Vehicle Lateral Dynamics

Path follower driver model

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Target

Simulate vehicle lateral dyncamics throug single track vehicle model plus path follower.

- Path follower model equations
    - Inputs, outputs, state vector
    - Axle forces description
    - Slip angles
- Assignments

Recall on Single track vehicle model

Vehicle in plane motion. One rigid body

3 dofs:

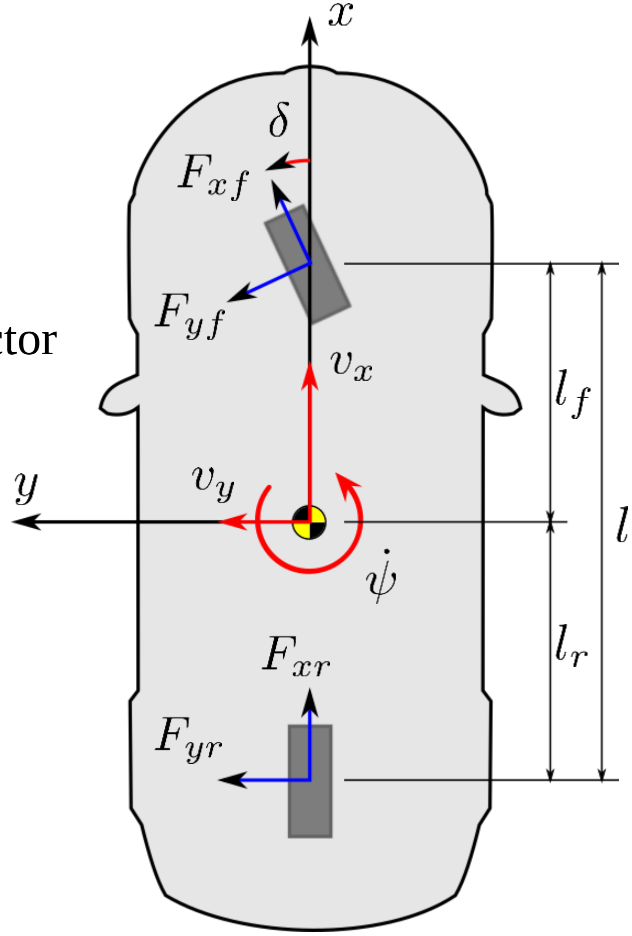
- Longitudinal speed
- Lateral speed
- Yaw rate

Imposing the longitudinal speed, the state vector becomes

$$\boldsymbol{x} = \begin{Bmatrix} v_y \\ \dot{\psi} \end{Bmatrix}$$

Remember the kinematic relationship

$$a_y = \dot{v}_y + v_x \dot{\psi}$$



Driver model for lateral dynamics

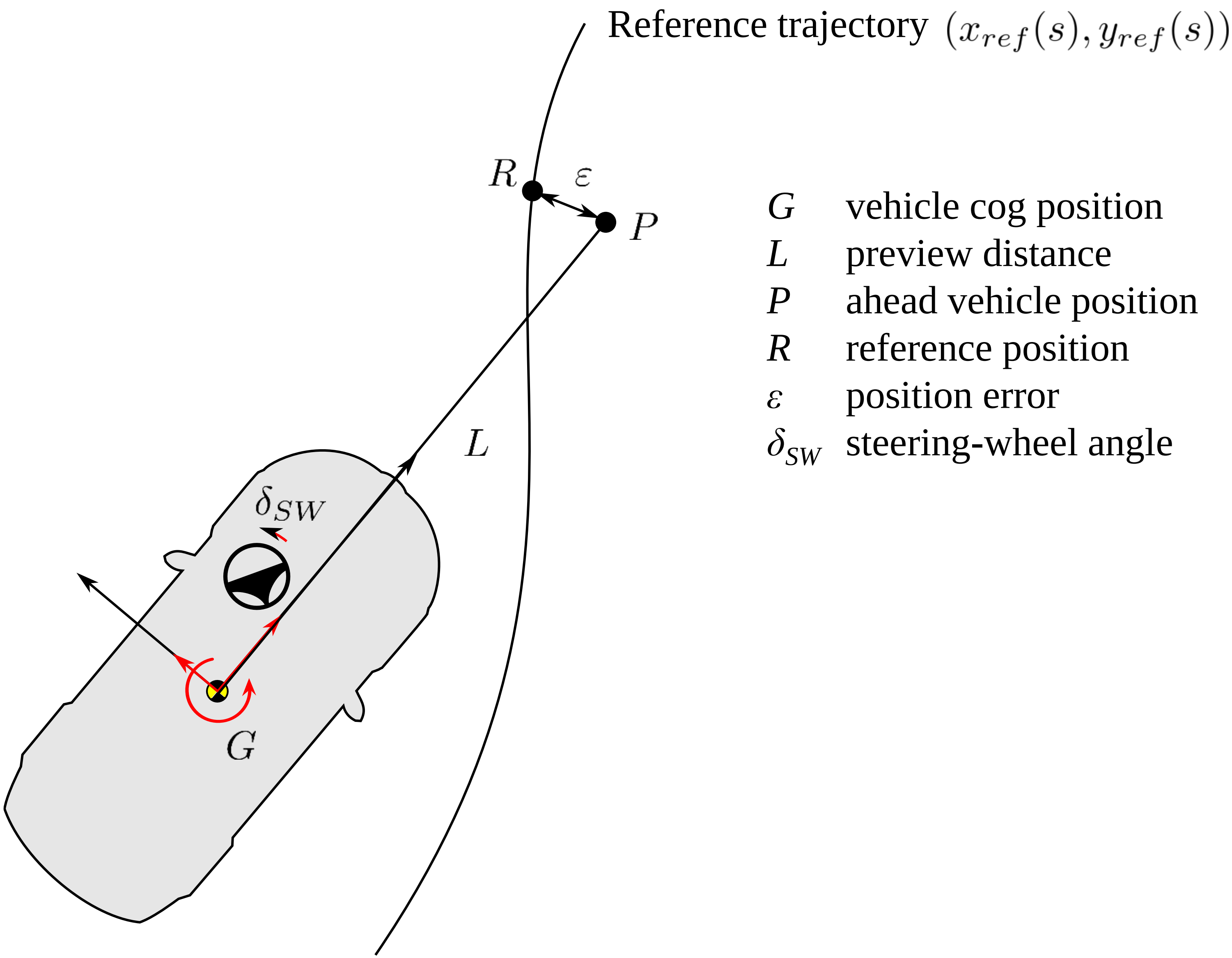
How to model a driver that follows a reference trajectory?

Path follower driver

- reference trajectory
- actual position of the vehicle
- Look ahead capability
- control law
- Time lag modeling

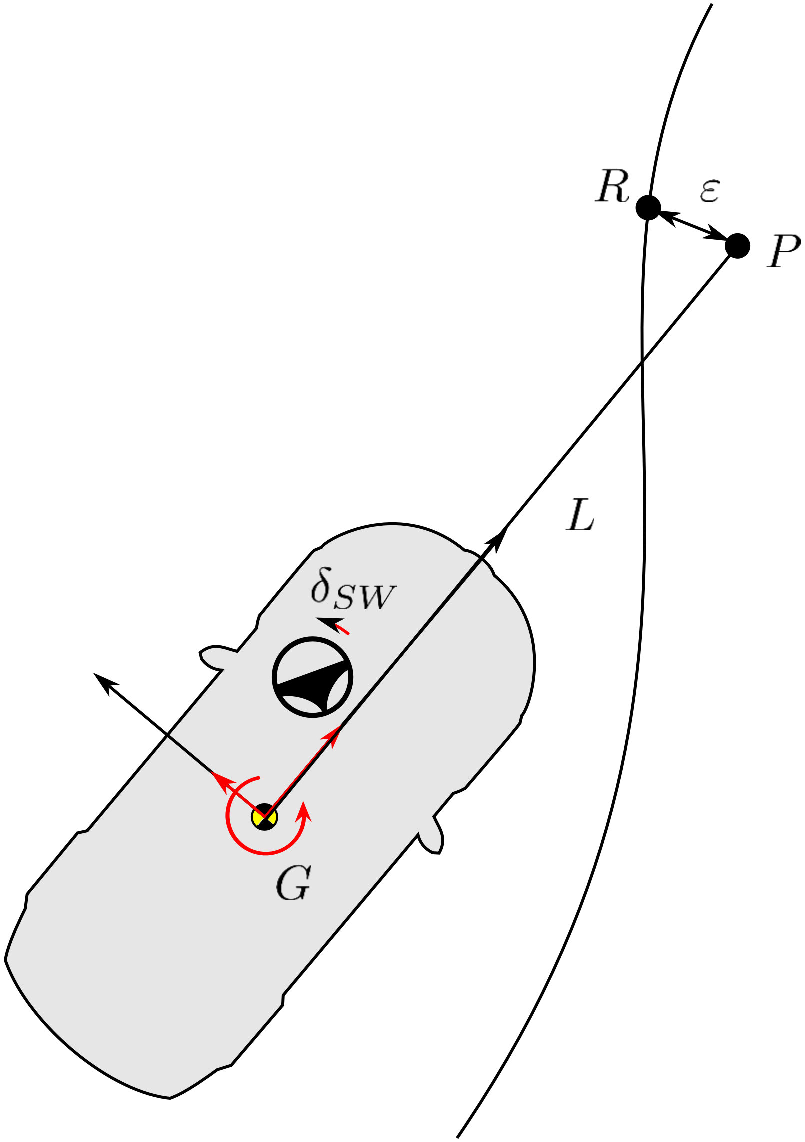
Lateral displacement error

Path follower



G	vehicle cog position
L	preview distance
P	ahead vehicle position
R	reference position
ε	position error
δ_{SW}	steering-wheel angle

Path follower – reference trajectory



Reference trajectory is the collection of points describing the track shape as a function of curvilinear abscissa *s*

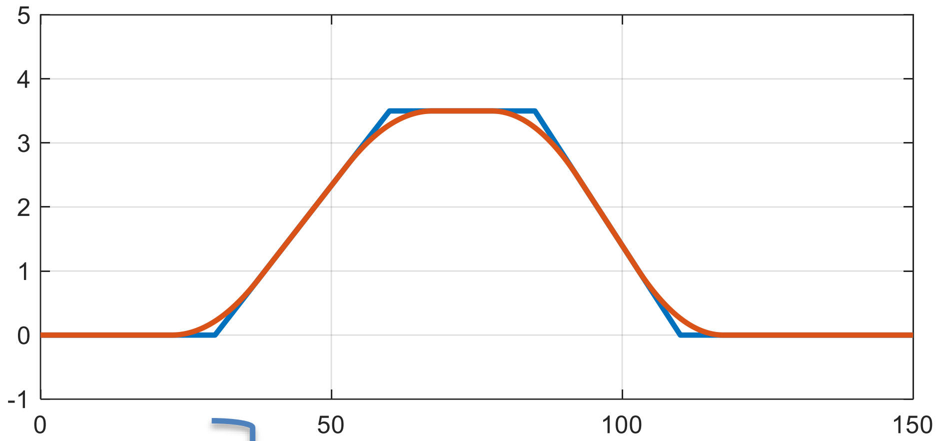
$$(x_{ref}(s), y_{ref}(s))$$

Curvilinear abscissa *s* represents the travelled distance thus

$$s = \int_0^t v dt$$

Path follower – reference trajectory

Reference trajectory of a double lane change

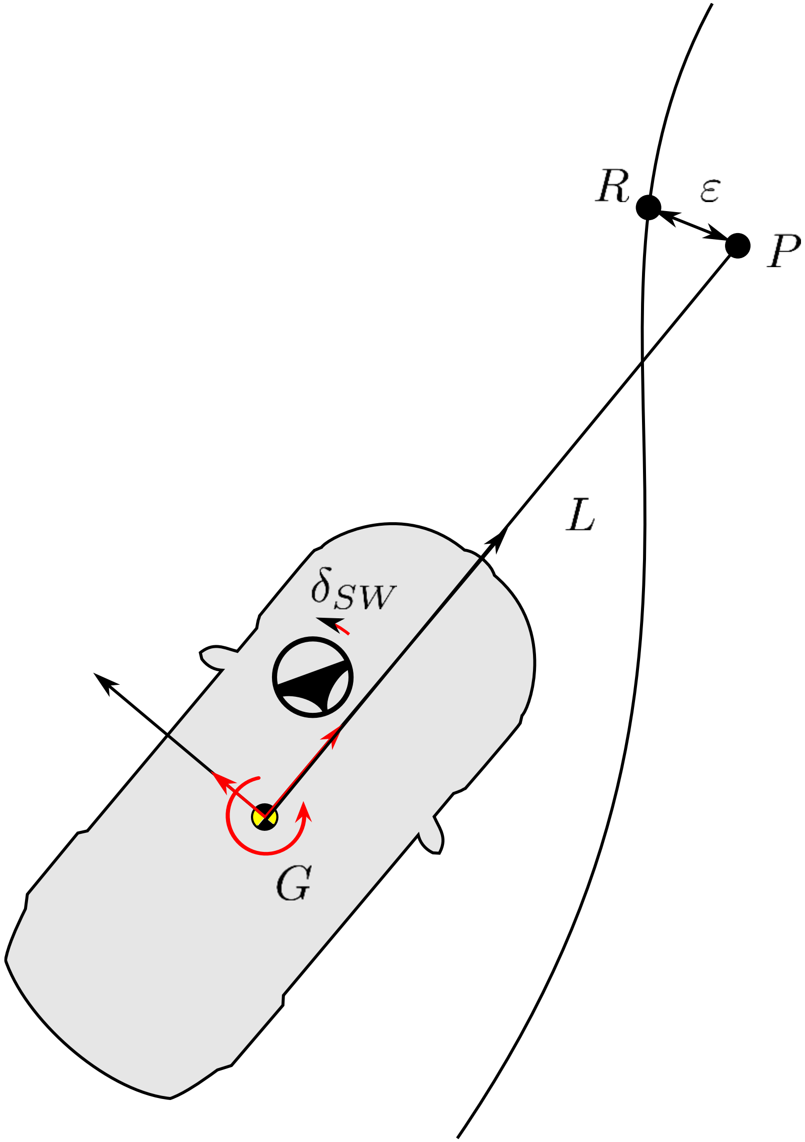



```
x  = [0 20 30 60 85 110 140 1000] ';
y  = [0 0 0  3.5  3.5  0 0 0]';
xx = [0:.1:x(end)]';
yy = interp1(x,y,xx);
yy = smooth(yy,150);
x = xx;
y = yy;

dx = diff(x);
dy = diff(y);
dx = [0; dx];
dy = [0; dy];
for ii=1:length(x)
   s(ii,1) = sum(sqrt(dx(1:ii).^2+dy(1:ii).^2));
end
ss = [0:.1:s(end)]';
xx = interp1(s,x,ss);
yy = interp1(s,y,ss);
traiettoria.s = ss;
traiettoria.x = xx;
traiettoria.y = yy;
```

Compute curvilinear abscissa of the reference path

Path follower – vehicle position



Vehicle position is obtained by integrating vehicle speed in absolute reference frame

$$X_G(t) = \int_0^t \dot{X}_G dt + X_{G0}$$

$$Y_G(t) = \int_0^t \dot{Y}_G dt + Y_{G0}$$

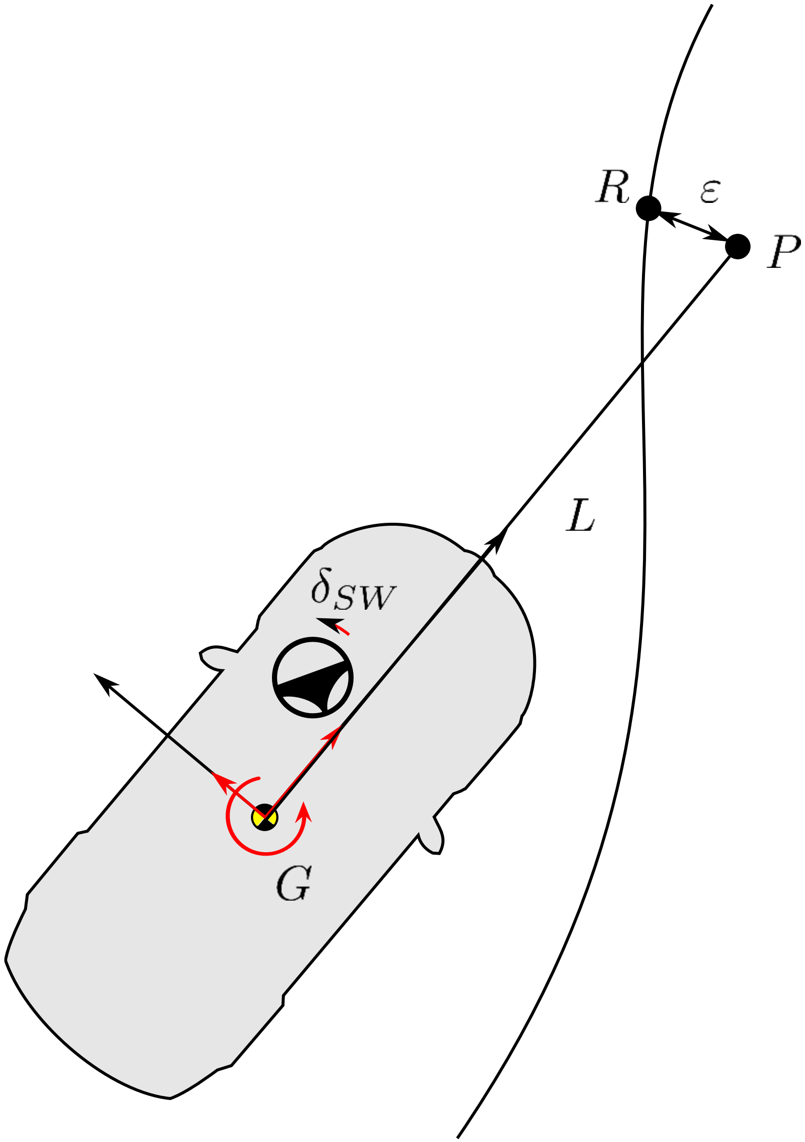
$$\psi(t) = \int_0^t \dot{\psi} dt + \psi_0$$

$$\dot{X}_G = v_x \cos\psi - v_y \sin\psi$$

$$\dot{Y}_G = v_x \sin\psi + v_y \cos\psi$$

$$s = \int_0^t v dt$$

Path follower – vehicle ahead position



The driver is ususally capable of anticipating the necessary maneuver. The position error can thus be computed according to a position in front of the vehicle (*P*) of a given distance *L*

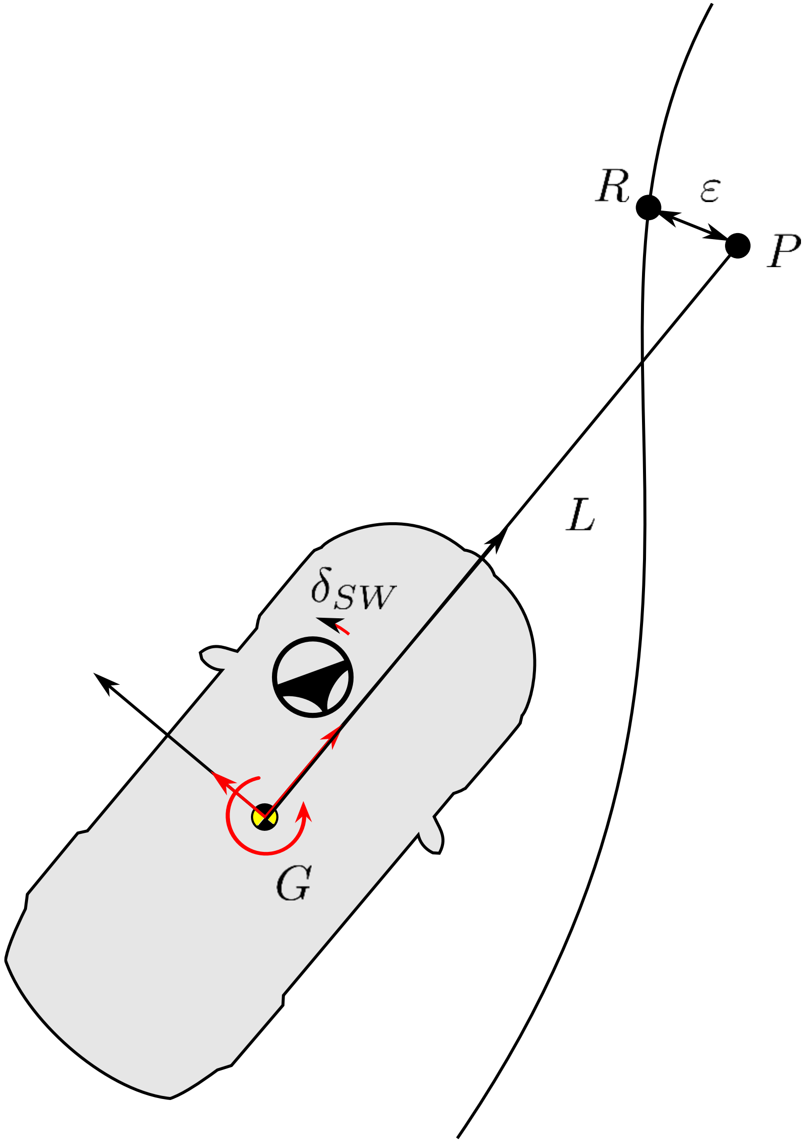
$$X_P = X_G + L\cos\psi$$
$$Y_P = Y_G + L\sin\psi$$

The preview distance *L* is not constant but changes with vehicle speed

$$L = v_x t_{driver} + L_0$$

t_{driver} is the driver characteristic time. **When going faster, the preview distance increases**

Path follower – position error



We need now to compute the R-P vector

$$(R - P) = (X_R - X_P)\boldsymbol{i} + (Y_R - Y_P)\boldsymbol{j}$$

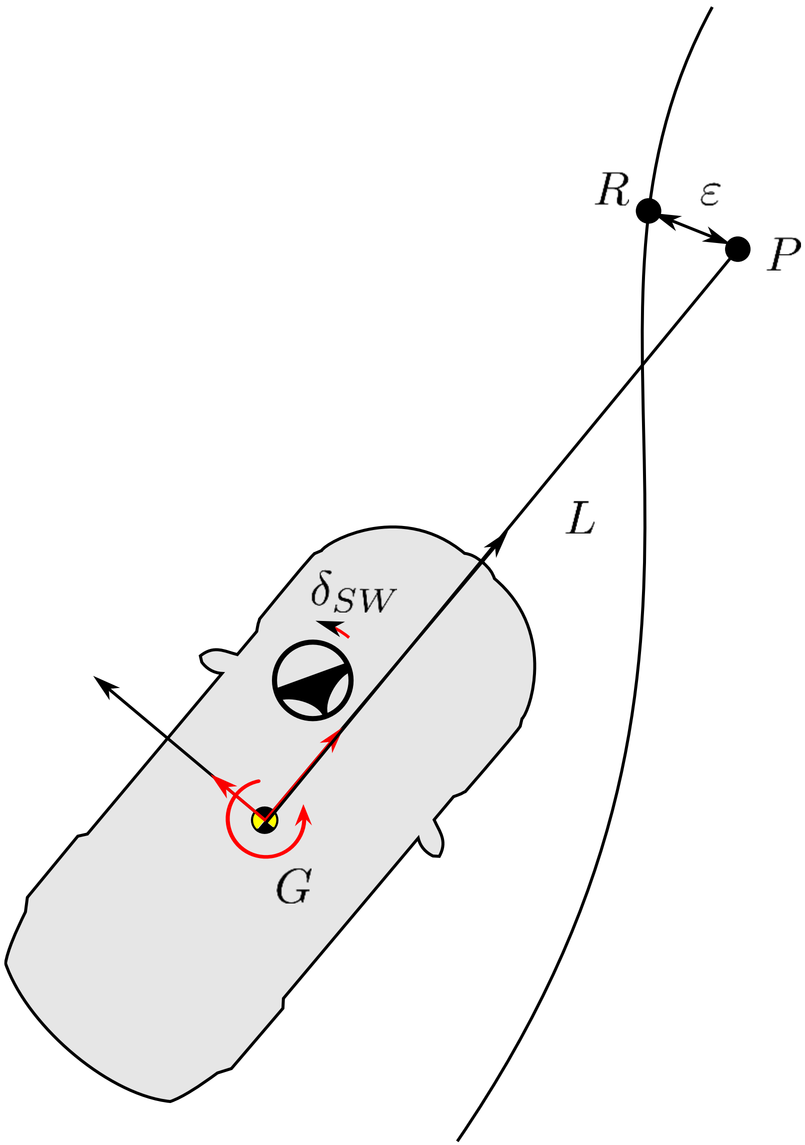
Transformed into vehicle reference frame

$$x_R = (X_R - X_P)\cos\psi + (Y_R - Y_P)\sin\psi$$
$$y_R = -(X_R - X_P)\sin\psi + (Y_R - Y_P)\cos\psi$$

The position error is then

$$\varepsilon = y_R$$

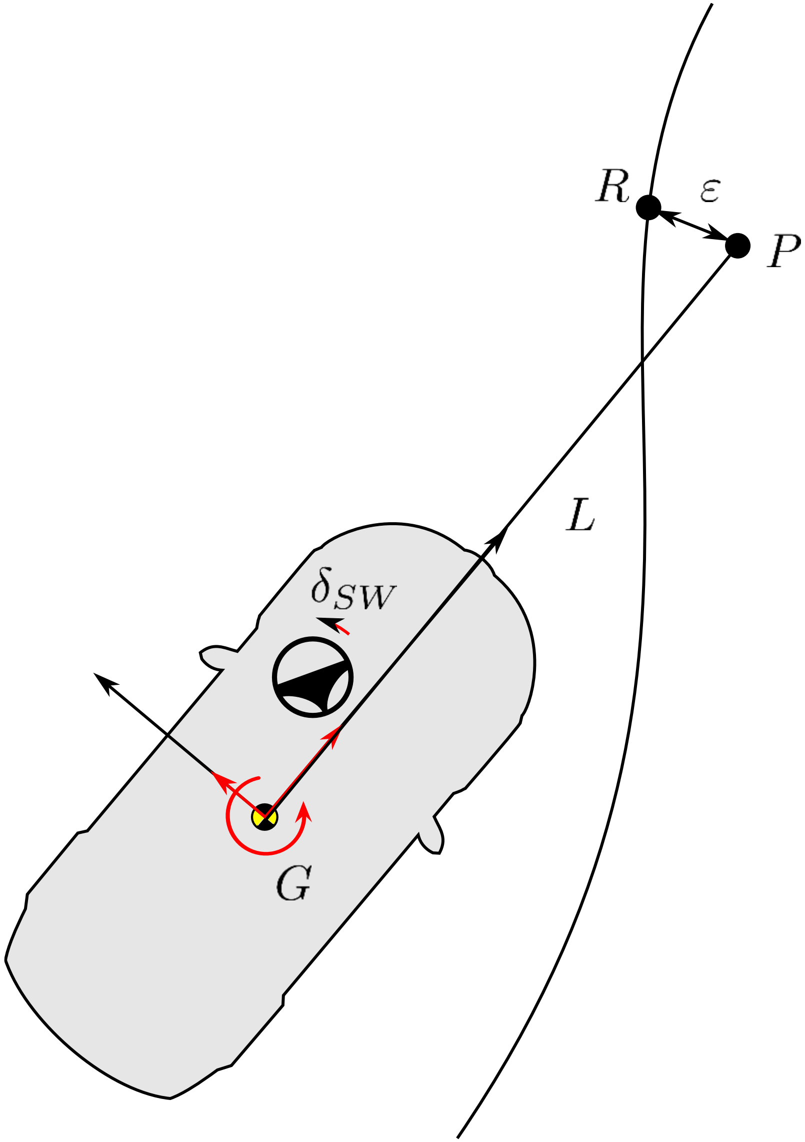
Path follower – control law



Steering wheel angle is thus proportional to the positioning error. A PD control law can be used

$$\hat{\delta}_{SW} = k_P \varepsilon + k_D \dot{\varepsilon}$$

Path follower – actuation time lag



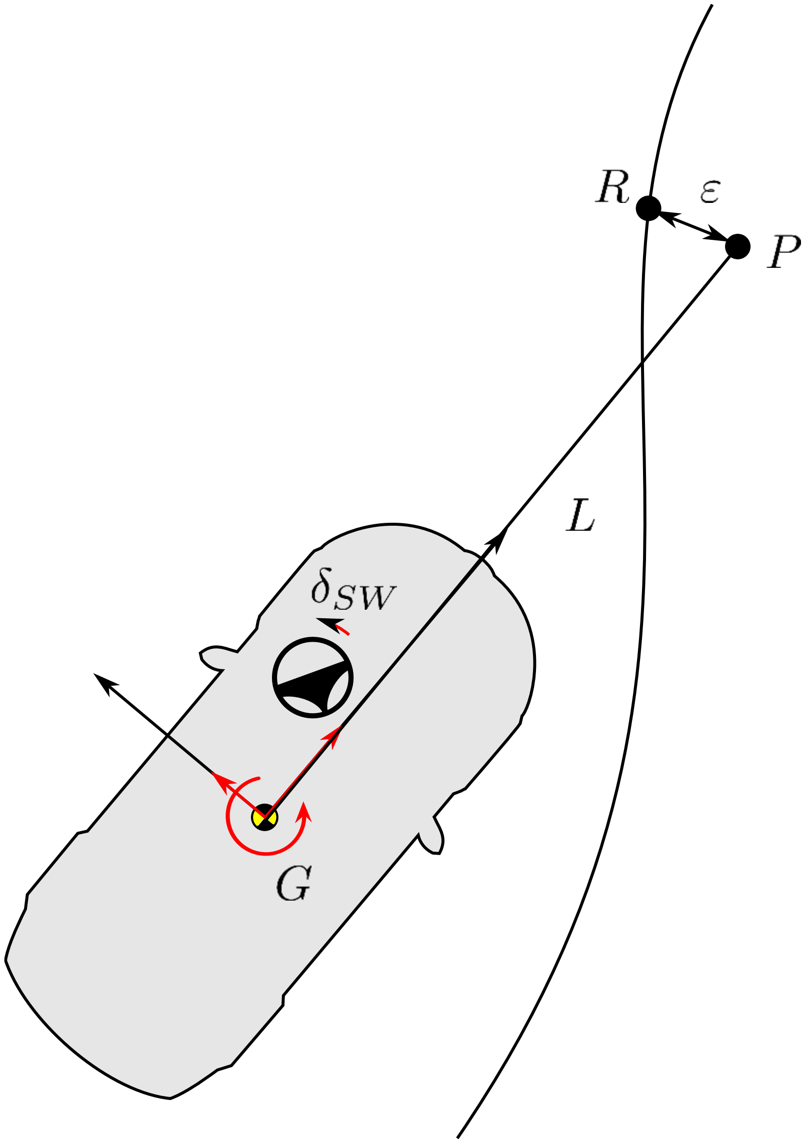
To reproduce the neuro-muscular time lag of the driver, a simple first order time lag tranfer function is used

$$TF = \frac{1}{\tau s + 1}$$

thus

$$\dot{\delta}_{SW} = \frac{1}{\tau_D}(\hat{\delta}_{SW} - \delta_{SW})$$

Path follower – actuation time lag



Consider the steering system ratio

$$\delta = \frac{1}{\tau_{SW}} \delta_{SW}$$

Assignements

Simulate following maneuvers:

- Double lane change maneuver

For each maneuver report the following graph

- Vy vs time
- Yaw rate vs time
- Sideslip angle vs time
- Lateral acceleration vs time
- Steering angle

Change vehicle speed and driver gains, analyse the different results obtained

Data


```
% Steering wheel ratio
T_SW = 18;

% path follower data
t_driver = 1.5;   % [s]
L0   = 0.5;       % [m]

kP   = 10;
kD   = 1;

TauD = 0.1;
```