Vehicle Lateral Dynamics

Single track vehicle model simulation

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Target

Simulate vehicle lateral dyncamics throug single track vehicle model.

- Single track model equations
    - Inputs, outputs, state vector
    - Vehicle equilibrium equations
    - Axle forces description
    - Slip angles
- Assignments

Single track vehicle model

Vehicle in plane motion. One rigid body

3 dofs:

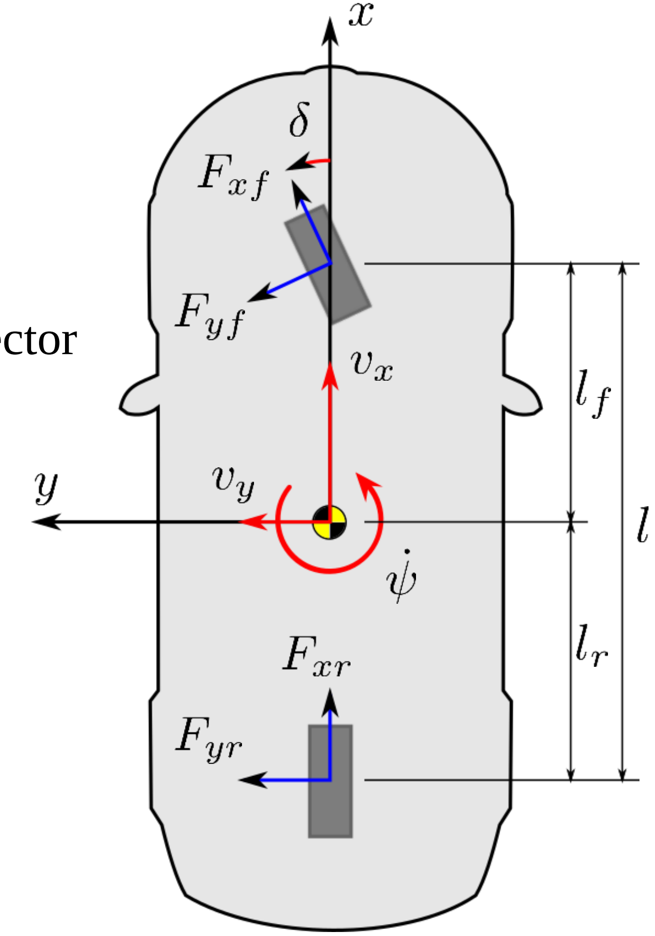
- Longitudinal speed
- Lateral speed
- Yaw rate

Imposing the longitudinal speed, the state vector becomes

$$\boldsymbol{x} = \begin{Bmatrix} v_y \\ \dot{\psi} \end{Bmatrix}$$

Remember the kinematic relationship

$$a_y = \dot{v}_y + v_x \dot{\psi}$$



Single track vehicle model

Contact forces are condensed so to have axle forces (single track).

Equilibrium equation thus read

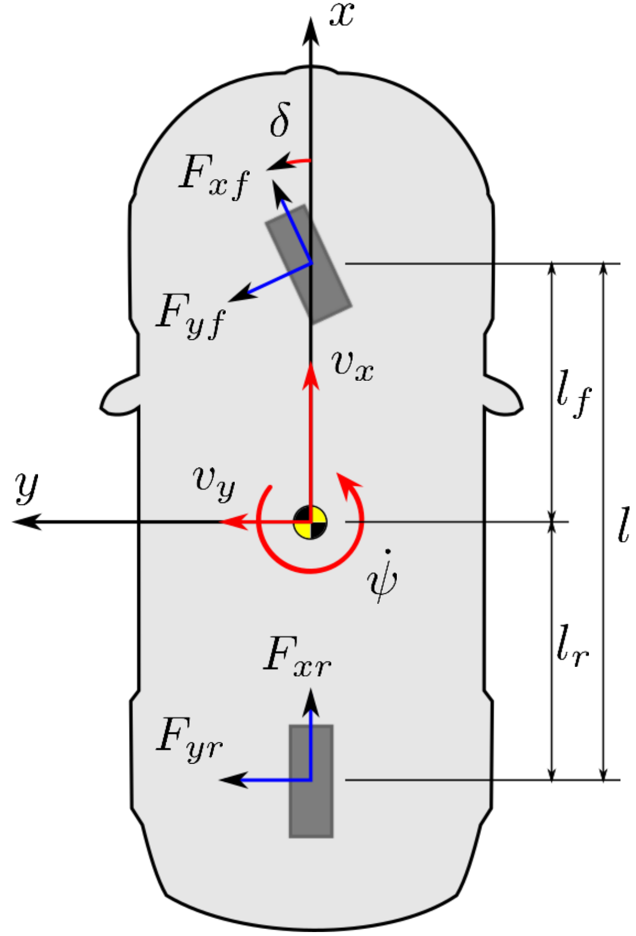
$$m a_y = F_{xf} \sin\delta + F_{yf} \cos\delta + F_{yr}$$
$$J_z \ddot{\psi} = (F_{xf} \sin\delta + F_{yf} \cos\delta) l_f + F_{yr} l_r$$

Inputs to the system are thus

- Vehicle longitudinal speed v_x
- Steering angle δ

The input vector is then

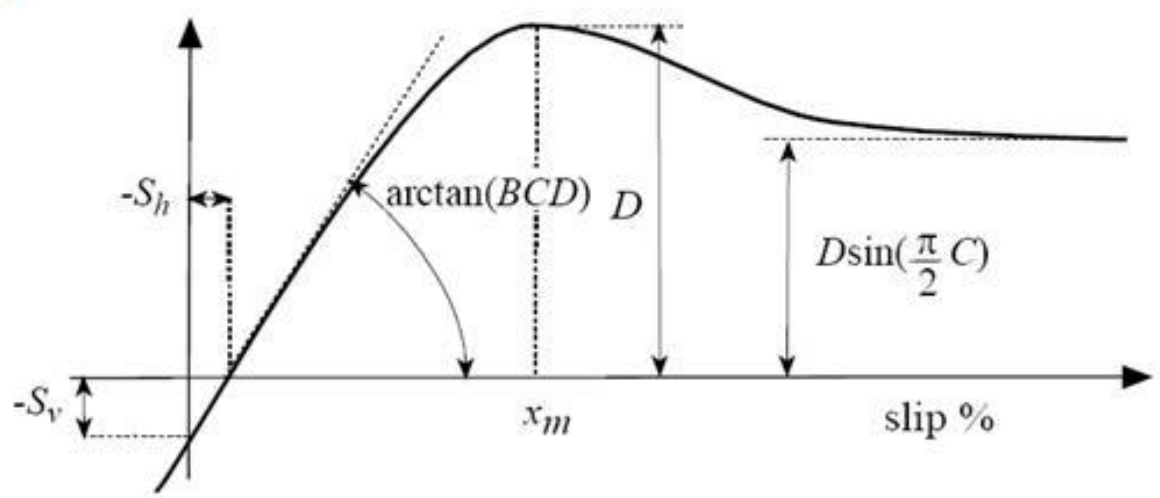
$$\boldsymbol{u} = \begin{Bmatrix} \delta \\ v_x \end{Bmatrix}$$



Contact cornering forces

Each force is then a state, while the steady-state contact forces are defined according to Pacejka Magic Formulae

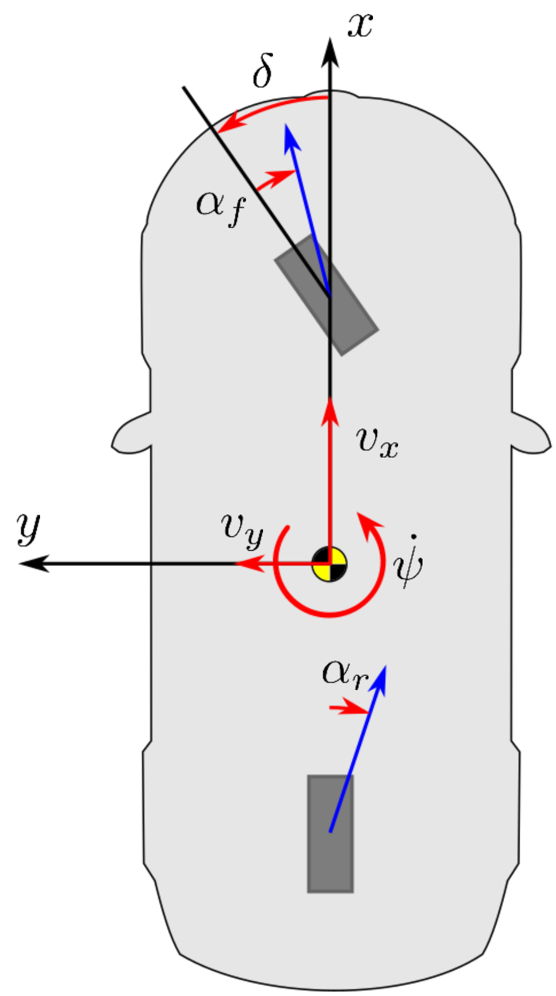
$$\hat{F}_{yi} = D_{yi} \sin\left\{C_{yi} \arctan\left[B_{yi}\alpha_i - E_i\left(B_{yi}\alpha_i - \arctan(B_{yi}\alpha_i)\right)\right]\right\}$$
$$D_{yi} = \mu_{yi} d_{yi} F_{zi}$$
$$B_{yi} = b_{yi}/\mu_{yi}$$



The slip angles in steady-state condition are computed according to the following equations

$$\hat{\alpha}_f = \delta - \arctan\left(\frac{v_y + \dot{\psi} l_f}{v_x}\right)$$

$$\hat{\alpha}_r = -\arctan\left(\frac{v_y - \dot{\psi} l_r}{v_x}\right)$$



Accounting for relaxation length effect

$$\frac{\lambda}{v_x}\dot{\alpha}_i + \alpha_i = \hat{\alpha}_i$$

We need to add slip angles to the state vector.

Contact cornering forces

The lateral friction coefficients depends on friction coefficient and longitudinal forces

$$\mu_{yi} = \sqrt{\mu^2 - \left(\frac{F_{xi}}{F_{zi}}\right)^2}$$

In this way it is possible to account for combined slip conditions in a simplified way, longitudinal forces are thus computed as follows

$$F_x = ma_x + \frac{1}{2}\rho C_x S v_x^2 + mgf_v$$

Which is divided among front and rear axle depending on traction (FWD, RWD, 4WD) and brake distribution

TRACTION

$$F_{xf} = \gamma_T F_x$$
$$F_{xr} = (1 - \gamma_T) F_x$$

BRAKING

$$F_{xf} = \gamma_B F_x$$
$$F_{xr} = (1 - \gamma_B) F_x$$

Other important vehicle quantities

To compute the vehicle trajectory we need to compute the speed component in absolute reference frame and integrate it in time to get the position

$$\dot{X}_G = v_x \cos\psi - v_y \sin\psi$$
$$\dot{Y}_G = v_x \sin\psi + v_y \cos\psi$$

$$X_G(t) = \int_0^t \dot{X}_G dt + X_{G0}$$
$$Y_G(t) = \int_0^t \dot{Y}_G dt + Y_{G0}$$
$$\psi(t) = \int_0^t \dot{\psi} dt + \psi_0$$

An important quantity for evaluating vehicle lateral dynamics is **sideslip angle**

$$\beta = \arctan\left(\frac{v_y}{v_x}\right)$$

Assignments

Simulate following maneuvers:

- Steering pad constant speed (at 3 different speeds)
- Steer step (at 2 speeds and 3 steer steps)

For each maneuver report the following graph

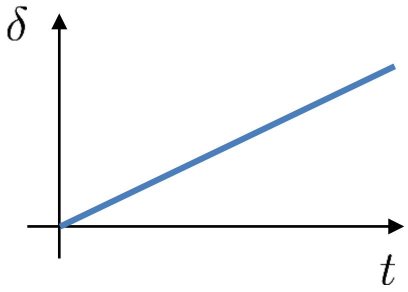
- Vy vs time
- Yaw rate vs time
- Sideslip angle vs time
- Lateral acceleration vs time
- Steering wheel angle vs time
- Steering wheel angle vs lateral acceleration
- Sideslip angle vs yaw rate

Data


```
% vehicle data
m    = 1582;                     % [kg]      vehicle mass
Jz   = 2210;                     % [kg m2] yaw moment of inertia
lf   = 0.977;                    % [m]       cog to front axle distance
lr   = 1.723;                    % [m]       cog to rear axle distance
l    = car.lf+car.lr;            % [m]       wheelbase
tau  = 13.1;                     % []        steer ratio
Rr   = 0.3;                      % [m]           wheels rolling radius
% resistance forces
fv   = .02;                      % []        rolling resistance coeff.
rho  = 1.2;                      % [kg/m3]   air density
Cd   = .3;                       % []        drag coefficient
S    = 2;                        % [m2]      front surface
% Magic Formula parameters
bf = 12;      % cornering factor
Cf = 1.3;     % asymptotic factor
df = 1;       % peak factor
Ef = -0.5;    % shape factor
br = 15;
Cr = 1.3;
dr = 1.1;
Er = -0.8;
L  = 2;       % relaxation length
% traction and braking ratios
gT = 1;       % 1 FWD, 0 RWD, 0.5 4WD
gB = 2/3;     %
```

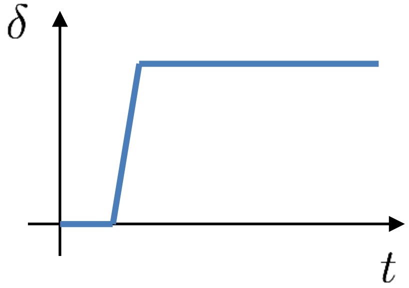
Maneuvers

Steering pad constant speed




```
delta          = [0 1]*20/180*pi;
delta_time     = [0 Tend];
```

Step steer maneuver




```
delta          = [0 0 1 1]*10/180*pi;
delta_time     = [0 0.5 0.6 Tend];
```