

FINAL YEAR PROJECT, DISSERTATION OR
PHYSICS EDUCATION REPORT

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1 Introduction

The endeavour to build a quantum computer holds the promise of solving computational problems which are currently intractable on classical computers. A particularly promising platform for this is the linear optical quantum computer model which in theory allows for scalable universal quantum computation.

2 Theory

2.1 On chip four wave mixing

2.2 Ring Resonators

2.3 Bistability

2.4 Self phase modulation

2.5 Schmidt Rank and Purity

3 Method

3.1 Coupling

3.2 Bistability

3.3 Joint Spectrum

3.4 $g^{(2)}(0)$

4 Results

4.1 Glassgow

4.2 Toshiba

4.3 a-Si

5 Discussion

6 Conclusion

References

- [1] Andreas Eckstein, Guillaume Boucher, Aristide Lematre, Pascal Filloux, Ivan Favero, Giuseppe Leo, John E. Sipe, Marco Liscidini, and Sara Ducci. High-resolution spectral characterization of two photon states via classical measurements. *Laser & Photonics Reviews*, 8(5):L76–L80, September 2014.

A Schmidt Number

A.1 Definition

Starting with some arbitrary state ψ :

$$|\psi\rangle = \sum_{i,j} \alpha(i,j) |i\rangle_A \otimes |j\rangle_B \quad (1)$$

The schmidt number K of this state measures the degree of entanglement. If $K = 1$ then you can find $|\psi\rangle = |\xi\rangle \otimes |\eta\rangle$ and for $K > 1$ you can find:

$$|\psi\rangle = \sum_i^K r_i |\xi_i\rangle_A \otimes |\eta_i\rangle_B \quad (2)$$

Note that $1 \leq K \leq D$ where D is the dimension of the system. The purity is the inverse of K so:

$$P = 1/K \quad (3)$$

An expression for K can be found using the density matrix for ψ :

$$\rho_{AB} = |\psi\rangle\langle\psi| = \sum_{i,j,k,l} \alpha(i,j) \alpha^*(k,l) |i\rangle\langle k| \otimes |j\rangle\langle l| \quad (4)$$

$$\rho_A = \text{Tr}_B(\rho_{AB}) = \sum_{i,j,k} \alpha(i,j) \alpha^*(k,j) |i\rangle\langle k| \quad (5)$$

$$\rho_A^2 = \sum_{i',j',k'} \sum_{i,j,k} \alpha(i,j) \alpha(k,j) \alpha^*(i',j') \alpha^*(k',j') |i\rangle\langle k| |i'\rangle\langle k'| \quad (6)$$

$$= \sum_{j',k'} \sum_{i,j,k} \alpha(i,j) \alpha^*(k,j) \alpha(k,j') \alpha^*(k',j') |i\rangle\langle k'| \quad (7)$$

$$\text{Tr}_A(\rho_A^2) = \sum_{i,j,k,j'} \alpha(i,j) \alpha^*(k,j) \alpha(k,j') \alpha^*(i,j') \quad (8)$$

$$(9)$$

For a unentangled ψ we know that $\text{Tr}_A(\rho_A^2) = 1$ For ψ entangled this will be smaller than 1 (proof comes from the property of the density operator that its eigenvalues are all smaller than 1). This fits the definition of the purity of a quantum state hence we can write:

$$P = \frac{1}{K} = \sum_{i,j,k,l} \alpha(i,j) \alpha^*(k,j) \alpha(k,l) \alpha^*(i,l) \quad (10)$$

A.2 Calculation from experimental data

A.2.1 Trace method

In the lab we can measure $|\phi(\omega_1, \omega_2)|^2$, here I outline how to extract the schmidt number from this set of values. Taking the positive square root of the matrix of values obtained from the lab you have a matrix \mathbf{f} given by:

$$\mathbf{f} = \sum_{\omega_1, \omega_2} \phi(\omega_1, \omega_2) |\omega_1\rangle\langle\omega_2| \quad (11)$$

(This seems to be some weird way of writing the wavefunction as a matrix, bare with me it turns out to be useful)

$$\mathbf{f}^\dagger \mathbf{f} = \sum_{\omega_1, \omega_2, \omega_3} \phi(\omega_1, \omega_2) \phi(\omega_3, \omega_2) |\omega_1\rangle \langle \omega_3| \quad (12)$$

$$(\mathbf{f}^\dagger \mathbf{f})^2 = \sum_{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6} \phi(\omega_1, \omega_2) \phi(\omega_3, \omega_2) \phi(\omega_4, \omega_5) \phi(\omega_6, \omega_5) |\omega_1\rangle \langle \omega_3| \omega_4\rangle \langle \omega_6| \quad (13)$$

$$(\mathbf{f}^\dagger \mathbf{f})^2 = \sum_{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6} \phi(\omega_1, \omega_2) \phi(\omega_3, \omega_2) \phi(\omega_3, \omega_5) \phi(\omega_6, \omega_5) |\omega_1\rangle \langle \omega_6| \quad (14)$$

$$\text{Tr} [(\mathbf{f}^\dagger \mathbf{f})^2] = \sum_{\omega_1, \omega_2, \omega_3, \omega_4} \phi(\omega_1, \omega_2) \phi(\omega_3, \omega_2) \phi(\omega_3, \omega_4) \phi(\omega_1, \omega_4) \quad (15)$$

I've done it this way because I wanted to figure out where the equation in [1] comes from. You can now see that equation 10 is of exactly the same form as $\text{Tr} [(\mathbf{f}^\dagger \mathbf{f})^2]$ (barring the conjugates but this is okay since ϕ is real.) Taking the parallel further it can be seen that equation 12 is of the form of a reduced density matrix. Here we must make sure to normalise to make sure this is a valid reduced density matrix. The normalisation is:

$$N = \text{Tr} [\mathbf{f}^\dagger \mathbf{f}] = \sum_{\omega_1, \omega_2} \phi(\omega_1, \omega_2)^2 \quad (16)$$

Giving:

$$\rho_A = \frac{\mathbf{f}^\dagger \mathbf{f}}{N} \quad (17)$$

We can then write:

$$\frac{1}{K} = \frac{\text{Tr} [(\mathbf{f}^\dagger \mathbf{f})^2]}{\text{Tr} [\mathbf{f}^\dagger \mathbf{f}]^2} \quad (18)$$