Department Of Physics

FINAL YEAR PROJECT, DISSERTATION OR PHYSICS EDUCATION REPORT

NAME:	Luka Milic
DEGREE COURSE:	Mathematics and Physics (MSci)
PROJECT TITLE:	Entanglement of photons pairs generated in silicon ring resonators
YEAR OF SUBMISSION:	2015
SUPERVISOR:	Damien Bonneau, Josh Silverstone and Mark Thompson
NUMBER OF WORDS:	341 (exclude appendices, references, captions and abstract)

Entanglement of photons pairs generated in silicon ring resonators

Luka Milic

April 6, 2015

Acknowledgements

Thank you Damien and Josh for your invaluable help throughout my project. Lizzy for working with me in the lab. Imad for your help and encouragement. Mark for giving me the opportunity and supervision. Raf for your help and Phil for supplying sarcasm.

Contents

1 Introduction

The endeavour to build a quantum computer holds the promise of solving computational problems which are currently intractable on classical computers. A particularly promising paradigm for this is the linear optical quantum computer (LOQC) model which in theory allows for scalable universal quantum computation. Work on LOQC can be done using bulk optics components but this quickly becomes impractical when the experiments need to be scaled up to more qubits. Integrated photonics is a solution to this problem and allows for experiments with more qubits in a much smaller space. Optical circuits can be implemented on such chips, popular materials are silicon-on-insulator (SOI) , lithium niobate and glass materials. Here we focus on SOI chips as they have many promising properties for the implementation of complex quantum optical circuits.

A key requirement for the full implementation of LOQC is a scalable, bright, deterministic and indistinguishable single photon source. Single photon sources in the SOI platform are typically made from the waveguide itself and use the spontaneous four-wave mixing which occurs in silicon due to the third order non-linearity to create a single photon pair. This report aims to develop a method of measuring the indistinguishably of the produced photons with a classical technique, exploiting stimulated four-wave mixing. This method collects a joint spectrum which is an estimation of the spectral shape of the two photons produced by the source. For a full description the Joint Spectral Amplitude JSA is the desired quantity, this is a full description of the wavefunction our the single photons emitted by the silicon ring resonators. However it is only within the scope of this work to measure the Joint Spectral Intensity, which is the absolute value squared the JSA. This gives an upper bound

The mission is therefore to develop a methodology to reconstruct these wavefunctions and hence engineer indistinguishable (high purity) single photon sources. In this work we performed such measurements on three SOI chips. The experimental work started with an initial proof of concept that one can collect joint spectrum data in the way desired. This was done on a chip supplied by Marc Sorel from Glassgow University. Then due to the fragility of these chip at high powers the experiment progressed to a chip manufactured by Toshiba. Finally in order to investigate a promising new material amorphous silicon chip was used for experiments.

In parallel techniques of analysing the output data are developed. Filtering techniques which remove noise are developed in order to make the data usable. A general framework is set out which aims to quantify the certainty in the measurements.

Finally we conclude that there is still much to be done in this area, proposing an outline for how to carry out effective measurements in the future.

2 Detailed Background and Theory

2.1 Integrated silicon photonics

Integrated silicon photonics is a promising new platform on which to conduct quantum information experiments.

2.2 Marco Liscid - Why its okay to use classical to probe quantum and an introduction to four wavemixing

Marco [?]

2.3 Ring Resonators

Ring resonators are used as single photon sources. However to understand their behaviour to first order no quantum mechanics is needed. Here are the 3 governing equations:

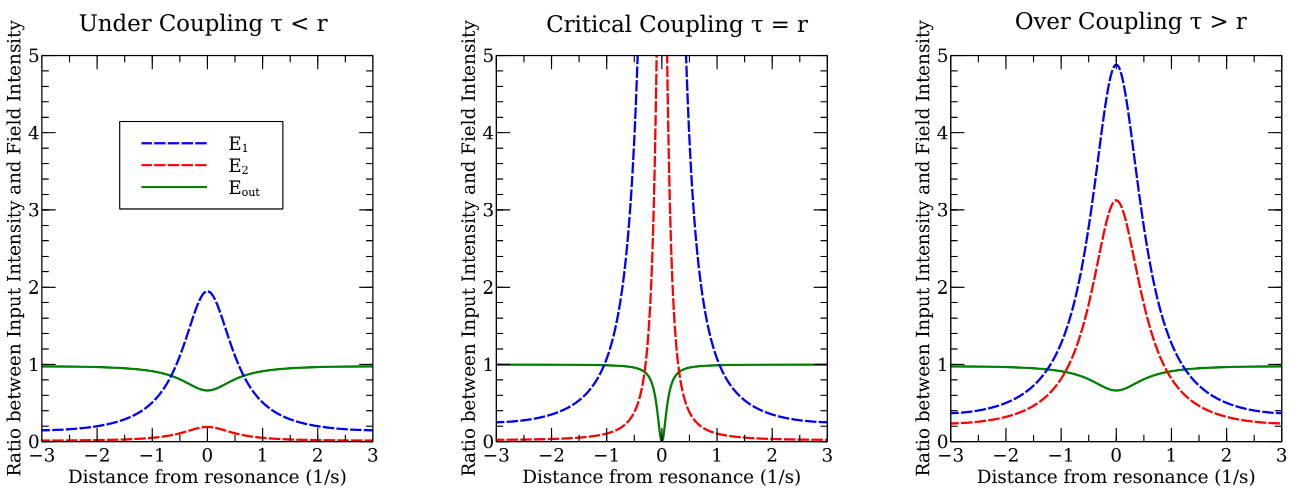

Figure 1: Notice how similar under and over coupling are to each other

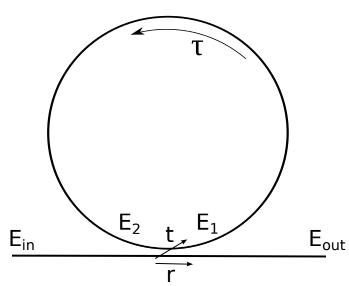

Figure 2: ahhh

$$\left|\frac{E_{out}}{E_0}\right|^2 = \frac{r^2 - 2r\tau\cos(\theta) + \tau^2}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{2.1}$$

$$\left|\frac{E_1}{E_0}\right|^2 = \frac{t^2}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{2.2}$$

$$\left|\frac{E_2}{E_0}\right|^2 = \tau^2 \left|\frac{E_1}{E_0}\right|^2 \tag{2.3}$$

2.4 Bistability

It can be experimentally observed that injecting power into a ring resonator will cause changes in the spectral position and shape of the resonance. Typically in silicon ring resonators the more power in the ring the more the resonance position is red-shifted by the thermo-optic effect [**?**]. A counter acting effect is carrier generation induced by two-photon absorption [1] which causes a blue-shift in the resonance position. This carrier generation process is much faster than the thermo-optic process so it more relevant to lasers with low repetition times.

The bi-stability effect is observed by changing the power injected into the ring resonator at a fixed wavelength. By steadily increasing the power of a monochromatic light source injected into the ring at a wavelength slightly higher than the resonance position λ_r of the ring, λ_r is increased (thermal effects dominate as the laser is a continuous wave and not pulsed). The shift in λ_r accelerates as more light is coupled into the ring and transmission falls as more light is coupled into the ring. The system is now in a different and stable state (assuming the injected light is not discontinued). With a low intensity probe it is now possible to map out the new position and shape of the resonance.

By doing the reverse experiment with the input power decreasing a similar phenomena is observed, however the sudden accelerating changing in resonance position is seen for a different power due to the ring coming from a different stable state.

Some knowledge of this effect is vital when planning experiments using high and variable powers, as one must take into account which state the ring is in. Further when automating equipment it may be vital to integrate knowledge of this into any scanning procedures.

2.5 Self phase modulation

2.6 Schmidt Rank and Purity

3 Method

3.1 Silicon Chips

3.1.1 Glassgow

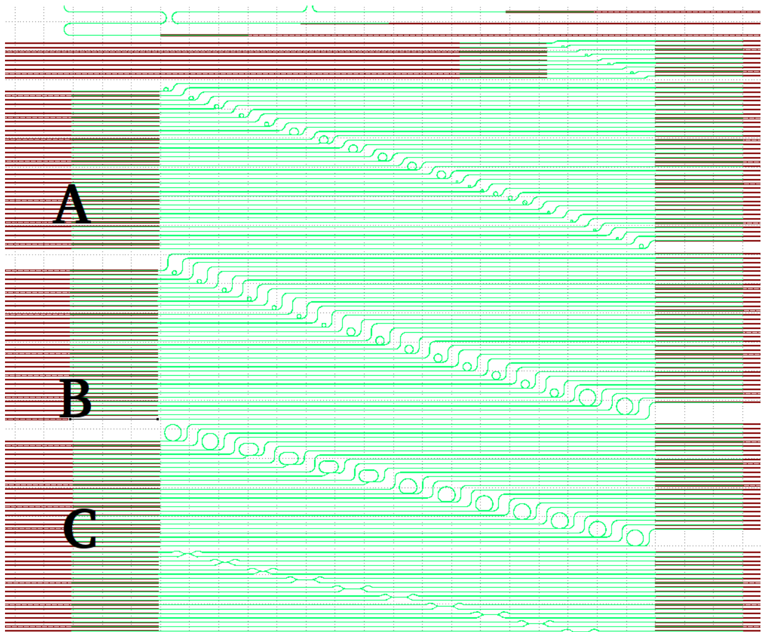

Figure 3: Glassgow test structure chip

3.1.2 Toshiba

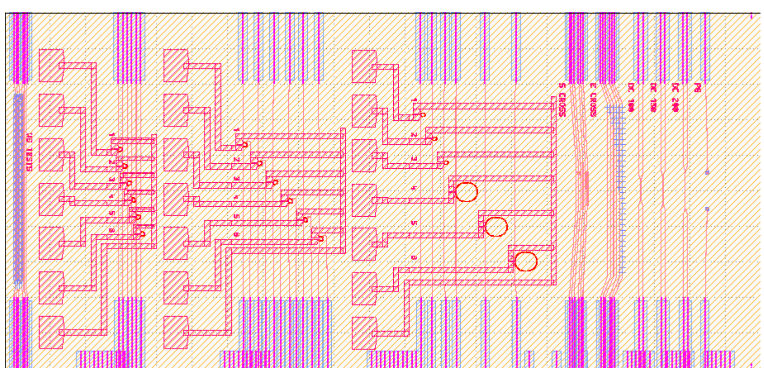

Figure 4: Glassgow test structure chip

3.1.3 a-Si

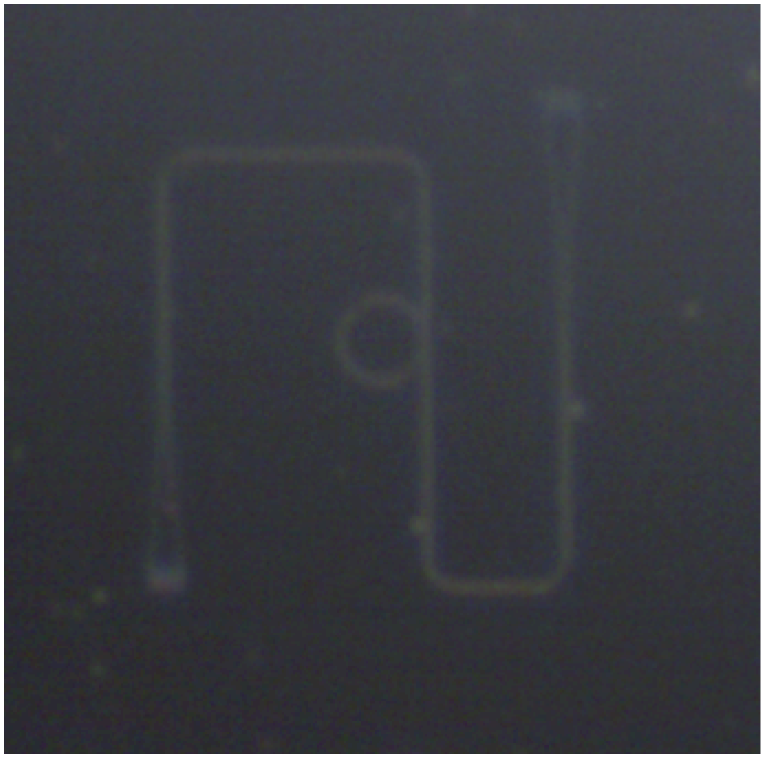

Figure 5: Glassgow test structure chip

3.2 Coupling

3.3 Joint Spectrum

3.4 What experiments can be done?

Assuming from the above that the procedure for collecting the JSI is fixed and fully understood the question that now needs to be answered is: what parameters can be reliably varied to change the JSI? One that we pursue and that forms the main part of this work is varying the power of the pump laser injected into the ring. This is of interest as it may

3.5 $g^{(2)}(0)$

3.6 Analysing Data

4 Results

4.1 Glassgow

This chip was used to do an initial proof of concept that the JSI of a ring resonator could be measured. Here the aim was to explore different ring geometries and develop an intuition on how to do the experiment.

A Pritel pulsed laser was used with a pulse duration of 2 ps, a FWHM of 1.0 nm with wavelength range 1530 nm to 1530 nm and a peak power of 100 W. Due to the laser sometimes destroying the side coupler a 3 dB attenuator was included.

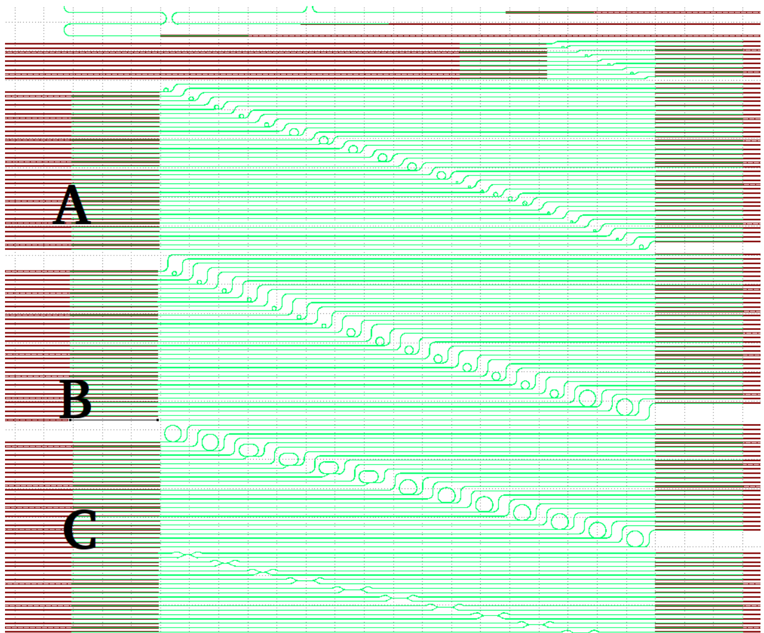

Figure 6: Glassgow test structure chip

4.2 Toshiba

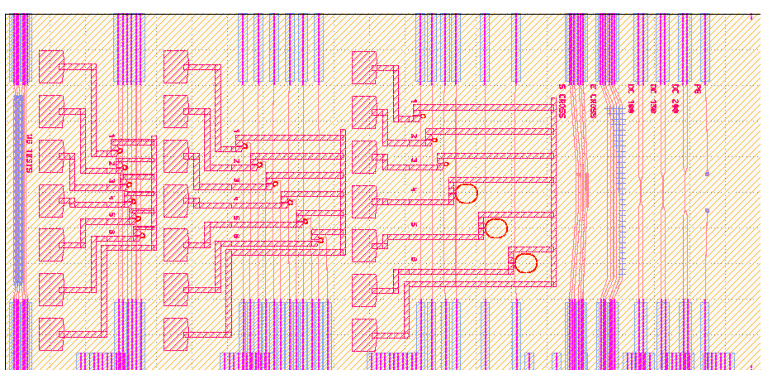

Figure 7: Glassgow test structure chip

4.2.1 Bistability Data

4.2.2 Pulse shaping

4.2.3 Power Scans

4.3 a-Si

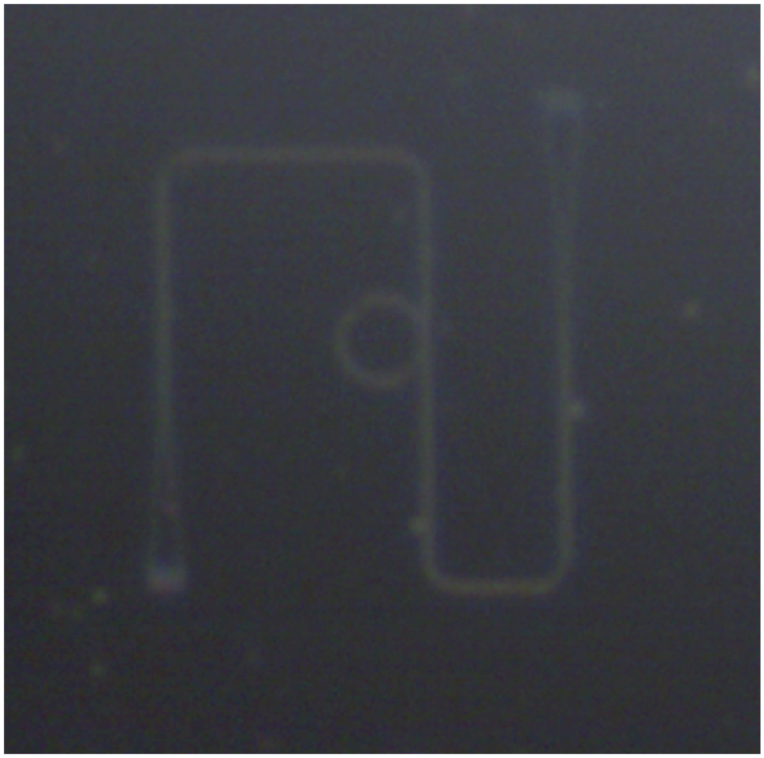

Figure 8: Glassgow test structure chip

5 Discussion

6 Conclusion

References

[1] Qianfan Xu and Michal Lipson. Carrier-induced optical bistability in silicon ring resonators. *Opt. Lett.*, 31(3):341–343, February 2006.

[2] Andreas Eckstein, Guillaume Boucher, Aristide Lematre, Pascal Filloux, Ivan Favero, Giuseppe Leo, John E. Sipe, Marco Liscidini, and Sara Ducci. High-resolution spectral characterization of two photon states via classical measurements. *Laser & Photonics Reviews*, 8(5):L76–L80, September 2014.

A Schmidt Number

A.1 Definition

Starting with some abitrary state ψ:

$$|\psi\rangle = \sum_{i,j} \alpha(i,j)|i\rangle_A \otimes |j\rangle_B \tag{A.1}$$

The schimdt number K of this state measures the degree of entanglement. If $K = 1$ then you can find $|\psi\rangle = |\xi\rangle \otimes |\eta\rangle$ and for $K > 1$ you can find:

$$|\psi\rangle = \sum_{i}^{K} r_i|\xi_i\rangle_A \otimes |\eta_i\rangle_B \tag{A.2}$$

Note that $1 \leq K \leq D$ where D is the dimension of the system. The purity is the inverse of K so:

$$P = 1/K \tag{A.3}$$

An expression for K can be found using the density matrix for ψ:

$$\rho_{AB} = |\psi\rangle\langle\psi| = \sum_{i,j,k,l} \alpha(i,j)\alpha^*(k,l)|i\rangle\langle k| \otimes |j\rangle\langle l| \tag{A.4}$$

$$\rho_A = \mathrm{Tr}_B(\rho_{AB}) = \sum_{i,j,k} \alpha(i,j)\alpha^*(k,j)|i\rangle\langle k| \tag{A.5}$$

$$\rho_A^2 = \sum_{i',j',k'}\sum_{i,j,k} \alpha(i,j)\alpha(k,j)\alpha^*(i',j')\alpha^*(k',j')|i\rangle\langle k|i'\rangle\langle k'| \tag{A.6}$$

$$= \sum_{j',k'}\sum_{i,j,k} \alpha(i,j)\alpha^*(k,j)\alpha(k,j')\alpha^*(k',j')|i\rangle\langle k'| \tag{A.7}$$

$$\mathrm{Tr}_A(\rho_A^2) = \sum_{i,j,k,j'} \alpha(i,j)\alpha^*(k,j)\alpha(k,j')\alpha^*(i,j') \tag{A.8}$$

$$\tag{A.9}$$

For a unentangled ψ we know that $\mathrm{Tr}_A(\rho_A^2) = 1$ For ψ entangled this will be smaller than 1 (proof comes from the property of the density operator that its eigenvalues are all smaller than 1). This fits the definiton of the purity of a quantum state hence we can write:

$$P = \frac{1}{K} = \sum_{i,j,k,l} \alpha(i,j)\alpha^*(k,j)\alpha(k,l)\alpha^*(i,l) \tag{A.10}$$

A.2 Calculation from experimental data

A.2.1 Trace method

In the lab we can measure $|\phi(\omega_1,\omega_2)|^2$, here I outline how to extract the schimdt number from this set of values. Taking the positive square root of the matrix of values obtained from the lab you have a matrix $\mathbf{f}$ given by:

$$\mathbf{f} = \sum_{\omega_1,\omega_2} \phi(\omega_1,\omega_2)|\omega_1\rangle\langle\omega_2| \tag{A.11}$$

(This seems to be some weird way of writing the wavefunction as a matrix, bare with me it turns out to be useful)

$$\mathbf{f}^\dagger\mathbf{f} = \sum_{\omega_1,\omega_2,\omega_3} \phi(\omega_1,\omega_2)\phi(\omega_3,\omega_2)|\omega_1\rangle\langle\omega_3| \tag{A.12}$$

$$(\mathbf{f}^\dagger\mathbf{f})^2 = \sum_{\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6} \phi(\omega_1,\omega_2)\phi(\omega_3,\omega_2)\phi(\omega_4,\omega_5)\phi(\omega_6,\omega_5)|\omega_1\rangle\langle\omega_3|\omega_4\rangle\langle\omega_6| \tag{A.13}$$

$$(\mathbf{f}^\dagger\mathbf{f})^2 = \sum_{\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6} \phi(\omega_1,\omega_2)\phi(\omega_3,\omega_2)\phi(\omega_3,\omega_5)\phi(\omega_6,\omega_5)|\omega_1\rangle\langle\omega_6| \tag{A.14}$$

$$\mathrm{Tr}\left[(\mathbf{f}^\dagger\mathbf{f})^2\right] = \sum_{\omega_1,\omega_2,\omega_3,\omega_4} \phi(\omega_1,\omega_2)\phi(\omega_3,\omega_2)\phi(\omega_3,\omega_4)\phi(\omega_1,\omega_4) \tag{A.15}$$

I've done it this way because I wanted to figure out where the equation in [2] comes from. You can now see that equation A.10 is of exactly the same form as Tr $\left[(\mathbf{f}^\dagger\mathbf{f})^2\right]$ (barring the conjugates but this is okay since ϕ is real.) Taking the parallel further it can be seen that equation A.12 is of the form of a reduced density matrix. Here we must make sure to normalise to make sure this is a valid reduced density matrix. The normalisation is:

$$N = \mathrm{Tr}\left[\mathbf{f}^\dagger\mathbf{f}\right] = \sum_{\omega_1,\omega_2} \phi(\omega_1,\omega_2)^2 \tag{A.16}$$

Giving:

$$\rho_A = \frac{\mathbf{f}^\dagger\mathbf{f}}{N} \tag{A.17}$$

We can then write:

$$\frac{1}{K} = \frac{\mathrm{Tr}\left[(\mathbf{f}^\dagger\mathbf{f})^2\right]}{\mathrm{Tr}\left[\mathbf{f}^\dagger\mathbf{f}\right]^2} \tag{A.18}$$

B Equipment Specifications

C Transfer matrix analysis of ring resonator cavities

A useful way to describe the spectral response of a ring resonator is by defining a transfer matrix. This 2×2 matrix relates the electric fields of the inputs and outputs of the waveguide and resonator. In our analysis we define the electric fields to be complex valued and also normalised to conserve energy, this condition implies the transfer matrix must be unitary.

First write down the obvious relations

$$E_{out} = r_1 E_{in} + t_1 E_2 \tag{C.1}$$

$$E_1 = r_2 E_2 + t_2 E_{in} \tag{C.2}$$

Define a matrix.

$$M = \begin{pmatrix} r_1 & t_1 \\ t_2 & r_2 \end{pmatrix} \tag{C.3}$$

Enforce unitarity

$$\begin{pmatrix} r_1 & t_1 \\ t_2 & r_2 \end{pmatrix} \begin{pmatrix} r_1^* & t_2^* \\ t_1^* & r_2^* \end{pmatrix} = \begin{pmatrix} |r_1|^2 + |t_1|^2 & r_1 t_2^* + r_2^* t_1 \\ r_2 t_1^* + r_1^* t_2 & |r_2|^2 + |t_2|^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{C.4}$$

Furthermore $\det M = 1$

$$r_1 r_2 - t_1 t_2 = 1 \tag{C.5}$$

so

$$r_2 = \frac{1 + t_1 t_2}{r_1} \tag{C.6}$$

Also $M^{-1} = M^*$

$$\begin{pmatrix} r_2 & -t_1 \\ -t_2 & r_1 \end{pmatrix} = \begin{pmatrix} r_1^* & t_1^* \\ t_2^* & r_2^* \end{pmatrix} \tag{C.7}$$

This tells us r_1 and r_2 must be real. So we can simplify the unitarity equation. So

$$t_1 = iT_1 \tag{C.8}$$

$$t_2 = iT_2 \tag{C.9}$$

Rewrite the interesting equations: Use also $r_2 = r_1^*$

$$\begin{pmatrix} |r_1|^2 + T_1^2 & -ir_1 T_2 + ir_1 T_1 \\ -ir_1^* T_1 + ir_1^* T_2 & |r_1|^2 + T_2^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{C.10}$$

Ok so $T_2 = T_1 = t$

$$\begin{pmatrix} |r_1|^2 + t^2 & 0 \\ 0 & |r_1|^2 + t^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{C.11}$$

$$|r_1|^2 = 1 - t^2 \tag{C.12}$$

$$|r_2|^2 = 1 - t^2 \tag{C.13}$$

d Basically I have a degree of freedom. $r_1 = e^{i\theta}r$ and $r_2 = e^{-i\theta}r$. Choose $\theta = 0$ for simplicity.

Now the matrix is:

$$\begin{pmatrix} r & it \\ it & r \end{pmatrix} \tag{C.14}$$

Now we add that

$$E_2 = \tau e^{i\theta} E_1$$

Recall the initial equations.

$$E_{out} = rE_{in} + itE_2 \tag{C.15}$$

$$E_1 = rE_2 + itE_{in} \tag{C.16}$$

$$E_1 = r\tau e^{i\theta} E_1 + itE_{in} \tag{C.17}$$

$$\frac{E_1}{E_0} = \frac{it}{1 - r\tau e^{i\theta}} \tag{C.18}$$

Okay so thats E_1 done.

$$\frac{E_1}{E_0} = \frac{it}{1 - r\tau e^{i\theta}} \tag{C.19}$$

$$\left|\frac{E_1}{E_0}\right|^2 = \frac{t^2}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{C.20}$$

For E_2 we have:

$$\frac{E_2}{E_0} = \frac{it\tau e^{i\theta}}{1 - r\tau e^{i\theta}} \tag{C.21}$$

$$\left|\frac{E_2}{E_0}\right|^2 = \frac{\tau^2 - \tau^2 r^2}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{C.22}$$

For E_{out} we have:

$$\frac{E_{out}}{E_0} = r + it\frac{it\tau e^{i\theta}}{1 - r\tau e^{i\theta}} \tag{C.23}$$

$$\frac{E_{out}}{E_0} = \frac{r - r^2\tau e^{i\theta} - t^2\tau e^{i\theta}}{1 - r\tau e^{i\theta}} \tag{C.24}$$

$$\left|\frac{E_{out}}{E_0}\right|^2 = \frac{(r - r^2\tau e^{i\theta} - t^2\tau e^{i\theta})(r - r^2\tau e^{-i\theta} - t^2\tau e^{-i\theta})}{(1 - r\tau e^{i\theta})(1 - r\tau e^{-i\theta})} \tag{C.25}$$

$$\left|\frac{E_{out}}{E_0}\right|^2 = \frac{(r(r - r^2\tau e^{-i\theta} - t^2\tau e^{-i\theta}) - r^2\tau e^{i\theta}(r - r^2\tau e^{-i\theta} - t^2\tau e^{-i\theta}) - t^2\tau e^{i\theta}(r - r^2\tau e^{-i\theta} - t^2\tau e^{-i\theta}))}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{C.26}$$

$$\left|\frac{E_{out}}{E_0}\right|^2 = \frac{r^2 - r^3\tau e^{-i\theta} - rt^2\tau e^{-i\theta} - r^3\tau e^{i\theta} + r^4\tau^2 + 2r^2\tau^2t^2 - rt^2\tau e^{i\theta} + t^4\tau^2}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{C.27}$$

$$\left|\frac{E_{out}}{E_0}\right|^2 = \frac{r^2 - e^{-i\theta}(r^3\tau + rt^2\tau) - e^{i\theta}(r^3\tau + rt^2\tau) + r^4\tau^2 + t^4\tau^2 + 2r^2\tau^2t^2}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{C.28}$$

$$\left|\frac{E_{out}}{E_0}\right|^2 = \frac{r^2 + 2(r^3\tau + rt^2\tau)\cos(\theta) + r^4\tau^2 + t^4\tau^2 + 2r^2\tau^2t^2}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{C.29}$$

$$\left|\frac{E_{out}}{E_0}\right|^2 = \frac{r^2 2(r^3\tau + r(1-r^2)\tau)\cos(\theta) + r^4\tau^2 + (1-r^2)^2\tau^2 + 2r^2\tau^2(1-r^2)}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{C.30}$$

$$\left|\frac{E_{out}}{E_0}\right|^2 = \frac{r^2 - 2(r^3\tau + r(1-r^2)\tau)\cos(\theta) + r^4\tau^2 + (1-2r^2+r^4)\tau^2 + 2r^2\tau^2 - 2r^4\tau^2}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{C.31}$$

$$\left|\frac{E_{out}}{E_0}\right|^2 = \frac{r^2 - 2r\tau\cos(\theta) + r^4\tau^2 + \tau^2 - 2r^2\tau^2 + r^4\tau^2 + 2r^2\tau^2 - 2r^4\tau^2}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{C.32}$$

FINAL RESULTS

$$\left|\frac{E_{out}}{E_0}\right|^2 = \frac{r^2 - 2r\tau\cos(\theta) + \tau^2}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{C.33}$$

$$\left|\frac{E_2}{E_0}\right|^2 = \frac{\tau^2 - \tau^2 r^2}{1 + r^2\tau^2 - 2r\tau cos(\theta)} = \tau^2\left|\frac{E_1}{E_0}\right|^2 \tag{C.34}$$

$$\left|\frac{E_1}{E_0}\right|^2 = \frac{t^2}{1 + r^2\tau^2 - 2r\tau cos(\theta)} \tag{C.35}$$