Intuitive Modelling and Formal Analysis of Collective Behaviour in Foraging Ants

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Goal

- Describe/design/reason about collective systems

How?

- Formulate hypotheses about
  - Individual behaviour
  - Interaction mechanisms (agent-agent, agent-environment)
- Check if collective features emerge with time + interactions

Wishlist

- Modelling languages that are
  - Agent-based
  - High-level
  - Intuitive (close to the domain of interest)
  - Formally defined

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  - Built on top of mature off-the-shelf solutions
  - Extensible

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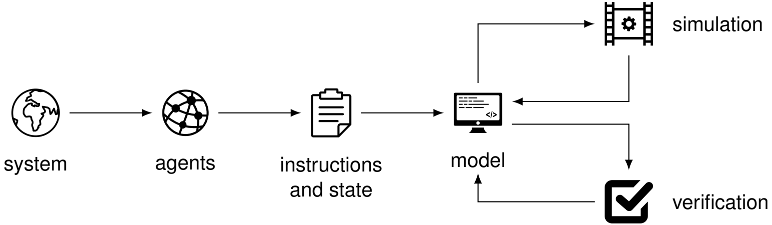
  - Extensible
- Effective methodologies to put all this at work

Our methodology



- Isolate features of agents & environment
- Come up with a high-level behavioural skeleton
- Flesh out the skeleton into a model
- Get feedback from simulation/verification
- Refine the model

Why?

- Well-known, extensively studied
- Several interesting mechanisms at play
  - Stigmergic (pheromone-based) interaction
  - Path integration

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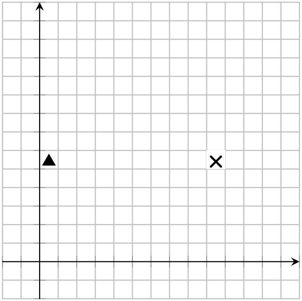
Our setting

- Arena: square grid of cells
- One cell contains food (×)
- One cell contains the nest (▲)
- Cells may be marked with pheromone



LAbS: System description

LAbS = simple, formal language for agent-based models

Parameters

size: Length of the sides of the arena

n: Number of ants (see line 4)

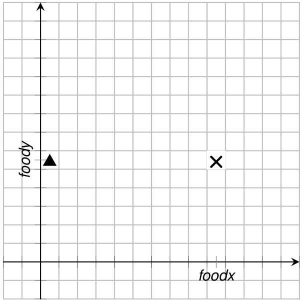
foodx, foody: Food cell coordinates

m, k: Related to ants' behaviour, initial state (coming soon)

Shared state

ph: 2-D array, tracks whether a cell is marked with pheromone

```
1 system {
2   extern = size, n, foodx, foody, m, k
3   environment = ph[size, size]: 0
4   spawn = Ant: n
5 }
```



Behaviour

- Explore surroundings for food
  - Exploration is random
  - But may be influenced by pheromone trail-following
- Bring found food to the nest
  - Dead reckoning (go back to the nest along a straight line)
  - Release pheromone along the way

Pheromone sensing

1. Sample two random cells within range m
2. If either cell is marked, move there;

   Otherwise move to a random cell within range

LAbS: Ant description

```
agent Ant {
  interface = x: 0..size; y: 0..size;
              nextX: 0; nextY: 0

  Behavior = Explore; GoHome; Behavior

  Explore =
    x ≠ foodx or y ≠ foody ⇒ (
      SmellPheromone; Move; Explore)

  Move =
    (nextX = x and nextY = y ⇒ {
      dX, dY ≔ [−m..m+1], [−m..m+1];
      nextX ← x+dX;
      nextY ← y+dY;
      nextX ← max(nextX, 0);
      nextY ← max(nextY, 0);
      nextX ← min(nextX, size−1);
      nextY ← min(nextY, size−1)
    });
    x, y ← nextX, nextY

  SmellPheromone = {
    dX ≔ [1..m+1];
    dY ≔ [1..m+1];
    testx1, testy1 ≔ min(x+dX, size−1), min(y+dY, size−1);
    testx2, testy2 ≔ max(x−dX, 0), max(y−dY, 0);

    nextX ← if ph[testx1, testy1] then testx1 else
            if ph[testx2, testy2] then testx2 else x;
    nextY ← if ph[testx1, testy1] then testy1 else
            if ph[testx2, testy2] then testy2 else y
  }

  GoHome =
    x ≠ 0 or y ≠ foody ⇒ ({
      ph[x,y] ⇐1;
      x ← max(0, x−1)
    }; GoHome)

}
```

Assumptions

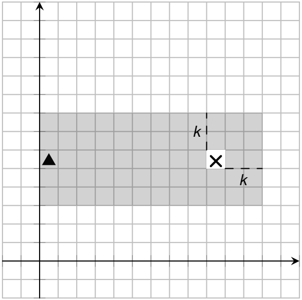
Additional constraints on the initial state

- At least one ant starts at the food location
- All the others start “far” from the shortest path (shaded area) between food and nest

LAbS: Quantified predicate in a separate section of the model

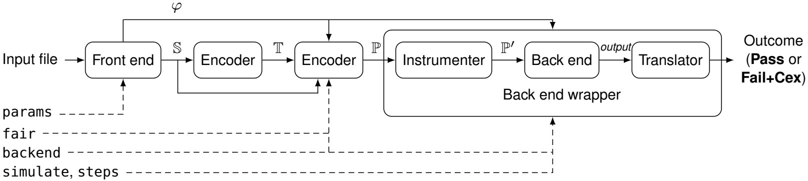
```
assume {
  FoodAnt = exists Ant a,
    (x of a = foodx) and (y of a = foody)

  FarFromThePath = forall Ant a,
    ((x of a = foodx) and (y of a = foody)) or
    (x of a > foodx + k) or
    (y of a > foody + k) or
    (y of a < foody − k)
}
```



A tool to verify/simulate LAbS models[1]

- Converts model into a symbolic intermediate representation
- Converts IR into imperative programs (here, sequential C)
- Reuses off-the-shelf analysis tools (here, SAT-based BMC[2])

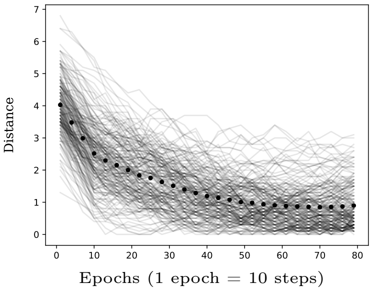


[1] `https://github.com/labs-lang/sliver`
[2] `https://www.cprover.org/cbmc`

Parameter values

size	Lenght of the arena's sides	20
foodx	Food *x*-coordinate	10
foody	Food *y*-coordinate	10
k	Initial distance from trail	2
n	Number of ants	10
m	Ants' movement range	1
B	Simulation bound	800
	Number of simulations	200

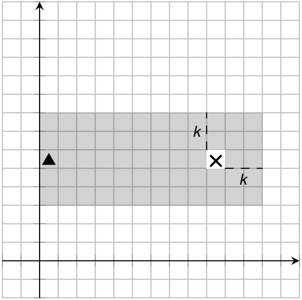
Average ant-trail distance



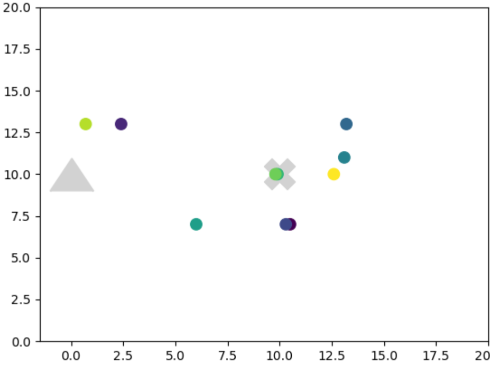
- Ants end up close to the pheromone trail in most simulations
- ... even though pheromone sensing is rather simple (nondeterministic, memoryless)

Now, let us specify that we would like *every* ant to be *within* the shaded region after a certain number of steps B

```
check {
  ShortestPath =
    after B forall Ant a,
    (x of a ≤ foodx + k) and
    (y of a ≥ foody − k) and
    (y of a ≤ foody + k)
}
```



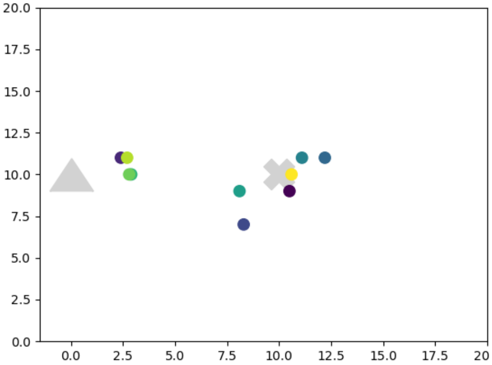
Verification results



(1 frame = 10 epochs = 100 steps)

Initial state: ant ● finds food

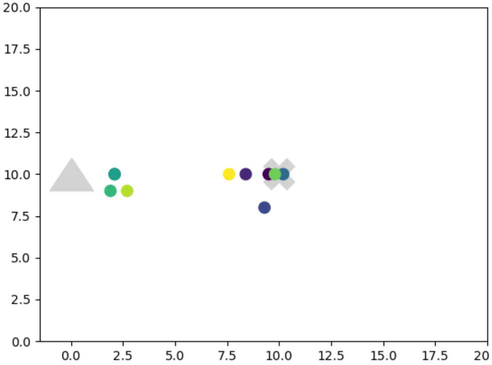
Verification results



(1 frame = 10 epochs = 100 steps)

Ant ● goes from ✕ towards ▲, leaves trail

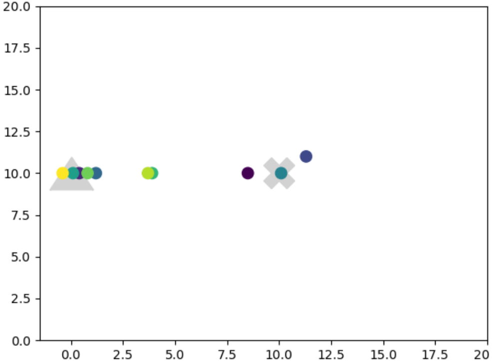
Verification results



(1 frame = 10 epochs = 100 steps)

Pheromone trail affects other ants

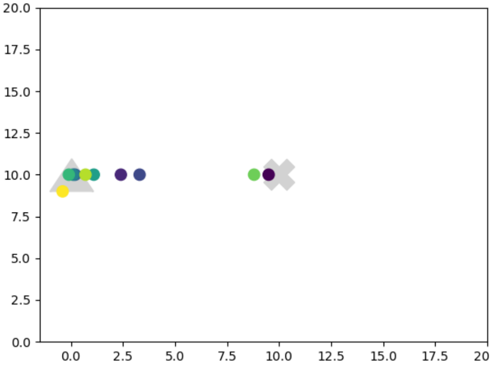
Verification results



(1 frame = 10 epochs = 100 steps)

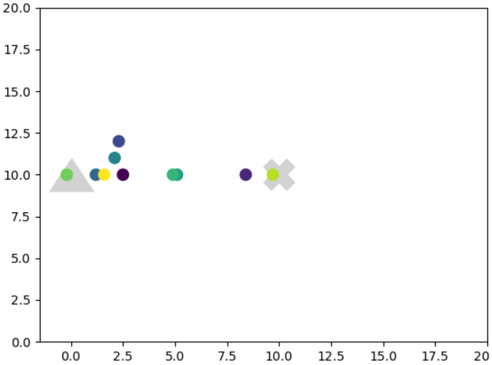
Several ants find food, go back to nest

Verification results



(1 frame = 10 epochs = 100 steps)

Ants (more or less) stay on track



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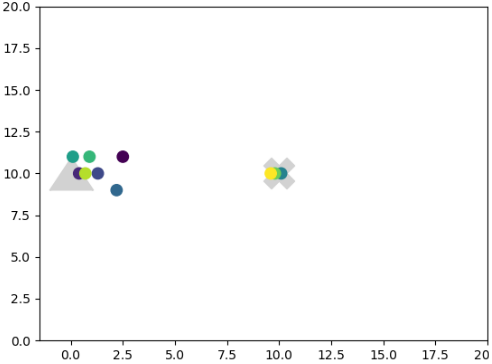

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Ants (more or less) stay on track

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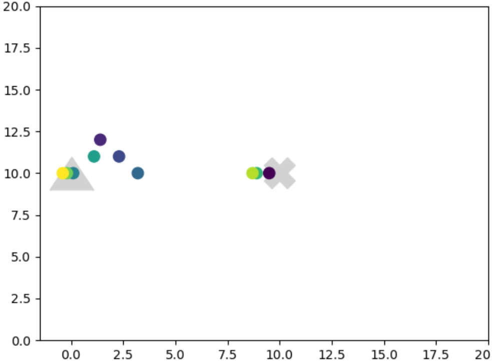
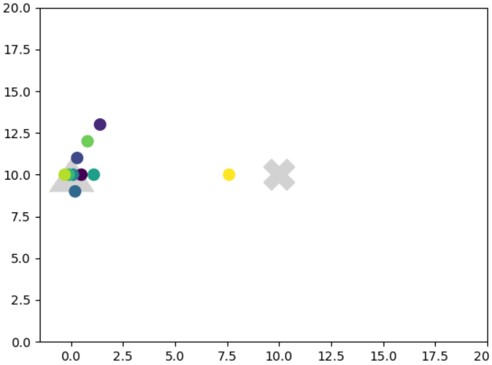


(1 frame = 10 epochs = 100 steps)

Ant ● starts straying from shortest path



(1 frame = 10 epochs = 100 steps)

Final state: ● is too far away

We can also use verification to generate “interesting” traces

Example. If *exactly one ant* starts at $\times$, can *every* ant end up close to the trail (after B steps)?

Verification (part 2)

We can also use verification to generate “interesting” traces

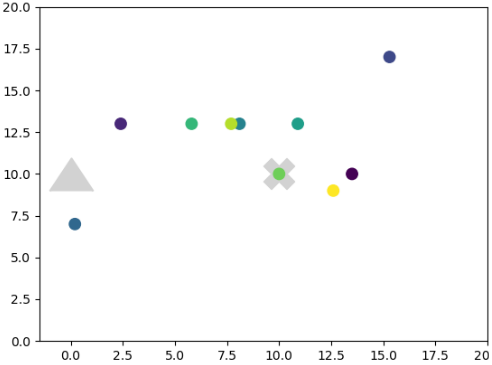
Example. If *exactly one ant* starts at ×, can *every* ant end up close to the trail (after *B* steps)?

Verify against the negation of the property:

```
assume {
  FoodAnt =
    exists–unique Ant a,
      (x of a = foodx) and
      (y of a = foody)

  FarFromThePath = ...
}
```

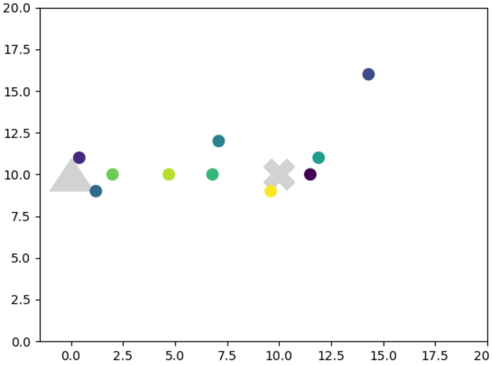
```
check {
  NegShortestPath =
  after B exists Ant a,
    (x of a > foodx + k) or
    (y of a < foody − k) or
    (y of a > foody + k)
}
```



(1 frame = 10 epochs = 100 steps)

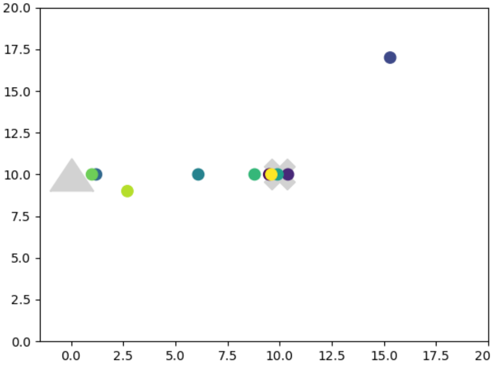
Initial state: ant ● finds food

Verification (part 2): Results



(1 frame = 10 epochs = 100 steps)

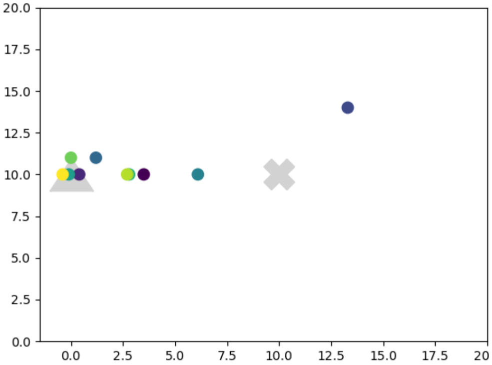
Ant ● goes from × towards ▲, leaves trail



(1 frame = 10 epochs = 100 steps)

Other ants explore arena, get on the trail

Verification (part 2): Results



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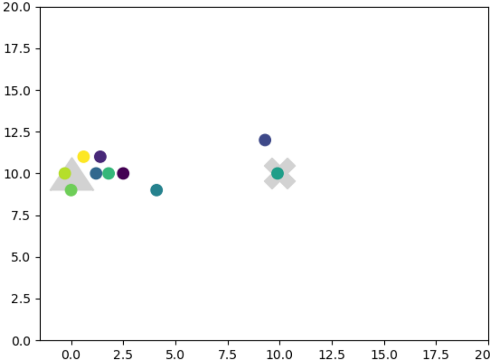
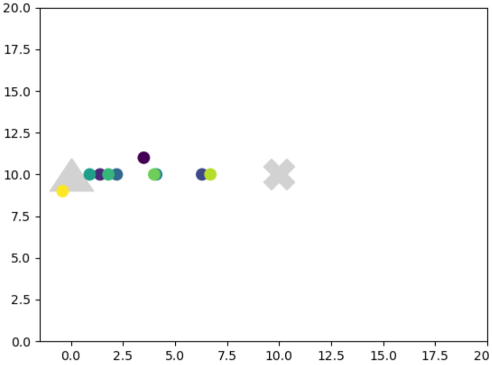


(1 frame = 10 epochs = 100 steps)

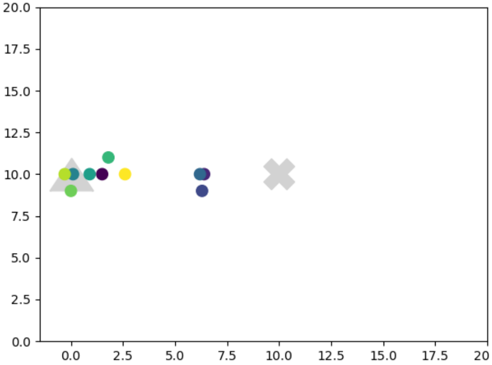
Ant ● starts getting closer



(1 frame = 10 epochs = 100 steps)

Ant ● starts getting closer

Verification (part 2): Results



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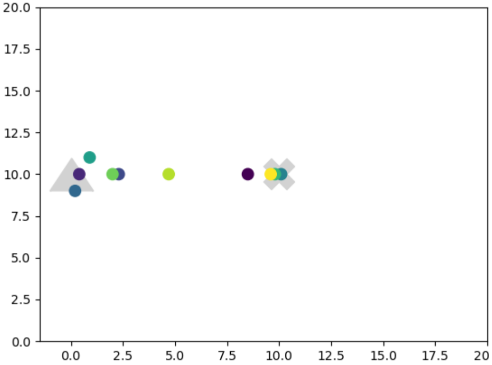

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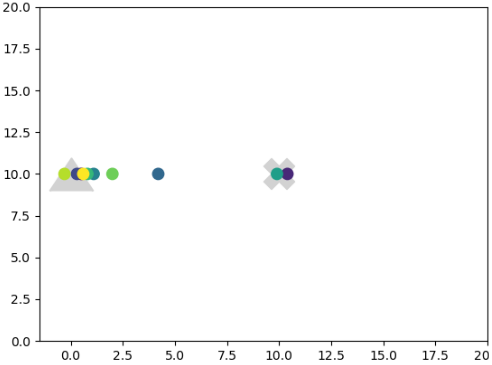


(1 frame = 10 epochs = 100 steps)

- Agent-based modelling of collective systems requires appropriate languages and tools
- These need to be supported by an adequate methodology
  - Gradual refinement of informal descriptions into formal models
  - Analysis-driven, iterative improvements to the model
- Simulation and exhaustive techniques complement each other

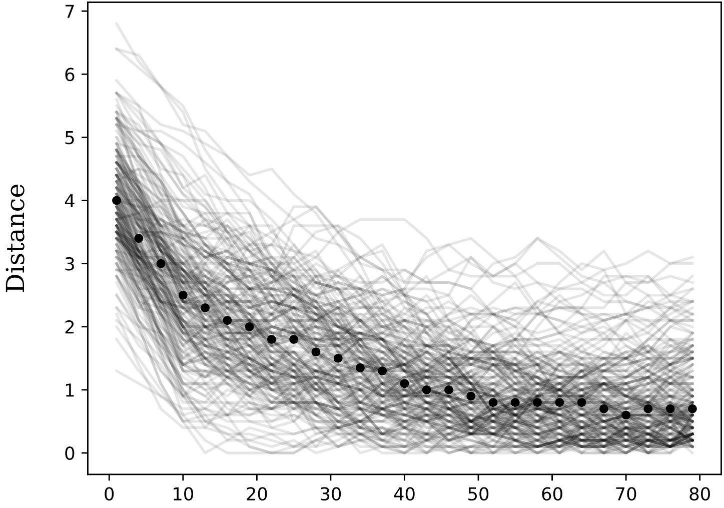
Future work

- Support more expressive properties (e.g., full LTL)
- Improve simulation/verification performance
- Implement runtime verification, statistical model checking, . . .
- Look for new case studies

Backup slides

Simulation results: Median distance

(Omitted from the paper)



Simulation results: Box plot
(Omitted from the paper)