

SAT & PSAT Must-Know Math Formulas

LOTLOUISCHO STEM CLUB

1 Conversions using Factor-Label Method

IMPORTANT! These kind of unit conversion questions **ALWAYS** appears on the exam. Apply the Factor-Label method in this example shown below.

An Amtrak Saluki train number 391 is traveling from Chicago Union Station to Champaign-Urbana Illinois Terminal Station at a speed of 60 miles per hour. How fast is the train traveling at meters per second? (1 mile = 1.609 km, 1 km = 1000 m, 1 hour = 60 minutes, and 1 minute = 60 seconds)

$$= \frac{60 \text{ miles}}{\text{hour}} \times \frac{1.609 \text{ kilometers}}{1 \text{ miles}} \times \frac{1000 \text{ meters}}{1 \text{ kilometers}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \times \frac{1 \text{ minutes}}{60 \text{ seconds}} = 26.82 \text{ meters per second}$$

2 Algebra – Linear Equations & Functions

y: Function or Graph

m: Slope

x: Variable

b: Y-Intercept

NOTE: X-intercept means when $y = 0$. Y-intercept means $x = 0$.

Standard Form $Ax + By = C$

$$\text{Slope} = -\frac{A}{B}$$

$$\text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-Intercept Form: $y = mx + b$

Point-slope Form: $y - y_1 = m(x - x_1)$

Average rate of change between $(a, f(a))$ and $(b, f(b))$ can be determined by

$$m = \frac{f(b) - f(a)}{b - a}$$

Let's say we have two lines $y_1 = m_1x + b_1$ and $y_2 = m_2x + b_2$. We can say that:

$m_1 = m_2$ **PARALLEL LINES (SAME SLOPE)**

$m_1 \cdot m_2 = -1$ **PERPENDICULAR LINES**

When you are given something like this:

$$\begin{aligned} ax + by &= c_1 \\ ax + by &= c_2 \end{aligned}$$

If $c_1 = c_2$, then there are infinite many solutions. If $c_1 \neq c_2$, then there are no solutions to the system of linear equations above.

Distance a vehicle or a person travels can be determined by

$$\mathbf{Distance = Velocity \times Time}$$

The distance d between two points (x_1, y_1) and (x_2, y_2) can be computed by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

and the midpoint M between two points can be determined by

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

3 Exponent Rules & Radicals

Be aware of MADSPM or in other words Mad Steve Pours Milkshake. What does this mean and what do we do with the exponents?

Multiplying Exponents mean **ADD**
 Dividing Exponents mean **SUBTRACT**
 Powering Exponents mean **MULTIPLY**

$$\begin{aligned} a^m \cdot a^n &= a^{m+n} \\ \frac{a^m}{a^n} &= a^{m-n} \\ (a^m)^n &= a^{m \cdot n} \\ (ab)^m &= a^m \cdot b^m \\ a^{-m} &= \frac{1}{a^m} \\ a^{1/n} &= \sqrt[n]{a} \\ a^{m/n} &= \sqrt[n]{a^m} \\ \sqrt{a} \cdot \sqrt{b} &= \sqrt{ab} \end{aligned}$$

y_0 : Initial value
 b : Growth/Decay Factor
 t : Time
 r : Rate
 n : Time period

$$\begin{aligned} y &= y_0 b^t \\ y &= y_0(1 \pm r)^t \quad (\text{Growth/decay model}) \\ A &= P \left(1 + \frac{r}{n} \right)^{nt} \quad (\text{Compound interest}) \\ A &= Pe^{rt} \quad (\text{Continuous growth/decay}) \end{aligned}$$

4 Quadratics & Polynomials

$$y = ax^2 + bx + c \quad (\text{Standard form})$$

$$y = a(x - h)^2 + k \quad (\text{Vertex form, vertex} = (h, k))$$

$$y = a(x - r_1)(x - r_2) \quad (\text{Factored form, roots } r_1, r_2)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (\text{Quadratic formula})$$

NOTE: These kind of questions shown below **ALWAYS** appear on the exam!

$$b^2 - 4ac > 0 \quad \text{TWO Real Solutions}$$

$$b^2 - 4ac = 0 \quad \text{ONE Real Solution}$$

$$b^2 - 4ac < 0 \quad \text{NO Real Solutions}$$

$$\text{Sum of solutions} = -\frac{b}{a}, \quad \text{Product of Solutions} = \frac{c}{a}$$

DISCLAIMER: If you are currently enrolled in AP Calculus, then you will know what this part is about. For those of you not enrolled in AP Calculus, this is a quick shortcut to determine the minimum or the maximum points on the quadratic equation $y = ax^2 + bx + c$.

$$\begin{aligned} y &= ax^2 + bx + c \\ \frac{dy}{dx} &= 2ax + b \\ \frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= 2ax + b = 0 \\ 2ax &= -b \\ x &= -\frac{b}{2a} \end{aligned}$$

Thus the minimum/maximum of $f(x) = ax^2 + bx + c$ is at $(-\frac{b}{2a}, f(-\frac{b}{2a}))$.

5 Factoring

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^2 - b^2 = (a - b)(a + b)$$

6 Complex Numbers

The canonical form for complex numbers is $a + bi$ where a is the real number and b is in the imaginary axis. The patterns shown below will repeat after four cycles.

$$\begin{aligned}i &= \sqrt{-1} \\i^2 &= -1 \\i^3 &= -i \\i^4 &= 1\end{aligned}$$

Now look what happens after we pass i^4 . Can you see a pattern?

$$\begin{aligned}i^5 &= \sqrt{-1} \\i^6 &= -1 \\i^7 &= -i \\i^8 &= 1\end{aligned}$$

7 Geometry

7.1 Circles

The canonical formula for circles is shown below:

$$(x - h)^2 + (y - k)^2 = r^2$$

where \mathbf{r} is the radius of the circle and (h, k) is the center of the circle. We can also compute:

$$\begin{aligned}\text{Area } A &= \pi r^2 \\ \text{Circumference } C &= 2\pi r \\ \text{Length of Arc } l &= r\theta \text{ or } l = \frac{n^\circ}{360^\circ} \times 2\pi r \\ \text{Area of Sector } S &= \frac{n^\circ}{360^\circ} \times \pi r^2\end{aligned}$$

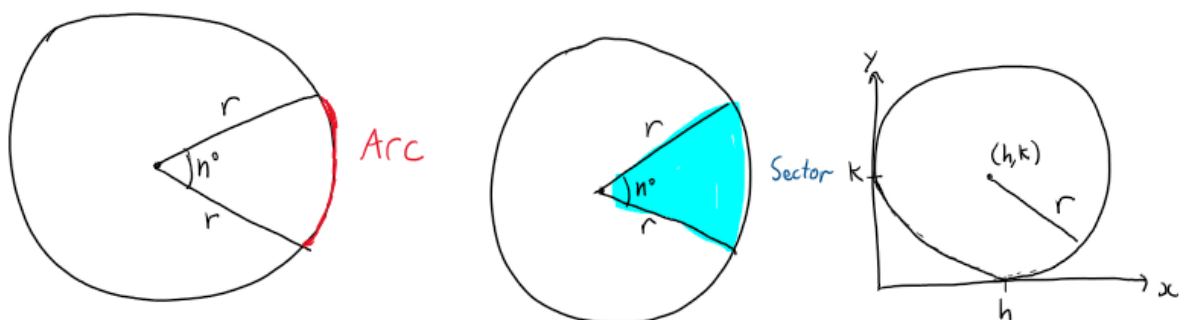


Figure 1: Diagrams of Circles and Properties.

7.2 Triangle Theorems

These are some important things to keep in mind about triangles.

Equilateral Triangle: ALL sides are EQUAL

Isosceles Triangle: Two sides are EQUAL

Scalene Triangle: ALL unequal sides

Acute angle means less than 90°

Right angle means 90°

Obtuse angle means greater than 90°

Sum of interior angles add up to 180°

The inequality theorem states that when you have sides a , b and c , then $a + b > c$.

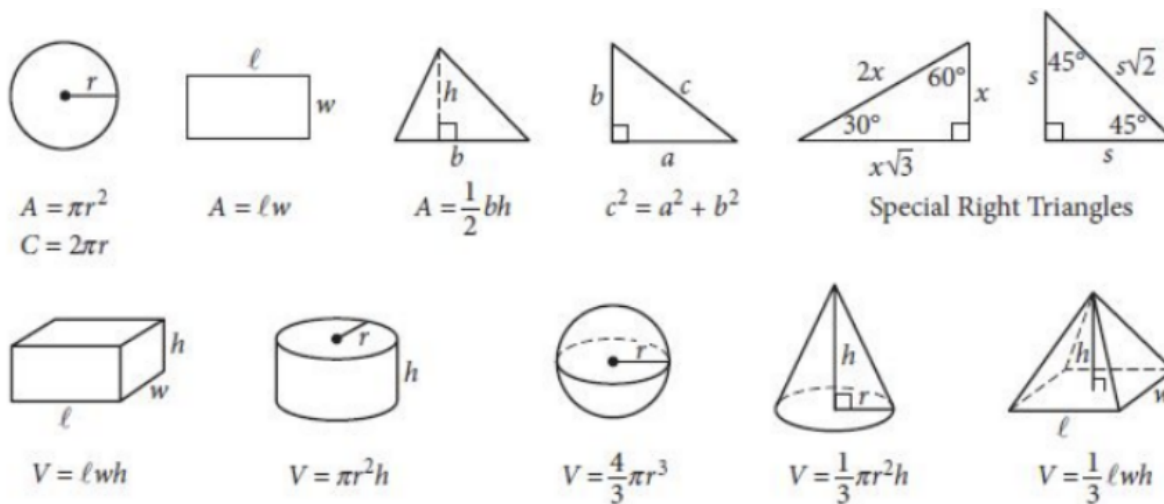
7.3 Triangle Congruency

Side-Side-Side (SSS): If all sides of one triangle is equal to another triangle, then congruent.

Side-Angle-Side (SAS): If two sides and the angle between them are equal, then congruent.

Angle-Side-Angle (ASA): If two angles and the side are equal, then congruent.

7.4 Common Formulas



The number of degrees of arc in a circle is 360.

The number of radians of arc in a circle is 2π .

The sum of the measures in degrees of the angles of a triangle is 180.

Figure 2: Geometry Formulas from Official College Board SAT and PSAT Exams.

7.4.1 Areas A

$$\begin{aligned}A_{Square} &= \text{Length}^2 \\A_{Rectangle} &= \text{Length} \times \text{Width} \\A_{Triangle} &= \frac{1}{2} \times \text{Base} \times \text{Height} \\ \text{Cube Surface Area} &= 6 \times \text{Length}^2 \\ \text{Sphere Surface Area} &= 4\pi r^2\end{aligned}$$

7.4.2 Perimeters P – Sum of All Sides of Shapes

$$\begin{aligned}P_{Square} &= 4 \times \text{Length} \\P_{Rectangle} &= 2 \times (\text{Length} + \text{Width}) \\P_{Triangle} &= a + b + c\end{aligned}$$

7.4.3 Volumes V

$$\begin{aligned}V_{Cube} &= \text{Length}^3 \\V_{\text{Rectangular Prism}} &= \text{Length} \times \text{Width} \times \text{Height} \\V_{Cylinder} &= \pi r^2 \times \text{Height} \\V_{Cone} &= \frac{1}{3} \pi r^2 \times \text{Height} \\V_{Sphere} &= \frac{4}{3} \pi r^3\end{aligned}$$

7.4.4 Pythagorean Theorem

$$a^2 + b^2 = c^2$$

where the sides of the triangle are a and b and the hypotenuse (longest side) is c .

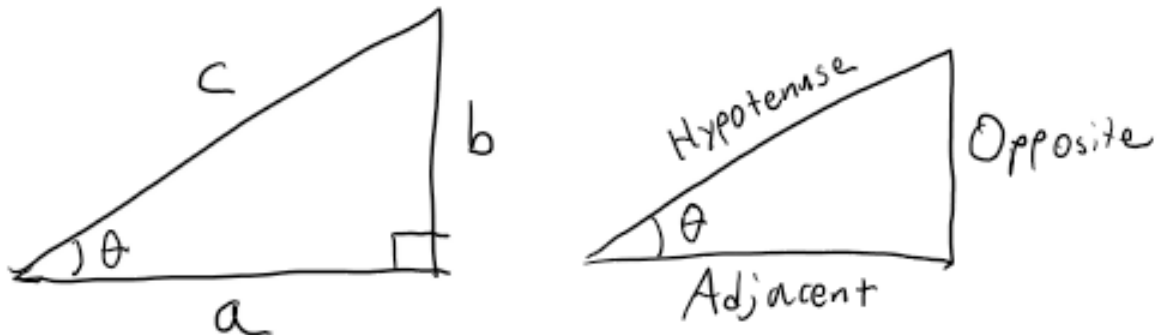


Figure 3: Diagrams of Right Triangles and Properties.

8 Trigonometry

Always remember **SOHCAHTOA** which means:

$$\begin{aligned}\sin \theta &= \frac{\text{Opposite}}{\text{Hypotenuse}} \\ \cos \theta &= \frac{\text{Adjacent}}{\text{Hypotenuse}} \\ \tan \theta &= \frac{\text{Opposite}}{\text{Adjacent}}\end{aligned}$$

Another thing to note is that when θ is in degrees:

$$\begin{aligned}\sin \theta &= \cos (90^\circ - \theta) \\ \cos \theta &= \sin (90^\circ - \theta) \\ \sin^2 \theta + \cos^2 \theta &= 1\end{aligned}$$

9 Lines

When lines intersect, the opposite angles are equal to each other!

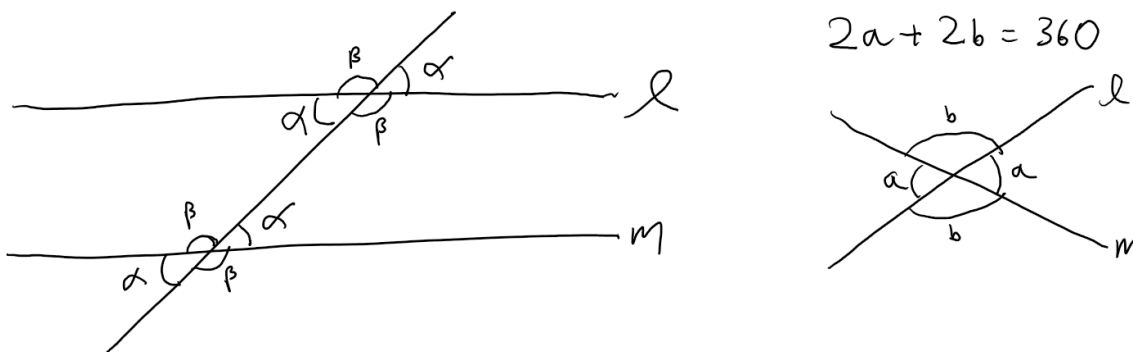


Figure 4: Diagrams of Lines and Properties.

10 Statistics and Data

$$\text{Mean (Average)} \mu = \frac{\text{Sum of Data}}{\text{Number of Data Points}}$$

Median = Middle value

Mode = Most Frequent Value

Range R = Max - Min

Standard Deviation σ = Spread of data and how far apart from mean value

Line of best fit: $y = mx + b$

$$\text{Percent} = \frac{\text{Part}}{\text{Whole}} \times 100\%$$

$$\text{Percent change: } \frac{\text{new-old}}{\text{old}} \times 100\%$$

I don't think this is tested but just for reference, the formulas for standard deviation are:

$$\text{Population } \sigma = \sum_{i=1}^n \sqrt{\frac{(x_i - \bar{x})^2}{n}}$$

$$\text{Sample } s = \sum_{i=1}^n \sqrt{\frac{(x_i - \bar{x})^2}{n-1}}$$

11 Probability and Counting

$$P = \frac{\text{Favorable Outcomes}}{\text{Total Outcomes}}$$

$$P(A \cap B) = P(A)P(B) \text{ (Independent Events)}$$

$$P(A \cup B) = P(A) + P(B) \text{ (Mutually Exclusive Events)}$$