Milestone 2: Algorithm Design

1. Milestone 1: Understanding the problems

GitHub Repository Link: https://github.com/loubnaB023/COP4533--FinalProject

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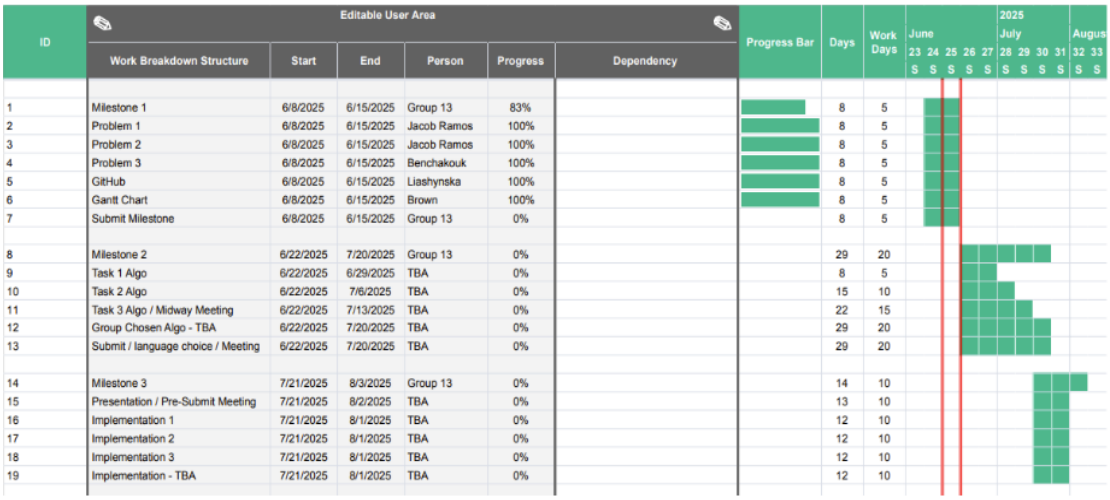
Member Roles:

- **Jacob Ramos**: Problem 1 & 2 completed both tasks (100% progress)
- **Loubna Benchakouk**: Problem 3 completed (100% progress)
- **Anhelina Liashynska**: GitHub setup
- **Dani Brown**: Gantt Chart
- Roles for upcoming milestones will be assigned later

Communication methods: We use Discord for regular communication and Google Docs for document collaboration.

Project Gantt Chart:

ID	Work Breakdown Structure	Start	End	Person	Progress	Dependency	Progress Bar	Days	Work Days
1	Milestone 1	6/8/2025	6/15/2025	Group 13	83%			8	5
2	Problem 1	6/8/2025	6/15/2025	Jacob Ramos	100%			8	5
3	Problem 2	6/8/2025	6/15/2025	Jacob Ramos	100%			8	5
4	Problem 3	6/8/2025	6/15/2025	Benchakouk	100%			8	5
5	GitHub	6/8/2025	6/15/2025	Liashynska	100%			8	5
6	Gantt Chart	6/8/2025	6/15/2025	Brown	100%			8	5
7	Submit Milestone	6/8/2025	6/15/2025	Group 13	0%			8	5
8	Milestone 2	6/22/2025	7/20/2025	Group 13	0%			29	20
9	Task 1 Algo	6/22/2025	6/29/2025	TBA	0%			8	5
10	Task 2 Algo	6/22/2025	7/6/2025	TBA	0%			15	10
11	Task 3 Algo / Midway Meeting	6/22/2025	7/13/2025	TBA	0%			22	15
12	Group Chosen Algo - TBA	6/22/2025	7/20/2025	TBA	0%			29	20
13	Submit / language choice / Meeting	6/22/2025	7/20/2025	TBA	0%			29	20
14	Milestone 3	7/21/2025	8/3/2025	Group 13	0%			14	10
15	Presentation / Pre-Submit Meeting	7/21/2025	8/2/2025	TBA	0%			13	10
16	Implementation 1	7/21/2025	8/1/2025	TBA	0%			12	10
17	Implementation 2	7/21/2025	8/1/2025	TBA	0%			12	10
18	Implementation 3	7/21/2025	8/1/2025	TBA	0%			12	10
19	Implementation - TBA	7/21/2025	8/1/2025	TBA	0%			12	10



Problem 1

We are given a matrix of stock prices where each row represents a different stock and each column will represent a different day. We calculate the maximum potential profit for each specific stock using a 1-based index.

Each stock/day combination maximum profit: (1,2,5,15) (2,1,3,9) (3,1,2,2) (4,2,5,7)

Answer: Stock/Day Combination Maximum Profit: (1,2,5,15)

Explanation:

- Stock 1 yields the maximum when bought on the 2nd day and sold on the 5th day for a profit of 15.
- Stock 2 yields the maximum profit when bought on day 1 and sold on day 3 yielding a profit of 9.
- Stock 3 yields the maximum potential profit when bought on day 1 and sold on day 2 yielding a profit of 2.
- Finally, stock 4 yields the maximum potential profit when bought on day 2 and sold on day 5 yielding a profit of 7.

Problem 2

We are given a matrix where each row represents a different stock and each column will represent a different day. We are given an integer k which will represent the maximum number of non-overlapping transactions permitted, in this case k = 3. For each transaction we must buy and sell one stock.

Answer: (4,1,2), (2,2,3), (1,3,5) total profit = 90

Explanation:

1. Stock 4: Buy on the 1st day at price 5, sell on the 2nd day at price 50 for a profit of 45.
2. Stock 2: Buy on the 2nd day at price 20, sell on the 3rd day at price 30 for a profit of 10.
3. Stock 1: Buy on the 3rd day at price 15, sell on the 5th day at price 50 for a profit of 35.
4. Total profit = 45 + 10 + 35 = 90

Problem 3

Problem Statement

We are given a matrix where each row represents a different stock and each column will represent a different day. Additionally, we are given an integer c which will represent a cooldown period where we cannot buy any stock for c days after selling any stock. If a stock is sold on day i, the next stock will not be eligible for purchase until day i + c + 1. For this example, c = 2.

Answer: (3,1,3), (3,6,7) total profit = 4 + 7 = 11

Explanation:

1. First transaction we buy stock 3 on day 1 and sell on day 3 for a profit of 4
2. Since the stock was sold on day 3 we cannot purchase another stock till day 6
3. On day 6, we buy stock 3 again and sell on day 7 for a profit of 7
4. The total profit is 11

Transaction rules:

1. We can only buy before we sell, and only once per transaction.

2. Resting period: after we sell on day j2 we need to wait until (j2+c+1) day to buy.

3. We can perform multiple transactions on any stock while following the cooldown rule.

4. Main objective is to maximize the total profit across all valid transactions.

Input:

We have a matrix A where each:

Row = one stock

Column = one day

A[i][j] = price of stock(i + 1) on day(j + 1)

Matrix A:

Day	1	2	3	4	5	6	7
Stock_1	7	1	5	3	6	8	9
Stock_2	2	4	3	7	9	1	8
Stock_3	5	8	9	1	2	3	10
Stock_4	9	3	4	8	7	4	1
Stock_5	3	1	5	8	9	6	4

Cooldown period: c = 2

To solve this problem we need to find all profitable transactions for each stock(row in the matrix)

1. Choose a buy day and then try all sell days that come after that buy day
2. For each(buy, sell) day, check if the price on the sell day is higher than the price on the buy day.
3. Keep just the profitable pairs(i, j, l)

Step 1: Identify All Possible Profitable Transactions

For each stock, we need to check all (buy, sell) pairs where buyDay < sellDay and profit > 0:

Stock 1: [7, 1, 5, 3, 6, 8, 9]

BuyDay	SellDay	BuyPrice	SellPrice	Profit	NextBuy	ValidTransaction
1	2	7	1	-6		
	3	7	5	-2		
	4	7	3	-4		
	5	7	6	-1		
	6	7	8	1	Day9(6+2+1)	No
	7	7	9	2	Day10(7+2+1)	No
2	3	1	5	4	Day6(3+2+1)	(6,7)
	4	1	3	2	Day7(4+2+1)	(7,7)
	5	1	6	5	Day8(5+2+1)	No
	6	1	8	7	Day9(6+2+1)	No
	7	1	9	8	Day10(7+2+1)	No
3	4	5	3	-2		
	5	5	6	1	Day8(5+2+1)	No
	6	5	8	3	Day9	No
	7	5	9	4	Day10	No
4	5	3	6	3	Day8	No
	6	3	8	5	Day9	No
	7	3	9	6	Day10	No
5	6	6	8	2	Day9	No
	7	6	9	3	Day10	No
6	7	8	9	1	Day10	No

From the table we see that the best combination for Stock 1: (2,7) with profit = 8

Stock 2: [2, 4, 3, 7, 9, 1, 8]

BuyDay	SellDay	BuyPrice	SellPrice	Profit	NextBuy	ValidTransaction
1	2	2	4	2	Day5(2+2+1)	(5, 6); (5, 7); (6, 7)
	3		3	1	Day6	(6, 7)
	4		7	5	Day7	(7, 7)
	5		9	7	Day8	No
	6		1	-1		
	7		8	6	Day10	No
2	3	4	3	-1		
	4		7	3	Day7	(7, 7)
	5		9	5	Day8	No
	6		1	-3		
	7		8	4	Day10	No
3	4	3	7	4	Day7	(7, 7)
	5		9	6	Day8	No
	6		1	-2		
	7		8	5	Day10	No
4	5	7	9	2	Day8	No
	6		1	-6		
	7		8	1	Day10	No
5	6	9	1	-8		
	7		8	-1		
6	7	1	8	7	Day9	No

Best single transaction for Stock 2: (1,5) with profit = 7

Stock 3: [5, 8, 9, 1, 2, 3, 10]

BuyDay	SellDay	BuyPrice	SellPrice	Profit	NextBuy	ValidTransaction
1	2	5	8	3	Day5	(5, 6); (5, 7); (6, 7)
	3		9	4	Day6	(6, 7)
	4		1	-4		
	5		2	-3		
	6		3	-2		
	7		10	5	Day10	No
2	3	8	9	1	Day6	(6, 7)
	4		1	-7		
	5		2	-6		
	6		3	-5		
	7		10	2	Day10	No
3	4	9	1	-8		
	5		2	-7		
	6		3	-6		

	7		10	1	Day10	No
4	5	1	2	1	Day8	No
	6		3	2	Day9	No
	7		10	9	Day10	No
5	6	2	3	1	Day9	No
	7		10	8	Day10	No
6	7	3	10	7	Day10	No

Best single transaction for Stock 3: (4,7) with profit = 9

Stock 4: [9, 3, 4, 8, 7, 4, 1]

BuyDay	SellDay	BuyPrice	SellPrice	Profit	NextBuy	ValidTransaction
1	2	9	3	-6		
	3		4	-5		
	4		8	-1		
	5		7	-2		
	6		4	-5		
	7		1	-8		
2	3	3	4	1	Day6(3+2+1)	(6, 7)
	4		8	5	Day7	(7, 7)
	5		7	4	Day8	No
	6		4	1	Day9	No
	7		1	-2		
3	4	4	8	4	Day7	(7, 7)
	5		7	3	Day8	No
	6		4	0	Day9	No
	7		1	-3		
4	5	8	7	-1		
	6		4	-4		
	7		1	-7		
5	6	7	4	-3		
	7		1	-6		
6	7	4	1	-3		

Best single transaction for Stock 4: (2,4) with profit = 5

Stock 5: [3, 1, 5, 8, 9, 6, 4]

BuyDay	SellDay	BuyPrice	SellPrice	Profit	NextBuy	ValidTransaction
1	2	3	1	-2		
	3		5	2	Day6(3+2+1)	(6, 7)
	4		8	5	Day7	(7, 7)
	5		9	6	Day8	No
	6		6	3	Day9	No
	7		4	1	Day10	No
2	3	1	5	4	Day6	(6, 7)
	4		8	7	Day7	(7, 7)
	5		9	8	Day8	No
	6		6	5	Day9	No
	7		4	3	Day10	No
3	4	5	8	3	Day7	(7, 7)
	5		9	4	Day8	No
	6		6	1	Day9	No
	7		4	-1		
4	5	8	9	1	Day8	No
	6		6	-2		
	7		4	-4		
5	6	9	6	-3		
	7		4	-5		
6	7	6	4	-2		

Best single transaction for Stock 5: (2,5) with profit = 8

Since we know the best individual transactions per stock. Now we check if we can combine some of them to build a valid sequence.

Starting with stock 1, the best transaction is: buy on day 2, sell on day 7 with profit = 8. After applying the cooldown rule the next valid buy day is day 10 but our max day is 7. Therefore, we can't combine it with any other transaction

- ⇨ Sequence (1, 2, 7) with total profit = 8

Stock 2: The best transaction is to buy on day 1 and sell on day 5 with profit = 7 and since the next buy day is day 8 we can't make an extra transaction.

- ⇨ Sequence (2, 1, 5) with total profit = 7

but we have another transaction with a smaller profit of 2 if we buy on day 1 and sell on day 2, after the resting period we can buy stock 3 on day 5, sell on day 7 with profit =8

- ⇨ Sequence (2, 1, 2), (3, 5, 7) with total profit = 10

Stock 3 we found that the best transaction is to buy on day 4, sell on day 7 with profit = 9 and since we need to wait for day10 (invalid) to make another transaction

- ⇨ Sequence (3, 4, 7) with total profit = 9

But if we buy on day 1 and sell on day 3 with profit = 4, we can combine it with Stock 2 on day 6 after the cooldown period, we buy on day 6 and sell on day 7 with profit = 7

- ⇨ Sequence (3, 1, 3), (2, 6, 7) with total profit = 11

Stock 4, we have the best profit = 5 if we buy on day 2 and sell on day 4, since the next valid buy day is day 7 and there is no available transaction starting day 7

- ⇨ Sequence (4, 2, 4) with total profit = 5

For Stock 5 the best transaction is when we buy on day 2 and sell on day 5 with profit = 8, after applying the cooldown rule, we don't get a valid day

- ⇨ Sequence (5, 2, 5) with total profit = 8

From the above, the maximum profit = 11 from the sequence (3, 1, 4), (2, 6, 7)

- ⇨ To achieve the maximum profit, buy 3rd stock on day 1, sell it on day 3. buy 2nd stock on day 6 and sell it on day 7 adhering to 2 days waiting period

2. Milestone 2: Algorithm Design

GitHub Repository Link: https://github.com/loubnaB023/COP4533--FinalProject

Individual submission:

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2.1 Task-1: Brute Force Algorithm for Problem1 $O(m \cdot n^2)$.

The goal is to find the maximum profit from a single buy/sell transaction on the same stock with one buy before one sell, only one transaction, and return stock index, buy day, sell day, and max profit.

Assumptions & variable definitions:

- m: number of stocks (rows in matrix A)
- n: number of days (columns in A)
- A transaction is defined as buying a stock on day j_1 and selling it later on day j_2, where $j_1 < j_2$.
- The transaction must be on the same stock (row).
- The result should be a tuple: (i, j_1, j_2, profit) where:
  - i: index of the chosen stock (1-based index)
  - j_1: day to buy
  - j_2: day to sell
  - profit = A[i][j_2] - A[i][j_1]

Pseudocode:

```
Algorithm MaxProfitBruteForce(A, m, n)

Input: matrix A(m × n) representing stock prices
Output: tuple (stock, buyDay, sellDay, profit) representing the best stock and days to buy/sell and max profit

Begin
    // --- initialize variables to store the best result found ---
    maxProfit ← 0
    bestStock ← 0
    bestBuyDay ← 0
    bestSellDay ← 0

    // --- try every possible stock ---
    for i ← 0 to m − 1 do
        // --- try every possible buy day ---
        for j1 ← 0 to n − 2 do
            // --- try every possible sell day after the buy day ---
            for j2 ← j1 + 1 to n − 1 do
                // --- calculate profit for the current transaction ---
                profit ← A[i][j2] − A[i][j1]
```

```
            // --- if this transaction gives higher profit, update the result ---
            if profit > maxProfit then
                maxProfit ← profit
                // 1-based indexing
                bestStock ← i + 1
                bestBuyDay ← j1 + 1
                bestSellDay ← j2 + 1
            end if
        end for
    end for
end for

// --- return result depending on whether any profit was made ---
if maxProfit = 0 then
    return (0, 0, 0, 0)   // no profitable transaction found
else
    return (bestStock, bestBuyDay, bestSellDay, maxProfit)
end if
End
```

The algorithm checks all possible pairs of days (j_1, j_2) for each stock to calculate potential profit, it loops over every stock (m stocks) and for each stock, compares every pair of buy/ sell days.
The algorithm keeps track of the max profit found so far and stores its stock index and days. If no profitable transaction exists (all negative or 0), it returns (0, 0, 0, 0).

2.2 Task-2: Greedy Algorithm for Problem1 O(m·n)

Assumptions & variable definitions:

- Only one transaction is allowed per stock
- Buy must occur before sell ($j_1 < j_2$)
- Each stock is evaluated independently
- Return (0, 0, 0, 0) if no profit is possible

Pseudocode:

```
Algorithm MaxProfitGreedySolution(A, m, n)

Input: matrix A(m × n) representing stock prices
Output: tuple (stock, buyDay, sellDay, profit) representing the best stock and days to buy/sell
and max profit

Begin
    // ---initialize variables to store best result ---
    maxProfit ← 0
    bestStock ← 0
```

```
    bestBuyDay ← 0
    bestSellDay ← 0

    // --- iterate through each stock ---
    for i ← 0 to m − 1 do
        // --- track the minimum price seen so far for this stock ---
        minPrice ← A[i][0]
        minDay ← 0

        // --- scan forward to find best day to sell ---
        for j ← 1 to n − 1 do
            // --- if selling today gives better profit, update result ---
            if A[i][j] − minPrice > maxProfit then
                maxProfit ← A[i][j] − minPrice
                bestStock ← i + 1
                bestBuyDay ← minDay + 1
                bestSellDay ← j + 1
            end if

            // --- If today's price is lower, update minPrice and minDay ---
            if A[i][j] < minPrice then
                minPrice ← A[i][j]
                minDay ← j
            end if
        end for
    end for

    // --- return result depending on whether profit was made ---
    if maxProfit = 0 then
        return (0, 0, 0, 0)     // no profitable transaction found
    else
        return (bestStock, bestBuyDay, bestSellDay, maxProfit)
    end if
End
```

In this greedy version of the algorithm, we optimize the search for the max profit from a single buy/ sell transaction. For each stock, the algorithm keeps the minimum price observed so far (minPrice) and the corresponding day (minDay). As it iterates through the remaining days, it computes the current potential profit by subtracting minPrice from the price on the current day. If this profit exceeds the previously recorded maxProfit, the algorithm updates the optimal transaction details (stock index, buy day, and sell day). If the current day's price is less than minPrice, it becomes the new minPrice. If no profitable transaction exists, the algorithm returns the default tuple (0, 0, 0, 0).

2.3 Task-3: Dynamic Programming Algorithm for Problem1 O(m·n)

Assumptions & variable definitions:

- m: number of stocks (rows in matrix A)
- n: number of days (columns in A)

- A transaction is defined as buying a stock on day j_1 and selling it later on day j_2, where $j_1 < j_2$.
- The transaction must be on the same stock (row).
- The result should be a tuple: (i, j_1, j_2, profit) where:
  - i: index of the chosen stock (1-based index)
  - j_1: day to buy
  - j_2: day to sell
  - profit = A[i][j_2] - A[i][j_1]

Pseudocode:

```
Algorithm MaxProfitDynamicProgramming(A, m, n)

Input: matrix A(m × n) representing stock prices

Output: tuple (stock, buyDay, sellDay, profit) representing the best stock and days to buy/sell
and max profit

Begin
    // --- initialize variables ---
    maxProfit ← 0
    bestStock ← 0
    bestBuyDay ← 0
    bestSellDay ← 0

    // --- loop through each stock ---
    for i ← 0 to m − 1 do
        minPrice ← A[i][0]     // minimum price seen so far for stock i
        minDay ← 0             // day when it occurred

        // --- loop through each day for this stock ---
        for j ← 1 to n − 1 do
            currentProfit ← A[i][j] − minPrice

            // --- check if this is the best profit so far ---
            if currentProfit > maxProfit then
                maxProfit ← currentProfit
                bestStock ← i + 1
                bestBuyDay ← minDay + 1
                bestSellDay ← j + 1
            end if

            // --- update minPrice if current day is cheaper ---
            if A[i][j] < minPrice then
                minPrice ← A[i][j]
                minDay ← j
            end if
        end for
    end for

    // --- result ---
```

```
    if maxProfit = 0 then
        return (0, 0, 0, 0)       // no profitable transaction found
    else
        return (bestStock, bestBuyDay, bestSellDay, maxProfit)
    end if
End
```

This algorithm uses dynamic programming logic to find the maximum profit from a single transaction per stock. For each stock, it keeps track of the minimum price seen so far (minPrice) and the corresponding day (minDay). As it scans through the days, it computes the profit of selling on the current day minus minPrice and updates the maximum profit and corresponding buy/sell days if it finds a better option. If no profit is possible, the algorithm returns (0, 0, 0, 0).

2.5 Task-5: Dynamic Programming Algorithm for Problem2 O(m·n·k)

Assumptions & variable definitions:

- m: number of stocks (rows in matrix A)
- n: number of days (columns in A)
- k: maximum number of transactions allowed
- A transaction consists of buying a stock on day j_1 and selling it on day j_2, where $j_1 < j_2$
- Transactions can involve different stocks
- Non-overlapping constraint: if the transaction ends on day d, next transaction can start on day d or later
- The result should be a sequence of tuples: $[(i_1, j_1, j_2), (i_2, j_3, j_4), ...]$ representing optimal transactions

Pseudocode:

```
Algorithm: MultiTransactionStockTrading(A, m, n, k)
Input:
    A[m × n]: matrix of stock prices (m stocks, n days)
    k: maximum number of transactions allowed
Output:
    list of transactions (stock, buyDay, sellDay) that maximize profit with at most k transactions

Begin

    // --- DP table to track max profit with t transactions up to day d ---
    dp[0..k][1..n] ← 0       // dp[t][d]: max profit with t transactions by day d

    // --- arrays to track our choices for reconstruction ---
    boughtStock[0..k][1..n]
    boughtDay[0..k][1..n]
    soldStock[0..k][1..n]
    didSell[0..k][1..n] ← false

    // --- initialize result container ---
    transactions ← empty list
```

```
// --- fill DP table ---
for t ← 1 to k do
    bestBuyProfit ← -A[1][1]          // best profit after buying stock 1 on day 1
    bestBuyStock ← 1
    bestBuyDay ← 1

    for day ← 2 to n do
        // --- case 1: no transaction today, carry forward profit ---
        noSellProfit ← dp[t][day - 1]

        // --- case 2: sell today ---
        maxSellProfit ← -∞
        bestSellStock ← -1

        for stock ← 1 to m do
            profit ← A[stock][day] + bestBuyProfit
            if profit > maxSellProfit then
                maxSellProfit ← profit
                bestSellStock ← stock
            end if
        end for

        // --- choose the better of the two options ---
        if maxSellProfit > noSellProfit then
            dp[t][day] ← maxSellProfit
            didSell[t][day] ← true
            soldStock[t][day] ← bestSellStock
            boughtStock[t][day] ← bestBuyStock
            boughtDay[t][day] ← bestBuyDay
        else
            dp[t][day] ← noSellProfit
            didSell[t][day] ← false
        end if

        // --- update best buy opportunity for future sells ---
        for stock ← 1 to m do
            newBuyProfit ← dp[t - 1][day] - A[stock][day]
            if newBuyProfit > bestBuyProfit then
                bestBuyProfit ← newBuyProfit
                bestBuyStock ← stock
                bestBuyDay ← day
            end if
        end for
    end for
end for

// --- backtrack to reconstruct the optimal transactions ---
currentDay ← n
currentTrans ← k
```

```
    while currentDay > 0 and currentTrans > 0 do
        if didSell[currentTrans][currentDay] = true then
            stock ← soldStock[currentTrans][currentDay]
            buyDay ← boughtDay[currentTrans][currentDay]
            transactions.prepend((stock, buyDay, currentDay))
            currentDay ← buyDay - 1
            currentTrans ← currentTrans - 1
        else
            currentDay ← currentDay - 1
        end if
    end while

    return transactions

End
```

This algorithm helps find the best way to make up to k profitable stock trades using prices from m stocks over n days. It builds a table dp[t][d] that keeps track of the highest possible profit at each day, for each number of transactions. At every step, it checks whether it's better to sell today or wait. It remembers when and which stock was bought and sold, so it can later figure out the best trades. In the end, it works backward through the table to list the best trades without any overlap.

2.6 Task-6: Dynamic Programming Algorithm for Problem3 $O(m \cdot n^2)$

The goal here is to maximize total profit from multiple stock transactions with a cooldown period c. After selling a stock on day j_2, the next allowed buy can only happen on day $j_2 + c + 1$ or later.

Assumptions & variable definitions:

Input Variables:

- A[1..m][1..n]: Matrix where A[i][j] represents price of stock i on day j
- m: Number of stocks
- n: Number of days
- c: Cooldown period

State Variables:

- Free[1..n]: Maximum profit on day i when not holding any stock (free to buy)
- Holding[1..n]: Maximum profit on day i when holding a stock (can sell)
- Cooldown[1..n]: Maximum profit on day i when in cooldown (just sold, cannot buy)

Tracking Variables:

- heldStock[1..n]: Which stock we're holding on each day (0 if not holding)
- purchaseDay[1..n]: When we bought the stock we're currently holding
- transactions[]: Final sequence of transactions (stock, buyDay, sellDay)

Pseudocode:

```
Algorithm StockTradingWithCooldown(A, m, n, c)

Input: matrix A(m × n) representing stock prices.
       cooldown period c
Output: list of transactions (stock, buyDay, sellDay) maximizing profit with cooldown constraint

Begin

    // --- DP arrays to track max profit for each state on each day ---
    Free ← -∞          // array [1..n]: not holding any stock, can buy
    Holding ← -∞       // array [1..n]: holding a stock, can sell
    Cooldown ← -∞      // array [1..n]: just sold, in cooldown period

    // --- containers to track our decisions ---
    heldStock ← 0          // array [1..n]: which stock we're holding each day
    purchaseDay ← 0        // array [1..n]: when we bought it

    // --- initialize result container ---
    transactions ← empty list

    // ---- Base case: Day 1, start with no money, no stocks, not in cooldown ---
    Free[1] ← 0            // we start free with 0 profit
    Holding[1] ← -∞        // can't be in hold without buying first
    Cooldown[1] ← -∞       // can't be in cooldown without selling first

    // --- fill the profit arrays day by day ---
    for day ← 2 to n do

        // --- state 1: we are free to buy today ---
        // stay free, do nothing, carry forward yesterday's profit
        Free[day] ← Free[day - 1]

        // cooldown period ended, we can be free again
        if day > c + 1 then          // cooldown lasts c days, so we are free after c + 1 days
            if Cooldown[day - 1] > Free[day] then
                Free[day] ← Cooldown[day - 1]
            end if
        end if

        // --- state 2: we are holding a stock today ---
        // we choose to keep holding the same stock from yesterday
        Holding[day] ← Holding[day - 1]
        heldStock[day] ← heldStock[day - 1]          // same stock
        purchaseDay[day] ← purchaseDay[day - 1]      // same purchase date

        // or we buy a new stock today only if we were free yesterday
        for stock ← 1 to m do
            profit ← Free[day - 1] - A[stock][day]    // subtract cost

            if profit > Holding[day] then
                Holding[day] ← profit
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            heldStock[day] ← stock          // remember which stock we bought
            purchaseDay[day] ← day          // remember when we bought it
        end if
    end for

    // --- state 3: we are in cooldown today ---
    // continue cooldown from yesterday
    Cooldown[day] ← Cooldown[day - 1]

    // sell our stock today and enter cooldown
    if heldStock[day - 1] > 0 then     // we were holding something yesterday
        stockToSell ← heldStock[day - 1]
        buyDay ← purchaseDay[day - 1]

        profit ← Holding[day - 1] + A[stockToSell][day]         // add sale price

        if profit > Cooldown[day] then
            Cooldown[day] ← profit
            // record the transaction during reconstruction phase
            transactions.add((stockToSell, buyDay, day))
        end if
    end if

end for

// --- identify the best final state and maximum profit ---
finalMaxProfit ← max(Free[n], Holding[n], Cooldown[n])

// which state gave us the best result?
finalState ← 0              // assuming we ended free

if Holding[n] ≥ Free[n] and Holding[n] ≥ Cooldown[n] then
    finalState ← 1          // we ended holding a stock
else if Cooldown[n] ≥ Free[n] then
    finalState ← 2      // we ended in cooldown
end if

// --- backtrack to find the actual transactions that led to optimal profit ---
transactions.clear()
currentDay ← n
currentState ← finalState

// --- walk backwards through our decisions to reconstruct the optimal path ---
while currentDay > 1 do

    if currentState = 0 then          // we are currently free
        // How did we get to the free state? Two possibilities:
        if currentDay > c + 1 and Free[currentDay] = Cooldown[currentDay - 1] then
            // we came from cooldown
            currentDay ← currentDay - 1
            currentState ← 2
        else
            // we stayed free from the previous day
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            currentDay ← currentDay - 1
            currentState ← 0
        end if

    else if currentState = 1 then   // we are currently holding
        // did we buy today or were we already holding?
        boughtToday ← false

        for stock ← 1 to m do
            if Holding[currentDay] = Free[currentDay - 1] - A[stock][currentDay] then
                boughtToday ← true    // found the stock we bought today
                break
            end if
        end for

        if boughtToday then
            // we bought today, so we were free yesterday
            currentDay ← currentDay - 1
            currentState ← 0
        else
            // we were already holding from yesterday
            currentDay ← currentDay - 1
            currentState ← 1
        end if

    else    // currentState = 2, we are in cooldown
        // did we sell today or were we already in cooldown?
        soldToday ← false

        if currentDay > 1 and heldStock[currentDay - 1] > 0 then
            stockSold ← heldStock[currentDay - 1]
            buyDay ← purchaseDay[currentDay - 1]

            // check if selling this stock today gave us our current profit
            if Cooldown[currentDay] = Holding[currentDay - 1] + A[stockSold][currentDay] then
                // if yes, we sold this stock today and record the transaction
                transactions.prepend((stockSold, buyDay, currentDay))
                soldToday ← true
                // we jump back to the day before we bought this stock
                currentDay ← buyDay - 1
                currentState ← 0
            end if
        end if

        if not soldToday then
            // we were already in cooldown from yesterday
            currentDay ← currentDay - 1
            currentState ← 2
        end if

    end if

end while
```

```
    return transactions

End
```

This algorithm helps to maximize the total profit by allowing multiple stock transactions while respecting a cooldown period c between trades. After selling a stock, we are not allowed to buy another until c days have passed. To manage this, the algorithm uses three dynamic programming arrays: Free, Holding, and Cooldown. Each array tracks the best profit we can achieve on a given day under a specific state. Free[day] represents the maximum profit if we are not holding any stock and are free to buy, Holding[day] keeps track of profits when we are currently holding a stock, and Cooldown[day] represents the profit when we are in a mandatory rest period after selling.

Each day, the algorithm evaluates whether to maintain the current state like continuing to hold a stock, or transition like buying or selling a stock. It computes the profit for each action and updates the respective state arrays accordingly. It also remembers which stock was bought or sold and on which day. After going through all the days, it performs a backtracking step, where it walks backward through the Free, Holding, and Cooldown arrays to reconstruct the exact series of buy and sell actions that led to the optimal profit.