DEPARTMENT OF PHYSICS

FINAL YEAR PROJECT, DISSERTATION OR PHYSICS EDUCATION REPORT

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The endeavour to build a quantum computer holds the promise of solving computational problems which are currently intractable on classical computers. This has driven many time [1]

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References

[1] Georg Harder, Vahid Ansari, Benjamin Brecht, Thomas Dirmeier, Christoph Marquardt, and Christine Silberhorn. An optimized photon pair source for quantum circuits. *Opt. Express*, 21(12):13975–13985, June 2013.

[2] Andreas Eckstein, Guillaume Boucher, Aristide Lematre, Pascal Filloux, Ivan Favero, Giuseppe Leo, John E. Sipe, Marco Liscidini, and Sara Ducci. High-resolution spectral characterization of two photon states via classical measurements. *Laser & Photonics Reviews*, 8(5):L76–L80, September 2014.

A Schmidt Number

A.1 Definition

Starting with some abitrary state ψ:

$$|\psi\rangle = \sum_{i,j} \alpha(i,j)|i\rangle_A \otimes |j\rangle_B \tag{1}$$

The schimdt number K of this state measures the degree of entanglement. If $K = 1$ then you can find $|\psi\rangle = |\xi\rangle \otimes |\eta\rangle$ and for $K > 1$ you can find:

$$|\psi\rangle = \sum_{i}^{K} r_i |\xi_i\rangle_A \otimes |\eta_i\rangle_B \tag{2}$$

Note that $1 \leq K \leq D$ where D is the dimension of the system. The purity is the inverse of K so:

$$P = 1/K \tag{3}$$

An expression for K can be found using the density matrix for ψ:

$$\rho_{AB} = |\psi\rangle\langle\psi| = \sum_{i,j,k,l} \alpha(i,j)\alpha^*(k,l)|i\rangle\langle k| \otimes |j\rangle\langle l| \tag{4}$$

$$\rho_A = \mathrm{Tr}_B(\rho_{AB}) = \sum_{i,j,k} \alpha(i,j)\alpha^*(k,j)|i\rangle\langle k| \tag{5}$$

$$\rho_A^2 = \sum_{i',j',k'} \sum_{i,j,k} \alpha(i,j)\alpha(k,j)\alpha^*(i',j')\alpha^*(k',j')|i\rangle\langle k|i'\rangle\langle k'| \tag{6}$$

$$= \sum_{j',k'} \sum_{i,j,k} \alpha(i,j)\alpha^*(k,j)\alpha(k,j')\alpha^*(k',j')|i\rangle\langle k'| \tag{7}$$

$$\mathrm{Tr}_A(\rho_A^2) = \sum_{i,j,k,j'} \alpha(i,j)\alpha^*(k,j)\alpha(k,j')\alpha^*(i,j') \tag{8}$$

$$\tag{9}$$

For a unentangled ψ we know that $\mathrm{Tr}_A(\rho_A^2) = 1$ For ψ entangled this will be smaller than 1 (proof comes from the property of the density operator that its eigenvalues are all smaller than 1). This fits the definiton of the purity of a quantum state hence we can write:

$$P = \frac{1}{K} = \sum_{i,j,k,l} \alpha(i,j)\alpha^*(k,j)\alpha(k,l)\alpha^*(i,l) \tag{10}$$

A.2 Calculation from experimental data

A.2.1 Trace method

In the lab we can measure $|\phi(\omega_1,\omega_2)|^2$, here I outline how to extract the schimdt number from this set of values. Taking the positive square root of the matrix of values obtained from the lab you have a matrix $\mathbf{f}$ given by:

$$\mathbf{f} = \sum_{\omega_1,\omega_2} \phi(\omega_1,\omega_2)|\omega_1\rangle\langle\omega_2| \tag{11}$$

(This seems to be some weird way of writing the wavefunction as a matrix, bare with me it turns out to be useful)

$$\mathbf{f}^\dagger \mathbf{f} = \sum_{\omega_1,\omega_2,\omega_3} \phi(\omega_1,\omega_2)\phi(\omega_3,\omega_2)|\omega_1\rangle\langle\omega_3| \tag{12}$$

$$(\mathbf{f}^\dagger \mathbf{f})^2 = \sum_{\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6} \phi(\omega_1,\omega_2)\phi(\omega_3,\omega_2)\phi(\omega_4,\omega_5)\phi(\omega_6,\omega_5)|\omega_1\rangle\langle\omega_3|\omega_4\rangle\langle\omega_6| \tag{13}$$

$$(\mathbf{f}^\dagger \mathbf{f})^2 = \sum_{\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6} \phi(\omega_1,\omega_2)\phi(\omega_3,\omega_2)\phi(\omega_3,\omega_5)\phi(\omega_6,\omega_5)|\omega_1\rangle\langle\omega_6| \tag{14}$$

$$\mathrm{Tr}\left[(\mathbf{f}^\dagger \mathbf{f})^2\right] = \sum_{\omega_1,\omega_2,\omega_3,\omega_4} \phi(\omega_1,\omega_2)\phi(\omega_3,\omega_2)\phi(\omega_3,\omega_4)\phi(\omega_1,\omega_4) \tag{15}$$

I've done it this way because I wanted to figure out where the equation in [2] comes from. You can now see that equation 10 is of exactly the same form as $\mathrm{Tr}\left[(\mathbf{f}^\dagger \mathbf{f})^2\right]$ (barring the conjugates but this is okay since ϕ is real.) Taking the parallel further it can be seen that equation 12 is of the form of a reduced density matrix. Here we must make sure to normalise to make sure this is a valid reduced density matrix. The normalisation is:

$$N = \mathrm{Tr}\left[\mathbf{f}^\dagger \mathbf{f}\right] = \sum_{\omega_1,\omega_2} \phi(\omega_1,\omega_2)^2 \tag{16}$$

Giving:

$$\rho_A = \frac{\mathbf{f}^\dagger \mathbf{f}}{N} \tag{17}$$

We can then write:

$$\frac{1}{K} = \frac{\mathrm{Tr}\left[(\mathbf{f}^\dagger \mathbf{f})^2\right]}{\mathrm{Tr}\left[\mathbf{f}^\dagger \mathbf{f}\right]^2} \tag{18}$$