

FINAL YEAR PROJECT, DISSERTATION OR
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Entanglement of photons pairs generated in silicon ring resonators

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1 Introduction

The endeavour to build a quantum computer holds the promise of solving computational problems which are currently intractable on classical computers. A particularly promising paradigm for this is the linear optical quantum computer (LOQC) model which in theory allows for scalable universal quantum computation. Work on LOQC can be done using bulk optics components but this quickly becomes impractical when the experiments need to be scaled up to more qubits. Integrated photonics is a solution to this problem and allows for experiments with more qubits in a much smaller space. Optical circuits can be implemented on such chips, popular materials are silicon-on-insulator (SOI), lithium niobate and glass materials. Here we focus on SOI chips as they have many promising properties for the implementation of complex quantum optical circuits.

A key requirement for the full implementation of LOQC is a scalable, bright, deterministic and indistinguishable single photon source. Single photon sources in the SOI platform are typically made from the waveguide itself and use the spontaneous four-wave mixing which occurs in silicon due to the third order non-linearity, creating a single photon pair. This report aims to develop a method of measuring the indistinguishability of the produced photons with a classical technique, exploiting stimulated four-wave mixing. This method collects a joint spectrum which is an estimation of the spectral shape of the two photons produced by the source. For a full description the Joint Spectral Amplitude JSA is the desired quantity, this is a full description of the wavefunction of the single photons emitted by the silicon ring resonators. However it is only within the scope of this work to measure the Joint Spectral Intensity, which is the absolute value squared of the JSA. This allows

The mission is therefore to develop a methodology to reconstruct these wavefunctions and hence engineer indistinguishable (high purity) single photon sources. In this work we performed such measurements on three SOI chips. The experimental work started with an initial proof of concept that one can collect joint spectrum data in the way desired. This was done on a chip supplied by Marc Sorel from Glasgow University. Then due to the fragility of these chips at high powers the experiment progressed to a chip manufactured by Toshiba. Finally in order to investigate a promising new material amorphous silicon chip was used for experiments.

In parallel techniques of analysing the output data are developed. Filtering techniques which remove noise are developed in order to make the data usable. A general framework is set out which aims to quantify the certainty in the measurements.

Finally we conclude that there is still much to be done in this area, proposing an outline for how to carry out effective measurements in the future.

2 Detailed Background and Theory

2.1 Integrated silicon photonics

Integrated silicon photonics is a promising new platform on which to conduct quantum information experiments.

2.2 Marco Liscid - Why its okay to use classical to probe quantum and an introduction to four wavemixing

Marco [1]

2.3 Ring Resonators

Ring resonators are used as single photon sources. However to understand their behaviour to first order no quantum mechanics is needed. Here are the 3 governing equations:

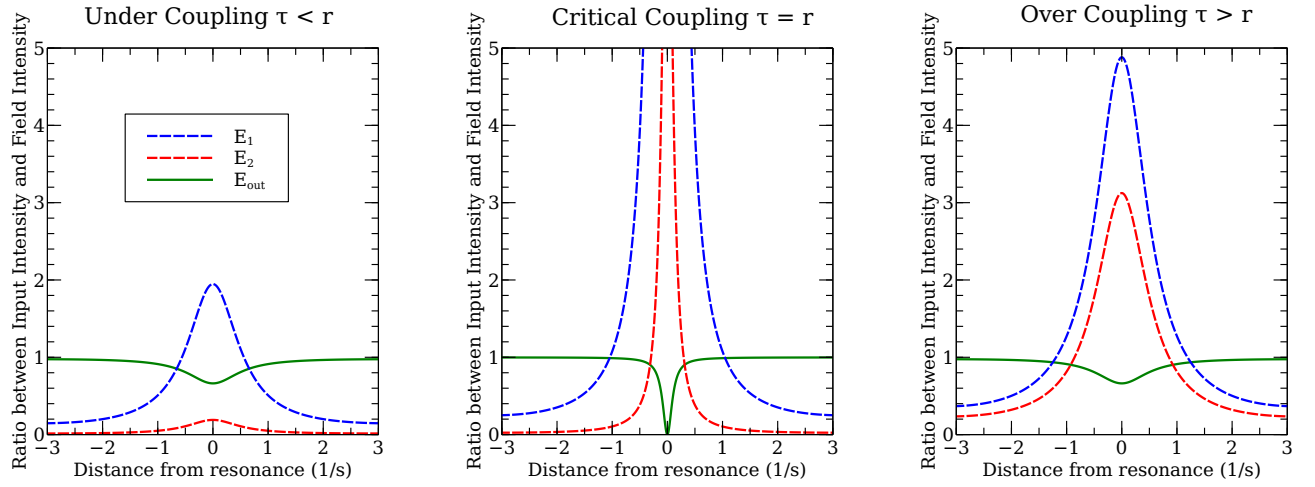


Figure 1: Notice how similar under and over coupling are to each other

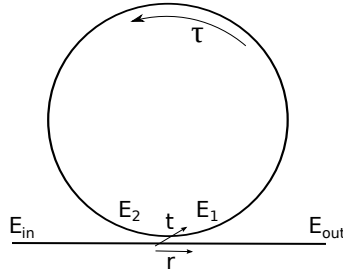


Figure 2: ahhh

$$\left| \frac{E_{out}}{E_0} \right|^2 = \frac{r^2 - 2r\tau \cos(\theta) + \tau^2}{1 + r^2\tau^2 - 2r\tau \cos(\theta)} \quad (2.1)$$

$$\left| \frac{E_1}{E_0} \right|^2 = \frac{t^2}{1 + r^2\tau^2 - 2r\tau \cos(\theta)} \quad (2.2)$$

$$\left| \frac{E_2}{E_0} \right|^2 = \tau^2 \left| \frac{E_1}{E_0} \right|^2 \quad (2.3)$$

2.4 Bistability

It can be experimentally observed that injecting power into a ring resonator will cause changes in the spectral position and shape of the resonance. Typically in silicon ring resonators the more power in the ring the more the resonance position is red-shifted by the thermo-optic effect [2]. Other effects are two photon absorption (TPA) and free carrier generation, these cause a blue shift [3].

2.5 Self phase modulation

2.6 Schmidt Rank and Purity

3 Method

3.1 Silicon Chips

3.2 Coupling

3.3 Joint Spectrum

3.4 $g^{(2)}(0)$

4 Results

4.1 Glassgow

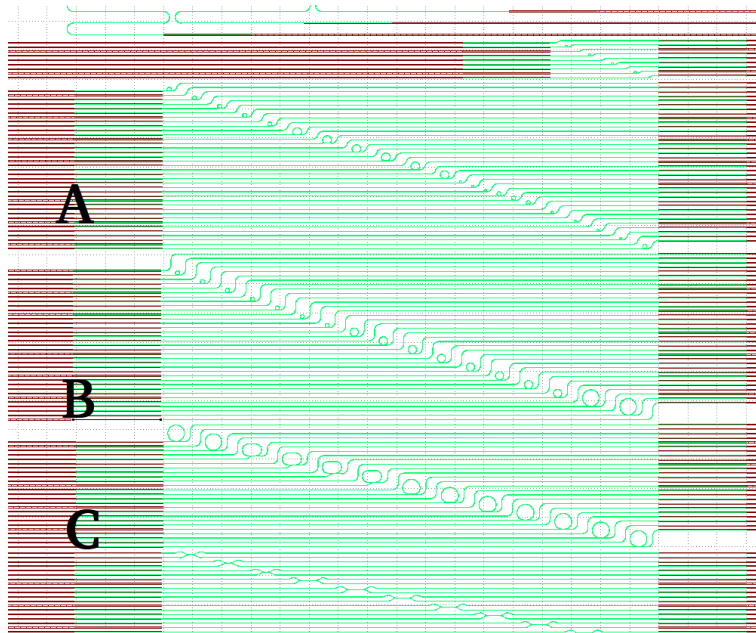


Figure 3: Glasgow test structure chip

4.2 Toshiba

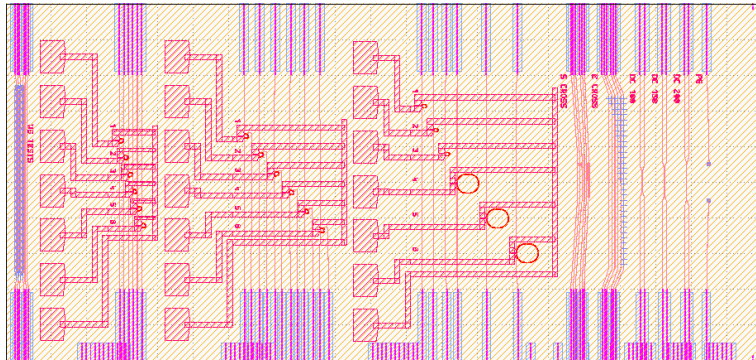


Figure 4: Glasgow test structure chip

4.2.1 Bistability Data

4.2.2 Pulse shaping

4.2.3 Power Scans

4.3 a-Si

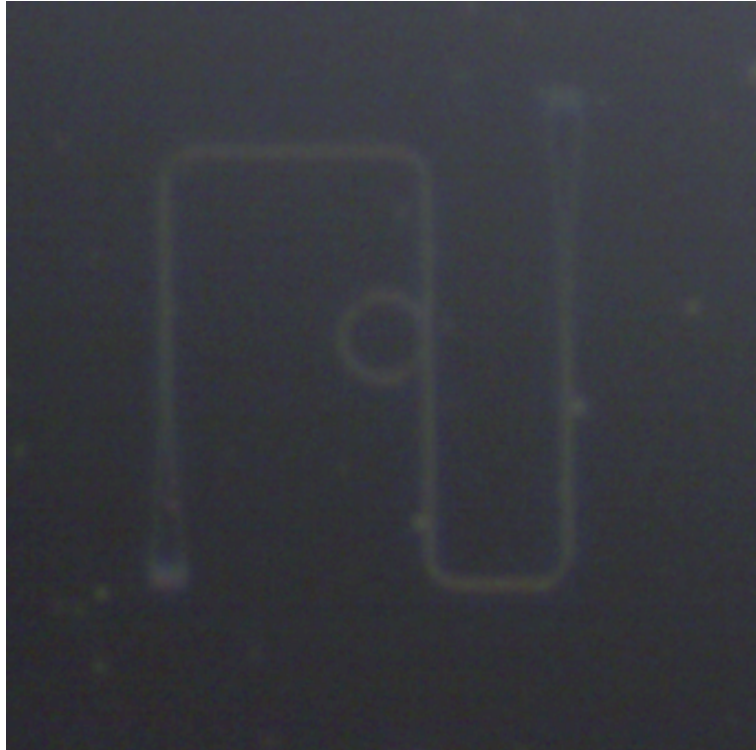


Figure 5: Glassgow test structure chip

5 Discussion

6 Conclusion

References

- [1] Lukas G. Helt, Marco Liscidini, and John E. Sipe. How does it scale? Comparing quantum and classical nonlinear optical processes in integrated devices. *J. Opt. Soc. Am. B*, 29(8):2199–2212, August 2012.
- [2] Vilson R. Almeida and Michal Lipson. Optical bistability on a silicon chip. *Opt. Lett.*, 29(20):2387–2389, October 2004.
- [3] Qianfan Xu and Michal Lipson. Carrier-induced optical bistability in silicon ring resonators. *Opt. Lett.*, 31(3):341–343, February 2006.
- [4] Andreas Eckstein, Guillaume Boucher, Aristide Lematre, Pascal Filloux, Ivan Favero, Giuseppe Leo, John E. Sipe, Marco Liscidini, and Sara Ducci. High-resolution spectral characterization of two photon states via classical measurements. *Laser & Photonics Reviews*, 8(5):L76–L80, September 2014.

A Schmidt Number

A.1 Definition

Starting with some arbitrary state ψ :

$$|\psi\rangle = \sum_{i,j} \alpha(i,j) |i\rangle_A \otimes |j\rangle_B \quad (\text{A.1})$$

The schmidt number K of this state measures the degree of entanglement. If $K = 1$ then you can find $|\psi\rangle = |\xi\rangle \otimes |\eta\rangle$ and for $K > 1$ you can find:

$$|\psi\rangle = \sum_i^K r_i |\xi_i\rangle_A \otimes |\eta_i\rangle_B \quad (\text{A.2})$$

Note that $1 \leq K \leq D$ where D is the dimension of the system. The purity is the inverse of K so:

$$P = 1/K \quad (\text{A.3})$$

An expression for K can be found using the density matrix for ψ :

$$\rho_{AB} = |\psi\rangle\langle\psi| = \sum_{i,j,k,l} \alpha(i,j) \alpha^*(k,l) |i\rangle\langle k| \otimes |j\rangle\langle l| \quad (\text{A.4})$$

$$\rho_A = \text{Tr}_B(\rho_{AB}) = \sum_{i,j,k} \alpha(i,j) \alpha^*(k,j) |i\rangle\langle k| \quad (\text{A.5})$$

$$\rho_A^2 = \sum_{i',j',k'} \sum_{i,j,k} \alpha(i,j) \alpha(k,j) \alpha^*(i',j') \alpha^*(k',j') |i\rangle\langle k| |i'\rangle\langle k'| \quad (\text{A.6})$$

$$= \sum_{j',k'} \sum_{i,j,k} \alpha(i,j) \alpha^*(k,j) \alpha(k,j') \alpha^*(k',j') |i\rangle\langle k'| \quad (\text{A.7})$$

$$\text{Tr}_A(\rho_A^2) = \sum_{i,j,k,j'} \alpha(i,j) \alpha^*(k,j) \alpha(k,j') \alpha^*(i,j') \quad (\text{A.8})$$

$$(\text{A.9})$$

For a unentangled ψ we know that $\text{Tr}_A(\rho_A^2) = 1$ For ψ entangled this will be smaller than 1 (proof comes from the property of the density operator that its eigenvalues are all smaller than 1). This fits the definition of the purity of a quantum state hence we can write:

$$P = \frac{1}{K} = \sum_{i,j,k,l} \alpha(i,j) \alpha^*(k,j) \alpha(k,l) \alpha^*(i,l) \quad (\text{A.10})$$

A.2 Calculation from experimental data

A.2.1 Trace method

In the lab we can measure $|\phi(\omega_1, \omega_2)|^2$, here I outline how to extract the schmidt number from this set of values. Taking the positive square root of the matrix of values obtained from the lab you have a matrix \mathbf{f} given by:

$$\mathbf{f} = \sum_{\omega_1, \omega_2} \phi(\omega_1, \omega_2) |\omega_1\rangle\langle\omega_2| \quad (\text{A.11})$$

(This seems to be some weird way of writing the wavefunction as a matrix, bare with me it turns out to be useful)

$$\mathbf{f}^\dagger \mathbf{f} = \sum_{\omega_1, \omega_2, \omega_3} \phi(\omega_1, \omega_2) \phi(\omega_3, \omega_2) |\omega_1\rangle \langle \omega_3| \quad (\text{A.12})$$

$$(\mathbf{f}^\dagger \mathbf{f})^2 = \sum_{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6} \phi(\omega_1, \omega_2) \phi(\omega_3, \omega_2) \phi(\omega_4, \omega_5) \phi(\omega_6, \omega_5) |\omega_1\rangle \langle \omega_3| \omega_4\rangle \langle \omega_6| \quad (\text{A.13})$$

$$(\mathbf{f}^\dagger \mathbf{f})^2 = \sum_{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6} \phi(\omega_1, \omega_2) \phi(\omega_3, \omega_2) \phi(\omega_3, \omega_5) \phi(\omega_6, \omega_5) |\omega_1\rangle \langle \omega_6| \quad (\text{A.14})$$

$$\text{Tr} [(\mathbf{f}^\dagger \mathbf{f})^2] = \sum_{\omega_1, \omega_2, \omega_3, \omega_4} \phi(\omega_1, \omega_2) \phi(\omega_3, \omega_2) \phi(\omega_3, \omega_4) \phi(\omega_1, \omega_4) \quad (\text{A.15})$$

I've done it this way because I wanted to figure out where the equation in [4] comes from. You can now see that equation A.10 is of exactly the same form as $\text{Tr} [(\mathbf{f}^\dagger \mathbf{f})^2]$ (barring the conjugates but this is okay since ϕ is real.) Taking the parallel further it can be seen that equation A.12 is of the form of a reduced density matrix. Here we must make sure to normalise to make sure this is a valid reduced density matrix. The normalisation is:

$$N = \text{Tr} [\mathbf{f}^\dagger \mathbf{f}] = \sum_{\omega_1, \omega_2} \phi(\omega_1, \omega_2)^2 \quad (\text{A.16})$$

Giving:

$$\rho_A = \frac{\mathbf{f}^\dagger \mathbf{f}}{N} \quad (\text{A.17})$$

We can then write:

$$\frac{1}{K} = \frac{\text{Tr} [(\mathbf{f}^\dagger \mathbf{f})^2]}{\text{Tr} [\mathbf{f}^\dagger \mathbf{f}]^2} \quad (\text{A.18})$$

B Joint Spectrum Code

```
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <time.h>
#include "TunicsInterface.h"
#include "OsaInterface.h"
#include "PiezoInterface.h"
#include "PowermeterInterface.h"
#include "Recoupling.h"
#include "VOAInterface.h"
#include "XTA50Interface.h"

// Turn off the laser and get a blank spectrum
void getBlankSpectrum(TunicsHandle laser, OsaHandle * osa, char * name, int amount);

// Prints out a 2D matrix representing the joint spectrum to file
void printJointSpectrumWavelengthData(char * filename, osaRawData ** spectrum, char delimiter, int number);

// Take spectral scan, main loop copied from the spectral scan program
int takeSpectralScan(float lmin, float lmax, float lstep, float attenuation, PowermeterHandle outmeter, PiezoHandle leftPiezo, PiezoHandle rightPiezo, float maximumZVoltage, float currentAttenuationLin, int JSAsToTake, int jsaNumber, int tunableFilterOnCW);

// Prints out a regular spectrum to file
void printSpectrum(char * filename, osaRawData * spectrum, char delimiter);

void takeAndPrintJointSpectrum(OsaHandle * osa,
                               TunicsHandle laser,
                               PowermeterHandle tapPowerMeter,
                               PowermeterHandle chipPowerMeter,
                               float osaStartWavelength_nm,
                               float osaEndWavelength_nm,
                               float osaSampleNumber,
                               float startWavelength,
                               float endWavelength,
                               float numberOfMeasurements,
                               float seedResolution,
                               int manageCoupling,
                               PiezoHandle leftPiezo,
                               PiezoHandle rightPiezo,
                               float maximumZVoltage,
                               float currentAttenuationLin,
                               int JSAsToTake,
                               int jsaNumber,
                               int tunableFilterOnCW);

XTA50System XTA50;

/* Modes of operation
1. Normal JSA
2. Tunable Filter on CW, Normal
```

```

        3. Normal Attenuation Scan
        4. Tunable Filter on CW, Attenuation Scan
        5. Tunable Filter on Pump, FWHM Scan
*/

int main (int argc, char** argv)
{
    if (argc != 31)
    {
        fprintf (stderr, "Error: Incorrect number of inputs. Expected 22, received %i.\n", argc-1);
        return 1;
    }

    int manageCoupling;
    float startWavelength, endWavelength, seedResolution, maximumZVoltage;
    char laserCom[256];
    char osaConnectionDescriptor[256];
    char leftPiezoCom[256];
    char rightPiezoCom[256];
    char chipPowermeterCom[256];
    char tapPowermeterCom[256];
    float cwLaserPower_mW = 0;
    float pumpLaserCurrent_mA = 0;
    float EDFACurrent_mA = 0;
    int scanType = 0;
    float startAttenuation, endAttenuation, attenuationStep;
    int cwAWGChannel, pumpAWGChannel;
    char chipName[256];
    float pumpWavelength;
    float TunableFilterCWOffset;
    float FWHMStart;
    float FWHMStep;
    float FWHMEnd;
    char tunableFilterCom[256];
    char VOACom[256];
    float ChipPowermeterAttenuation;
    float TapPowermeterAttenuation;

    sscanf(argv[1], "%f", &startWavelength);
    sscanf(argv[2], "%f", &endWavelength);
    sscanf(argv[3], "%f", &seedResolution);
    sscanf(argv[4], "%i", &manageCoupling);
    sscanf(argv[5], "%f", &maximumZVoltage);
    strcpy(laserCom,argv[6]);
    strcpy(osaConnectionDescriptor,argv[7]);
    strcpy(leftPiezoCom,argv[8]);
    strcpy(rightPiezoCom,argv[9]);
    strcpy(chipPowermeterCom,argv[10]);
    strcpy(tapPowermeterCom,argv[11]);
    sscanf(argv[12], "%f", &cwLaserPower_mW);

```



```

sscanf(argv[13], "%f", &pumpLaserCurrent_mA);
sscanf(argv[14], "%f", &EDFACurrent_mA);
sscanf(argv[15], "%d", &scanType);
sscanf(argv[16], "%f", &startAttenuation);
sscanf(argv[17], "%f", &endAttenuation);
sscanf(argv[18], "%f", &attenuationStep);
sscanf(argv[19], "%d", &cwAWGChannel);
sscanf(argv[20], "%d", &pumpAWGChannel);
strcpy(chipName,argv[21]);
sscanf(argv[22], "%f", &pumpWavelength);
sscanf(argv[23], "%f", &TunableFilterCWOffset);
sscanf(argv[24], "%f", &FWHMStart);
sscanf(argv[25], "%f", &FWHMStep);
sscanf(argv[26], "%f", &FWHMEnd);
strcpy(tunableFilterCom,argv[27]);
strcpy(VOACom,argv[28]);
sscanf(argv[29], "%f", &ChipPowermeterAttenuation);
sscanf(argv[30], "%f", &TapPowermeterAttenuation);

printf("min = %f\nmax= %f\nstep = %f\n", startWavelength,endWavelength,seedResolution);
VOAHandle voaHandle = InitVOA(VOACom);

// Set up devices
OsaHandle * osa = InitOsa(osaConnectionDescriptor);

//osaRawData * spec = osaGetSpectrum(osa);
//printSpectrum("OSASpectrum950nm.txt", spec, ' ');
//return 0;

/* CW Laser */
TunicsHandle laser = InitTunics(laserCom);
PowermeterHandle chipPowerMeter = InitPowermeter(chipPowermeterCom);
PowermeterHandle tapPowerMeter = InitPowermeter(tapPowermeterCom);

/* Piezo for side coupling */
PiezoHandle leftPiezo;
PiezoHandle rightPiezo;
if(manageCoupling == 1)
{
    leftPiezo = InitPiezo(leftPiezoCom);
    rightPiezo = InitPiezo(rightPiezoCom);
}

/* Start the tunable filter */
XTA50_SerialCom_type XTA50Vars;
int useTunableFilterForCW = 0;
if(scanType == 2 || scanType == 3 || scanType == 5)
{
    sprintf(XTA50Vars.COMPort,tunableFilterCom);
    InitXTA50(XTA50Vars);
}

```

```

        if(scanType == 2 || scanType == 4)
        {
            useTunableFilterForCW = 1;
        }
    }

    // CW LasertapPowerMeter
    int numberOfMeasurements = (int)ceil((endWavelength-startWavelength)/seedResolution)+1;

    // Get important data form OSA
    float * tempArray;
    tempArray = getNumericalSettings(osa,"DCA?",3); // Get wavelength range and sample number
    float osaStartWavelength_nm = tempArray[0];
    float osaEndWavelength_nm = tempArray[1];
    float osaSampleNumber = (int)tempArray[2];

    tempArray = getNumericalSettings(osa,"RES?",1); // Get OSA resolution
    float osaResolution_nm = tempArray[0];

    char * VBW = getTextSetting(osa,"VBW?"); // Get OSA video bandwidth
    char * HDR = getTextSetting(osa,"DRG?"); // Get dynamic range mode

    // Create a big file with as much information about the joint spectrum as possible
    // Open files
    FILE * infoFile = fopen("info.txt", "w");
    fprintf(infoFile, "#General Information\n");
    fprintf(infoFile, "Chip : %s\n",chipName);
    // Get the time
    time_t current_time;
    char* c_time_string;
    current_time=time(NULL);
    c_time_string = ctime(&current_time);
    fprintf(infoFile, "Timestamp: ");
    fprintf(infoFile, c_time_string);
    fprintf(infoFile, "\n");
    fprintf(infoFile, "Notes : \n");
    fprintf(infoFile, "Recoupling enabled: %d\n\n", manageCoupling);
    fprintf(infoFile, "#CW Laser Parameters\n");
    fprintf(infoFile, "AWG Channel : %d\n",cwAWGChannel);
    fprintf(infoFile, "Start wavelength (nm): %f\n",startWavelength);
    fprintf(infoFile, "End wavelength (nm) : %f\n",endWavelength);
    fprintf(infoFile, "Wavelength step (nm) : %f\n",seedResolution);
    fprintf(infoFile, "Sample number : %d\n",numberOfMeasurements);
    fprintf(infoFile, "Laser power (mW): %f\n\n",cwLaserPower_mW);

    fprintf(infoFile, "#Pump laser parameters\n");
    fprintf(infoFile, "AWG Channel : %d\n",pumpAWGChannel);
    fprintf(infoFile, "Wavelength (nm) : %f\n",pumpWavelength);
    fprintf(infoFile, "Current (mA): %f\n\n",pumpLaserCurrent_mA);

    fprintf(infoFile, "#EDFA parameters\n");

```

```

fprintf(infoFile, "Current (mA): %d\n\n",EDFACurrent_mA);

fprintf(infoFile, "# OSA Settings \n");
fprintf(infoFile, "Start Wavelength (nm) : %f\n",osaStartWavelength_nm);
fprintf(infoFile, "End Wavelength (nm) : %f\n",osaEndWavelength_nm);
fprintf(infoFile, "Sample Number : %f\n",osaSampleNumber);
fprintf(infoFile, "Resolution (nm) : %f\n",osaResolution_nm);
fprintf(infoFile, "Video Bandwidth : %s",VBW);
fprintf(infoFile, "Dynamic Range Mode: : %s\n",HDR);

SetTunicsEmission (laser,1);
if(useTunableFilterForCW)
{
    SetLambda(startWavelength-0.08);
}
SetTunicsWavelength(laser,startWavelength);
float initialChipPower = 10.0*log10(MeasurePowermeter(chipPowerMeter)) + 30 + ChipPowermeterAttenuation;
float initialTapPower = 10.0*log10(MeasurePowermeter(tapPowerMeter)) + 30 + TapPowermeterAttenuation;

fprintf(infoFile,"#Powermeter initial readings\n");
fprintf(infoFile, "Before Chip (dBm): %f\n",initialTapPower);
fprintf(infoFile, "After Chip (dBm): %f\n",initialChipPower);
fprintf(infoFile, "Loss (dBm): %f\n\n",initialTapPower-initialChipPower);

system("MKDIR osaSpectrumsWithNoCW");
system("MKDIR jointSpectrums");
system("MKDIR powerLogs");
//system("MKDIR spectralScans");

/* Buffer for filenames */
char str[500];
// Normal JSA
// Tunable Filter on CW, Normal
if(scanType == 1 || scanType == 2)
{
    getBlankSpectrum(laser,osa,"osaSpectrumsWithNoCW/noSeedSpectrumBefore",5);
    SetTunicsEmission(laser,1);
    //takeSpectralScan(startWavelength,endWavelength,seedResolution,0,chipPowerMeter,tapPowerMeter,1);
    takeAndPrintJointSpectrum(osa,
        laser,
        tapPowerMeter,
        chipPowerMeter,
        osaStartWavelength_nm,
        osaEndWavelength_nm,
        osaSampleNumber,
        startWavelength,
        endWavelength,
        numberOfMeasurements,
        seedResolution,
        manageCoupling,
        leftPiezo,

```

```

        rightPiezo,
        maximumZVoltage,
        0,
        1,
        0,
        useTunableFilterForCW);
    getBlankSpectrum(laser,osa,"osaSpectrumsWithNoCW/noSeedSpectrumAfter",5);
    SetTunicsEmission(laser,1);
}
// Attenuation Scan
// Tunable Filter on CW, Attenuation Scan
if(scanType == 3 || scanType == 4)
{
    fprintf(infoFile,"#Attenuator Settings\n");
    fprintf(infoFile, "Attenuation enabled?: %d\n",scanType);
    fprintf(infoFile, "Start Attenuation: %f\n",startAttenuation);
    fprintf(infoFile, "End Attenuation: %f\n",endAttenuation);
    fprintf(infoFile, "Attenuation Step: %f\n\n",attenuationStep);
    fprintf(infoFile,"#Attenuations\nno. watts DB Tap Power Chip Power\n");
    int numberOfJSAsToTake = (int)((endAttenuation - startAttenuation)/attenuationStep) + 1;
    float currentTapPower;
    float currentChipPower;
    SetDVA_attenuation(voaHandle,10*log10(startAttenuation));
    int i;
    for(i = 0; i < numberOfJSAsToTake; ++i)
    {
        sprintf(str,"osaSpectrumsWithNoCW/noSeedSpectrumBefore_%i_",i);

        getBlankSpectrum(laser,osa,str,5);
        SetTunicsEmission(laser,1);

        float currentAttenuationLin = startAttenuation + attenuationStep*(float)i;
        float currentAttenuation_dB = -10 * log10 (1-currentAttenuationLin);
        SetAttenuationLin(voaHandle,currentAttenuationLin);

        printf("Trying to set attenuator to %f dB\n", currentAttenuation_dB);
        currentTapPower = 10.0*log10(MeasurePowermeter(tapPowerMeter)) + 30 + TapPowermeterAttenuation;
        currentChipPower = 10.0*log10(MeasurePowermeter(chipPowerMeter)) + 30 + ChipPowermeterAttenuation;
        fprintf(infoFile, "%d %f %f %f %f\n",i,currentAttenuationLin,currentAttenuation_dB,currentTapPower,currentChipPower);
        //takeSpectralScan(startWavelength,endWavelength,seedResolution,currentAttenuation_dB,chipPower);
        takeAndPrintJointSpectrum(osa,
            laser,
            tapPowerMeter,
            chipPowerMeter,
            osaStartWavelength_nm,
            osaEndWavelength_nm,
            osaSampleNumber,
            startWavelength,
            endWavelength,
            numberOfMeasurements,
            seedResolution,

```

```

        manageCoupling,
        leftPiezo,
        rightPiezo,
        maximumZVoltage,
        currentAttenuationLin,
        numberOfJSAsToTake,
        i,
        useTunableFilterForCW);

if(manageCoupling == 1)
{
    if(GetPiezoVoltage(leftPiezo,'z') < maximumZVoltage)
    {
        printf("Recoupling left piezo.\n");
        RecoupleDynamic(chipPowerMeter,leftPiezo,100);
    }
    else
    {
        printf("LEFT Z MAX REACHED!");
    }
    if(GetPiezoVoltage(rightPiezo,'z') < maximumZVoltage)
    {
        printf("Recoupling right piezo.\n");
        RecoupleDynamic(chipPowerMeter,rightPiezo,100);
    }
    else
    {
        printf("RIGHT Z MAX REACHED!");
    }
}

}

sprintf(str,"osaSpectrumsWithNoCW/noSeedSpectrumBefore_%i_",i);
getBlankSpectrum(laser,osa,str,5);
SetTunicsEmission(laser,1);
}

//Tunable Filter on Pump, FWHM Scan
if(scanType == 5)
{
    for(int i = 0; i < (FWHMEnd-FWHMStart)/FWHMStep + 1; ++i)
    {
        float FWHM = FWHMStart + (float)i*FWHMStep;
        SetFWHM(FWHM);
        printf("FWHM = %f\n", FWHM);

        sprintf(str,"osaSpectrumsWithNoCW/noSeedSpectrumBefore_%i_",i);
        getBlankSpectrum(laser,osa,str,5);
        SetTunicsEmission(laser,1);

        fprintf(infoFile, "%d %f %f\n",i,FWHM,MeasurePowermeter(tapPowerMeter),MeasurePowermeter(chip

```

```

        takeAndPrintJointSpectrum(osa,
                                laser,
                                tapPowerMeter,
                                chipPowerMeter,
                                osaStartWavelength_nm,
                                osaEndWavelength_nm,
                                osaSampleNumber,
                                startWavelength,
                                endWavelength,
                                numberOfMeasurements,
                                seedResolution,
                                manageCoupling,
                                leftPiezo,
                                rightPiezo,
                                maximumZVoltage,
                                0,
                                1,
                                i,
                                useTunableFilterForCW);
        sprintf(str,"osaSpectrumsWithNoCW/noSeedSpectrumAfter_%i_",i);
        getBlankSpectrum(laser,osa,str,5);
        SetTunicsEmission(laser,1);
    }
    if(manageCoupling == 1)
    {
        if(GetPiezoVoltage(leftPiezo,'z') < maximumZVoltage)
        {
            printf("Recoupling left piezo.\n");
            RecoupleDynamic(chipPowerMeter,leftPiezo,100);
        }
        else
        {
            printf("LEFT Z MAX REACHED!");
        }
        if(GetPiezoVoltage(rightPiezo,'z') < maximumZVoltage)
        {
            printf("Recoupling right piezo.\n");
            RecoupleDynamic(chipPowerMeter,rightPiezo,100);
        }
        else
        {
            printf("RIGHT Z MAX REACHED!");
        }
    }
    fclose(infoFile);
}

// Record chip transmission
fprintf(infoFile,"#Powermeter final readings\n");
if(useTunableFilterForCW)
{

```

```

        SetLambda(startWavelength-0.08);
    }
    SetTunicsWavelength(laser,startWavelength);
    float chip = 10.0*log10(MeasurePowermeter(chipPowerMeter)) + 30 + ChipPowermeterAttenuation;
    float tap = 10.0*log10(MeasurePowermeter(tapPowerMeter)) + 30 + TapPowermeterAttenuation;
    fprintf(infoFile, "Before Chip (dBm): %f\n",tap);
    fprintf(infoFile, "After Chip (dBm): %f\n",chip);
    fprintf(infoFile, "Loss (dBm): %f\n\n",tap-chip);
    fclose(infoFile);

    //Disable the laser
    SetTunicsEmission(laser,0);

    if(manageCoupling == 1)
    {
        //Retract the piezos (Hopefully zero is backwards)
        RampToVoltage(leftPiezo,0.00,'z');
        RampToVoltage(rightPiezo,0.00,'z');
    }

    /* Close all the things */
    ClosePowermeter(tapPowerMeter);
    ClosePowermeter(chipPowerMeter);
    if(manageCoupling == 1)
    {
        ClosePiezo(leftPiezo,0);
        ClosePiezo(rightPiezo,0);
    }
    CloseTunics(laser);
    osaWrite(osa,"SRT"); // Tell the osa to keep taking spectrums
    //CloseOsa(osa);
    CloseXTA50();
    return 0;
}

void getBlankSpectrum(TunicsHandle laser, OsaHandle * osa, char * name, int amount)
{
    char str[500];
    SetTunicsEmission(laser,0);
    /* Get a spectrum after experiment, might come in handy */
    printf("Taking initial spectrum without seed laser.\n");
    osaRawData * spectrumWithoutSeedLaser;
    for(int i = 0; i < amount; ++i)
    {
        spectrumWithoutSeedLaser = osaGetSpectrum(osa);
        sprintf(str,"%s%d%s",name,i,".txt");
        printSpectrum(str,spectrumWithoutSeedLaser,',');
    }
}

```

```

void printSpectrum(char * filename, osaRawData * spectrum, char delimiter)
{
    FILE * fp = fopen(filename, "w");
    float * singleSpectrum = parseRawDataIntoFloatArray(spectrum,spectrum->osaSampleNumber,'\n');
    for(int i = 0; i < spectrum->osaSampleNumber; ++i)
    {
        fprintf(fp,"%e %e\n", spectrum->osaStartWavelength + i*(spectrum->osaEndWavelength - spectrum->osaStartWavelength), singleSpectrum[i]);
    }

    fclose(fp);
}

void printJointSpectrumWavelengthData(char * filename, osaRawData ** spectrum, char delimiter, int numberOfSeedReadings)
{
    /* Print out the wavelengths to files (Probably pointless) */
    #define BUFLLEN 128

    char buffer[BUFLLEN];
    strcpy(buffer,filename);
    strcpy(buffer,strtok(buffer, "."));

    char osaFilename[BUFLLEN];
    strcpy(osaFilename,buffer);

    char seedFilename[BUFLLEN];
    strcpy(seedFilename,buffer);

    strcat(osaFilename,"_osaWavelengths.txt");
    strcat(seedFilename,"_seedWavelengths.txt");

    FILE * osaWavelengths = fopen(osaFilename, "w");
    FILE * seedWavelengths = fopen(seedFilename, "w");

    for(int i = 0; i < spectrum[0]->osaSampleNumber; ++i)
    {
        float value = spectrum[0]->osaStartWavelength + (float)i*(spectrum[0]->osaEndWavelength - spectrum[0]->osaStartWavelength);
        fprintf(osaWavelengths, "%e\n", value);
    }

    for(int i = 0; i < spectrum[0]->numberOfSeedReadings; ++i)
    {
        float value = spectrum[i]->seedWavelength;
        fprintf(seedWavelengths, "%e\n", value);
    }

    fclose(osaWavelengths);
    fclose(seedWavelengths);
}

void takeAndPrintJointSpectrum(OsaHandle * osa,

```



```

        TunicsHandle laser,
        PowermeterHandle tapPowerMeter,
        PowermeterHandle chipPowerMeter,
        float osaStartWavelength_nm,
        float osaEndWavelength_nm,
        float osaSampleNumber,
        float startWavelength,
        float endWavelength,
        float numberOfMeasurements,
        float seedResolution,
        int manageCoupling,
        PiezoHandle leftPiezo,
        PiezoHandle rightPiezo,
        float maximumZVoltage,
        float currentAttenuationLin,
        int JSAsToTake,
        int i,
        int tunableFilterOnCW)
{

    float currentAttenuation_dB = -10 * log10 (1-currentAttenuationLin);

    // 2 Dimensional data
    osaRawData ** spectrum = NULL;

    // Alloc the data
    spectrum = malloc(sizeof(osaRawData)*numberOfMeasurements);
    char str[256];
    sprintf(str,"jointSpectrums/jsa_%d.txt",i);
    FILE * dataFile = fopen(str, "w");
    sprintf(str,"jointSpectrums/normalised_jsa_%d.txt",i);
    FILE * normalisedDataFile = fopen(str, "w");
    sprintf(str,"powerLogs/powerLog_%d.txt",i);
    FILE * powerFile = fopen(str, "w");

    fprintf(dataFile,"xrange %e %e\nyrange %e %e\ninvertrows\n", osaStartWavelength_nm, osaEndWavelength_nm,
    fprintf(normalisedDataFile,"xrange %e %e\nyrange %e %e\ninvertrows\n", osaStartWavelength_nm, osaEndWavelength_nm,

    float wavelength = startWavelength;

    float minsLeft = 0.0f;
    for(int i = 0; i < numberOfMeasurements; ++i)
    {
        int startTime = time(0);
        /* Set CW laser wavelength */
        if(tunableFilterOnCW)
        {
            SetLambda(wavelength-0.08);
        }
        SetTunicsWavelength(laser,wavelength);
    }
}

```

```

// Record the readings on the power meters
float tapPower = MeasurePowermeter(tapPowerMeter)*100; //++20 accounts for the tapping
float chipPower = MeasurePowermeter(chipPowerMeter)*(100.0/99.0);
fprintf(powerFile,"%e,%e,%e\n",wavelength,tapPower,chipPower);
printf("Current wavelength %.3f\n", wavelength);
printf("Percentage: %.1f%\n", (double)100*i/(numberOfMeasurements-1));
printf("Attenuation %.1f (dB)\n", currentAttenuation_dB);
printf("Taking Spectrum...\n");

spectrum[i] = osaGetSpectrum(osa);

float normalisation = tapPower;
if(normalisation < 0)
{
    printf("Normalisation was negative, something is probably wrong with the set up!\n");
}

for(int j = 0; j < osaSampleNumber; ++j)
{
    fprintf(dataFile,"%e", (spectrum[i]->floatData)[j]);
    fprintf(normalisedDataFile,"%e", (spectrum[i]->floatData)[j]/normalisation);
    if(j != osaSampleNumber - 1)
    {
        fprintf(dataFile, " ");
        fprintf(normalisedDataFile, " ");
    }
}
fprintf(dataFile,"\n");
fprintf(normalisedDataFile,"\n");

/* Transfer CW laser settings to the spectrum struct */
spectrum[i]->seedWavelength = wavelength;
spectrum[i]->seedStartWavelength = startWavelength;
spectrum[i]->seedEndWavelength = endWavelength;
spectrum[i]->numberOfSeedReadings = (int)numberOfMeasurements;

/* Update the wavelength for the next loop */
wavelength += seedResolution;

int endTime = time(0);
minsLeft = (float)(endTime-startTime)*(float)(numberOfMeasurements-i+1+6)/(float)60;
float totalTimeLeft = minsLeft*(float)(JSAsToTake-i);
if(JSAsToTake > 1)
{
    printf("This scan should finish in %.2f mins.\n The experiment should finish in %.2f mins.\n");
}
else
{
    printf("This scan should finish in %.2f mins.\n\n", minsLeft);
}

```

```

    }

    fclose(dataFile);
    fclose(normalisedDataFile);

    /* Data Formatting */
    printJointSpectrumWavelengthData("jsa.txt", spectrum, '\n', numberOfMeasurements);

    /* Freeing data and closing devices */
    for(int i = 0; i < numberOfMeasurements; ++i)
    {
        free(spectrum[i]->data);
        free(spectrum[i]->floatData);
        free(spectrum[i]);
    }

    free(spectrum);
}

int takeSpectralScan(float lmin, float lmax, float lstep, float attenuation, PowermeterHandle outmeter, P
{
    char str[256];
    sprintf(str, "spectralScans/spectralScan%8.3f-%8.3f_att_%.3f.csv", lmin, lmax, attenuation);
    FILE * fp = fopen(str, "w");

    //Move to start wavelength and enable laser
    SetTunicsWavelength (laser, lmin);
    SetTunicsEmission (laser, 1);
    Sleep (100);

    //Scan
    printf ("Scanning from %8.3f to %8.3f in steps of %.3f.\n", lmin, lmax, lstep);
    float l = 0;
    double p1, p2;
    for (int i = 0; l < lmax; i++)
    {
        l = lmin + i*lstep;

        //SetTunicsWavelengthFast (laser, l, DWELL_TIME);
        SetTunicsWavelength (laser, l);

        p1 = MeasurePowermeter (outmeter); // Measure output
        p2 = MeasurePowermeter (outmeter2); // Measure output

        fprintf (fp, "%e, %e, %e\n", l, p1, p2);
    }

    fclose(fp);
}

```

```

return 1;
}

```

C Transfer matrix analysis of ring resonator cavities

A useful way to describe the spectral response of a ring resonator is by defining a transfer matrix. This 2×2 matrix relates the electric fields of the inputs and outputs of the waveguide and resonator. In our analysis we define the electric fields to be complex valued and also normalised to conserve energy, this condition implies the transfer matrix must be unitary.

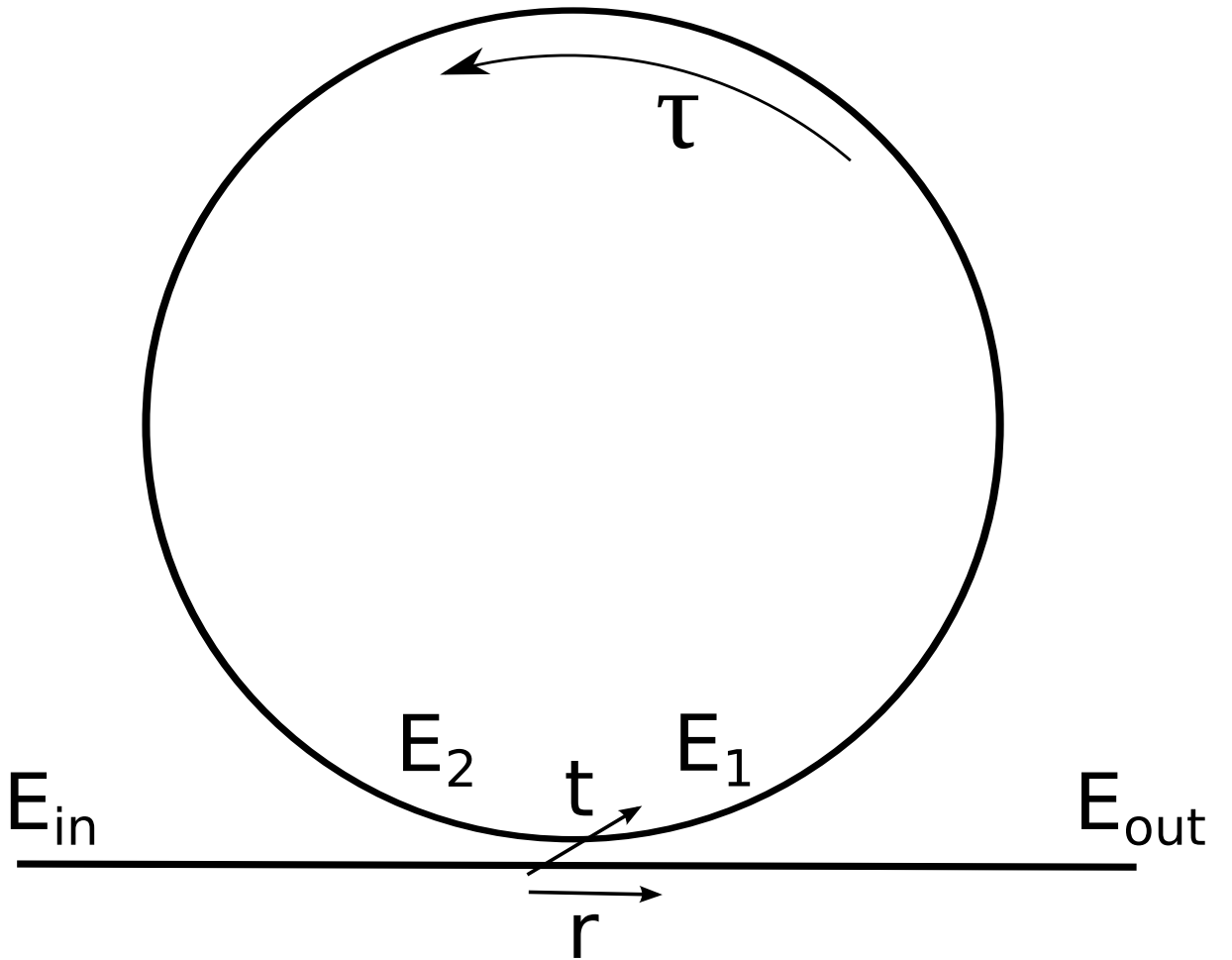


Figure 6: ahhh

First write down the obvious relations

$$E_{out} = r_1 E_{in} + t_1 E_2 \quad (C.1)$$

$$E_1 = r_2 E_2 + t_2 E_{in} \quad (C.2)$$

Define a matrix.

$$M = \begin{pmatrix} r_1 & t_1 \\ t_2 & r_2 \end{pmatrix} \quad (C.3)$$

Enforce unitarity

$$\begin{pmatrix} r_1 & t_1 \\ t_2 & r_2 \end{pmatrix} \begin{pmatrix} r_1^* & t_2^* \\ t_1^* & r_2^* \end{pmatrix} = \begin{pmatrix} |r_1|^2 + |t_1|^2 & r_1 t_2^* + r_2^* t_1 \\ r_2 t_1^* + r_1^* t_2 & |r_2|^2 + |t_2|^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (C.4)$$

Furthermore $\det M = 1$

$$r_1 r_2 - t_1 t_2 = 1 \quad (C.5)$$

so

$$r_2 = \frac{1 + t_1 t_2}{r_1} \quad (C.6)$$

Also $M^{-1} = M^*$

$$\begin{pmatrix} r_2 & -t_1 \\ -t_2 & r_1 \end{pmatrix} = \begin{pmatrix} r_1^* & t_1^* \\ t_2^* & r_2^* \end{pmatrix} \quad (C.7)$$

This tells us r_1 and r_2 must be real. So we can simplify the unitarity equation. So

$$t_1 = iT_1 \quad (C.8)$$

$$t_2 = iT_2 \quad (C.9)$$

Rewrite the interesting equations: Use also $r_2 = r_1^*$

$$\begin{pmatrix} |r_1|^2 + T_1^2 & -ir_1 T_2 + ir_1 T_1 \\ -ir_1^* T_1 + ir_1^* T_2 & |r_1|^2 + T_2^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (C.10)$$

Ok so $T_2 = T_1 = t$

$$\begin{pmatrix} |r_1|^2 + t^2 & 0 \\ 0 & |r_1|^2 + t^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (C.11)$$

$$|r_1|^2 = 1 - t^2 \quad (C.12)$$

$$|r_2|^2 = 1 - t^2 \quad (C.13)$$

d Basically I have a degree of freedom. $r_1 = e^{i\theta} r$ and $r_2 = e^{-i\theta} r$. Choose $\theta = 0$ for simplicity.

Now the matrix is:

$$\begin{pmatrix} r & it \\ it & r \end{pmatrix} \quad (C.14)$$

Now we add that

$$E_2 = \tau e^{i\theta} E_1$$

Recall the initial equations.

$$E_{out} = r E_{in} + it E_2 \quad (C.15)$$

$$E_1 = rE_2 + itE_{in} \quad (C.16)$$

$$E_1 = r\tau e^{i\theta} E_1 + itE_{in} \quad (C.17)$$

$$\frac{E_1}{E_0} = \frac{it}{1 - r\tau e^{i\theta}} \quad (C.18)$$

Okay so that's E_1 done.

$$\frac{E_1}{E_0} = \frac{it}{1 - r\tau e^{i\theta}} \quad (C.19)$$

$$\left| \frac{E_1}{E_0} \right|^2 = \frac{t^2}{1 + r^2\tau^2 - 2r\tau\cos(\theta)} \quad (C.20)$$

For E_2 we have:

$$\frac{E_2}{E_0} = \frac{it\tau e^{i\theta}}{1 - r\tau e^{i\theta}} \quad (C.21)$$

$$\left| \frac{E_2}{E_0} \right|^2 = \frac{\tau^2 - \tau^2 r^2}{1 + r^2\tau^2 - 2r\tau\cos(\theta)} \quad (C.22)$$

For E_{out} we have:

$$\frac{E_{out}}{E_0} = r + it \frac{it\tau e^{i\theta}}{1 - r\tau e^{i\theta}} \quad (C.23)$$

$$\frac{E_{out}}{E_0} = \frac{r - r^2\tau e^{i\theta} - t^2\tau e^{i\theta}}{1 - r\tau e^{i\theta}} \quad (C.24)$$

$$\left| \frac{E_{out}}{E_0} \right|^2 = \frac{(r - r^2\tau e^{i\theta} - t^2\tau e^{i\theta})(r - r^2\tau e^{-i\theta} - t^2\tau e^{-i\theta})}{(1 - r\tau e^{i\theta})(1 - r\tau e^{-i\theta})} \quad (C.25)$$

$$\left| \frac{E_{out}}{E_0} \right|^2 = \frac{(r(r - r^2\tau e^{-i\theta} - t^2\tau e^{-i\theta}) - r^2\tau e^{i\theta}(r - r^2\tau e^{-i\theta} - t^2\tau e^{-i\theta}) - t^2\tau e^{i\theta}(r - r^2\tau e^{-i\theta} - t^2\tau e^{-i\theta}))}{1 + r^2\tau^2 - 2r\tau\cos(\theta)} \quad (C.26)$$

$$\left| \frac{E_{out}}{E_0} \right|^2 = \frac{r^2 - r^3\tau e^{-i\theta} - rt^2\tau e^{-i\theta} - r^3\tau e^{i\theta} + r^4\tau^2 + 2r^2\tau^2t^2 - rt^2\tau e^{i\theta} + t^4\tau^2}{1 + r^2\tau^2 - 2r\tau\cos(\theta)} \quad (C.27)$$

$$\left| \frac{E_{out}}{E_0} \right|^2 = \frac{r^2 - e^{-i\theta}(r^3\tau + rt^2\tau) - e^{i\theta}(r^3\tau + rt^2\tau) + r^4\tau^2 + t^4\tau^2 + 2r^2\tau^2t^2}{1 + r^2\tau^2 - 2r\tau\cos(\theta)} \quad (C.28)$$

$$\left| \frac{E_{out}}{E_0} \right|^2 = \frac{r^2 + 2(r^3\tau + rt^2\tau)\cos(\theta) + r^4\tau^2 + t^4\tau^2 + 2r^2\tau^2t^2}{1 + r^2\tau^2 - 2r\tau\cos(\theta)} \quad (C.29)$$

$$\left| \frac{E_{out}}{E_0} \right|^2 = \frac{r^2 2(r^3\tau + r(1 - r^2)\tau)\cos(\theta) + r^4\tau^2 + (1 - r^2)^2\tau^2 + 2r^2\tau^2(1 - r^2)}{1 + r^2\tau^2 - 2r\tau\cos(\theta)} \quad (C.30)$$

$$\left| \frac{E_{out}}{E_0} \right|^2 = \frac{r^2 - 2(r^3\tau + r(1 - r^2)\tau)\cos(\theta) + r^4\tau^2 + (1 - 2r^2 + r^4)\tau^2 + 2r^2\tau^2 - 2r^4\tau^2}{1 + r^2\tau^2 - 2r\tau\cos(\theta)} \quad (C.31)$$

$$\left| \frac{E_{out}}{E_0} \right|^2 = \frac{r^2 - 2r\tau\cos(\theta) + r^4\tau^2 + \tau^2 - 2r^2\tau^2 + r^4\tau^2 + 2r^2\tau^2 - 2r^4\tau^2}{1 + r^2\tau^2 - 2r\tau\cos(\theta)} \quad (C.32)$$

FINAL RESULTS

$$\left| \frac{E_{out}}{E_0} \right|^2 = \frac{r^2 - 2r\tau\cos(\theta) + \tau^2}{1 + r^2\tau^2 - 2r\tau\cos(\theta)} \quad (C.33)$$

$$\left|\frac{E_2}{E_0}\right|^2 = \frac{\tau^2 - \tau^2 r^2}{1 + r^2 \tau^2 - 2r\tau \cos(\theta)} = \tau^2 \left|\frac{E_1}{E_0}\right|^2 \quad (\text{C.34})$$

$$\left|\frac{E_1}{E_0}\right|^2 = \frac{t^2}{1 + r^2 \tau^2 - 2r\tau \cos(\theta)} \quad (\text{C.35})$$