symbol  $S_2$ , and no other nonterminals whose name begins with "S."  $S_1$  and  $S_2$  appear on the right side of no productions. Also, no nonterminal appears in both  $G_1$  and  $G_2$ .

We wish to combine the symbols and productions of  $G_1$  and  $G_2$  to form a new grammar  $G$ , whose language is the union of the languages of  $G_1$  and  $G_2$ . The start symbol of  $G$  will be  $S$ . All productions and symbols of  $G_1$  and  $G_2$  will be symbols and productions of  $G$ . Which of the following sets of productions, added to those of  $G$ , is guaranteed to make  $L(G)$  be  $L(G_1) \cup L(G_2)$ ?

- a)  $S \rightarrow S_1S_2$
- b)  $S \rightarrow S_1, S_1 \rightarrow S_2, S_2 \rightarrow \varepsilon$
- c)  $S \rightarrow S_1 \mid S_3, S_3 \rightarrow S_2$
- d)  $S \rightarrow S_1S_3, S_3 \rightarrow S_2$

Answer submitted: **c)**

You have answered the question correctly.

## Question Explanation:

Each of the choices involves only  $S$ ,  $S_1$ ,  $S_2$ , and in some cases other nonterminals whose names begin with "S" and that therefore are known not to appear in  $G_1$  or  $G_2$ . As a result, we need only to look at the strings involving  $S_1$  and  $S_2$  only, that are derivable from  $S$ , using the new productions. In order for  $L(G)$  to be  $L(G_1) \cup L(G_2)$ ,

it is necessary and sufficient that the strings involving only symbols  $S_1$  and  $S_2$  that are derived from  $S$  using only the additional productions be exactly the two strings  $\{S_1, S_2\}$ .

For example, adding  $S \rightarrow S_1 \mid S_2$  obviously has this property. So does the set of productions

$$S \rightarrow S_1$$

$$S_1 \rightarrow S_2$$

Note that if  $S_1$  could appear on the right side of productions of  $G_1$ , then this choice would not work --- derivations of  $G_2$  could suddenly appear in the middle of derivations that should be in  $G_1$  only.

$$S \rightarrow S_1 \mid S_3$$

$$S_3 \rightarrow S_2$$

5. If  $h$  is the homomorphism defined by  $h(a) = 0$  and  $h(b) = \varepsilon$ , which of the following strings is in  $h^{-1}(000)$ ?
- a) abbba
  - b) baabbbabb
  - c) babab
  - d) abbbabaab

Answer submitted: **b)**

You have answered the question correctly.

## Question Explanation:

Since there are three 0's in  $h(w)$ ,  $w$  must have exactly three  $a$ 's. It can have any number of  $b$ 's, since  $h(b) = \varepsilon$ . Of the choices, only baabbbabb has exactly three  $a$ 's.

6. Apply the CYK algorithm to the input ababaa and the grammar:

$$S \rightarrow AB \mid BC$$

$$A \rightarrow BA \mid a$$

$$B \rightarrow CC \mid b$$

$$C \rightarrow AB \mid a$$

Compute the table of entries  $X_{i,j}$  = the set of nonterminals that derive positions  $i$

through  $j$ , inclusive, of the string ababaa. Then, identify a true assertion about one of the  $X_{ij}$ 's in the list below.

- a)  $X_{23} = \{A\}$
- b)  $X_{26} = \{S,A\}$
- c)  $X_{26} = \{B\}$
- d)  $X_{36} = \{S,A,C\}$

Answer submitted: **b)**

You have answered the question correctly.

Question Explanation:

Here is the table:

B						
SAC	SA					
B	B	SA				
B	SC	B	-			
SC	SA	SC	SA	B		
AC	B	AC	B	AC	AC	
a	b	a	b	a	a	