Phase Space Reconstruction Analysis Report for Beehive Dynamics
Entry 3 - Module 5

Group 6: Djourdan Johnson, Jacuqot Qiu, Lotte Michels, Nawat Nawati Azhati, Nuo Xu, Xuelin Wei

Abstract

This report presents the phase space reconstruction analysis of three variables—temperature, humidity, and hive power—from beehive sensor data (tag number 202204, July–August 2020). The analysis follows the methodology of Module 5, focusing on single-variable phase space reconstruction to understand the dynamics of each variable. We explore the deterministic and periodic nature of these variables using Takens' Delay Embedding Theorem (Takens, 1981), correlation dimension estimation, and surrogate data methods.

1 Introduction

Understanding the dynamics of beehive environmental variables is crucial for monitoring hive health. This study applies phase space reconstruction to three time series—temperature, humidity, and hive power—from a beehive sensor dataset (tag number 202204, July–August 2020). Following Module 5, we use single-variable phase space reconstruction to investigate the deterministic and periodic nature of these variables. The analysis employs Takens' Delay Embedding Theorem (Takens, 1981) to reconstruct the phase space, estimates correlation dimensions to assess dynamic complexity, and uses surrogate data to evaluate result robustness (Module 7: Surrogate Methods).

2 Methods

We applied single-variable phase space reconstruction to each variable using the following steps:

- **Data Preprocessing**: Removed duplicate timestamps, subsampled every 8th point to reduce computational load, and retained original trends without detrending.
- **Exploratory Visualization**: Plotted time series, pairwise scatter plots, and a 3D vector field with density to observe trends and relationships.
- **Phase Space Reconstruction**: Used Takens' Delay Embedding Theorem (Takens, 1981) to reconstruct the phase space with the `buildTakens` function.
- **Parameter Selection**: Initially estimated time delay (τ) using Average Mutual Information (AMI) and embedding dimension (M) with Cao's method (Cao, 1997), then optimized via sensitivity analysis.
- **Correlation Dimension**: Calculated the correlation dimension to quantify dynamic complexity.
- **Surrogate Data**: Generated surrogate data to test robustness (Module 7: Surrogate Methods). A significantly lower original dimension compared to the surrogate indicates deterministic dynamics.
- **Visualization**: Generated phase space plots to visualize trajectories.

3 Results

3.1 Data Overview and Preprocessing

We analyzed three time series from the beehive sensor data:

- **Temperature**: Range 3.93°C to 37.53°C, mean 31.80°C, standard deviation 6.31233.
- **Humidity**: Range 43.47% to 80.60%, mean 62.04%, standard deviation 7.836648.
- **Hive Power**: Range -10.000 to 31.197, mean 8.732, standard deviation 10.05023.

Preprocessing:

- Removed duplicate timestamps based on `published_at`.
- Subsampled the data by taking every 8th point (reducing from ~8000 to ~1000 points) to lower computational load, retaining main dynamic features.
- Did not detrend the data to preserve long-term dynamics (e.g., daily cycles).

Exploratory Visualization:

- **Time Series Plot**: Showed possible daily cycles in temperature and humidity, with hive power exhibiting burst-like variability (Figure 1).
- **Pairwise Scatter Plots**: Revealed an inverse relationship between temperature and humidity, and variable patterns in hive power (Figures 2, 3, and 4).
- **Vector Field and Density Plot**: Visualized dynamic flow in 3D, using change rates as a density proxy (Figure 5).

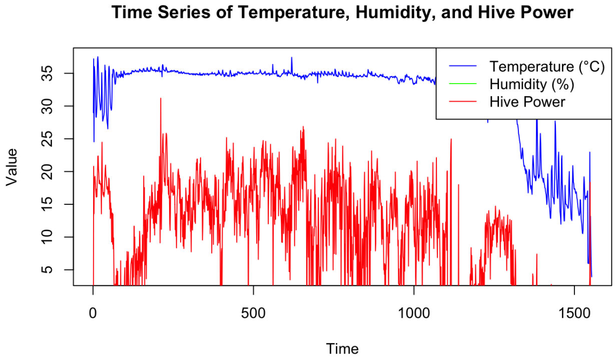

Figure 1: Time Series Plot of Temperature, Humidity, and Hive Power

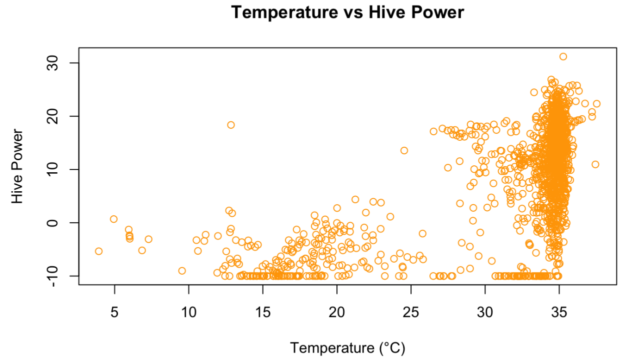

Figure 3: Temperature vs. Hive Power Scatter Plot

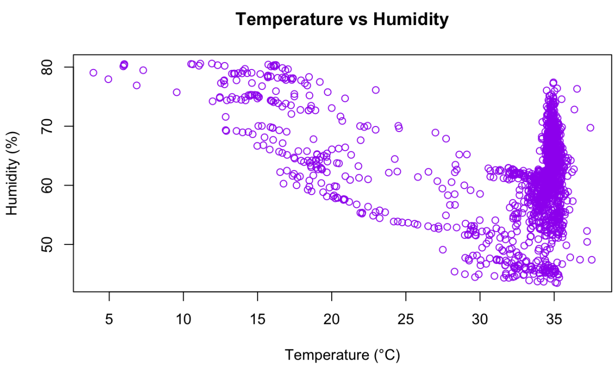

Figure 2: Temperature vs. Humidity Scatter Plot

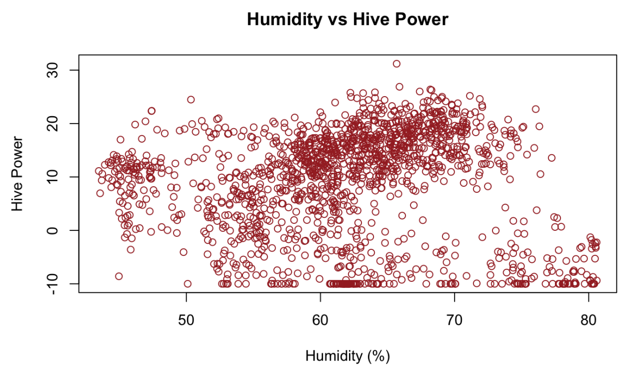

Figure 4: Humidity vs. Hive Power Scatter Plot

3.2 Phase Space Reconstruction

Initial parameter estimation used AMI for τ and Cao's method for M:

- Temperature: $\tau = 38, M = 8$, Correlation Dimension = 3.198562 (Surrogate = 3.725026).
- Humidity: $\tau = 13, M = 9$, Correlation Dimension = 7.530213 (Surrogate = 3.695639).
- Hive Power: $\tau = 1, M = 3$, Correlation Dimension = 1.309729 (Surrogate = 2.698470).

Sensitivity analysis tested τ (2, 4, 6, 8) and M (2, 4, 6):

- Temperature: Dimensions ranged from 1.015674 to 2.377713 (surrogate: 1.256028 to 3.771640), lower at smaller τ and M (Figures 6 and 7).
- Humidity: Dimensions ranged from 1.866193 to 5.684184 (surrogate: 1.968508 to 5.870703), often higher than surrogate at larger M, suggesting noise (Figures 8 and 9).
- Hive Power: Dimensions ranged from 1.384295 to 5.404614 (surrogate: 1.710832 to 5.417492), lower at smaller τ and M, indicating deterministic dynamics (Figures 10 and 11).

Optimized parameters were selected where the correlation dimension was lowest and significantly below the surrogate:

- Temperature: $\tau = 2, M = 6$
- Humidity: $\tau = 2, M = 2$
- Hive Power: $\tau = 2, M = 2$

Final results with optimized parameters:

- **Temperature**: Dimensions: 1544 × 6. Correlation Dimension: 1.995383 (Surrogate: 3.718860). Deterministic dynamics confirmed (Figure 12).
- **Humidity**: Dimensions: 1552 × 2. Correlation Dimension: 1.866193 (Surrogate: 1.978875). Small difference suggests possible noise (Figure 13).
- **Hive Power**: Dimensions: 1552 × 2. Correlation Dimension: 1.384295 (Surrogate: 1.717435). Deterministic dynamics with possible periodicity (Figure 14).

Hive Power Periodicity Attempt: Tested $\tau = 360, M = 3$, but the plot showed irregular lines rather than a closed loop (Figure 15).

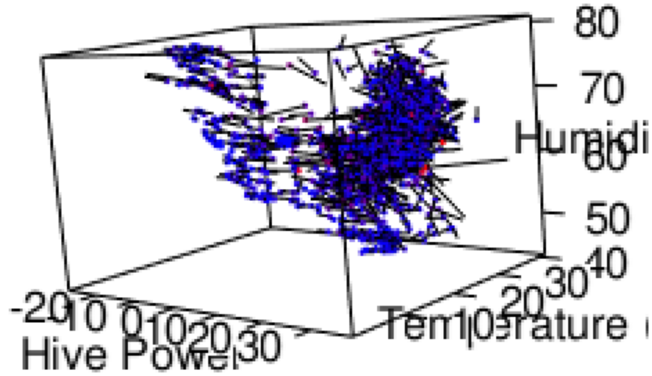

Figure 5: Vector Field and Density Plot (Temperature vs. Humidity vs. Hive Power)

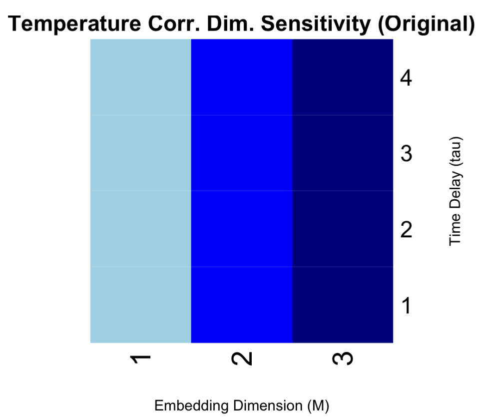

Figure 6: Sensitivity Analysis Heatmap for Temperature (Original)

4 Discussion

Temperature exhibits deterministic dynamics with a low-dimensional structure (1.995383), consistent with Module 4 attractors (32-33°C). Humidity's dimension (1.866193) suggests deterministic dynamics but possible noise (close to surrogate 1.978875). Hive Power's dimension (1.384295) indicates deterministic dynamics, with a value near 1 suggesting periodicity (Limit Cycle, Module 5, P6). However, its phase space plot did not show a smooth closed trajectory, possibly due to quasi-periodicity (multiple frequencies, Module 5, P7) or mixed dynamics (periodic baseline with non-periodic bursts). Future work includes refining τ for hive power, reducing noise in humidity, and exploring multi-view embedding.

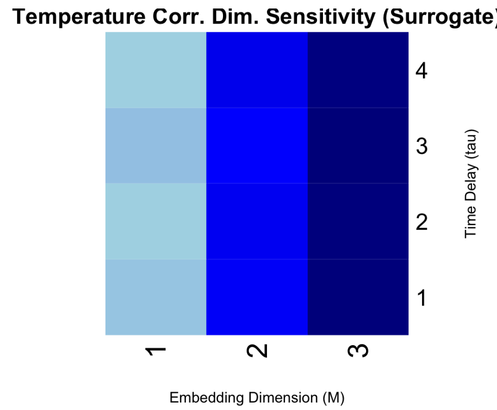

Figure 7: Sensitivity Analysis Heatmap for Temperature (Surrogate)

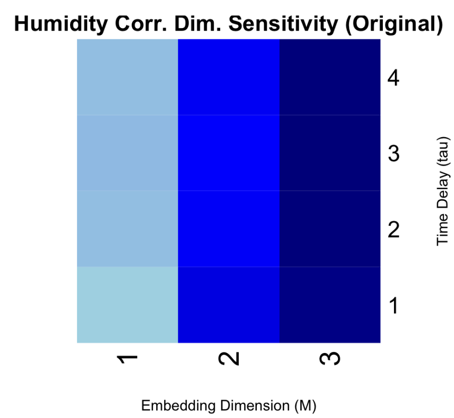

Figure 8: Sensitivity Analysis Heatmap for Humidity (Original)

References

Takens, F. (1981). Detecting strange attractors in turbulence. In *Dynamical Systems and Turbulence, Warwick 1980* (pp. 366–381). Springer.

Cao, L. (1997). Practical method for determining the minimum embedding dimension of a scalar time series. *Physica D: Nonlinear Phenomena*, 110(1-2), 43–50.

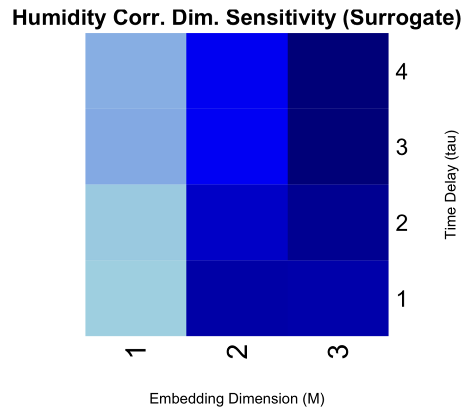

Figure 9: Sensitivity Analysis Heatmap for Humidity (Surrogate)

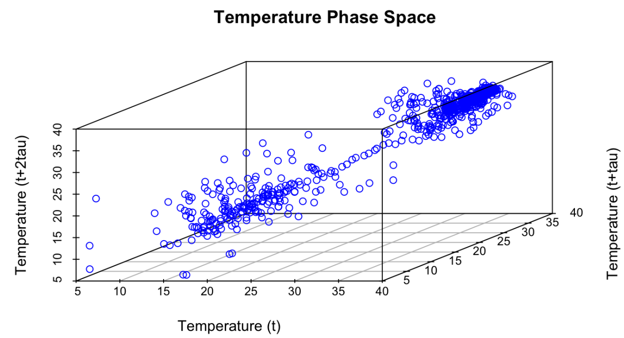

Figure 12: Temperature Phase Space Plot (tau = 2, M = 6, 3D)

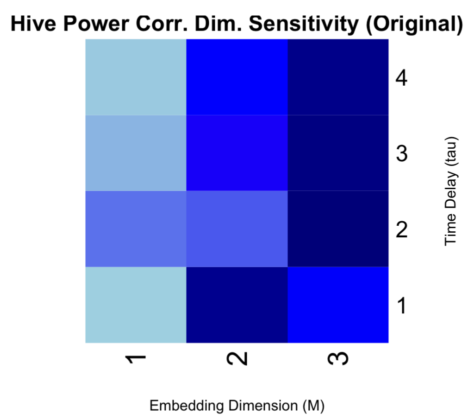

Figure 10: Sensitivity Analysis Heatmap for Hive Power (Original)

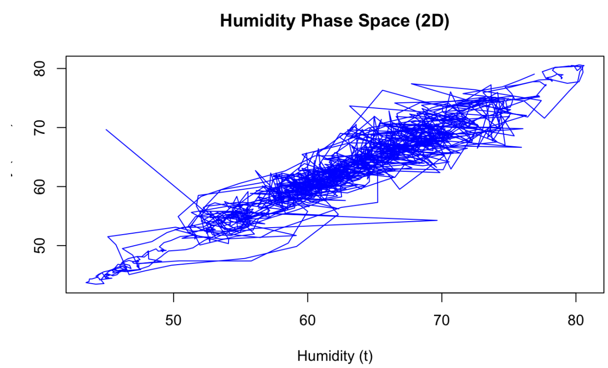

Figure 13: Humidity Phase Space Plot (tau = 2, M = 2, 2D)

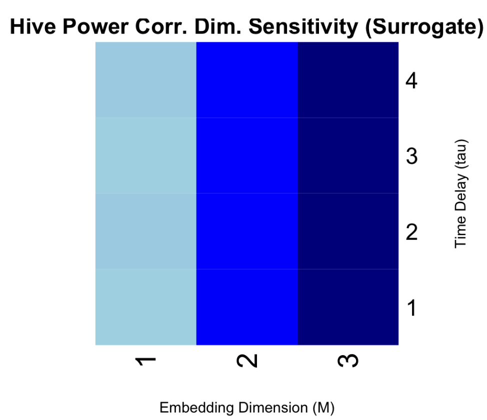

Figure 11: Sensitivity Analysis Heatmap for Hive Power (Surrogate)

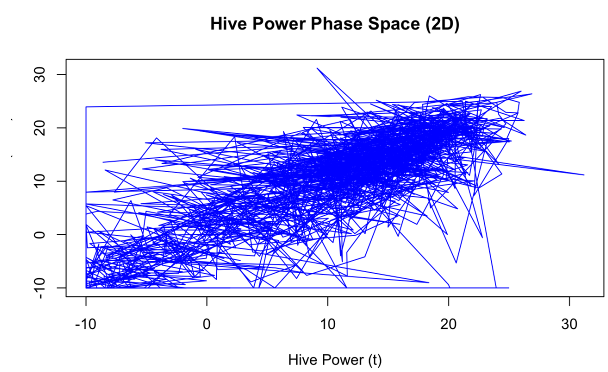

Figure 14: Hive Power Phase Space Plot (tau = 2, M = 2, 2D)

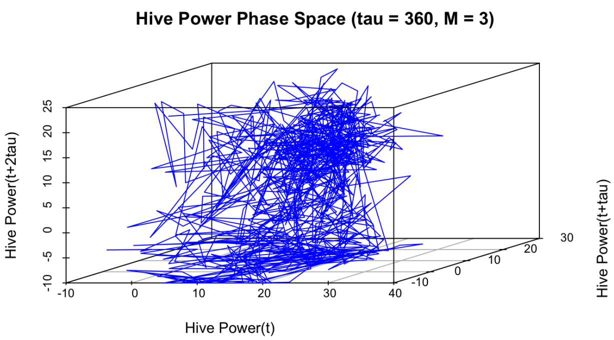

Figure 15: Hive Power Phase Space Plot (tau = 360, M = 3, 3D, First 200 Points)