

# Multivariate Recurrence Analysis

## ePortfolio Entry 4 - Module 6

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### 1 Introduction

In this project, we conducted a detailed recurrence-based time series analysis of Hive 202204, focusing on the summer segment of the dataset collected from the MSPB database (Zhu et al., 2023). Our objective was to investigate the temporal dynamics between acoustic and thermal environmental data recorded inside the hive.

We primarily employed Recurrence Quantification Analysis (RQA) and Classical Cross Recurrence Quantification Analysis (CRQA) to explore the behavioral dynamics of the hive during the summer (Wallot & Leonardi, 2018; Wallot, 2019), and we compared the differences in RQA results before and after detrending. Principal Component Analysis (PCA) and K-means clustering were also used as complementary methods to classify temporal dynamic patterns.

We focused on three key signals—temperature, hive power, and humidity—all of which are indicative of colony activity levels and stress conditions.

The code used for the computations and visualizations in this entry can be found [here](#).

### 2 Data Preprocessing

Given that the RQA method does not require assumptions of distributional form or stationarity and is robust to outliers, we first removed duplicate timestamps, eliminated missing values, and standardized all three variables (Wallot & Leonardi, 2018). We also verified that the three variables shared a consistent sampling frequency, aligned the time series accordingly, and excluded extreme outliers. Due to irregular sampling and a high proportion of missing values at the beginning and end of the recordings (Zhu et al., 2023), we removed these time segments to ensure data quality.

Short gaps within the selected time window were handled automatically using `.dropna()`, and since all variables were uniformly resampled at 15-minute intervals, these gaps were smoothed without disrupting temporal continuity or synchrony.

As a result, the cleaned dataset retained sufficient temporal integrity to support valid CRQA analysis.

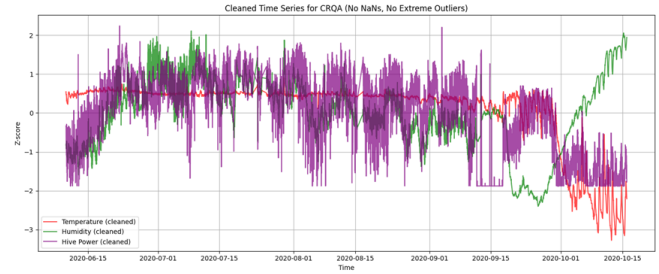


Figure 1: Cleaned Time Series for CRQA

### 3 Univariate RQA

To characterize the internal dynamics of Hive, we conducted univariate RQA analyses on each of the individual signal variables. We employed embedding parameters recommended by the Delayed Mutual Information (DMI) and False Nearest Neighbors (FNN) methods (Wallot & Mønster, 2018). We obtained the following RQA measures:

Signal	m	r
temperature_z	47	6
humidity_z	42	6
hive_power_z	20	7

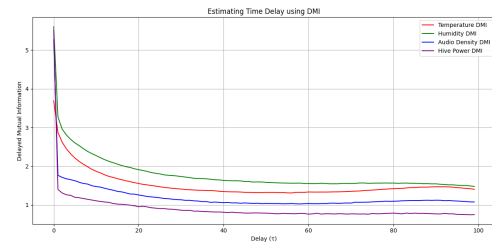


Figure 2: Estimating Time Delay using DMI

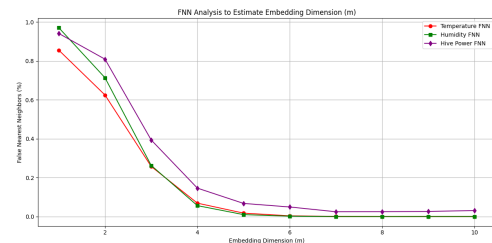


Figure 3: FNN Analysis to Estimate Embedding Dimension

	temperature_z	humidity_z	hive_power_z
RR	0.179877	0.098953	0.108894
DET	0.981573	0.925347	0.717501
L	17.33446	8.568472	3.889255
L_max	2500	2822	526
DIV	0.0004	0.000354	0.001901
L_entr	3.48672	2.613604	1.741879
LAM	0.987129	0.955045	0.809494
TT	26.17579	13.08573	5.335198
V_max	1101	783	553
V_entr	3.737406	2.950859	2.103107
W	126.9318	117.0331	38.05462
W_max	11104	10583	11057
W_div	9.00E-05	9.40E-05	9.00E-05
W_entr	3.714394	3.715002	3.074755
DET/RR	5.456905	9.351356	6.588961
LAM/DET	1.00566	1.032094	1.128214

Table: RQA Summary (Variables as Rows)

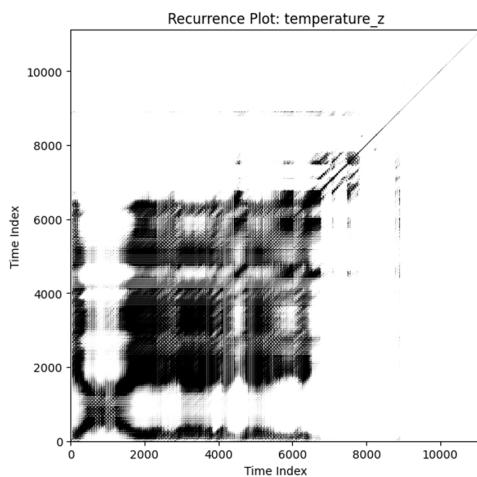


Figure 4: Recurrence Plot: Temp<sub>z</sub>

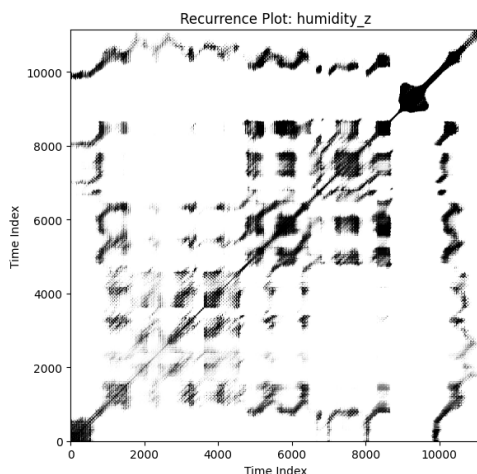


Figure 5: Recurrence Plot: humidity\_z

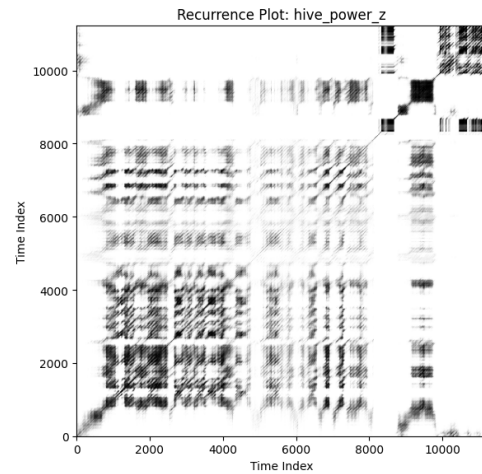


Figure 6: Recurrence Plot: hive\_power\_z

The recurrence plot of temperature is densely populated and well-structured, with repeated motifs and clear diagonals, indicating strong self-similarity and stable cyclic behavior. This is supported by its RQA metrics: RR = 17.99%, DET = 98.16%, LAM = 98.71%. This suggests temperature is a highly regulated and stable variable in the hive.

In contrast, the recurrence plot of humidity is sparser, with irregular bands and fewer diagonals. Although DET is relatively high (92.53%), the lower RR (9.90%) indicates fewer recurring states. A dense region in the latter part suggests external influences or regime shifts. Humidity shows some temporal coherence but is more susceptible to environmental fluctuations. The recurrence plot of hive\_power is most fragmented, lacking clear patterns, with high variability. This is reflected in its metrics: DET = 71.75%, RR = 10.89%. This aligns with its role as a proxy for behavioral fluctuations in hive activity. Overall, the variables show a stability hierarchy: temperature, humidity, hive power. This reflects differing levels of regulation and sensitivity to external factors.

To avoid long-term trends obscuring dynamic structure, we detrended all signals. As shown in rolling means, temperature declines sharply after mid-September, while humidity and hive\_power fluctuate throughout. If unremoved, these trends could distort recurrence metrics or inflate determinism. Detrending improves the accuracy of recurrence analysis by focusing on short-term structure.

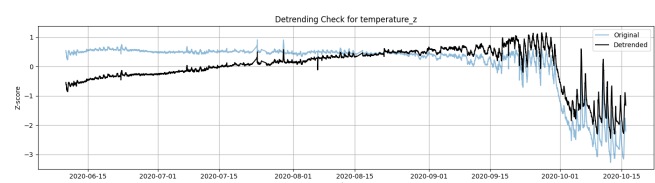


Figure 7: Detrending Check for temperature\_z

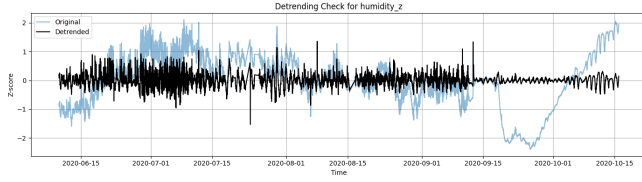


Figure 8: Detrending Check for humidity\_z

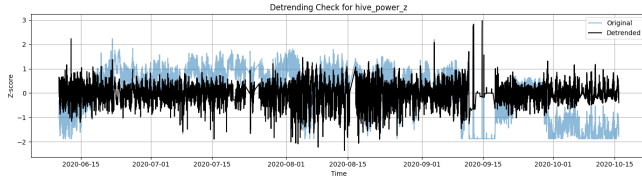


Figure 9: Detrending Check for hive\_power\_z

	[H]		
	temperature_z	humidity_z	hive_power_z
RR	0.034926	0.819903	0.488145
DET	0.980671	0.981251	0.862554
L	17.68371	17.6754	5.224522
L_max	2500	2822	526
DIV	0.0004	0.000354	0.001901
L_entr	3.351397	3.14273	2.157038
LAM	0.986793	0.988008	0.90282
TT	26.01152	28.42331	7.449154
V_max	1011	2822	680
V_entr	3.672196	3.415477	2.490389
W	694.8218	7.853371	8.309424
W_max	11104	7373	8503
W_div	9.00E-05	0.000136	0.000118
W_entr	4.412869	2.447347	2.433745
DET/RR	28.07891	1.196789	1.767003
LAM/DET	1.006242	1.006886	1.046682

Table: Detrended RQA Summary

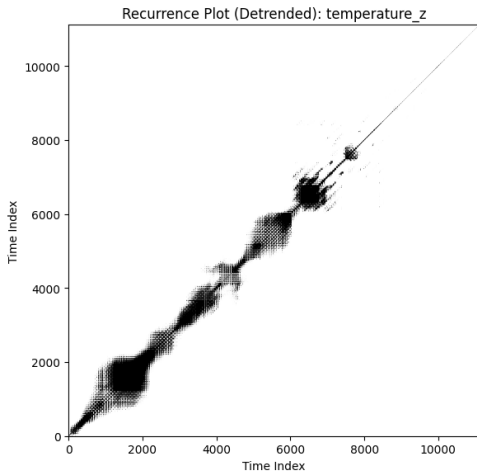


Figure 10: Recurrence Plot (Detrended): temperature\_z

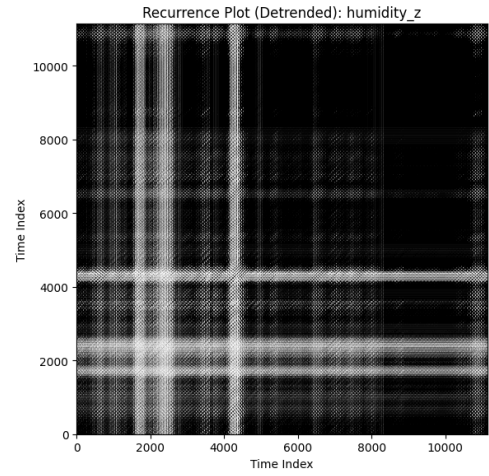


Figure 11: Recurrence Plot (Detrended): humidity\_z

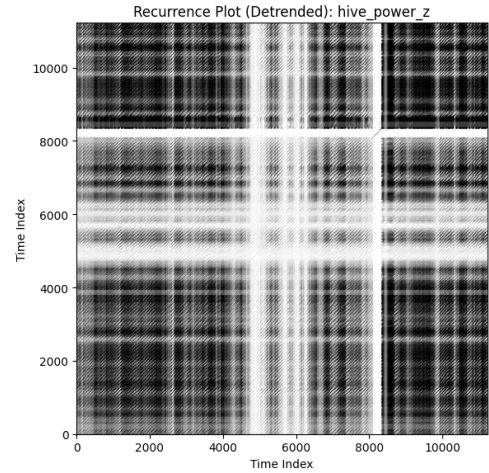


Figure 12: Recurrence Plot (Detrended): hive\_power\_z

To extract genuine dynamical patterns, we repeated the RQA after removing low-frequency trends. The results show that detrending removed nonstationary components while preserving or enhancing meaningful recurrence features. For temperature, RR dropped (17.99 to 3.49%) while DET and LAM remained high (98.07%, 98.68%), indicating genuine cyclic structure. For humidity, the recurrence became highly repetitive (RR: 81.99%), with DET and LAM staying high, suggesting enhanced regularity and periodicity. For hive\_power, RR rose to 48.81% and DET increased from 71.75% to 86.26%, revealing latent behavioral structures. Notably, even with a fixed radius of 0.7, the high RR values for humidity and hive power indicate pronounced short-term recurrence after detrending, potentially reflecting cyclic regulation or collective behavioral rhythms.

Overall, detrending improved the interpretability of recurrence plots by emphasizing short-term structural dynamics.

## 4 CRQA Results

For CRQA analysis we selected the parameters  $m=7$  and  $\epsilon=33$ . To ensure that the recurrence matrix captured sufficient

recurrence without becoming overly saturated, we conducted a sweep analysis of RR across different radius values. The optimal radius was found to be 0.6, where the RR fell within the interpretable range of 2–10

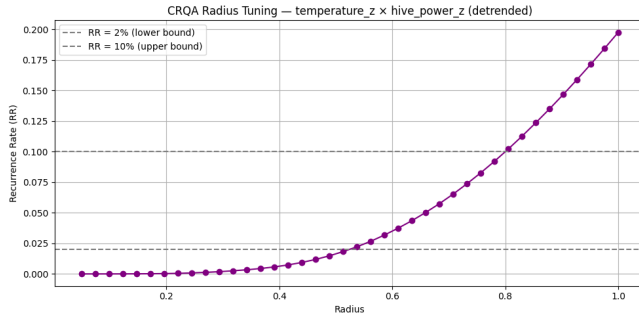


Figure 13: CRQA Radius Tuning

We obtained the following key metrics:

CRQA Metrics	Value
Recurrence rate (RR)	0.099899
Determinism (DET)	0.700436
Average diagonal line length (L)	3.905308
Longest diagonal line length (L_max)	50
Divergence (DIV)	0.02
Entropy diagonal lines (L_entr)	1.768985
Laminarity (LAM)	0.700744
Trapping time (TT)	3.916318

CRQA Metrics

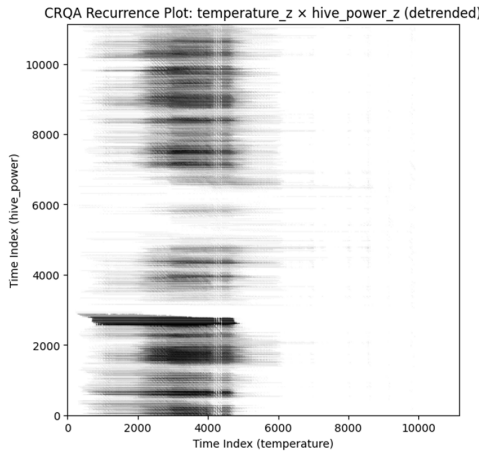


Figure 14: CRQA Recurrence Plot: temperature\_z x hive\_power\_z (detrended)

To assess the dynamic relationship between hive temperature and acoustic activity, we performed CRQA on the detrended temperature and hive\_power signals. The cross-recurrence plot shows horizontal banding in the lower and middle regions, indicating that hive power recurs in alignment with temperature at specific time lags. The recurrence rate (RR = 9.99%) suggests moderate shared state space, while determinism (DET = 70.04%) indicates short-lived coordination patterns. Low diagonal length ( $L = 3.91$ ) and entropy, along

with  $LAM = 70.07\%$  and  $TT = 3.92$ , suggest that alignment is local and episodic.

In sum, CRQA reveals structured but non-persistent coupling between temperature and hive acoustic signals.

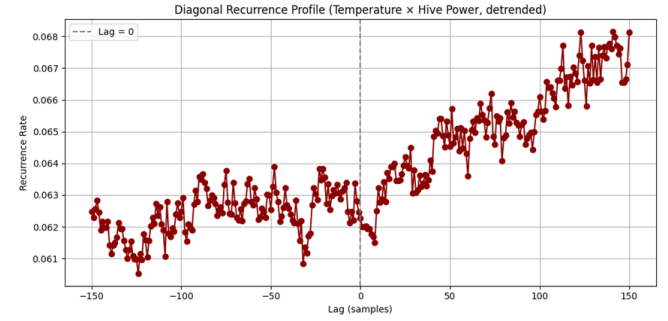


Figure 15: Diagonal Recurrence Profile

To further examine the temporal relationship between hive temperature and acoustic activity, we computed a Diagonal Recurrence Profile (DRP) based on the cross-recurrence matrix of the detrended temperature and hive\_power signals.

The DRP quantifies the recurrence rate (RR) along diagonals offset by varying time lags, thereby revealing whether one signal systematically precedes or follows the other in terms of shared dynamic states.

The DRP curve, constructed with a maximum lag of 150 samples, exhibits a clear upward trend toward positive lags, indicating that hive power aligns more frequently with earlier states in temperature. In other words, temperature changes tend to precede shifts in hive acoustic energy. This asymmetric profile implies that temperature may be a leading indicator of behavioral changes rather than occurring concurrently.

## 5 Windowed CRQA and Local DRP

To assess how the dynamic coupling between hive temperature and acoustic activity evolves over time, a windowed CRQA was performed on the detrended temperature and hive\_power signals.

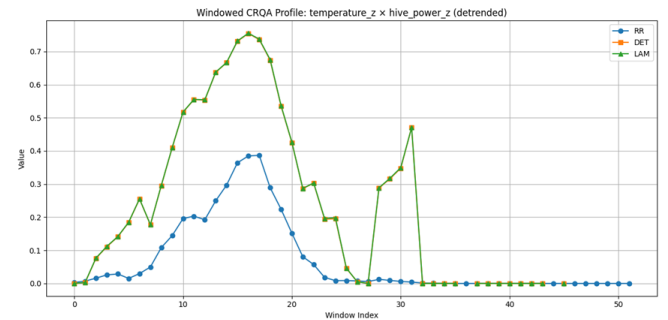


Figure 16: Windowed CRQA Profile: temperature\_z x hive\_power\_z (detrended)

The signals were segmented into overlapping windows, and CRQA metrics—including recurrence rate (RR), determinism (DET), and laminarity (LAM)—were computed within each window to capture temporal variations in coordination.



The resulting profile reveals a clear phase of elevated interaction between window indices 10 and 20. RR peaks near window 17 (0.39), and both DET and LAM exceed 0.75, indicating strongly structured joint dynamics.

In contrast, beyond this phase—especially after window index 30—all three metrics decline sharply, suggesting a breakdown in coordination between the signals. A secondary peak appears near window 31, but it is brief and less coherent.

These results indicate that the coupling between temperature and hive acoustic energy is not constant, but fluctuates, with a distinct phase of strong synchronization likely reflecting coordinated behavioral or regulatory activity within the hive.

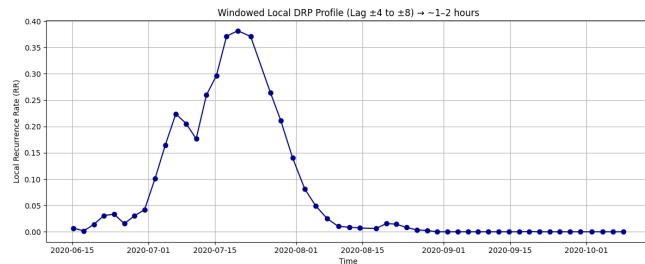


Figure 17: Windowed Local DRP Profile (Lag  $\pm 4$  to  $\pm 8$ )  $\rightarrow$  1–2 hours

To examine the temporal evolution of short-lag coupling between hive temperature and acoustic activity, a windowed local diagonal recurrence profile (local DRP) was computed using a lag range of  $\pm 4$  to  $\pm 8$  samples (1–2 hours). This profile captures localized recurrence rates (RR) across successive time windows, identifying when short-lag alignment is strong or weak.

The resulting profile reveals a distinct peak in recurrence around mid-July 2020, with RR peaking at 0.38, indicating strong short-lag temporal alignment. This peak is preceded by a gradual increase from late June and followed by a steady decline through August. After early August, the RR drops to near zero and remains flat through September and October, suggesting the short-term relationship faded. This analysis shows that short-lag coordination between temperature and hive power is temporally localized, occurring mainly in early to mid-summer. The strong recurrence likely reflects coordinated thermoregulation or collective behaviors that dissipate with seasonal changes or hive condition shifts.

## 6 PCA and KMeans Clustering

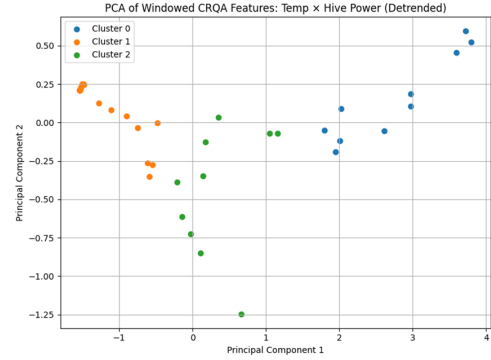


Figure 18: PCA of Windowed CRQA Features

To identify distinct temporal regimes in the interaction dynamics between hive temperature and hive acoustic energy, we performed k-means clustering on windowed CRQA features (RR, DET, LAM) extracted from detrended signals.

The resulting clusters were visualized using Principal Component Analysis (PCA), and their feature-wise means are summarized in the accompanying table.

Three distinct clusters emerged: Cluster 0 (blue, 10 windows): High recurrence (RR = 0.2786), moderate DET and LAM (0.636), indicating strong and structured coordination.

Cluster 1 (orange, 24 windows): Minimal RR (0.0081), nearly zero DET and LAM (0.047), corresponding to uncoordinated or decoupled dynamics.

Cluster 2 (green, 10 windows): Moderate RR (0.0602), intermediate DET and LAM (0.34), possibly reflecting transitional behavior between states.

These findings confirm that the temperature–hive power relationship is temporally structured, fluctuating between phases of strong synchrony, weak coupling, and transitions.

The clustering highlights discrete dynamical states in the hive system and their temporal evolution.

Cluster	RR	DET	LAM	PC1	PC2
0	0.2786	0.636	0.637	2.7414	0.1547
1	0.0081	0.0471	0.0475	-1.2775	0.1188
2	0.0602	0.3396	0.3403	0.3246	-0.4399

Table: Cluster Summary of Windowed CRQA Features

## 7 Conclusion

This study applied Recurrence Quantification Analysis (RQA), Cross-Recurrence Quantification Analysis (CRQA), Diagonal Recurrence Profile (DRP), and their multivariate extensions to systematically investigate the dynamic coupling between hive temperature and acoustic energy.

We found that determinism (DET) and recurrence rate (RR) were the most informative indicators, effectively differentiating between periodic regulation, decoupled states, and transitional dynamics.

Windowed analysis and clustering revealed three discrete dynamical phases—synchronization, decoupling, and transition—showing that coupling is temporally localized rather than persistent.

DRP results indicated that temperature changes tend to precede acoustic activity, suggesting temperature as a potential leading indicator of behavioral coordination.

In conclusion, recurrence-based methods not only revealed the complex nonlinear coupling between hive temperature and acoustic energy, but also provided an effective framework for detecting dynamic state transitions in ecological behavioral systems.

## References

1. Analyzing multivariate dynamics using cross-recurrence quantification analysis (CRQA), diagonal-cross-recurrence profiles (DCRP), and multidimensional recurrence quantification analysis (MdRQA): A tutorial in R. *Frontiers in Psychology*, 9, 2232.
2. Calculation of average mutual information (AMI) and false-nearest neighbors (FNN) for the estimation of embedding parameters of multidimensional time series in Matlab. *Frontiers in Psychology*, 9, 1679.
3. Multimodal sensor data of honey bee colonies: From univariate statistics to complex systems analysis. *Patterns*, 4(6), 100779.