



## Gradiance Online Accelerated Learning

Zayd

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Help

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Questions about languages classes NP and above, based on Sections 10.1, 11.1, 11.2, and 11.3 of HMU.

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1. Suppose there are three languages (i.e., problems), of which we know the following:

1.  $L_1$  is in P.
2.  $L_2$  is NP-complete.
3.  $L_3$  is not in NP.

Suppose also that we do not know anything about the resolution of the "P vs. NP" question; for example, we do not know definitely whether  $P=NP$ . Classify each of the following languages as (a) Definitely in P, (b) Definitely in NP (but perhaps not in P and perhaps not NP-complete) (c) Definitely NP-complete (d) Definitely not in NP:

- $L_1 \cup L_2$ .
- $L_1 \cap L_2$ .
- $L_2 c L_3$ , where  $c$  is a symbol not in the alphabet of  $L_2$  or  $L_3$  (i.e., the *marked concatenation* of  $L_2$  and  $L_3$ , where there is a unique marker symbol between the strings from  $L_2$  and  $L_3$ ).
- The complement of  $L_3$ .

Based on your analysis, pick the correct, definitely true statement from the list below.

- a) The complement of  $L_3$  is definitely not NP-complete.
- b)  $L_1 \cap L_2$  is definitely not in NP.
- c)  $L_2 c L_3$  is definitely in NP.
- d)  $L_1 \cap L_2$  is definitely in NP.

Answer submitted: **d)**

You have answered the question correctly.

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Question Explanation:

Let  $L = L_1 \cup L_2$ . It is possible that  $L_1$  is empty; that is certainly one language in P. If so, then  $L = L_2$ , and  $L$  is NP-complete. On the other hand, suppose  $L_1$  is all strings over the alphabet of  $L_2$  --- another language in P. Then  $L = L_1$ , a language in P. Since we don't know whether  $P = NP$ , we cannot conclude  $L$  is definitely in P and we cannot conclude that  $L$  is definitely NP-complete.

On the other hand, we can conclude definitely that  $L$  is in NP. A nondeterministic, polynomial-time algorithm for  $L$  starts by applying a polynomial-time algorithm for  $L_1$  to its input, and if the result is negative applies the NP-recognizer for  $L_2$  to the same input. It accepts the input if either recognizer accepts.

The argument for  $L = L_1 \cap L_2$  is essentially the same, although now  $L$  is in P if  $L_1$  is empty and  $L$  is NP-complete if  $L_1$  is all strings over the alphabet of  $L_2$ . Also, the nondeterministic polynomial-time algorithm for  $L$  works by trying the polynomial-time recognizer for  $L_1$  first, and only accepting if that recognizer accepts and the NP-recognizer for  $L_2$  also accepts.

Now, consider  $L = L_2 c L_3$ . Suppose  $L$  had an NP-recognizer. Let  $x$  be some string in  $L_2$  (since  $L_2$  is NP-complete, and we do not know that  $P = NP$ , we can be sure  $L_2$  is not empty, so  $x$  exists). Then there is a nondeterministic polynomial-time algorithm for  $L_3$  that works as follows. Take input  $w$ , and test it for membership in  $L_3$  by feeding  $xw$  to the NP-recognizer for  $L$ . Respond exactly as this

recognizer responds. Since we know  $x$  is in  $L_2$ , the recognizer for  $L$  accepts  $xw$  if and only if  $w$  is in  $L_3$ . We now have an NP-recognizer for  $L_3$ , but we were told that  $L_3$  is *not* in NP. Thus, our assumption that  $L$  is in NP must be false.

Last, let  $L$  be the complement of  $L_3$ . If  $L$  is in P, then the complement of  $L$ , which is  $L_3$ , is also in P. But we know  $L_3$  is not even in NP. We do not even know that  $L$  is in NP; for example,  $L_3$  could be an undecidable problem. On the other hand,  $L$  could be NP-complete. For example, the complement of the problem SAT is not known to be in NP. But it is possible that  $L_3$  is the complement of SAT and therefore  $L = \text{SAT}$ , a known NP-complete problem.

2. The classes of languages P and NP are closed under certain operations, and not closed under others, just like classes such as the regular languages or context-free languages have closure properties. Decide whether P and NP are closed under each of the following operations.

1. Union.
2. Intersection.
3. Intersection with a regular language.
4. Concatenation.
5. Kleene closure (star).
6. Homomorphism.
7. Inverse homomorphism.

Then, select from the list below the true statement.

- a) P is not closed under intersection with a regular language.
- b) NP is not closed under intersection.
- c) NP is not closed under homomorphism.
- d) P is not closed under union.

Answer submitted: **c)**

You have answered the question correctly.

Question Explanation:

Both P and NP are closed under each of these operations, except for homomorphism. To see why neither class is closed under homomorphism, start with a very hard language  $L$ , say one that requires time  $2^{2^n}$ . Make it easy by appending to each word of length  $n$  exactly  $2^{2^n}$   $c$ 's, where  $c$  is a new symbol. That is, let  $L' = Lc^{2^{2^n}}$ . Then  $L'$  is surely in P and NP. But  $L$  is  $h(L')$ , if  $h$  is the homomorphism that sends  $c$  to  $\epsilon$  and is the identity on all symbols of  $L$ . If P or NP were closed under homomorphism, then  $L$  would be in NP, which it is not.

Here are the constructions that show P and NP closed under the other six operations:

1.  $L_1$  [union]  $L_2$ : Apply the tests for membership in  $L_1$  and then for membership in  $L_2$ . Accept if either accepts. If  $L_1$  and  $L_2$  are in P, then both tests are polynomial, so the entire process is polynomial. If  $L_1$  and  $L_2$  are in NP, then there is a nondeterministic polynomial algorithm for the entire process.
2.  $L_1 \cap L_2$ : The argument is the same, but accept only if both tests accept.
3. Intersection with a regular set: The argument is the same as (2), since a regular language is surely in P and NP.
4.  $L_1L_2$ : If  $L_1$  and  $L_2$  are in NP, just guess the point on the input where the string in  $L_2$  begins, and apply the nondeterministic polynomial tests to the two parts of the input. If  $L_1$  and  $L_2$  are in P, we have to try systematically all possible breakpoints between the prefix of the input that is in  $L_1$  and the suffix that is in  $L_2$ . If the tests for  $L_1$  and  $L_2$  are polynomial, say  $p(n)$  and  $q(n)$  running time, then we must apply each at most  $n$  times on input of length  $n$ . Since  $n(p(n)+q(n))$  is a polynomial, we can recognize  $L_1L_2$  in polynomial time.
5.  $L^*$ : If  $L$  is in NP, guess all the places on the input where strings in  $L$  end. Run the nondeterministic polynomial-time test on each segment. Since there are at most  $n$  segments, the whole process is nondeterministic polynomial. If  $L$  is in P, we again must be systematic. For each pair of input positions  $i$  and  $j$ , run the polynomial-time test to see whether positions  $i$  through  $j$  of the input is in  $L$ . If  $L$  has a  $p(n)$ -time algorithm, then this process takes  $O(n^2p(n))$  time, a polynomial. Then, use a dynamic programming algorithm, taking  $O(n^3)$  time, to determine whether positions  $i$  through  $j$  can be composed of  $1, 2, \dots, n$  strings in  $L$ . At the end, we only care about whether positions 1 through  $n$  can be composed of some number of strings

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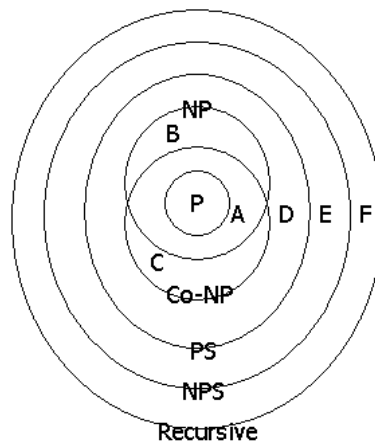
in  $L$ .

6.  $h^{-1}(L)$ : Any homomorphism  $h$  can only expand the length of the string to which it is applied by a constant factor. To recognize  $h^{-1}(L)$ , apply  $h$  to the input  $w$ , and see whether  $h(w)$  is in  $L$ . If there is a polynomial-time, or nondeterministic polynomial-time test for membership in  $L$ , this test will, in the same order of magnitude time complexity tell us whether  $w$  is in  $h^{-1}(L)$ .

3. Consider the following problems:

1. SP (Shortest Paths): given a weighted, undirected graph with nonnegative integer edge weights, given two nodes in that graph, and given an integer limit  $k$ , determine whether the length of the shortest path between the nodes is  $k$  or less.
2. WHP (Weighted Hamilton Paths): given a weighted, undirected graph with nonnegative integer edge weights, and given an integer limit  $k$ , determine whether the length of the shortest Hamilton path in the graph is  $k$  or less.
3. TAUT (Tautologies): given a propositional boolean formula, determine whether it is true for all possible truth assignments to its variables.
4. QBF (Quantified Boolean Formulas): given a boolean formula with quantifiers for-all and there-exists, such that there are no free variables, determine whether the formula is true.

In the diagram below are seven regions, P and A through F.



Place each of the four problems in its correct region, on the assumption that NP is equal to neither P nor co-NP nor PS.

- a) Problem QBF is in region F.
- b) Problem WHP is in region B.
- c) Problem SP is in region C.
- d) Problem WHP is in region P.

Answer submitted: **b)**

You have answered the question correctly.

Question Explanation:

SP is in P. You can use Dijkstra's algorithm to solve the problem in quadratic time.

WHP is NP-complete. It is a generalization of Hamilton-path, but the weights don't affect the nondeterministic-polynomial-time "guessing" algorithm to solve it. On the assumption that NP is neither P nor co-NP, WHP must be in region B.

TAUT is essentially the complement of SAT. Since SAT is NP-complete, TAUT must be in region C,

unless NP is equal to one of P or co-NP (which we assume not to be the case).

QBF is complete for PS. Thus, assuming PS is not NP, QBF is in D.

4. Let us denote a problem X as NP-Easy if it is polynomial-time reducible to some problem Y that is in NP. Let us denote as NP-Equivalent, the class of problems that are both NP-Easy and NP-Hard. Let A,B,C,D and E be problems such that A is NP-Hard, B is NP-Complete, C is NP-Equivalent, D is NP-Easy and E is in NP. Which of the following statements is TRUE?
- If C is in P then  $P=NP$ .
  - If B is in P then so is A.
  - If A is in P then E may or may not be in P
  - If D is in P then  $P=NP$

Answer submitted: **b)**

Your answer is incorrect.

For every language L in NP, there is a polynomial-time reduction from L to A. Hence there is a polynomial-time reduction from B to A. However A could still take exponential time. In that case the algorithm for A combined with the algorithm that reduces B to A together will not give a polynomial-time algorithm for B, but existence of other polynomial-time algorithms for B is not inconsistent with this fact. Relevant reading includes most of Section 10.1 (p. 426).

#### Question Explanation:

The question is based on definitions of NP, NP-hardness, NP-completeness, and Theorems 10.4 and 10.5 in Section 10.1.6, p. 434--435. Some key facts we are using include:

- If a problem A reduces to a problem B, then A is at least as hard as B.
- Since A is NP-Hard, for every language L in NP, there is a polynomial-time reduction from L to A.
- Since C is NP-Easy, it implies that C is reducible in polynomial time to another problem F in NP. F is reducible in polynomial time to A. So C is polynomial-time reducible to A. A similar reasoning holds for D.
- Since A is NP-Hard, every problem in NP reduces to A in polynomial time. If A is in P, all such problems R can be solved in polynomial time using the algorithm for reducing R to A combined with the algorithm for A. This would imply that  $P = NP$ .
- We denote a problem X as NP-Easy if it is polynomial-time reducible to some problem Y that is in NP. Then the fact that Y is in NP, combined with the reduction, implies that X is also in NP. Moreover, if X is NP-equivalent, then X is also NP-Hard. Hence the notions of NP-Equivalence and NP-Completeness are the same.

The correct choice is: **a)**

5. There is a Turing transducer  $T$  that transforms problem P1 into problem P2.  $T$  has one read-only input tape, on which an input of length  $n$  is placed.  $T$  has a read-write scratch tape on which it uses  $O(S(n))$  cells.  $T$  has a write-only output tape, with a head that moves only right, on which it writes an output of length  $O(U(n))$ . With input of length  $n$ ,  $T$  runs for  $O(T(n))$  time before halting. You may assume that each of the upper bounds on space and time used are as tight as possible.

A given combination of  $S(n)$ ,  $U(n)$ , and  $T(n)$  may:

- Imply that  $T$  is a polynomial-time reduction of P1 to P2.
- Imply that  $T$  is NOT a polynomial-time reduction of P1 to P2.
- Be impossible; i.e., there is no Turing machine that has that combination of tight bounds on the space used, output size, and running time.

What are all the constraints on  $S(n)$ ,  $U(n)$ , and  $T(n)$  if  $T$  is a polynomial-time reducer? What are the constraints on feasibility, even if the reduction is not polynomial-time? After working out these constraints, identify the true statement from the list below.

- $S(n) = n^2$ ;  $U(n) = n^3$ ;  $T(n) = n^4$  is not physically possible.
- $S(n) = n$ ;  $U(n) = n^2$ ;  $T(n) = n \log_2 n$  is a polynomial-time reduction
- $S(n) = n^2$ ;  $U(n) = n^2$ ;  $T(n) = n!$  is not physically possible.
- $S(n) = n^2$ ;  $U(n) = n^3$ ;  $T(n) = n^4$  is a polynomial-time reduction

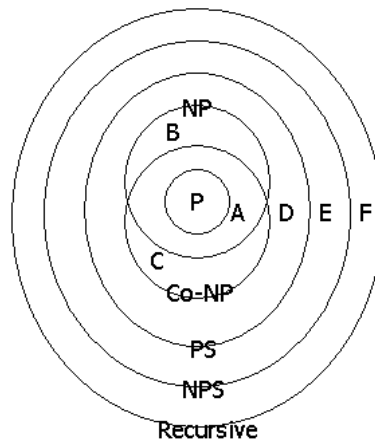
Answer submitted: **d)**

You have answered the question correctly.

Question Explanation:

There is one constraint on  $T$  being a polynomial-time reduction:  $T(n)$  must be a polynomial. However, there are also some constraints on feasibility. First, in time  $T(n)$ ,  $T$  cannot write more than  $T(n)$  cells on either its scratch or output tape. Thus,  $S(n)$  and  $U(n)$  must both be  $O(T(n))$ . Second, if  $T(n)$  is larger than  $O(n \log S(n) c^{S(n)})$  for any constant  $c$ , then  $T$  must repeat an "ID" consisting of the state, scratch-tape contents, and head positions on the input and scratch tapes.

6. In the diagram below we see certain complexity classes (represented as circles or ovals) and certain regions labeled A through F that represent the differences of some of these complexity classes.



The state of our knowledge regarding the existence of problems in the regions A-F is imperfect. In some cases, we know that a region is nonempty, and in other cases we know that it is empty. Moreover, if  $P=NP$ , then we would know more about the emptiness or nonemptiness of some of these regions, but still would not know everything.

Decide what we know about the regions A-F currently, and also what we would know if  $P=NP$ . Then, identify the true statement from the list below.

- It is not known whether region C is empty.
- Region A is definitely empty.
- Region B is definitely not empty.
- Region B is definitely empty.

Answer submitted: **b)**

Your answer is incorrect.

If A is empty, then  $P = NP \cap \text{co-NP}$ . What do we know about this question? See Section 11.1 (p. 484).

Question Explanation:

Since we do not know whether  $P=NP$ , or whether  $NP=\text{co-NP}$  (i.e., whether NP is closed under complementation), we do not know whether any of A, B, or C is empty. If we know  $P=NP$ , then surely A and B are empty. But if  $P=NP$ , then  $\text{co-NP}=NP=P$ , since P is closed under complementation. Thus, C would be empty as well.

Savitch's theorem tells us that  $PS=NPS$ ; i.e., region E is empty. We also know that there are arbitrarily complex recursive languages, so region F is definitely *not* empty. Finally, we do not know about region D, since it is open whether  $PS=NP$  or even  $PS=P$ . And even if we knew  $P=NP$ , we would still not be sure whether or not  $PS=NP=P$ .

The correct choice is: **a)**

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