SAT & PSAT Must-Know Math Formulas

LOTLOUISCHO STEM CLUB

1 Conversions using Factor-Label Method

IMPORTANT! These kind of unit conversion questions ALWAYS appears on the exam. Apply the Factor-Label method in this example shown below.

An Amtrak Saluki train number 391 is traveling from Chicago Union Station to Champaign-Urbana Illinois Terminal Station at a speed of 60 miles per hour. How fast is the train traveling at meters per second? (1 mile = 1.609 km, 1 km = 1000 m, 1 hour = 60 minutes, and 1 minute = 60 seconds)

$= \frac{\text{60 miles}}{\text{hour}} \times \frac{\text{1.609 kilometers}}{\text{1 miles}} \times \frac{\text{1000 meters}}{\text{1 kilometers}} \times \frac{\text{1 hour}}{\text{60 minutes}} \times \frac{\text{1 minutes}}{\text{60 seconds}} = 26.82$ meters per second

2 Algebra – Linear Equations & Functions

y: Function or Graph
m: Slope
x: Variable
b: Y-Intercept

NOTE: X-intercept means when $y = 0$. Y-intercept means $x = 0$.

Standard Form $Ax + By = C$
Slope $= -\frac{A}{B}$
Slope m $= \frac{y_2 - y_1}{x_2 - x_1}$
Slope-Intercept Form: $y = mx + b$
Point-slope Form: $y - y_1 = m(x - x_1)$

Average rate of change between (a,f(a)) and (b, f(b)) can be determined by

$$m = \frac{f(b) - f(a)}{b - a}$$

Let's say we have two lines $y_1 = m_1x + b_1$ and $y_2 = m_2x + b_2$. We can say that:

$m_1 = m_2$ **PARALLEL LINES (SAME SLOPE)**
$m_1 \cdot m_2 = -1$ **PERPENDICULAR LINES**

When you are given something like this:

$$ax + by = c_1$$
$$ax + by = c_2$$

If $c_1 = c_2$, then there are infinite many solutions. If $c_1 \neq c_2$, then there are no solutions to the system of linear equations above.

Distance a vehicle or a person travels can be determined by

$$\textbf{Distance} = \textbf{Velocity} \times \textbf{Time}$$

The distance d between two points (x_1, y_1) and (x_2, y_2) can be computed by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

and the midpoint M between two points can be determined by

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

3 Exponent Rules & Radicals

Be aware of MADSPM or in other words Mad Steve Pours Milkshake. What does this mean and what do we do with the exponents?

Multiplying Exponents mean **ADD**
Dividing Exponents mean **SUBTRACT**
Powering Exponents mean **MULTIPLY**

$$a^m \cdot a^n = a^{m+n}$$
$$\frac{a^m}{a^n} = a^{m-n}$$
$$(a^m)^n = a^{m \cdot n}$$
$$(ab)^m = a^m \cdot b^m$$
$$a^{-m} = \frac{1}{a^m}$$
$$a^{1/n} = \sqrt[n]{a}$$
$$a^{m/n} = \sqrt[n]{a^m}$$
$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

y_0: Initial value
b: Growth/Decay Factor
t: Time
r: Rate
n: Time period

$$y = y_0 b^t$$
$$y = y_0(1 \pm r)^t \quad \text{(Growth/decay model)}$$
$$A = P\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(Compound interest)}$$
$$A = Pe^{rt} \quad \text{(Continuous growth/decay)}$$

4 Quadratics & Polynomials

$$y = ax^2 + bx + c \quad \text{(Standard form)}$$
$$y = a(x-h)^2 + k \quad \text{(Vertex form, vertex} = (h,k))$$
$$y = a(x-r_1)(x-r_2) \quad \text{(Factored form, roots } r_1, r_2)$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \text{(Quadratic formula)}$$

NOTE: These kind of questions shown below ALWAYS appear on the exam!

$$b^2 - 4ac > 0 \textbf{ TWO Real Solutions}$$

$$b^2 - 4ac = 0 \textbf{ ONE Real Solution}$$

$$b^2 - 4ac < 0 \textbf{ NO Real Solutions}$$

$$\textbf{Sum of solutions } = -\frac{b}{a}, \textbf{ Product of Solutions } = \frac{c}{a}$$

DISCLAIMER: If you are currently enrolled in AP Calculus, then you will know what this part is about. For those of you not enrolled in AP Calculus, this is a quick shortcut to determine the minimum or the maximum points on the quadratic equation $y = ax^2 + bx + c$.

$$\begin{gathered} y = ax^2 + bx + c \\ \tfrac{dy}{dx} = 2ax + b \\ \tfrac{dy}{dx} = 0 \\ \tfrac{dy}{dx} = 2ax + b = 0 \\ 2ax = -b \\ x = -\tfrac{b}{2a} \end{gathered}$$

Thus the minimum/maximum of $f(x) = ax^2 + bx + c$ is at $(-\frac{b}{2a}, f(-\frac{b}{2a}))$.

5 Factoring

$$\begin{gathered} a^2 + 2ab + b^2 = (a+b)^2 \\ a^2 - 2ab + b^2 = (a-b)^2 \\ a^2 - b^2 = (a-b)(a+b) \end{gathered}$$

6 Complex Numbers

The canonical form for complex numbers is $a + bi$ where a is the real number and b is in the imaginary axis. The patterns shown below will repeat after four cycles.

$$i = \sqrt{-1}$$
$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$

Now look what happens after we pass i^4. Can you see a pattern?

$$i^5 = \sqrt{-1}$$
$$i^6 = -1$$
$$i^7 = -i$$
$$i^8 = 1$$

7 Geometry

7.1 Circles

The canonical formula for circles is shown below:

$$(x - h)^2 + (y - k)^2 = r^2$$

where **r** is the radius of the circle and (h, k) is the center of the circle. We can also compute:

$$\text{Area } A = \pi r^2$$
$$\text{Circumference } C = 2\pi r$$
$$\text{Length of Arc } l = r\theta \text{ or } l = \frac{n^\circ}{360^\circ} \times 2\pi r$$
$$\text{Area of Sector } S = \frac{n^\circ}{360^\circ} \times \pi r^2$$

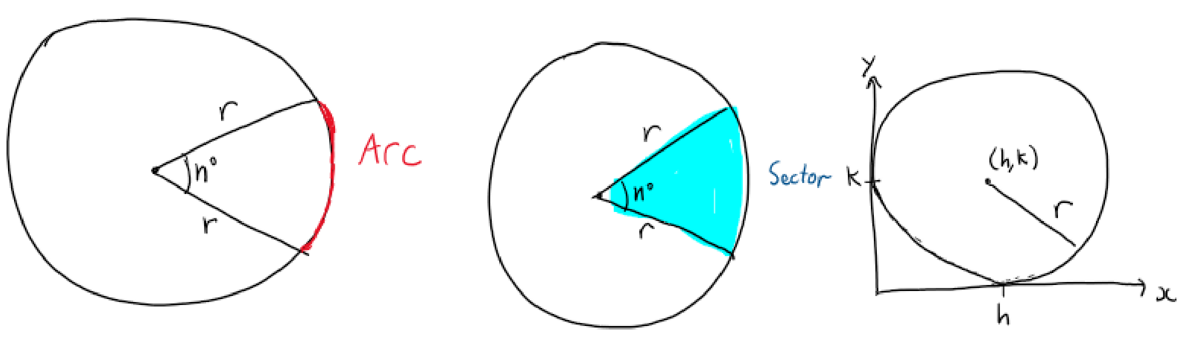

Figure 1: Diagrams of Circles and Properties.

7.2 Triangle Theorems

These are some important things to keep in mind about triangles.

Equilateral Triangle: ALL sides are EQUAL
Isosceles Triangle: Two sides are EQUAL
Scalene Triangle: ALL unequal sides
Acute angle means less than 90°
Right angle means 90°
Obtuse angle means greater than 90°
Sum of interior angles add up to 180°
The inequality theorem states that when you have sides a, b and c, then $a + b > c$.

7.3 Triangle Congruency

Side-Side-Side (SSS): If all sides of one triangle is equal to another triangle, then congruent.
Side-Angle-Side (SAS): If two sides and the angle between them are equal, then congruent.
Angle-Side-Angle (ASA): If two angles and the side are equal, then congruent.

7.4 Common Formulas

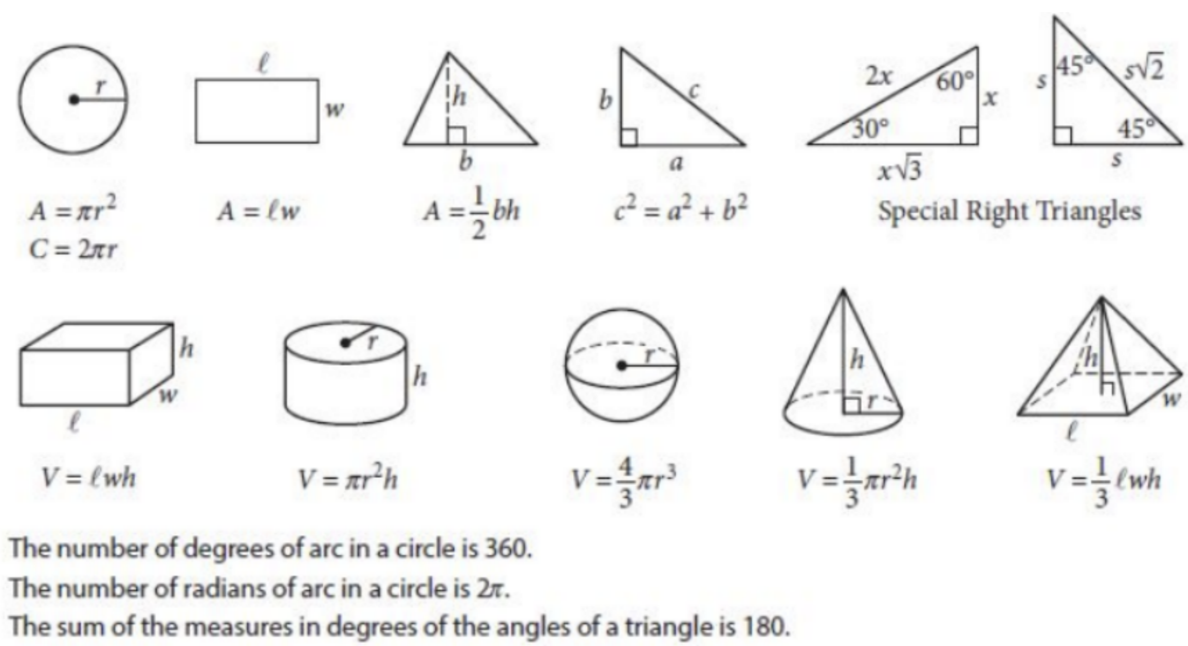

Figure 2: Geometry Formulas from Official College Board SAT and PSAT Exams.

7.4.1 Areas A

$$A_{Square} = \mathbf{Length}^2$$
$$A_{Rectangle} = \text{Length} \times \text{Width}$$
$$A_{Triangle} = \frac{1}{2} \times \text{Base} \times \text{Height}$$
$$\text{Cube Surface Area} = 6 \times \text{Length}^2$$
$$\text{Sphere Surface Area} = 4\pi r^2$$

7.4.2 Perimeters P – Sum of All Sides of Shapes

$$P_{Square} = 4 \times \text{Length}$$
$$P_{Rectangle} = 2 \times (\text{Length} + \text{Width})$$
$$P_{Triangle} = a + b + c$$

7.4.3 Volumes V

$$V_{\text{Cube}} = \text{Length} \times \text{Width} \times \text{Height}$$
$$V_{\text{Rectangular Prism}} = \text{Length} \times \text{Width} \times \text{Height}$$
$$V_{\text{Cylinder}} = \pi r^2 \times \text{Height}$$
$$V_{\text{Cone}} = \frac{1}{3}\pi r^2 \times \text{Height}$$
$$V_{\text{Sphere}} = \frac{4}{3}\pi r^3$$

7.4.4 Pythagorean Theorem

$$a^2 + b^2 = c^2$$

where the sides of the triangle are a and b and the hypotenuse (longest side) is c.

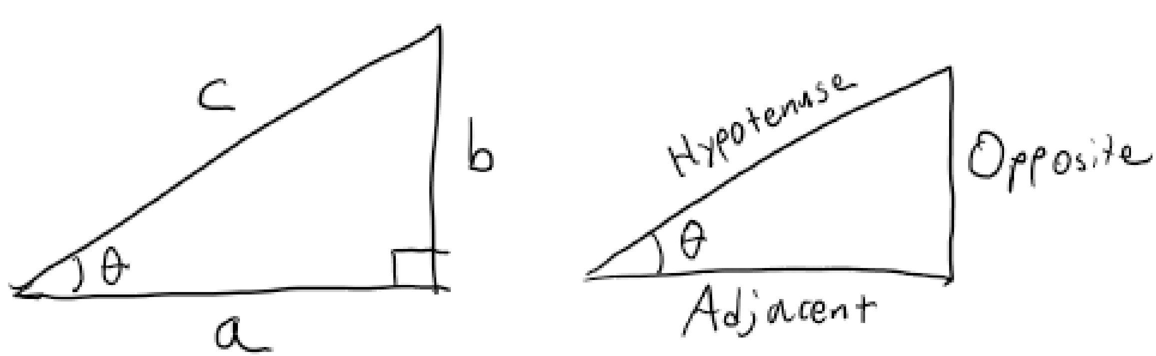

Figure 3: Diagrams of Right Triangles and Properties.

8 Trigonometry

Always remember **SOHCAHTOA** which means:

$$\sin\theta = \frac{\textbf{Opposite}}{\textbf{Hypotenuse}}$$
$$\cos\theta = \frac{\textbf{Adjacent}}{\textbf{Hypotenuse}}$$
$$\tan\theta = \frac{\textbf{Opposite}}{\textbf{Adjacent}}$$

Another thing to note is that when θ is in degrees:

$$\sin\theta = \cos(90^\circ - \theta)$$
$$\cos\theta = \sin(90^\circ - \theta)$$
$$\sin^2\theta + \cos^2\theta = 1$$

9 Lines

When lines intersect, the opposite angles are equal to each other!

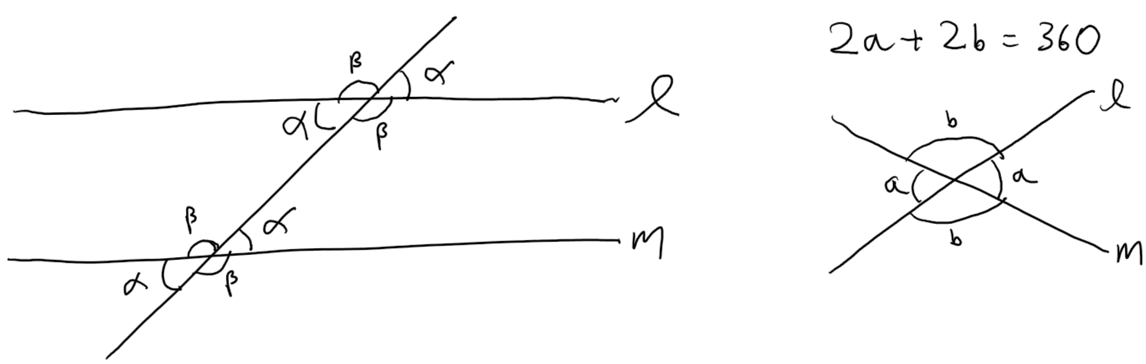

Figure 4: Diagrams of Lines and Properties.

10 Statistics and Data

$$\text{Mean (Average) } \mu = \frac{\text{Sum of Data}}{\text{Number of Data Points}}$$
$$\text{Median} = \text{Middle value}$$
$$\text{Mode} = \text{Most Frequent Value}$$
$$\text{Range R} = \text{Max - Min}$$
$$\text{Standard Deviation } \sigma = \text{Spread of data and how far apart from mean value}$$
$$\text{Line of best fit: } \quad y = mx + b$$

$$\text{Percent} = \frac{Part}{Whole} \times 100\%$$
$$\text{Percent change: } \frac{\text{new}-\text{old}}{\text{old}} \times 100\%$$

I don't think this is tested but just for reference, the formulas for standard deviation are:

$$\textbf{Population } \sigma = \sum_{i=1}^{n} \sqrt{\frac{(x_i - \bar{x})^2}{n}}$$
$$\textbf{Sample } s = \sum_{i=1}^{n} \sqrt{\frac{(x_i - \bar{x})^2}{n-1}}$$

11 Probability and Counting

$$\text{P} = \frac{\text{Favorable Outcomes}}{\text{Total Outcomes}}$$
$$P(A \cap B) = P(A)P(B) \textbf{ (Independent Events)}$$
$$P(A \cup B) = P(A) + P(B) \textbf{ (Mutually Exclusive Events)}$$