DM geo 1

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EXERCICE 1:

-Proving that the curve is of class C^1:

-as the curve is defined by the functions:
x(t)=t-sin(t)
and
y(t)=1-cos(t)
and we know that both are of class C^∞ then they are of class C^1

-Finding g'(t) and studying the regularity of the curve:

x(t) =t-sin(t) $\longrightarrow$ x'(t)=1-cos(t)
-y(t)=1-cos(t) $\longrightarrow$ y'(t)=sin(t)
-the curve is regular if x'(t)=0 and y'(t) $\neq$ 0
set x'(t)=0 $\longrightarrow$ 1-cos(t)=0 $\longrightarrow$ t=2kπ
but since y'(t)= sin(t) , and sin(2kπ)=0, then at t= (2kπ)
g(t) = (0,0) so , the curve is not regular.

-Studying points where the tangent is horizontal:

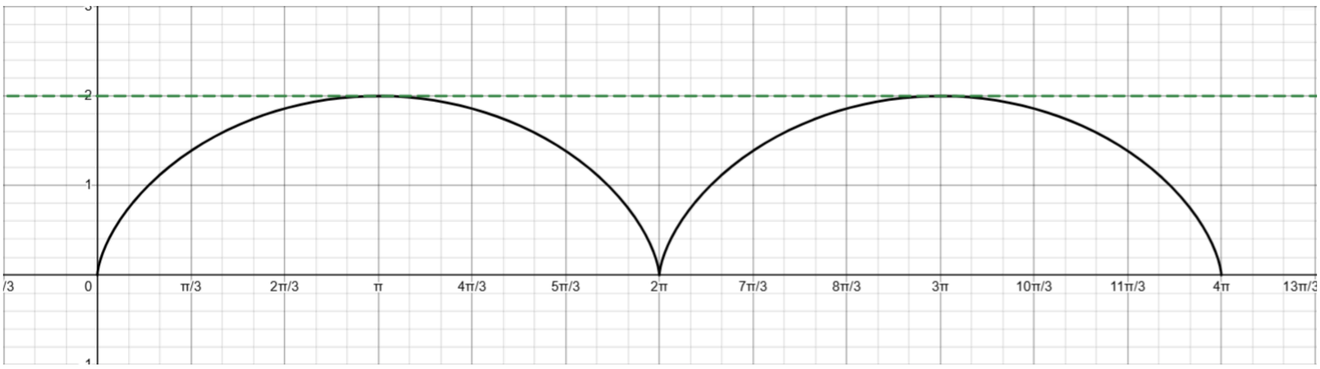
-for the tangent to be horizontal , we need: y'(t)= 0 but also x'(t)$\neq$ 0
set y'(t)=0 $\longrightarrow$ sin(t)=0 $\longrightarrow$ t=kπ
-but x'(t)=0 for t=2kπ ;
the tangent is horizontal at only odd multiples of π
meaning the tangent is horizontal at only t=(2k+1)π

-Studying the variation of x(t) and y(t):

-for x(t);
we have x'(t)= 1-cos(t) , and we know that:
-1 cos (t) 1 $\longrightarrow$ 0 1-cos (t) 2
so it's always positive ; meaning x(t) is increasing from 0 to 2π
-for y(t);
we have y'(t)=sin(t) , and we know that it's positive from 0 to π and negative from π to 2π
-for g(t);
so our important points are t=0 , t=π and t=2π
g(0)=(0,0)
g(π)=(π,2)
g(2π)=(2π,0)

-The shape of the curve and its type:

-Through digital plotting we notice that the parametric curve described is a cycloidal shape



EXERCICE 2:

-The support of the curve :

-we have g(t)=(t^2,t^4) ;
so :

$$x(t) = t^2 \, y(t) = t^4$$

-we notice that :

$$y = (t^2)^2 \longrightarrow y = x^2$$

and since x is defined by t^2 ; y=x^2 is defined on $0\,\infty$
so it's trace is that of a parabola x^2 from 0 to ∞

-Regularity of the curve :

-g(t)=(t^2,t^4) $\longrightarrow$ g'(t) = (2t,4t)
a curve is regular if g'(t)$\neq$(0,0), so we test the functions x'(t)=0 and y'(t)=0

$$x'(t) = 2t \longrightarrow t = 0$$

$$y'(t) = 4t \longrightarrow t = 0$$

-so at the point t=0 the curve is irregular , and regular at all other points, and the only singular point is at g(t)=(0,0) for t=0

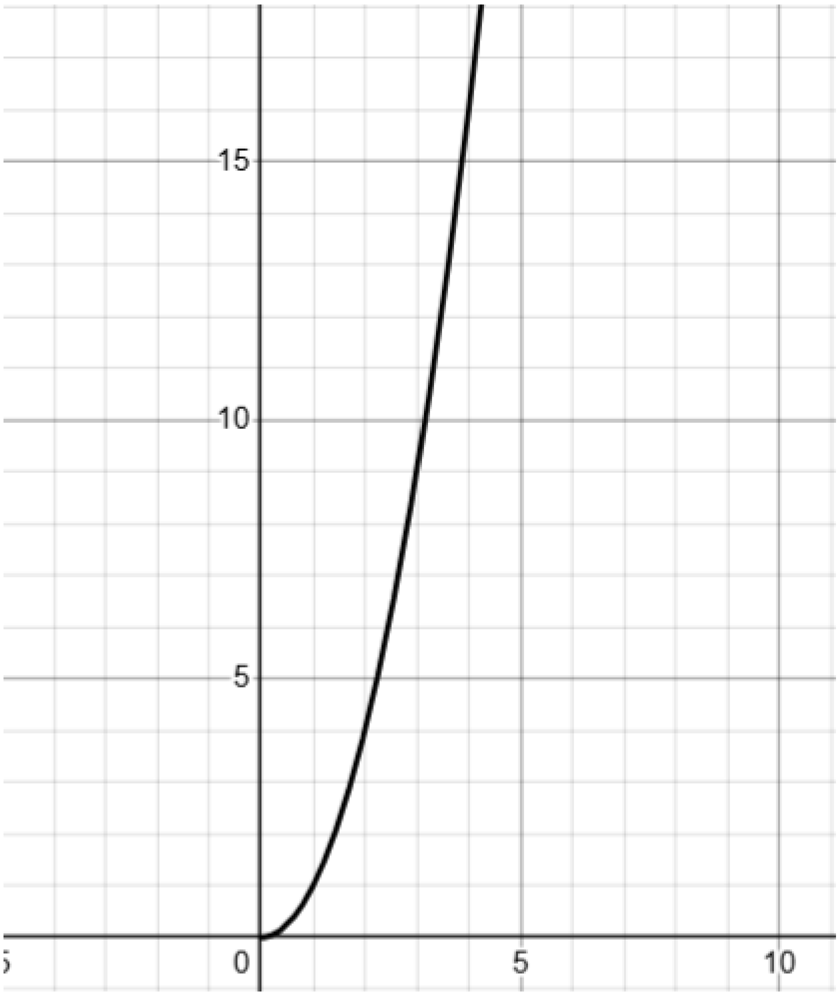
- The existance of a tangent at t=0 :

-we know the support of our parametric curve is that of y=x^2 function which is a parabola; from that we can deduce that it's tangent at t=0 is the tangent of a parabola at y=0 and that is known to be the horizontal line at y=0; and that is the axis x.

-The behaviour of the curve at t=0:

-since the curve describes a square function y=x^2 , and we have the tangent at y=0 being the axis x we can say that the curve is solely above the axis x and is convex

-The shape of our curve :



EXERCICE 3:

-The regularity of the curve :

-we must first find the derivative g(t):

$$x(t) = e^t cos(t) \longrightarrow x'(t) = e^t(cos(t) - sin(t))$$

$$y(t) = e^t sin(t) \longrightarrow y'(t) = e^t(cos(t) + sin(t))$$

we need to set x'(t)=0 :

$$e^t(cos(t) - sin(t) = 0 \longrightarrow sin(t) - cos(t) = 0 \longrightarrow tan(t) = 1$$

which gives us t = $\frac{\pi}{4}$
and since y'($\frac{\pi}{4}$)$\neq$ 0
the curve is regular for all points

-Finding the support of the curve

-by setting calculating x^2+y^2 we find :

$$x(t)^2 + y(t)^2 = e^{2t}(cos(t) + sin(t))^2 \longrightarrow x^2 + y^2 = e^{2t}$$

-Deducing that the curve is spiral

EXERCICE 4:

-The regularity of the curve:

-first we must find the derivatives of g_1(t) and g_2(t)
g_1'(t)=(1,2t)
and
g_2'(t)=(2t,1)
-since the parameter of x in g_1'(t) is always 1 , and the parameter of y in g_2'(t) is always 1 the curve is regular over [0;1]

-The length of the curves g_1,g_2 and their relation :

-we have the formula for the length of a curve :

$$_a (\frac{x}{t})^2 + (\frac{y}{t})^2$$

and since we notice that g_1 and g_2 are the same just with the parameters of x and y flipped , we can deduce that L1 and L2 are the same
L_1= $_{10} 1^2 + 4t^2$

-Deduction from the comparison :

since we deduced earlier tha the g_1 and g_2 curves are the same just with parameters x(t) and y(t) swapped , we can say that the graph of g_2 is the same one of g_1 just with the axies swapped , and this is further suppoted by the following graph of the parameters