



## Gradiance Online Accelerated Learning

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- [Home Page](#)
- [Assignments Due](#)
- [Progress Report](#)
- [Handouts](#)
- [Tutorials](#)
- [Homeworks](#)
- [Lab Projects](#)
- [Log Out](#)

Help

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Questions about languages classes NP and above, based on Sections 10.1, 11.1, 11.2, and 11.3 of HMU.

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1. Let us denote a problem  $X$  as NP-Easy if it is polynomial-time reducible to some problem  $Y$  that is in NP. Let us denote as NP-Equivalent, the class of problems that are both NP-Easy and NP-Hard. Let  $A, B, C, D$  and  $E$  be problems such that  $A$  is NP-Hard,  $B$  is NP-Complete,  $C$  is NP-Equivalent,  $D$  is NP-Easy and  $E$  is in NP. Which of the following statements is TRUE?
- If  $C$  is in  $P$  then so is  $A$ .
  - If  $P=NP$  then  $C$  is in  $P$ .
  - If  $P=NP$  then  $A$  is in  $P$ .
  - If  $D$  is in  $P$  then so is  $A$ .

Answer submitted: **c)**

Your answer is incorrect.

Choice Explanation: Since  $A$  is NP-Hard, for every  $L$  in NP, there is a polynomial-time reduction to  $A$ . But  $A$  could still take exponential time. In that case the algorithm for  $A$  combined with the algorithm that reduces  $L$  to  $A$  together will not give a polynomial-time algorithm for  $L$ , but existence of other polynomial-time algorithms for  $L$  is not inconsistent with this fact. Relevant reading includes most of Section 10.1 (p. 426).

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Question Explanation:

The question is based on definitions of NP, NP-hardness, NP-completeness, and Theorems 10.4 and 10.5 in Section 10.1.6, p. 434--435. Some key facts we are using include:

1. If a problem  $A$  reduces to a problem  $B$ , then  $A$  is at least as hard as  $B$ .
2. Since  $A$  is NP-Hard, for every language  $L$  in NP, there is a polynomial-time reduction from  $L$  to  $A$ .
3. Since  $C$  is NP-Easy, it implies that  $C$  is reducible in polynomial time to another problem  $F$  in NP.  $F$  is reducible in polynomial time to  $A$ . So  $C$  is polynomial-time reducible to  $A$ . A similar reasoning holds for  $D$ .
4. Since  $A$  is NP-Hard, every problem in NP reduces to  $A$  in polynomial time. If  $A$  is in  $P$ , all such problems  $R$  can be solved in polynomial time using the algorithm for reducing  $R$  to  $A$  combined with the algorithm for  $A$ . This would imply that  $P = NP$ .
5. We denote a problem  $X$  as NP-Easy if it is polynomial-time reducible to some problem  $Y$  that is in NP. Then the fact that  $Y$  is in NP, combined with the reduction, implies that  $X$  is also in NP. Moreover, if  $X$  is NP-equivalent, then  $X$  is also NP-Hard. Hence the notions of NP-Equivalence and NP-Completeness are the same.

The correct choice is: **b)**

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2. There is a Turing transducer  $T$  that transforms problem  $P_1$  into problem  $P_2$ .  $T$  has one read-only input tape, on which an input of length  $n$  is placed.  $T$  has a read-write scratch tape on which it uses  $O(S(n))$  cells.  $T$  has a write-only output tape, with a head that moves only right, on which it writes an output of length  $O(U(n))$ . With input of length  $n$ ,  $T$  runs for  $O(T(n))$  time before halting. You may assume that each of the upper bounds on space and time used are as tight as possible.

A given combination of  $S(n)$ ,  $U(n)$ , and  $T(n)$  may:

1. Imply that  $T$  is a polynomial-time reduction of P1 to P2.
2. Imply that  $T$  is NOT a polynomial-time reduction of P1 to P2.
3. Be impossible; i.e., there is no Turing machine that has that combination of tight bounds on the space used, output size, and running time.

What are all the constraints on  $S(n)$ ,  $U(n)$ , and  $T(n)$  if  $T$  is a polynomial-time reducer? What are the constraints on feasibility, even if the reduction is not polynomial-time? After working out these constraints, identify the true statement from the list below.

- a)  $S(n) = n^2$ ;  $U(n) = n^3$ ;  $T(n) = n^4$  is a polynomial-time reduction
- b)  $S(n) = \log n$ ;  $U(n) = n$ ;  $T(n) = n^2$  is not physically possible.
- c)  $S(n) = n^2$ ;  $U(n) = n^3$ ;  $T(n) = n^4$  is possible, but not a polynomial-time reduction.
- d)  $S(n) = n$ ;  $U(n) = n^2$ ;  $T(n) = 2^n$  is not physically possible.

Answer submitted: **b)**

Your answer is incorrect.

There are two reasons a combination might be impossible. Perhaps the running time is so large that the Turing machine cannot run that long without entering a loop because it repeats an ID (combination of state, head positions on the input and scratch tapes, and scratch-tape contents). Or the running time is so small that the Turing machine cannot possibly use that many cells of the scratch or output tapes. The conditions for a reduction to be polynomial-time are in Section 10.1.5 (p. 433).

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Question Explanation:

There is one constraint on  $T$  being a polynomial-time reduction:  $T(n)$  must be a polynomial. However, there are also some constraints on feasibility. First, in time  $T(n)$ ,  $T$  cannot write more than  $T(n)$  cells on either its scratch or output tape. Thus,  $S(n)$  and  $U(n)$  must both be  $O(T(n))$ . Second, if  $T(n)$  is larger than  $O(n \log S(n) c^{S(n)})$  for any constant  $c$ , then  $T$  must repeat an "ID" consisting of the state, scratch-tape contents, and head positions on the input and scratch tapes.

The correct choice is: **a)**

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3. The classes of languages P and NP are closed under certain operations, and not closed under others, just like classes such as the regular languages or context-free languages have closure properties. Decide whether P and NP are closed under each of the following operations.

1. Union.
2. Intersection.
3. Intersection with a regular language.
4. Concatenation.
5. Kleene closure (star).
6. Homomorphism.
7. Inverse homomorphism.

Then, select from the list below the true statement.

- a) P is closed under homomorphism.
- b) NP is not closed under union.
- c) P is closed under Kleene closure.
- d) NP is not closed under concatenation.

Answer submitted: **c)**

You have answered the question correctly.

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Question Explanation:

Both P and NP are closed under each of these operations, except for homomorphism. To see why neither class is closed under homomorphism, start with a very hard language  $L$ , say one that requires time  $2^{2^n}$ . Make it easy by appending to each word of length  $n$  exactly  $2^{2^n}$   $c$ 's, where  $c$  is a new symbol. That is, let  $L' = Lc^{2^{2^n}}$ . Then  $L'$  is surely in P and NP. But  $L$  is  $h(L')$ , if  $h$  is the homomorphism that sends  $c$  to  $\epsilon$  and is the identity on all symbols of  $L$ . If P or NP were closed under homomorphism, then  $L$  would be in NP, which it is not.

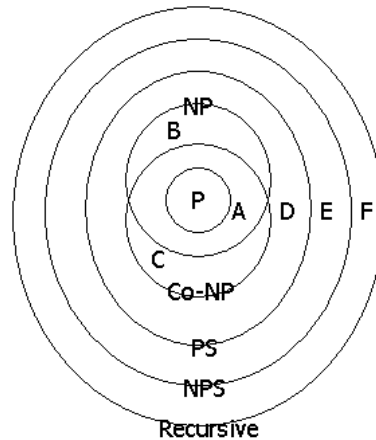
Here are the constructions that show P and NP closed under the other six operations:

1.  $L_1 \cup L_2$ : Apply the tests for membership in  $L_1$  and then for membership in  $L_2$ . Accept if either accepts. If  $L_1$  and  $L_2$  are in P, then both tests are polynomial, so the entire process is polynomial. If  $L_1$  and  $L_2$  are in NP, then there is a nondeterministic polynomial algorithm for the entire process.
2.  $L_1 \cap L_2$ : The argument is the same, but accept only if both tests accept.
3. Intersection with a regular set: The argument is the same as (2), since a regular language is surely in P and NP.
4.  $L_1 L_2$ : If  $L_1$  and  $L_2$  are in NP, just guess the point on the input where the string in  $L_2$  begins, and apply the nondeterministic polynomial tests to the two parts of the input. If  $L_1$  and  $L_2$  are in P, we have to try systematically all possible breakpoints between the prefix of the input that is in  $L_1$  and the suffix that is in  $L_2$ . If the tests for  $L_1$  and  $L_2$  are polynomial, say  $p(n)$  and  $q(n)$  running time, then we must apply each at most  $n$  times on input of length  $n$ . Since  $n(p(n)+q(n))$  is a polynomial, we can recognize  $L_1 L_2$  in polynomial time.
5.  $L^*$ : If  $L$  is in NP, guess all the places on the input where strings in  $L$  end. Run the nondeterministic polynomial-time test on each segment. Since there are at most  $n$  segments, the whole process is nondeterministic polynomial. If  $L$  is in P, we again must be systematic. For each pair of input positions  $i$  and  $j$ , run the polynomial-time test to see whether positions  $i$  through  $j$  of the input is in  $L$ . If  $L$  has a  $p(n)$ -time algorithm, then this process takes  $O(n^2 p(n))$  time, a polynomial. Then, use a dynamic programming algorithm, taking  $O(n^3)$  time, to determine whether positions  $i$  through  $j$  can be composed of  $1, 2, \dots, n$  strings in  $L$ . At the end, we only care about whether positions  $1$  through  $n$  can be composed of some number of strings in  $L$ .
6.  $h^{-1}(L)$ : Any homomorphism  $h$  can only expand the length of the string to which it is applied by a constant factor. To recognize  $h^{-1}(L)$ , apply  $h$  to the input  $w$ , and see whether  $h(w)$  is in  $L$ . If there is a polynomial-time, or nondeterministic polynomial-time test for membership in  $L$ , this test will, in the same order of magnitude time complexity tell us whether  $w$  is in  $h^{-1}(L)$ .

4. Consider the following problems:

1. SP (Shortest Paths): given a weighted, undirected graph with nonnegative integer edge weights, given two nodes in that graph, and given an integer limit  $k$ , determine whether the length of the shortest path between the nodes is  $k$  or less.
2. WHP (Weighted Hamilton Paths): given a weighted, undirected graph with nonnegative integer edge weights, and given an integer limit  $k$ , determine whether the length of the shortest Hamilton path in the graph is  $k$  or less.
3. TAUT (Tautologies): given a propositional boolean formula, determine whether it is true for all possible truth assignments to its variables.
4. QBF (Quantified Boolean Formulas): given a boolean formula with quantifiers for-all and there-exists, such that there are no free variables, determine whether the formula is true.

In the diagram below are seven regions, P and A through F.



Place each of the four problems in its correct region, on the assumption that NP is equal to neither P nor co-NP nor PS.

- a) Problem TAUT is in region B.
- b) Problem QBF is in region E.
- c) Problem WHP is in region B.
- d) Problem QBF is in region C.

Answer submitted: **a)**

Your answer is incorrect.

Hint: Consider the relationship between TAUT and the problem SAT. What do the known facts about SAT tell us about the complexity of TAUT? The complexity of SAT is in Section 10.2.3 (p. 440).

#### Question Explanation:

SP is in P. You can use Dijkstra's algorithm to solve the problem in quadratic time.

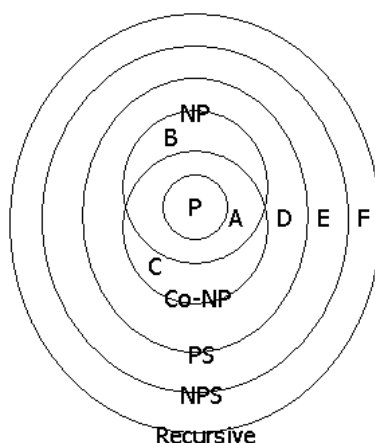
WHP is NP-complete. It is a generalization of Hamilton-path, but the weights don't affect the nondeterministic-polynomial-time "guessing" algorithm to solve it. On the assumption that NP is neither P nor co-NP, WHP must be in region B.

TAUT is essentially the complement of SAT. Since SAT is NP-complete, TAUT must be in region C, unless NP is equal to one of P or co-NP (which we assume not to be the case).

QBF is complete for PS. Thus, assuming PS is not NP, QBF is in D.

The correct choice is: **c)**

5. In the diagram below we see certain complexity classes (represented as circles or ovals) and certain regions labeled A through F that represent the differences of some of these complexity classes.



The state of our knowledge regarding the existence of problems in the regions A-F is imperfect. In some cases, we know that a region is nonempty, and in other cases we know that it is empty. Moreover, if  $P=NP$ , then we would know more about the emptiness or nonemptiness of some of these regions, but still would not know everything.

Decide what we know about the regions A-F currently, and also what we would know if  $P=NP$ . Then, identify the true statement from the list below.

- Region D is definitely not empty.
- If  $P=NP$ , it would still not be known whether region E is empty.
- Region E is definitely empty.
- If  $P=NP$ , it would still not be known whether region A is empty.

Answer submitted: **c)**

You have answered the question correctly.

#### Question Explanation:

Since we do not know whether  $P=NP$ , or whether  $NP=co-NP$  (i.e., whether NP is closed under complementation), we do not know whether any of A, B, or C is empty. If we know  $P=NP$ , then surely A and B are empty. But if  $P=NP$ , then  $co-NP=NP=P$ , since P is closed under complementation. Thus, C would be empty as well.

Savitch's theorem tells us that  $PS=NPS$ ; i.e., region E is empty. We also know that there are arbitrarily complex recursive languages, so region F is definitely *not* empty. Finally, we do not know about region D, since it is open whether  $PS=NP$  or even  $PS=P$ . And even if we knew  $P=NP$ , we would still not be sure whether or not  $PS=NP=P$ .

6. Suppose there are three languages (i.e., problems), of which we know the following:

- $L_1$  is in P.
- $L_2$  is NP-complete.
- $L_3$  is not in NP.

Suppose also that we do not know anything about the resolution of the "P vs. NP" question; for example, we do not know definitely whether  $P=NP$ . Classify each of the following languages as (a) Definitely in P, (b) Definitely in NP (but perhaps not in P and perhaps not NP-complete) (c) Definitely NP-complete (d) Definitely not in NP:

- $L_1 \cup L_2$ .
- $L_1 \cap L_2$ .
- $L_2 c L_3$ , where c is a symbol not in the alphabet of  $L_2$  or  $L_3$  (i.e., the *marked concatenation* of  $L_2$  and  $L_3$ , where there is a unique marker symbol between the strings from  $L_2$  and  $L_3$ ).
- The complement of  $L_3$ .

Based on your analysis, pick the correct, definitely true statement from the list below.

- a) The complement of  $L_3$  is definitely in NP.
- b)  $L_1 \cap L_2$  is definitely in NP.
- c)  $L_1 \cap L_2$  is definitely NP-complete.
- d)  $L_2 \cup L_3$  is definitely NP-complete.

Answer submitted: **b)**

You have answered the question correctly.

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#### Question Explanation:

Let  $L = L_1 \cup L_2$ . It is possible that  $L_1$  is empty; that is certainly one language in P. If so, then  $L = L_2$ , and  $L$  is NP-complete. On the other hand, suppose  $L_1$  is all strings over the alphabet of  $L_2$  --- another language in P. Then  $L = L_1$ , a language in P. Since we don't know whether  $P = NP$ , we cannot conclude  $L$  is definitely in P and we cannot conclude that  $L$  is definitely NP-complete.

On the other hand, we can conclude definitely that  $L$  is in NP. A nondeterministic, polynomial-time algorithm for  $L$  starts by applying a polynomial-time algorithm for  $L_1$  to its input, and if the result is negative applies the NP-recognizer for  $L_2$  to the same input. It accepts the input if either recognizer accepts.

The argument for  $L = L_1 \cap L_2$  is essentially the same, although now  $L$  is in P if  $L_1$  is empty and  $L$  is NP-complete if  $L_1$  is all strings over the alphabet of  $L_2$ . Also, the nondeterministic polynomial-time algorithm for  $L$  works by trying the polynomial-time recognizer for  $L_1$  first, and only accepting if that recognizer accepts and the NP-recognizer for  $L_2$  also accepts.

Now, consider  $L = L_2 \cup L_3$ . Suppose  $L$  had an NP-recognizer. Let  $x$  be some string in  $L_2$  (since  $L_2$  is NP-complete, and we do not know that  $P = NP$ , we can be sure  $L_2$  is not empty, so  $x$  exists). Then there is a nondeterministic polynomial-time algorithm for  $L_3$  that works as follows. Take input  $w$ , and test it for membership in  $L_3$  by feeding  $xw$  to the NP-recognizer for  $L$ . Respond exactly as this recognizer responds. Since we know  $x$  is in  $L_2$ , the recognizer for  $L$  accepts  $xw$  if and only if  $w$  is in  $L_3$ . We now have an NP-recognizer for  $L_3$ , but we were told that  $L_3$  is *not* in NP. Thus, our assumption that  $L$  is in NP must be false.

Last, let  $L$  be the complement of  $L_3$ . If  $L$  is in P, then the complement of  $L$ , which is  $L_3$ , is also in P. But we know  $L_3$  is not even in NP. We do not even know that  $L$  is in NP; for example,  $L_3$  could be an undecidable problem. On the other hand,  $L$  could be NP-complete. For example, the complement of the problem SAT is not known to be in NP. But it is possible that  $L_3$  is the complement of SAT and therefore  $L = \text{SAT}$ , a known NP-complete problem.

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