

Energy savings for UAV flight in unsteady gusting conditions through trajectory optimization

Lou Grimaud

Illinois Institute of Technology

July 2014

Table of Contents

1. Introduction
2. The trajectory optimization problem
 - Dynamic soaring
 - Neutral energy loop
 - Implementation and validation
 - Quasi-steady aerodynamic model results
3. The unsteady pitching aerodynamic model
 - Experimental setup
 - The Goman and Khrabrov model
 - Determination and validation of the model
4. Unsteady trajectory optimization
 - Time constant equivalence
 - Gust duration dependency
 - Phase results
5. Conclusion

Table of Contents

1. Introduction
2. The trajectory optimization problem
 - Dynamic soaring
 - Neutral energy loop
 - Implementation and validation
 - Quasi-steady aerodynamic model results
3. The unsteady pitching aerodynamic model
 - Experimental setup
 - The Goman and Khrabrov model
 - Determination and validation of the model
4. Unsteady trajectory optimization
 - Time constant equivalence
 - Gust duration dependency
 - Phase results
5. Conclusion

Introduction and motivations

Table of Contents

1. Introduction
2. The trajectory optimization problem
 - Dynamic soaring
 - Neutral energy loop
 - Implementation and validation
 - Quasi-steady aerodynamic model results
3. The unsteady pitching aerodynamic model
 - Experimental setup
 - The Goman and Khrabrov model
 - Determination and validation of the model
4. Unsteady trajectory optimization
 - Time constant equivalence
 - Gust duration dependency
 - Phase results
5. Conclusion

Different types of soaring

Figure : something about soaring

Spatial wind gradients

- ▶ Thermal updrafts
- ▶ Horizontal wind gradients

Temporal wind gradients

- ▶ Natural turbulences
- ▶ Building and natural feature wake

Defining the energy extraction problem

What is an “optimal trajectory”?

- ▶ Maximum energy at the end of the cycle
- ▶ Maximizing the energy gain at each instant of the cycle
- ▶ *Minimize the energy input needed for sustainable flight*

The neutral energy loop

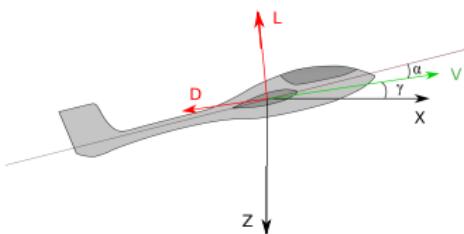
Finding the minimal wind gust that allows to maintain altitude and speed over a gust.

Aircraft model¹

Newton-Euler equations of motion:

$$\begin{aligned} m\ddot{x} &= -L' \cdot \sin(\gamma) + D' \cdot \cos(\gamma) \\ m\ddot{z} &= L' \cdot \cos(\gamma) - D' \cdot \sin(\gamma) - m \cdot g \end{aligned}$$

Lissaman's non-dimensional variables



- ▶ Velocities with V^* the optimal glide speed
- ▶ Time with $T = \frac{V^*}{g}$
- ▶ Lift and drag coefficients $L = \frac{C_l}{C_l^*}$ $D = \frac{C_d}{C_d^*}$
- ▶ Dynamic pressure $Q = \frac{L'}{MgL} = \frac{\rho V^2 C_l^*}{2Mg}$

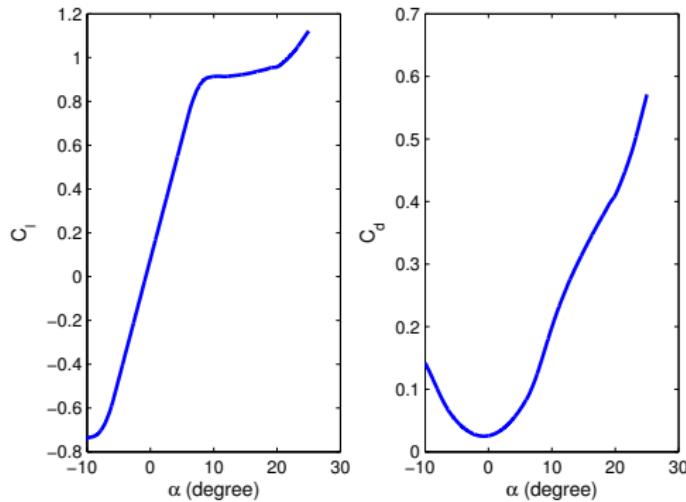
$$Q = (U - U_g)^2 + (W - W_g)^2$$

$$\begin{aligned} \frac{dU}{dT} &= -LQ \cdot \sin(\gamma) + DQ \cdot \cos(\gamma) \\ \frac{dW}{dT} &= LQ \cdot \cos(\gamma) - DQ \cdot \sin(\gamma) - 1 \end{aligned}$$

¹ Lissaman P and Patel C. Neutral energy cycles for a vehicle in sinusoidal and turbulent vertical gusts. 45th AIAA meeting, 863, 2007

Quasi-steady lift and drag model

- ▶ NACA0009 characteristic
 - ▶ Lissaman's quadratic drag



$$D = \frac{1+L^2}{2G^*}$$

Figure : Simplified lift and drag for the NACA0009 airfoil

Wind profiles

We define three different wind profiles:

- ▶ Vertical wind gust:

$$\begin{aligned}W_g &= W_a \cdot \sin(2\pi T) \\U_g &= 0\end{aligned}$$

- ▶ Horizontal wind gust:

$$\begin{aligned}W_g &= 0 \\U_g &= W_a \cdot \cos(2\pi T)\end{aligned}$$

- ▶ Combined wind gust:

$$\begin{aligned}W_g &= W_a \cdot \sin(2\pi T) \\U_g &= W_a \cdot \cos(2\pi T + \varphi)\end{aligned}$$

Optimization algorithm

- ▶ State vector

$$x = \begin{bmatrix} \dots \\ X_i \\ Z_i \\ U_i \\ W_i \\ L_i/\alpha_i \\ \dots \\ W_a \end{bmatrix} \quad i \in [1, N]$$

- ▶ Cost function: W_a
- ▶ Constraints: equations of motion, limits on the range of angles of attack allowed, neutral energy loop conditions.

Constraints formulation

- ▶ Equations of motion (with Simpson's discrete integral)

$$y_i = \begin{bmatrix} X_i \\ Z_i \\ U_i \\ W_i \end{bmatrix} \quad \dot{y}_i = \begin{bmatrix} U_i \\ W_i \\ -L_i Q_i \cdot \sin(\gamma_i) + D_i Q_i \cdot \cos(\gamma_i) \\ L_i Q_i \cdot \cos(\gamma_i) - D_i Q_i \cdot \sin(\gamma_i) - 1 \end{bmatrix}$$

$$0 = y_{k+1} - y_k - \frac{1}{6}(y_k + 4y_m + y_{k+1})\delta t \quad \forall i \in [1, N-1]$$

- ▶ Neutral energy loop
- ▶ Limits in range of C_l

$$\begin{aligned} Z_1 &= Z_N & L_1 &= L_N \\ W_1 &= W_N & U_1 &= U_N \end{aligned}$$

$$L_{\min} \leq L_i \leq L_{\max}$$

$$\begin{aligned} W_2 - W_1 &= W_N - W_{N-1} \\ U_2 - U_1 &= U_N - U_{N-1} \end{aligned}$$

Comparison with Lissaman's results

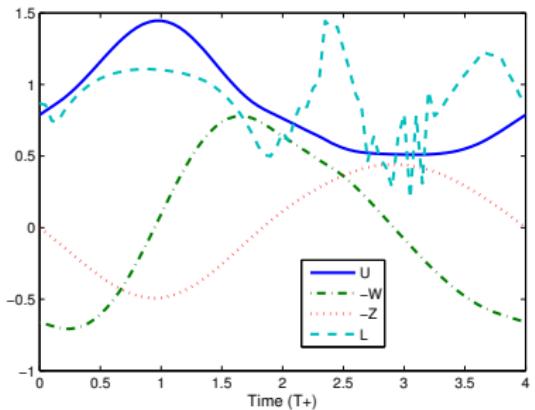


Figure : Optimization results for a $4T$ long vertical gust

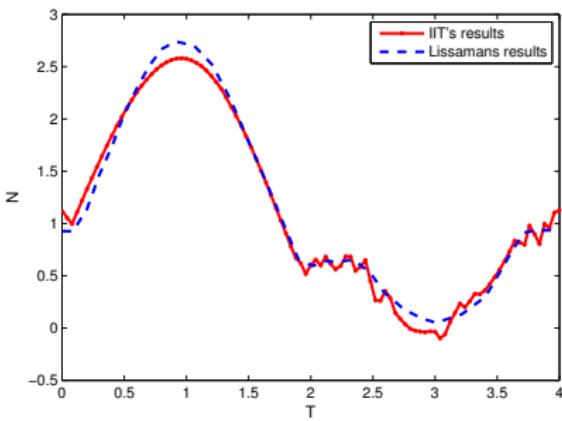


Figure : Comparison with Lissaman's non-dimensional normal force N for a $4T$ long vertical gust

Quasi-steady lift to drag model

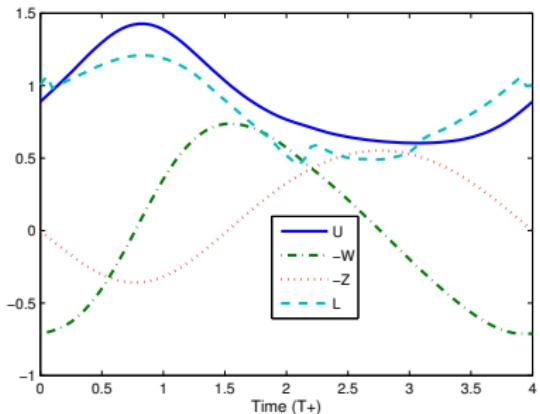


Figure : $4T$ long vertical gust for the NACA0009 airfoil, $W_a = 0.205$

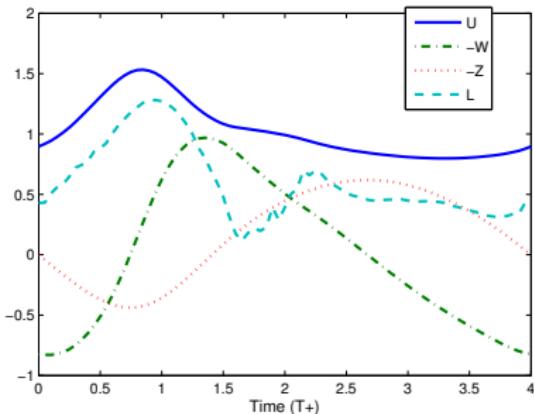


Figure : $4T$ long combined gust for the NACA0009 airfoil, $W_a = 0.387$

T_g dependency

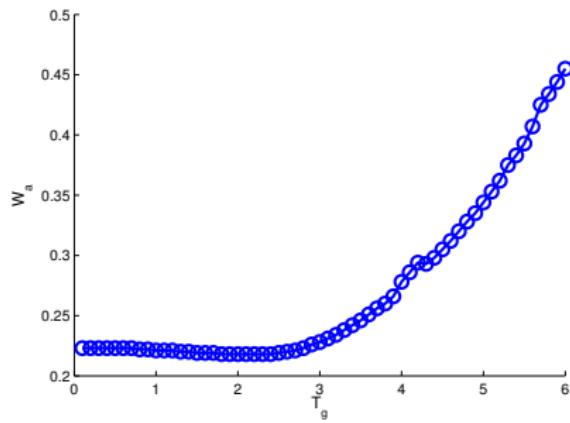


Figure : Influence of gust duration on the minimum gust amplitude for vertical gusts

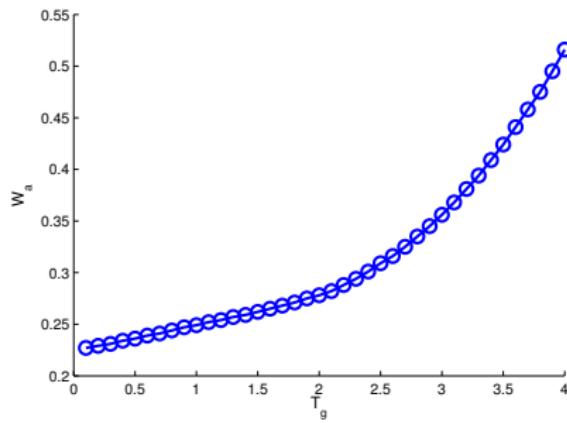


Figure : Influence of gust duration on the minimum gust amplitude for combined gusts

Difference between short and long gusts

We can see that there is tipping point around $T_g = 2.5$

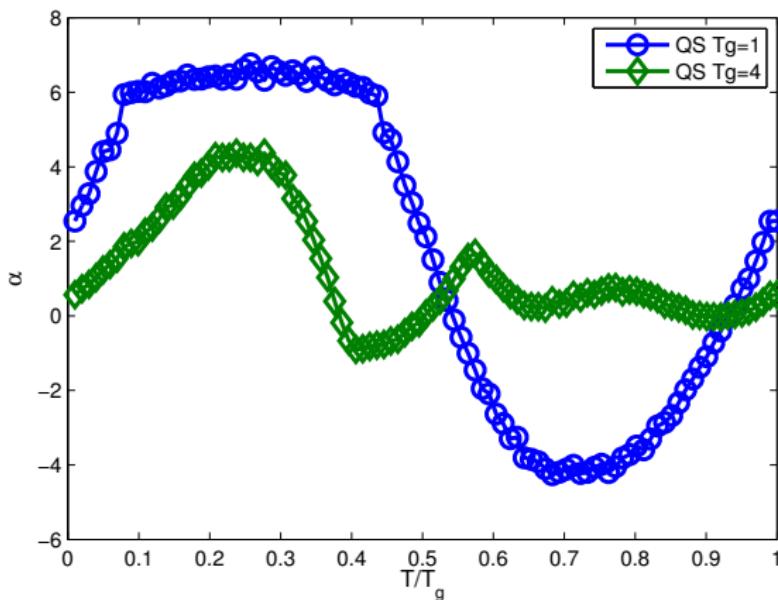


Figure : Difference between short and long gust angle of attack profile for combined gusts

Angle of attack limitation

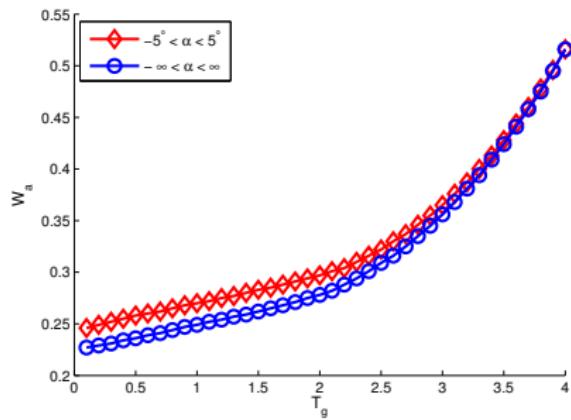


Figure : Difference in performance for combined wind gusts if no high angle of attack are allowed

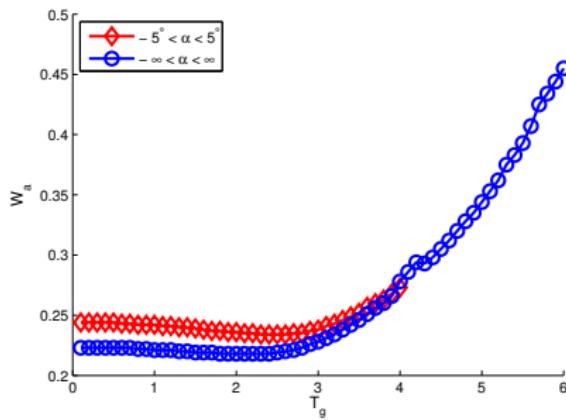


Figure : Difference in performance for vertical wind gusts if no high angle of attack are allowed

Phase influence

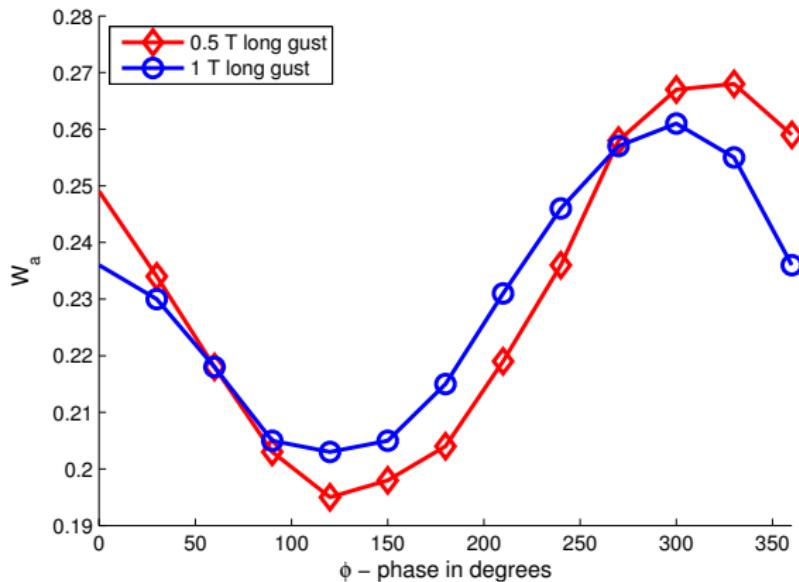


Figure : Influence of the phase between the components of the combined gust

Table of Contents

1. Introduction
2. The trajectory optimization problem
 - Dynamic soaring
 - Neutral energy loop
 - Implementation and validation
 - Quasi-steady aerodynamic model results
3. The unsteady pitching aerodynamic model
 - Experimental setup
 - The Goman and Khrabrov model
 - Determination and validation of the model
4. Unsteady trajectory optimization
 - Time constant equivalence
 - Gust duration dependency
 - Phase results
5. Conclusion

Pitching mechanism and experimental conditions

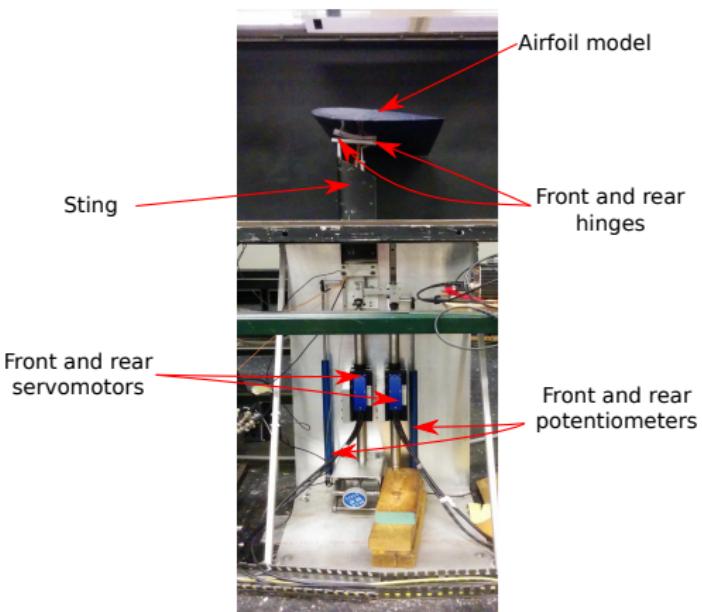


Figure : Airfoil model inside the wind tunnel

Experimental conditions

- ▶ Free stream velocity: 3 m/s
 - ▶ Airfoil: NACA0009
 - ▶ Reynolds number 50000

Controller and data acquisition

- ▶ Angle of attack controlled by simulink® and two servomotors
 - ▶ Servos position measured by two linear potentiometers
 - ▶ Piezoelectric force balance (NANO17) to measure the forces on the airfoil

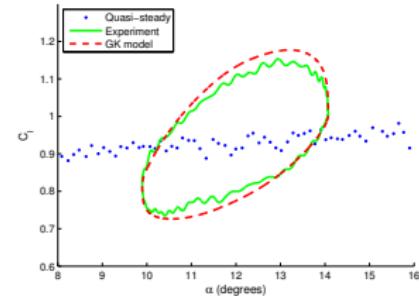
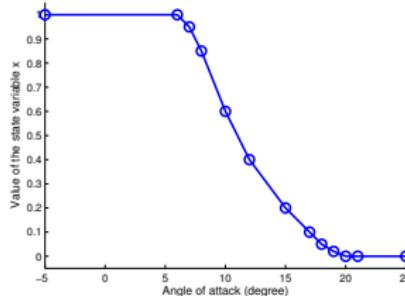
The GK model concept

The Goman and Khrabrov model² - a non-linear state space model

$$\begin{aligned} C_l &= f(\alpha, x(\alpha)) \\ C_d &= g(\alpha, x(\alpha)) \\ \tau_1 \frac{dx}{dt} + x &= x_0(\alpha - \tau_2 \dot{\alpha}) \end{aligned}$$

- ▶ Lift and drag model
- ▶ Non-linear state map
- ▶ Time constants τ_1 and τ_2

$$\begin{aligned} C_l &= 2\pi\alpha(0.6x + 0.4) + C_{l0} \\ C_d &= \frac{((2-x)C_l)^2}{G_{\max}} + C_{d0} \end{aligned}$$



²Goman M and Khrabrov A. *Journal of Aircraft*, 31(5):1109–1115, 1994.

Quasi-steady map and state variable

$$C_l(\alpha, x) = 2\pi \cdot \alpha(0.6x + 0.4) + C_{l0}$$

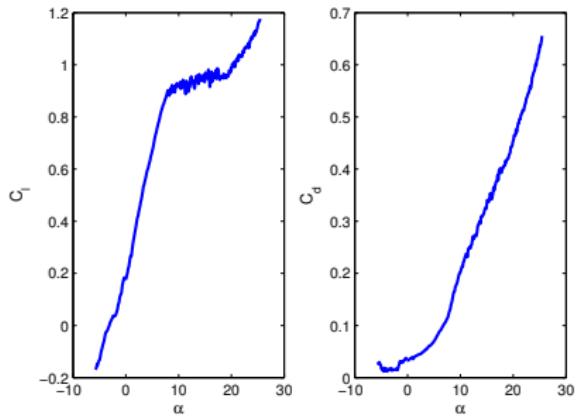


Figure : Lift and drag coefficient in the quasi-steady case

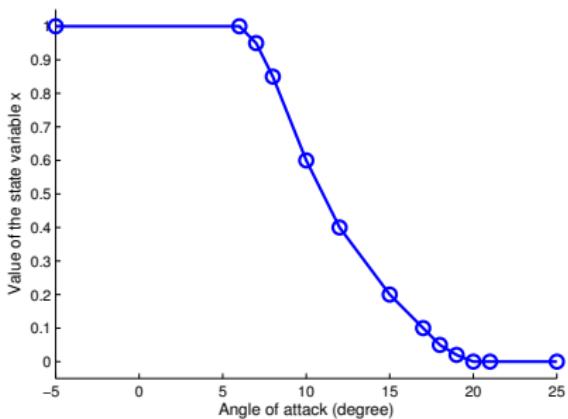


Figure : Quasi-steady profile for the state variable x

Time constant determination

Periodic sinusoidal pitching at different frequencies $k = \pi \frac{cf}{u}$

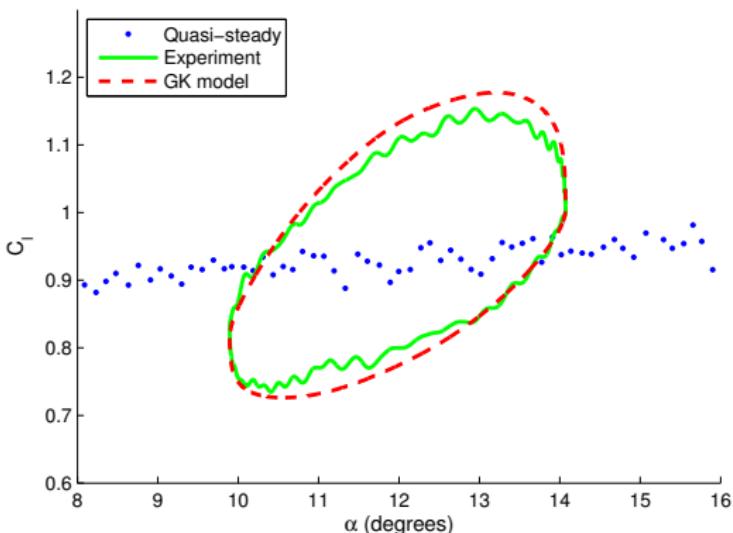


Figure : Comparison of experimental lift coefficient and model prediction after tuning of the time constant at $k = 0.128$

We find $\tau_1 = 3.1t^+$ and $\tau_2 = 4.29t^+$ (with $t^+ = \frac{c}{u}$)

Comparison with periodic measurements - Lift

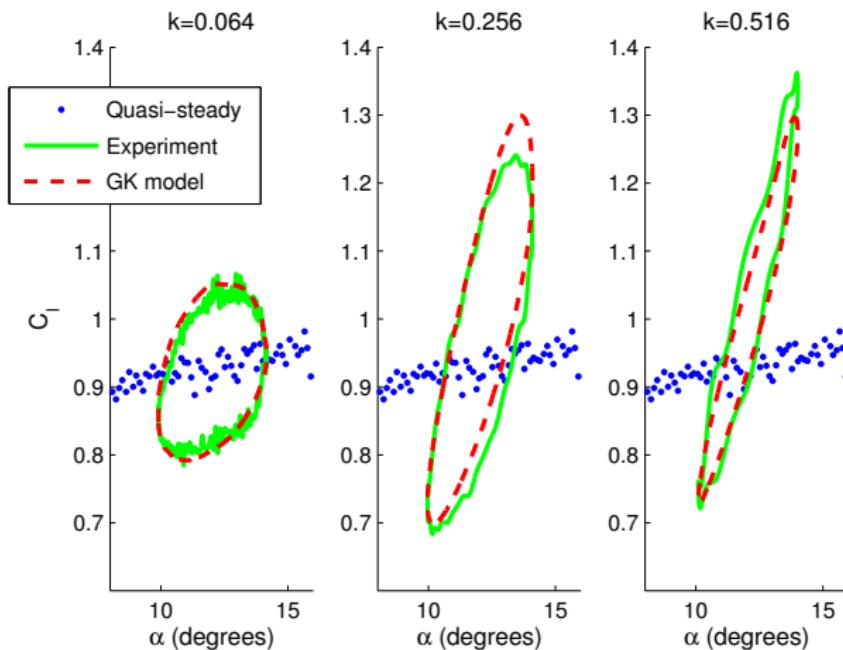


Figure : Lift measurement and prediction during sinusoidal pitching around 12 degree

Comparison with periodic measurements - Drag

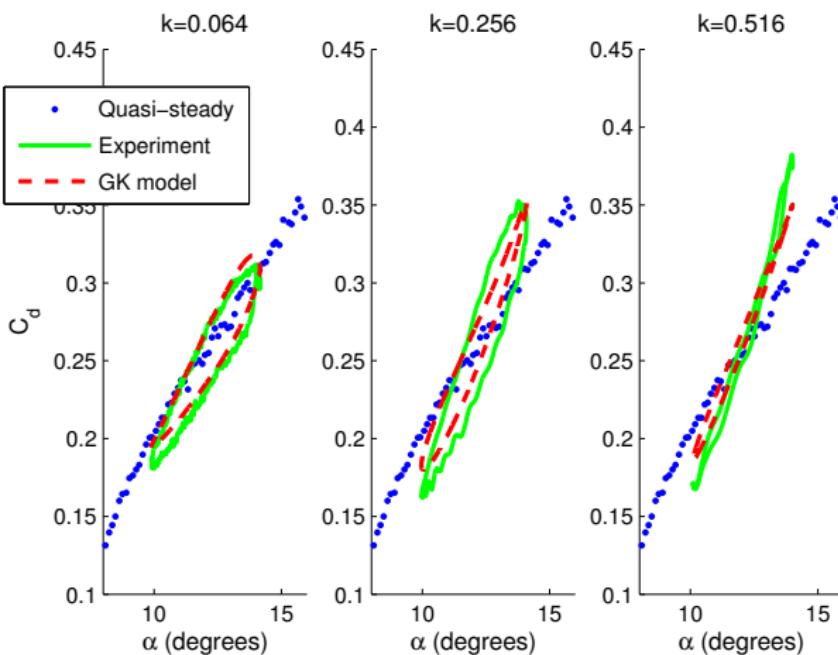


Figure : drag measurement and prediction during sinusoidal pitching around 12 degree

Pseudo-random case - Lift

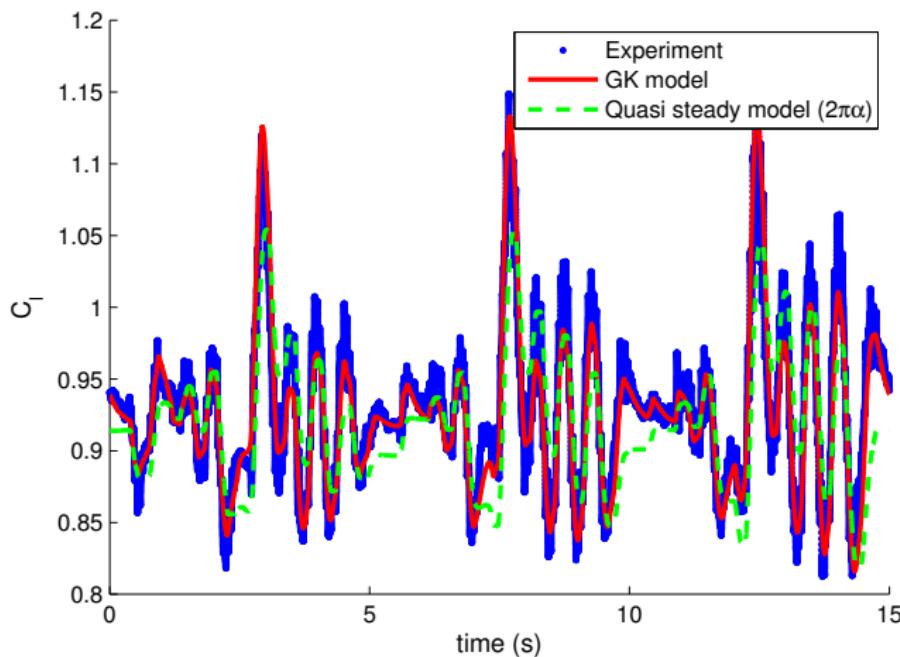


Figure : Unsteady effects of random pitching on the lift

Pseudo-random case - Drag

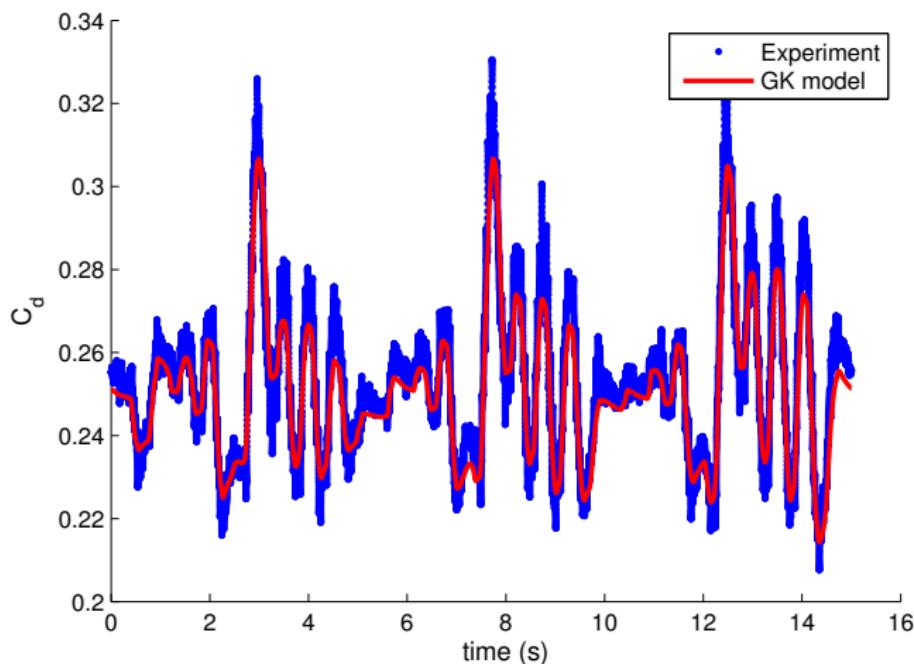


Figure : Unsteady effects of random pitching on the drag

Table of Contents

1. Introduction
2. The trajectory optimization problem
 - Dynamic soaring
 - Neutral energy loop
 - Implementation and validation
 - Quasi-steady aerodynamic model results
3. The unsteady pitching aerodynamic model
 - Experimental setup
 - The Goman and Khrabrov model
 - Determination and validation of the model
4. Unsteady trajectory optimization
 - Time constant equivalence
 - Gust duration dependency
 - Phase results
5. Conclusion

Froude number equivalence

Gliding vehicle time scale

$$T = \frac{g}{V^*}$$

Airfoil time scale

$$t^+ = \frac{c}{u}$$

For a vehicle flying at V^*

$$Fr = \frac{T}{t^+} = \frac{V^{*2}}{g \cdot c}$$

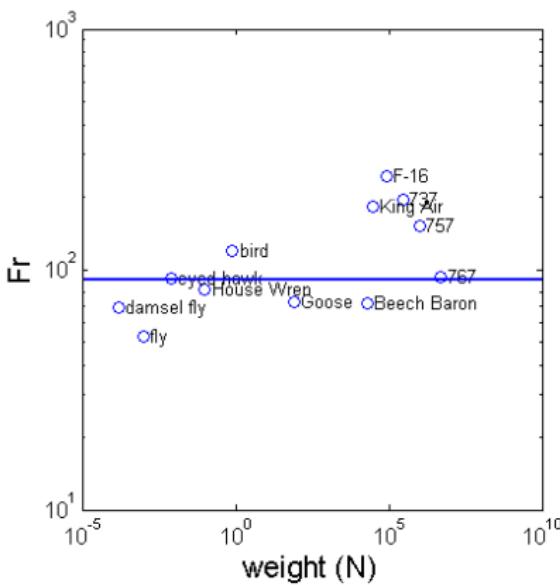


Figure : T to t+ ratio for various flying objects

Difference with the quasi-steady model optimization

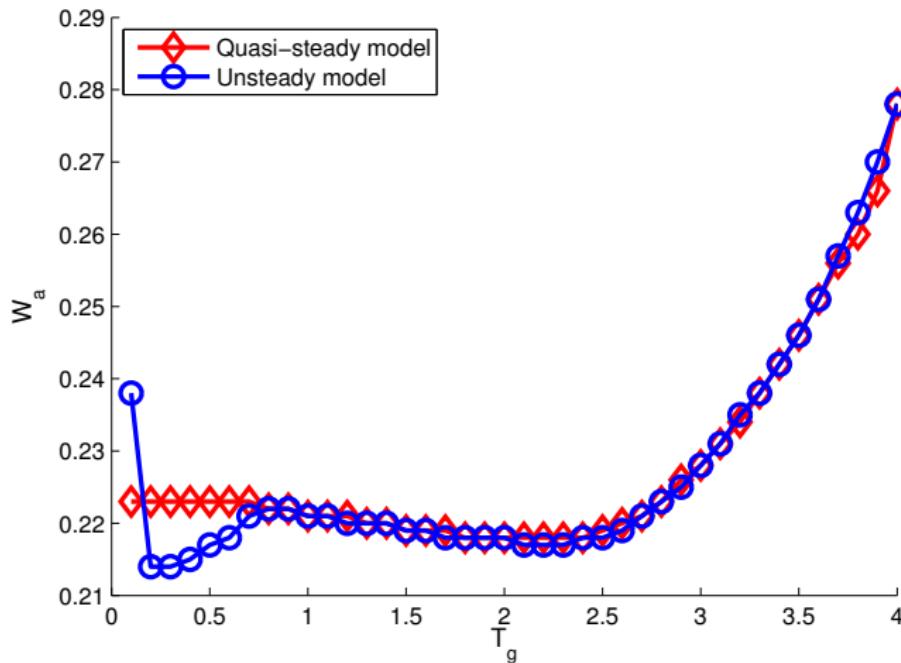


Figure : Performance difference between quasi-steady and unsteady model for vertical gusts

Difference with the quasi-steady model optimization

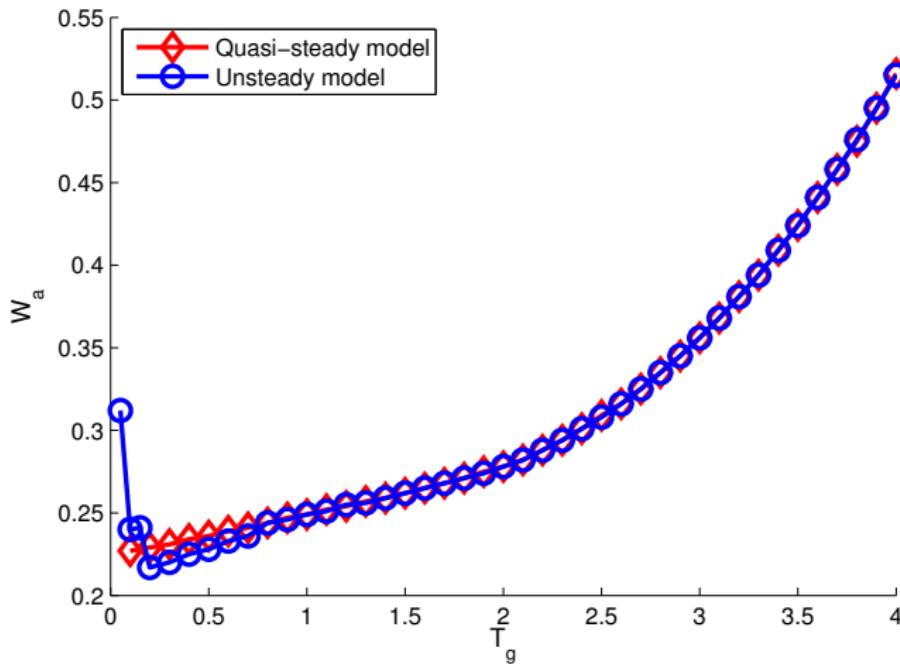


Figure : Performance difference between quasi-steady and unsteady model for combined gusts

A closer look at good performing short gusts

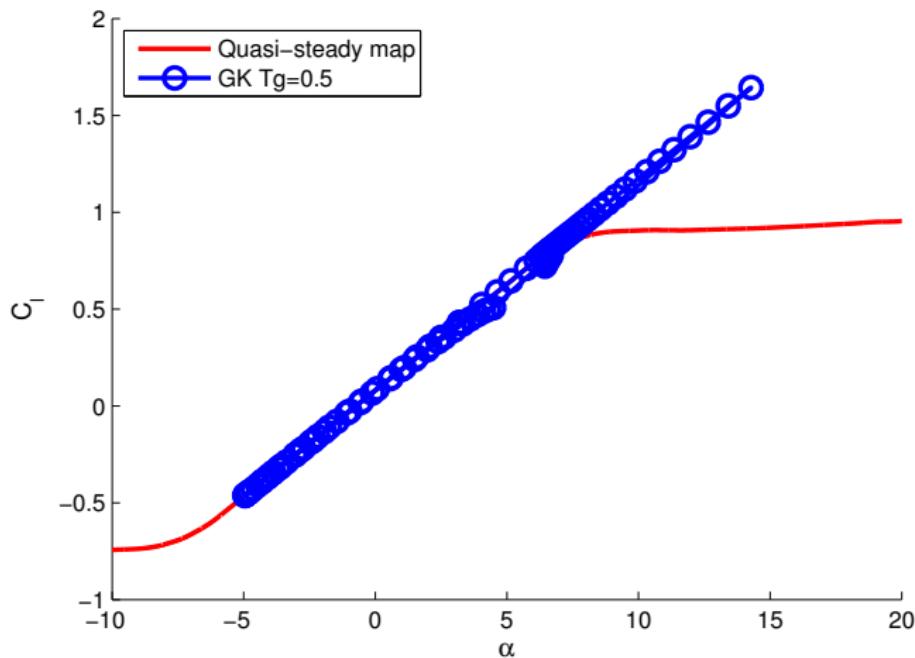


Figure : Lift coefficient versus angle of attack for 0.5T long vertical wind gusts with the unsteady model

A closer look at good performing short gusts

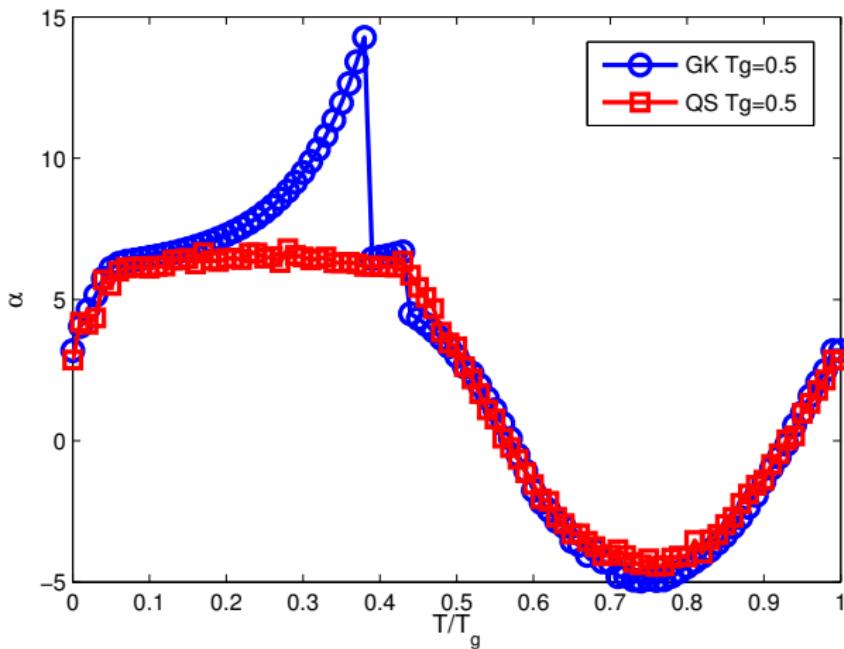


Figure : Angle of attack for short vertical gusts with the quasi-steady (QS) and unsteady (GK) model

Difference around $T_g \leq 0.2$

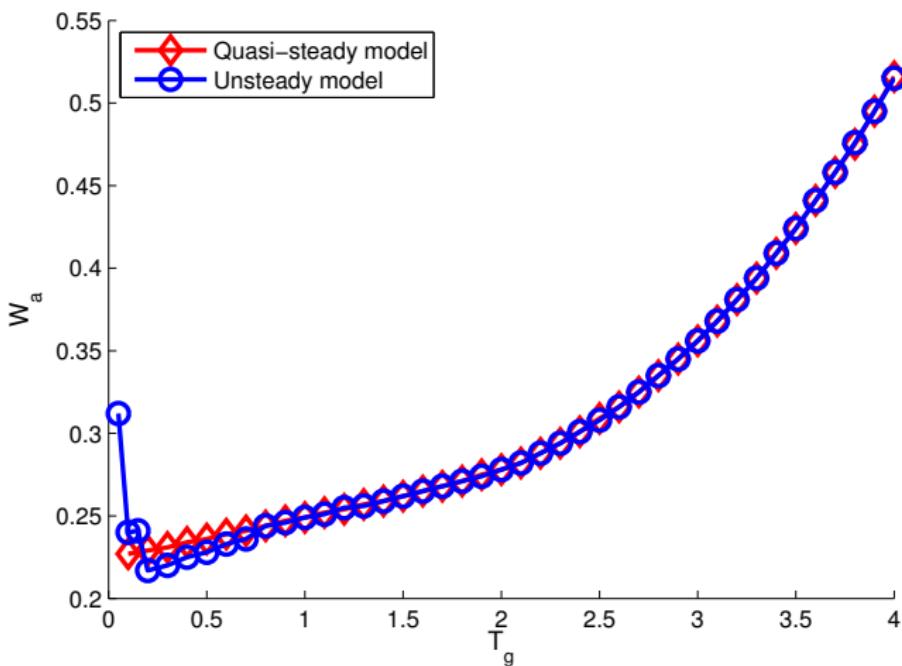


Figure : Performance difference between quasi-steady and unsteady model for combined gusts

Difference around $T_g \leq 0.2$

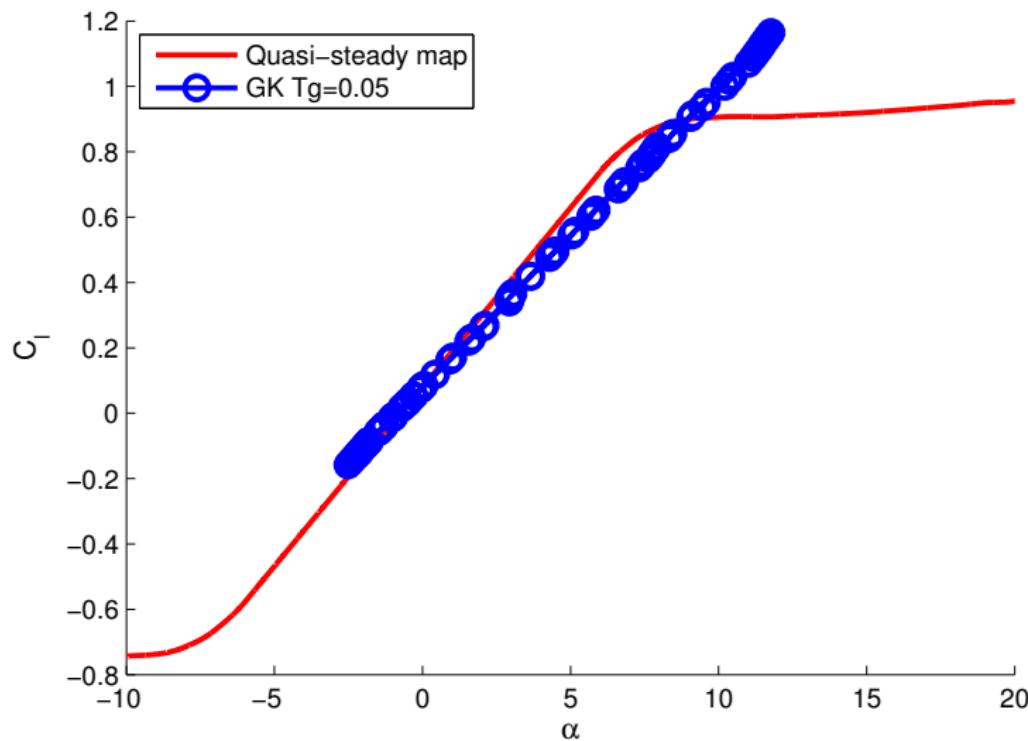


Figure : Lift coefficient versus angle of attack for a $0.05T$ long combined

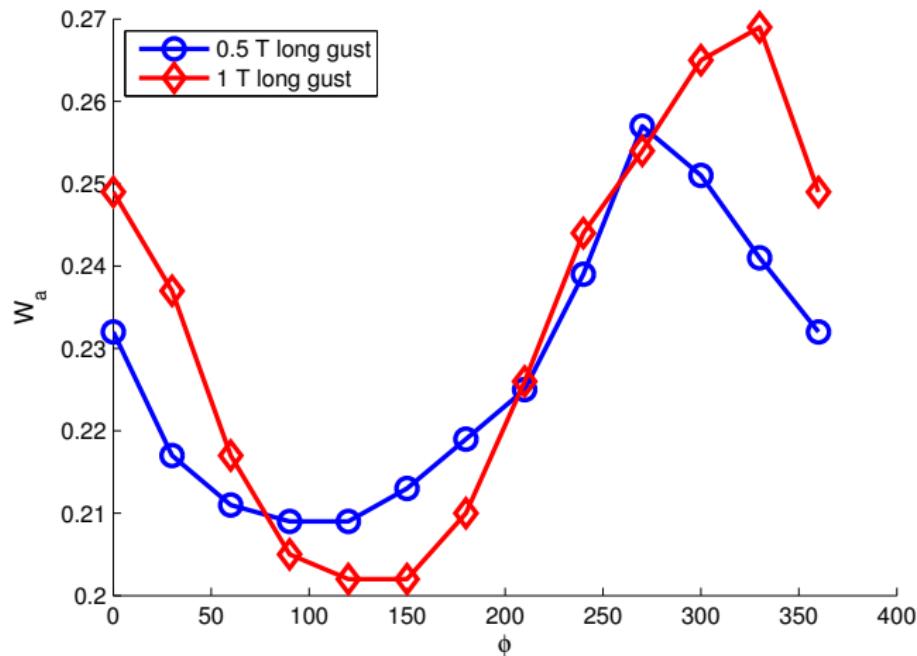


Figure : Influence of the phase between the components of the combined gust in the unsteady model case

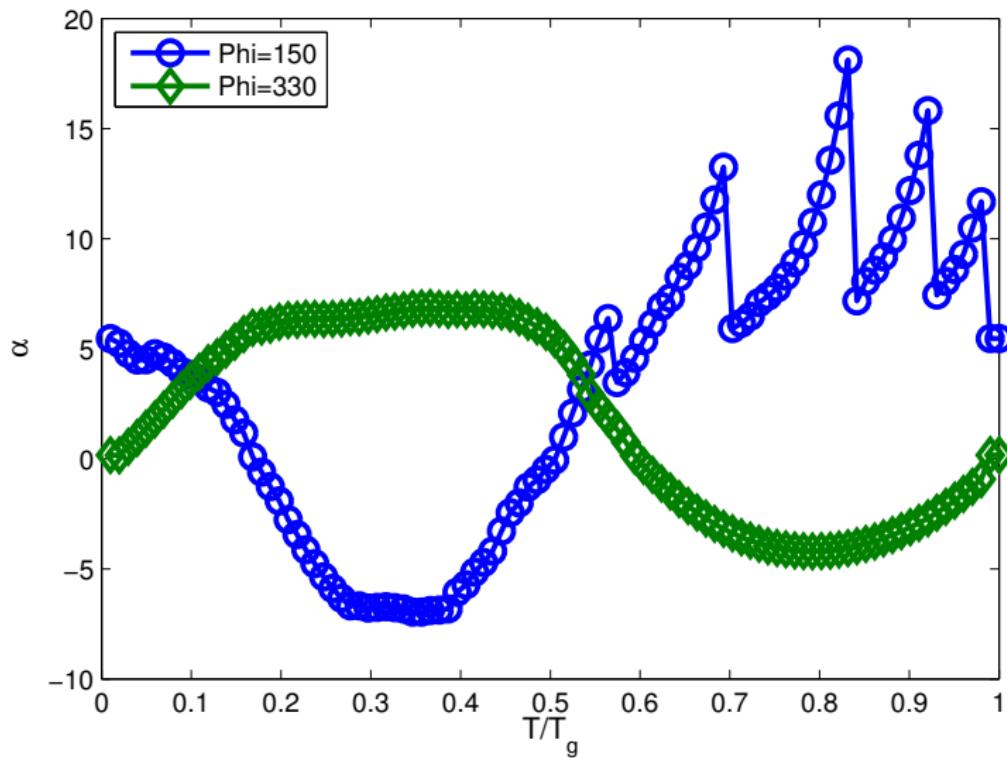


Figure : Angle of attack profile for different phase angle when $T_g = 1$

Table of Contents

1. Introduction
2. The trajectory optimization problem
 - Dynamic soaring
 - Neutral energy loop
 - Implementation and validation
 - Quasi-steady aerodynamic model results
3. The unsteady pitching aerodynamic model
 - Experimental setup
 - The Goman and Khrabrov model
 - Determination and validation of the model
4. Unsteady trajectory optimization
 - Time constant equivalence
 - Gust duration dependency
 - Phase results
5. Conclusion

GK model prediction:

- ▶ Accurate prediction of lift and *drag* for arbitrary pitch motion
- ▶ The drag coefficient shares the same state variable as the lift
- ▶ The model is fast enough to be used for optimization algorithms

Trajectory optimization

- ▶ Neutral energy flight is possible through combinations of vertical and horizontal gusts
- ▶ Vehicle and airfoil time scale are related through the Froude number
- ▶ Unsteady aerodynamic effects are seen for gusts shorter than $0.7T$
- ▶ The unsteady effects are beneficial for $T_g \in [0.2, 0.7]$ as they let the airfoil achieve higher C_l values
- ▶ The unsteady effects are detrimental for T_g smaller than 0.2, where the “angle of attack lag” becomes significant