



Zayd

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Submission number: 69045
Submission certificate: FC060796
Submission time: 2014-03-15 20:51:59 PST (GMT - 8:00)

Number of questions: 14
Positive points per question: 3.0
Negative points per question: 1.0
Your score: 38

These questions, based on Section 5.1 of HMU, are not in either of the other two homeworks on CFG's, but are available for use.

Help

1. Here are eight simple grammars, each of which generates an infinite language of strings. These strings tend to look like alternating a 's and b 's, although there are some exceptions, and not all grammars generate all such strings.

1. $S \rightarrow abS \mid ab$
2. $S \rightarrow SS \mid ab$
3. $S \rightarrow aB; B \rightarrow bS \mid a$
4. $S \rightarrow aB; B \rightarrow bS \mid b$
5. $S \rightarrow aB; B \rightarrow bS \mid ab$
6. $S \rightarrow aB \mid b; B \rightarrow bS$
7. $S \rightarrow aB \mid a; B \rightarrow bS$
8. $S \rightarrow aB \mid ab; B \rightarrow bS$

The initial symbol is S in all cases. Determine the language of each of these grammars. Then, find, in the list below, the pair of grammars that define the same language.

- a) $G1: S \rightarrow aB, B \rightarrow bS, B \rightarrow ab$
 $G2: S \rightarrow SS, S \rightarrow ab$
- b) $G1: S \rightarrow abS, S \rightarrow ab$
 $G2: S \rightarrow SS, S \rightarrow ab$
- c) $G1: S \rightarrow aB, B \rightarrow bS, B \rightarrow a$
 $G2: S \rightarrow aB, B \rightarrow bS, B \rightarrow b$
- d) $G1: S \rightarrow aB, B \rightarrow bS, B \rightarrow b$
 $G2: S \rightarrow aB, B \rightarrow bS, S \rightarrow b$

Answer submitted: **b)**

You have answered the question correctly.

2. Identify in the list below a sentence of length 6 that is generated by the grammar $S \rightarrow (S)S \mid \varepsilon$

- a) $) ()) ($
- b) $(() ()$
- c) $) () (($
- d) $)) ((($

Answer submitted: **b)**

You have answered the question correctly.

3. Let L be the language of all strings of a's and b's such that no prefix (proper or not) has more b's than a's. Let G be the grammar with productions

$$S \rightarrow aS \mid aSbS \mid \varepsilon$$

To prove that $L = L(G)$, we need to show two things:

1. If $S \Rightarrow^* w$, then w is in L .
2. If w is in L , then $S \Rightarrow^* w$.

We shall consider only the proof of (1) here. The proof is an induction on n , the number of steps in the derivation $S \Rightarrow^* w$. Here is an outline of the proof, with reasons omitted. You need to supply the reasons.

Basis:

- 1) If $n=1$, then w is ε because _____.
- 2) w is in L because _____.
- Induction:
- 3) Either (a) $S \Rightarrow aS \Rightarrow^{n-1} w$ or (b) $S \Rightarrow aSbS \Rightarrow^{n-1} w$ because _____.
- 4a) In case (a), $w = ax$, and $S \Rightarrow^{n-1} x$ because _____.
- 5a) In case (a), x is in L because _____.
- 6a) In case (a), w is in L because _____.
- 4b) In case (b), w can be written $w = aybz$, where $S \Rightarrow^p y$ and $S \Rightarrow^q z$ for some p and q less than n because _____.
- 5b) In case (b), y is in L because _____.
- 6b) In case (b), z is in L because _____.
- 7b) In case (b), w is in L because _____.

For which of the steps above the appropriate reason is contained in the following argument:

"The following two statements are true

- (i) if string x has no prefix with more b's than a's, then neither does string ax ,
- (ii) if strings y and z are such that no prefix has more b's than a's, then neither does string $aybz$."

- a) 1
- b) 6a
- c) 5a
- d) 3

Answer submitted: **b)**

You have answered the question correctly.

4. Let L be the language of all strings of a's and b's such that no prefix (proper or not) has more b's than a's. Let G be the grammar with productions

$$S \rightarrow aS \mid aSbS \mid \varepsilon$$

To prove that $L = L(G)$, we need to show two things:

1. If $S \Rightarrow^* w$, then w is in L .
2. If w is in L , then $S \Rightarrow^* w$.

We shall consider only the proof of (1) here. The proof is an induction on n , the number of steps in the derivation $S \Rightarrow^* w$. Here is an outline of the proof, with reasons omitted. You need to supply the reasons.

Basis:

1)

If $n=1$, then w is ϵ because _____.

2)

w is in L because _____.

Induction:

3)

Either (a) $S \Rightarrow aS \Rightarrow^{n-1} w$ or (b) $S \Rightarrow aSbS \Rightarrow^{n-1} w$ because _____.

4a)

In case (a), $w = ax$, and $S \Rightarrow^{n-1} x$ because _____.

5a)

In case (a), x is in L because _____.

6a)

In case (a), w is in L because _____.

4b)

In case (b), w can be written $w = aybz$, where $S \Rightarrow^p y$ and $S \Rightarrow^q z$ for some p and q less than n because _____.

5b)

In case (b), y is in L because _____.

6b)

In case (b), z is in L because _____.

7b)

In case (b), w is in L because _____.

For which of the steps above the appropriate reason is contained in the following argument:

"All n -step derivations of w produce either ϵ (for $n=1$) or use one of the productions with at least one nonterminal in the body (for $n > 1$). In case the production $S \rightarrow aS$ is used, then $w=ax$ with x being produced by a $(n-1)$ -step derivation. In case the production $S \rightarrow aSbS$ is used then $w=aybz$ with y and z being produced by derivations with number of steps less than n ."

a) 5a

b) 4b

c) 5b

d) 6b

Answer submitted: **b)**

You have answered the question correctly.

5. Consider the grammar G with start symbol S :

$S \rightarrow bS \mid aA \mid b$

$A \rightarrow bA \mid aB$

$B \rightarrow bB \mid aS \mid a$

Which of the following is a word in $L(G)$?

a) abbbaaababaa

b) babbbabaaaa

c) ababbbbbb

d) ababbbbbb

Answer submitted: **b)**

You have answered the question correctly.

6. Consider the grammars:

$G_1: S \rightarrow AB, A \rightarrow aAA \mid \epsilon, B \rightarrow abBB \mid \epsilon$

$G_2: S \rightarrow CB, C \rightarrow aCC \mid aC \mid a, B \rightarrow abBB \mid abB \mid ab$

$G_3: S \rightarrow CB \mid C \mid B \mid \epsilon, C \rightarrow aCC \mid aC \mid a, B \rightarrow abBB \mid abB \mid ab$

$G_4: S \rightarrow aSR \mid \epsilon, A \rightarrow aA \mid \epsilon, R \rightarrow abR \mid \epsilon$

$G_4: S \rightarrow aSb|a, A \rightarrow aA|a, B \rightarrow aB|a$
 $G_5: S \rightarrow ASB|AB, A \rightarrow aA|a, B \rightarrow abB|ab$
 $G_6: S \rightarrow ASB|aab, A \rightarrow aA|a, B \rightarrow abB|ab$

Describe the language of each of these grammars. Then, identify from the list below a pair of grammars that define the same language?

- a) G_3 and G_4
- b) G_1 and G_5
- c) G_1 and G_2
- d) G_3 and G_6

Answer submitted: **a)**

You have answered the question correctly.

7. Which of the following grammars derives a subset L_s of the language: $L = \{x \mid \text{(i) } x \text{ contains } a \text{ and } c \text{ in proportion } 4:3, \text{ (ii) } x \text{ does not begin with } c \text{ and (iii) there are no two consecutive } c\text{'s}\}$ such that L_s is missing at most a finite number of strings from L .

- a) $S \rightarrow acacaca, S \rightarrow SaSaSaScSaScSaS$
- b) $S \rightarrow \epsilon, S \rightarrow SaScSaScSaScSaS$
- c) $S \rightarrow acacaca, S \rightarrow SaScSaScSaScSaS, S \rightarrow SaSaSaScSaScSa$
- d) $S \rightarrow \epsilon, S \rightarrow acacaScSaS$

Answer submitted: **b)**

You have answered the question correctly.

8. Which of the following pairs of grammars define the same language?

- a) $G_1: S \rightarrow AB, A \rightarrow aAA|\epsilon, B \rightarrow bBB|\epsilon$
 $G_2: S \rightarrow AB|A|B|\epsilon, A \rightarrow aAA|aA|a, B \rightarrow bBB|bB|b$
- b) $G_1: S \rightarrow AB|a, A \rightarrow b, B \rightarrow b$
 $G_2: S \rightarrow a$
- c) $G_1: S \rightarrow AB, A \rightarrow aAA|\epsilon, B \rightarrow baB|\epsilon$
 $G_2: S \rightarrow CB|C|B, C \rightarrow aCC|aC|a, B \rightarrow baBB|baB|ba$
- d) $G_1: S \rightarrow SaScSa|aca|\epsilon$
 $G_2: S \rightarrow SaSAaS|aca, A \rightarrow cS|\epsilon$

Answer submitted: **a)**

You have answered the question correctly.

9. Programming languages are often described using an extended form of context-free grammar, where curly brackets are used to denote a construct that can repeat 0, 1, 2, or any number of times. For example, $A \rightarrow B\{C\}D$ says that an A can be replaced by a B and a D , with any number of C 's (including 0) between them. This notation does not allow us to describe anything but context-free languages, since an extended production can always be replaced by several conventional productions.

Suppose a grammar has the extended production:

$A \rightarrow 0\{B\}1$

Convert this extended production to conventional productions. Identify, from the list below, the conventional productions that are equivalent to the extended production above.

- a) $A \rightarrow 0A_11$
 $A_1 \rightarrow B \mid \epsilon$

- b) $A \rightarrow 0A_11$
 $A_1 \rightarrow A_1B \mid \varepsilon$
 $A \rightarrow 0A_11$
- c) $A_1 \rightarrow A_1B \mid B$
- d) $A \rightarrow 0A_11$
 $A_1 \rightarrow BA_1 \mid B$

Answer submitted: a)

Your answer is incorrect.

With these productions, we can derive zero or one occurrence of B, but there is no way to derive more than one occurrence, e.g., 0BB1 or 0BBB1, from A. A possible aid is to examine the rules for derivations in Section 5.1.3 (p. 175).

10. Consider the grammar $G1: S \rightarrow \varepsilon, S \rightarrow aS, S \rightarrow aSbS$ and the language L that contains exactly those strings of a's and b's such that every prefix has at least as many a's as b's. We want to prove the claim: $G1$ generates all strings in L.

We take the following inductive hypothesis to prove the claim:

For $n < k$, $G1$ generates every string of length n in L.

To prove the inductive step we argue as follows:

"For each string w in L either _____ (a1) or _____ (a2) holds. In both cases we use the inductive hypothesis and one of the rules to show that string w can be generated by the grammar. In the first case we use rule $S \rightarrow aS$ and in the second case we use rule $S \rightarrow aSbS$."

Which phrases can replace the _____ so that this argument is correct?

- a) a1: w can be written as $w=aw'bw''$ where for both w' and w'' it holds that each prefix has as many a's as b's.
a2: each prefix has more a's than b's.
- b) a1: each prefix has equal number of a's and b's.
a2: there is a b in string w such that the part of the string until the b belongs in L by inductive hypothesis and the part after this b belongs in L by inductive hypothesis.
- c) a1: each prefix has more a's than b's.
a2: w can be written as $w=aw'bw''$ where for both w' and w'' it holds that each prefix has as many a's as b's.
- d) a1: each prefix has equal number of b's and a's.
a2: w can be written as $w=aw'bw''$ where for both w' and w'' it holds that each prefix has as many a's as b's.

Answer submitted: c)

You have answered the question correctly.

11. Consider the grammar G and the language L:

$G: S \rightarrow AB \mid a \mid abC, A \rightarrow b, C \rightarrow abC \mid c$

$L: \{w \mid w \text{ a string of a's, b's, and c's with an equal number of a's and b's}\}.$

Grammar G does not define language L. To prove, we use a string that either is produced by G and not contained in L or is contained in L but is not produced by G. Which string can be used to prove it?

- a) a
b) cabaca
c) ababa
d) abac

Answer submitted: **a)**

You have answered the question correctly.

12. Consider the following languages and grammars. $G_1: S \rightarrow aA|aS, A \rightarrow ab$

$G_2: S \rightarrow abS|aA, A \rightarrow a$

$G_3: S \rightarrow Sa|AB, A \rightarrow aA|a, B \rightarrow b$

$G_4: S \rightarrow aS|b$

$L_1: \{a^i b \mid i=1,2,\dots\}$

$L_2: \{(ab)^i aa \mid i=0,1,\dots\}$

$L_3: \{a^i b \mid i=2,3,\dots\}$

$L_4: \{a^i b a^j \mid i=1,2,\dots, j=0,1,\dots\}$

$L_5: \{a^i b \mid i=0,1,\dots\}$

Match each grammar with the language it defines. Then, identify a correct match from the list below.

a) G_3 defines L_4 .

b) G_4 defines L_4 .

c) G_3 defines L_3 .

d) G_2 defines L_3 .

Answer submitted: **a)**

You have answered the question correctly.

13. Which of the following grammars derives a subset of the language:

$\{x \mid x \text{ contains } a \text{ and } c \text{ in proportion } 4:3 \text{ and there are no two consecutive } c\text{'s}\}$?

a) $S \rightarrow acacaca \mid S \rightarrow SaScSaScSaScSaS \mid S \rightarrow SaSaSaScSaScSaS$

b) $S \rightarrow \varepsilon \mid S \rightarrow SaScSaScSaScSaS$

c) $S \rightarrow \varepsilon \mid S \rightarrow SaScSaScSaScSa$

d) $S \rightarrow \varepsilon \mid S \rightarrow aScScaSaScSaS$

Answer submitted: **c)**

You have answered the question correctly.

14. Consider the grammar G_1 :

$S \rightarrow \varepsilon \mid aS \mid aSbS$

Which of the following is correct (for a choice to be correct, all propositions must be correct)?

a) For any word w with every prefix having at least as many a 's as b 's, there is a unique b in w such that w can be written as $aw'bw''$ --- hence w can be generated from shorter words using the production $S \rightarrow aSbS$.

b) The string $aaabbbabababba$ is not generated by the grammar.

c) a) G_1 generates all and only the strings of a 's and b 's such that every string has at least as many a 's as b 's. b) The inductive hypothesis to prove it is: For $n < k$, it holds: Any word in G_1 of length n , is such that all its prefixes contain more a 's than b 's or as many a 's as b 's.

d) The string $aabbab$ is generated by a unique parse tree.

Answer submitted: **a)**

Answer submitted: 

You have answered the question correctly.

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