CS154

Name: _____ Date: _____

Note: The purpose of the following questions is:

• *Enhance learning*	• *Summarized points*	• *Analyze abstract ideas*

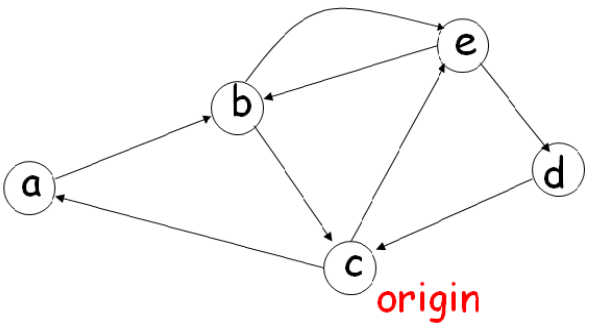
Class 1: Mathematical Preliminaries

Sets:

1. [Slide 2,3] What is set? Give example of *finite set* and *infinite set.*
2. [Slide 5] Define and give example of *Universal Set*.
3. [Slide 6] What is the Union, Intersection and Difference of these two sets: A = { 1, 2, 3 } , B = { 2, 3, 4, 5},
4. [Slide 7] Universal set = {1, …, 7} and A = { 1, 2, 3 }, what is the complement of A
5. [Slide 8] What is the complement of { odd integers }
6. [Slide 9] What is DeMorgan's Laws
7. [Slide 10] What is Empty, Null Set?
8. [Slide 11] What is subset, give an example.
9. [Slide 12] What is Disjoint sets, give an example
10. [Slide 13] Define Set Cardinality, what is set size?
11. [Slide 14] What is powerset of S = { a, b, c }?
12. [Slide 15] If A = { 2, 4 }, B = { 2, 3, 5 }, what is the Cartesian product of A and B
13. [Slide 16] Define the domain and range of a function, give an example.
14. [Slide 17, 18] Give an example of the following equivalence relations: Reflexive, Symmetric and Transitive.
15. [Slide 19] What is equivalence classes, give an example.

Graphs:

16. [Slide 20] What is a directed graph, give an example and show the Nodes (Vertices) and the Edges.
17. [Slide 21] Give example of Labeled Graph
18. [Slide 22] Define Walk in graph, give an example.
19. [Slide 23] Define Path in graph, give an example.
20. [Slide 24] Define Cycle in graph, give an example.
21. [Slide 25] What is Euler Tour
22. [Slide 26] What is Hamiltanian Cycle, give an example
23. [Slide 27-31] How to find all simple paths in the following graph
24. [Slide 32-34] Define the following: Trees, root, leaf, height, Bin



Proof Techniques

25. [Slide 36,37] Explain the steps of prove by induction technique.
26. [Slide 38-41] A binary tree is a tree in which no parent can have more than two children. Prove by induction that a binary tree of height n at most 2^n leaves
27. [Slide 43] Explain the steps of proof by contradiction technique.
28. [Slide 44-45] A rational number is a number that can be expressed as the ratio of two integers n and m so that n and m have no common factor. A real number that is not rational is said to be irrational. Show by using proof by contradiction that $\sqrt{2}$ is irrational.

Languages

29. [Slide 47-48] What is a *Language*, and what is *String*, give an example of each.
30. [Slide 49-50] Give an example for concatenating two strings and the reverse of a string.
31. [Slide 51-52] Define the Length of a string, give an example.
32. [Slide 53] What is an Empty String?
33. [Slide 54] What is Substring?
34. [Slide 55] What is the Prefix and Suffix of a string.
35. [Slide 57] What is the * operation?
36. [Slide 58] What is the + operation?
37. [Slide 61] Give an example of string in the language $L = \{a^n b^n : n \geq 0\}$
38. [Slide 62] Operations on Languages:

$$\{a, ab, aaaa\} \cup \{bb, ab\} =$$
$$\{a, ab, aaaa\} \cap \{bb, ab\} =$$
$$\{a, ab, aaaa\} - \{bb, ab\} =$$

39. [Slide 63] Consider the Language $L^R = \{w^R : w \in L\}$

    Then:

$$\{ab, aab, baba\}^R =$$

40. [Slide 64] Concatenation definition: $L_1 L_2 = \{xy : x \in L_1, y \in L_2\}$

    Example:

$$\{a, ab, ba\}\{b, aa\} =$$

41. [Slide 65] Definition: $L^n = \underbrace{LL\cdots L}_{n}$

    Example:

$$\{a, b\}^3 = \{a, b\}\{a, b\}\{a, b\} =$$

    Special case:

$$L^0 = \{\lambda\}$$

$$\{a, bba, aaa\}^0 =$$

42. [Slide 66] if the Language: $L = \{a^n b^n : n \geq 0\}$

    Then:

$$L^2 =$$

43. [Slide 67] Star-Closure (Kleene *)

    Definition: $L^* = L^0 \cup L^1 \cup L^2 \cdots$

    Example:

$$\{a, bb\}^* = \left\{ \begin{array}{l} \lambda, \\ a, bb, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array} \right\}$$

44. [Slide 68] Positive Closure

    Definition:

$$L^+ = L^1 \cup L^2 \cup \cdots$$
$$= L^* - \{\lambda\}$$

    Example:

$$\{a, bb\}^+ = \left\{ \begin{array}{l} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array} \right\}$$

Reading:

An Introduction to Formal Language and Automata, Peter Linz, 5th edition, Sec 1.1, 1.2

Selected Exercises:

Section 1.1: 5, 6, 8, 13, 17, 30, 33

Section 1.2: 2, 4, 5, 11(d)