

# Energy savings for UAV flight in unsteady gusting conditions through trajectory optimization

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2. The trajectory optimization problem
  - Dynamic soaring
  - Neutral energy loop
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# Introduction and motivations

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# Different types of soaring

Figure : something about soaring

## Spatial wind gradients

- ▶ Thermal updrafts
- ▶ Horizontal wind gradients

## Temporal wind gradients

- ▶ Natural turbulences
- ▶ Building and natural feature wake

# Defining the energy extraction problem

What is an “optimal trajectory”?

- ▶ Maximum energy at the end of the cycle
- ▶ Maximizing the energy gain at each instant of the cycle
- ▶ *Minimize the energy input needed for sustainable flight*

The neutral energy loop

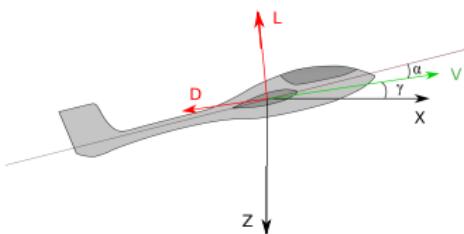
Finding the minimal wind gust that allows to maintain altitude and speed over a gust.

# Aircraft model<sup>1</sup>

Newton-Euler equations of motion:

$$\begin{aligned} m\ddot{x} &= -L' \cdot \sin(\gamma) + D' \cdot \cos(\gamma) \\ m\ddot{z} &= L' \cdot \cos(\gamma) - D' \cdot \sin(\gamma) - m \cdot g \end{aligned}$$

## Lissaman's non-dimensional variables



- ▶ Velocities with  $V^*$  the optimal glide speed
- ▶ Time with  $T = \frac{V^*}{g}$
- ▶ Lift and drag coefficients  $L = \frac{C_l}{C_l^*}$   $D = \frac{C_d}{C_d^*}$
- ▶ Dynamic pressure  $Q = \frac{L'}{MgL} = \frac{\rho V^2 C_l^*}{2Mg}$   

$$Q = (U - U_g)^2 + (W - W_g)^2$$

$$\begin{aligned} \frac{dU}{dT} &= -LQ \cdot \sin(\gamma) + DQ \cdot \cos(\gamma) \\ \frac{dW}{dT} &= LQ \cdot \cos(\gamma) - DQ \cdot \sin(\gamma) - 1 \end{aligned}$$

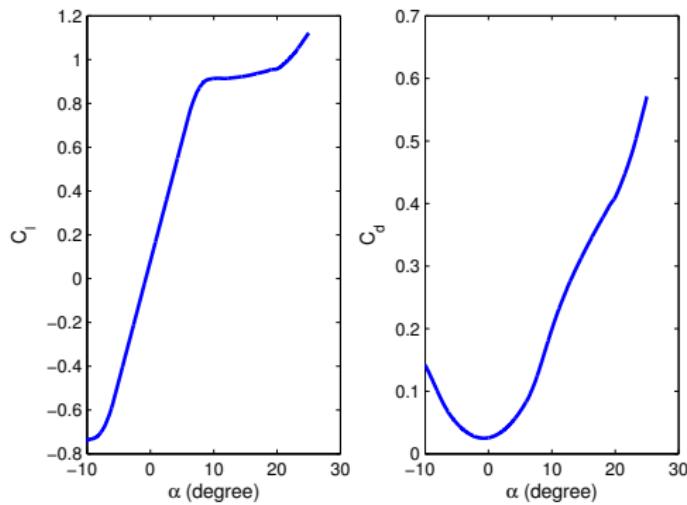
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<sup>1</sup> Lissaman P and Patel C. Neutral energy cycles for a vehicle in sinusoidal and turbulent vertical gusts. 45th AIAA meeting, 863, 2007

# Quasi-steady lift and drag model

► NACA0009 characteristic

► Lissaman's quadratic drag



$$D = \frac{1+L^2}{2G^*}$$

Figure : Simplified lift and drag for the NACA0009 airfoil

# Wind profiles

We define three different wind profiles:

- ▶ Vertical wind gust:

$$W_g = W_a \cdot \sin\left(2\pi \frac{T}{T_g}\right)$$
$$U_g = 0$$

- ▶ Horizontal wind gust:

$$W_g = 0$$
$$U_g = W_a \cdot \cos\left(2\pi \frac{T}{T_g}\right)$$

- ▶ Combined wind gust:

$$W_g = W_a \cdot \sin\left(2\pi \frac{T}{T_g}\right)$$
$$U_g = W_a \cdot \cos\left(2\pi \frac{T}{T_g} + \varphi\right)$$

# Optimization algorithm - a minimization problem

- ▶ State vector

$$x = \begin{bmatrix} \dots \\ X_i \\ Z_i \\ U_i \\ W_i \\ L_i/\alpha_i \\ \dots \\ W_a \end{bmatrix} \quad i \in [1, N]$$

- ▶ Cost function:  $W_a$
- ▶ Constraints: equations of motion, limits on the range of angles of attack allowed, neutral energy loop conditions.

# Constraints formulation

- ▶ Equations of motion (with Simpson's discrete integral)

$$y_i = \begin{bmatrix} X_i \\ Z_i \\ U_i \\ W_i \end{bmatrix} \quad \dot{y}_i = \begin{bmatrix} U_i \\ W_i \\ -L_i Q_i \cdot \sin(\gamma_i) + D_i Q_i \cdot \cos(\gamma_i) \\ L_i Q_i \cdot \cos(\gamma_i) - D_i Q_i \cdot \sin(\gamma_i) - 1 \end{bmatrix}$$

$$0 = y_{k+1} - y_k - \frac{1}{6}(y_k + 4y_m + y_{k+1})\delta t \quad \forall i \in [1, N-1]$$

- ▶ Neutral energy loop
- ▶ Limits in range of  $C_l$

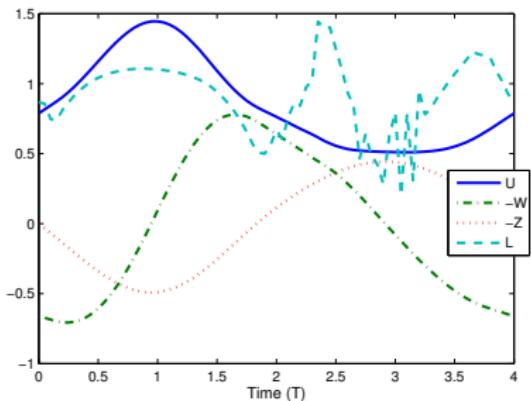
$$\begin{aligned} Z_1 &= Z_N & L_1 &= L_N \\ W_1 &= W_N & U_1 &= U_N \end{aligned}$$

$$\begin{aligned} L_{\min} &\leq L_i \leq L_{\max} \\ \alpha_{\max} &\leq \alpha_i \leq \alpha_{\max} \end{aligned}$$

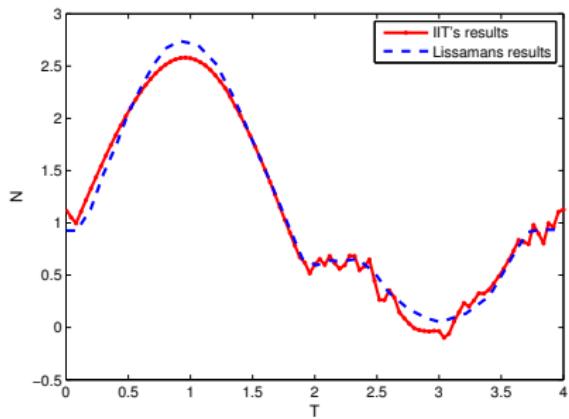
$$\begin{aligned} W_2 - W_1 &= W_N - W_{N-1} \\ U_2 - U_1 &= U_N - U_{N-1} \end{aligned}$$

# Comparison with Lissaman's results - Quadratic model

Lissaman found a gust amplitude  $W_a = 0.128$  for this case



**Figure :** Optimization results for a  $4T$  long vertical gust  $W_a = 0.129$



**Figure :** Comparison with Lissaman's non-dimensional normal force  $N$  for a  $4T$  long vertical gust

# Quasi-steady lift to drag model - NACA009

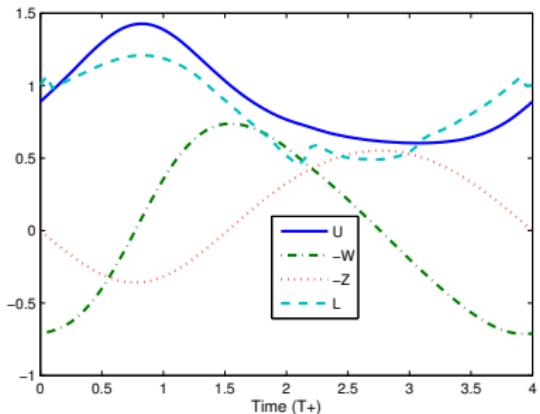


Figure :  $4T$  long vertical gust for the NACA0009 airfoil,  $W_a = 0.205$

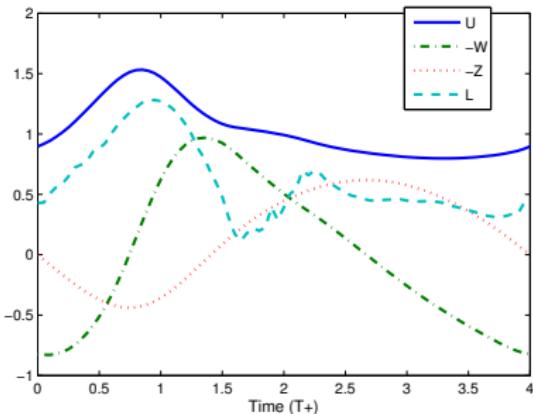


Figure :  $4T$  long combined gust for the NACA0009 airfoil,  $W_a = 0.387$

# T<sub>g</sub> dependency

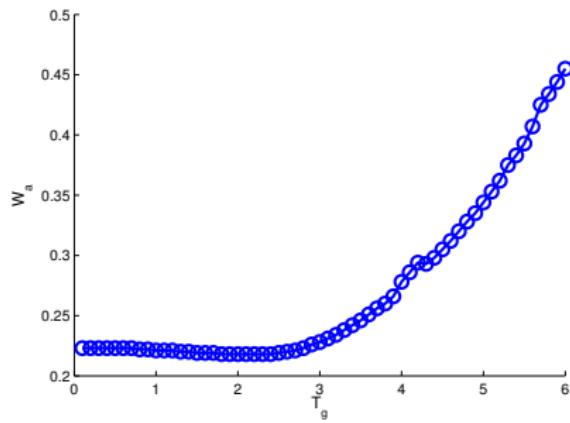


Figure : Influence of gust duration on the minimum gust amplitude for vertical gusts

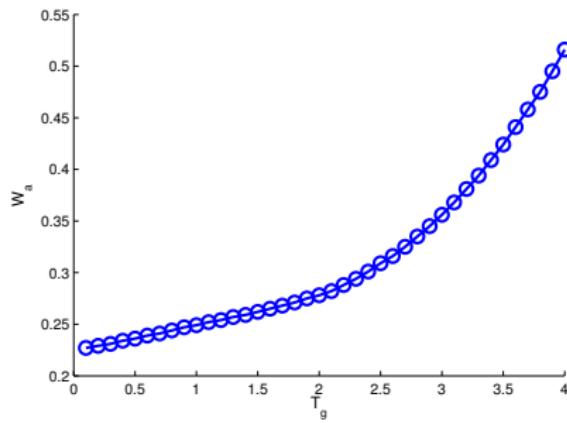


Figure : Influence of gust duration on the minimum gust amplitude for combined gusts

# Difference between short and long gusts

We can see that there is tipping point around  $T_g = 2.5$

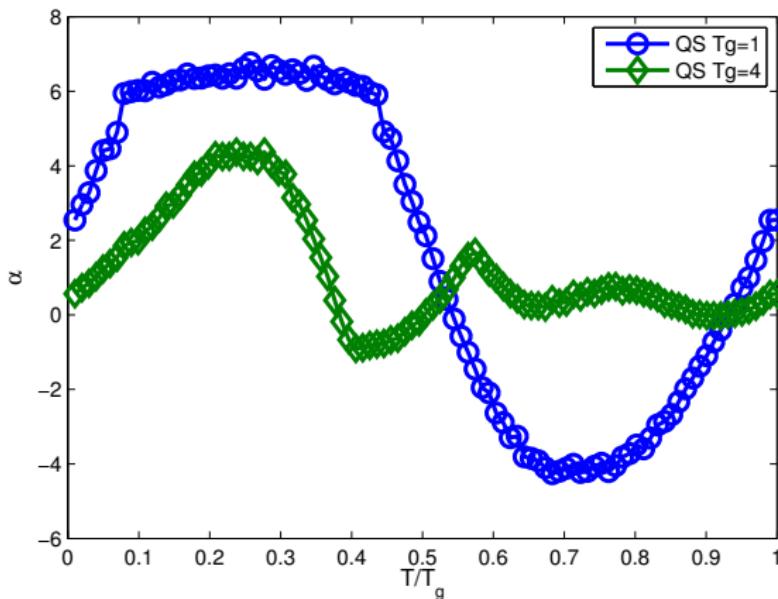
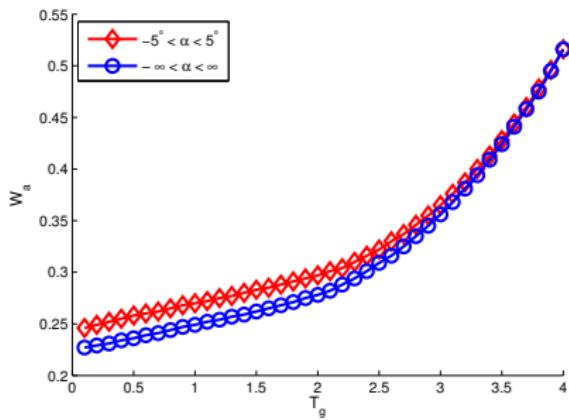
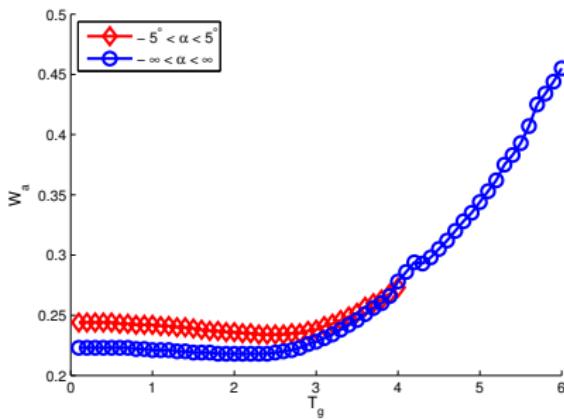


Figure : Difference between short and long gust angle of attack profile for combined gusts

# Angle of attack limitation



**Figure :** Difference in performance for combined wind gusts if no high angle of attack are allowed



**Figure :** Difference in performance for vertical wind gusts if no high angle of attack are allowed

# Phase influence

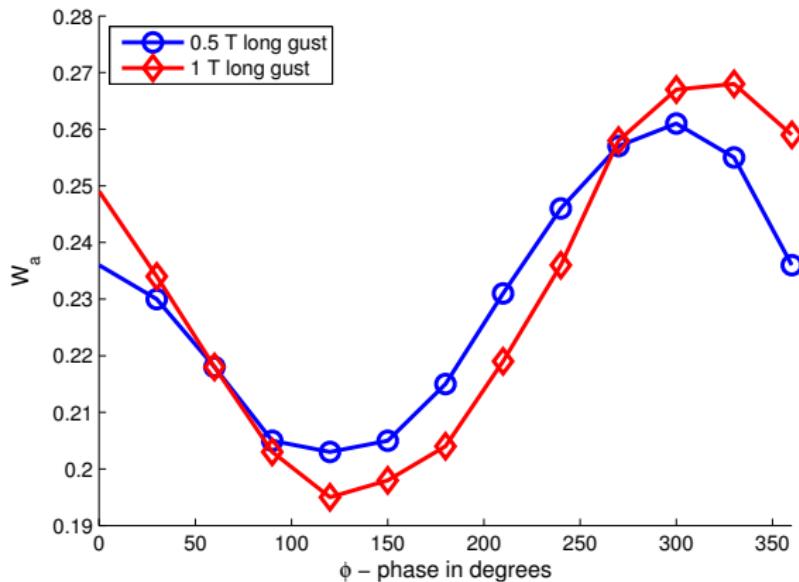


Figure : Influence of the phase between the components of the combined gust

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# Pitching mechanism and experimental conditions

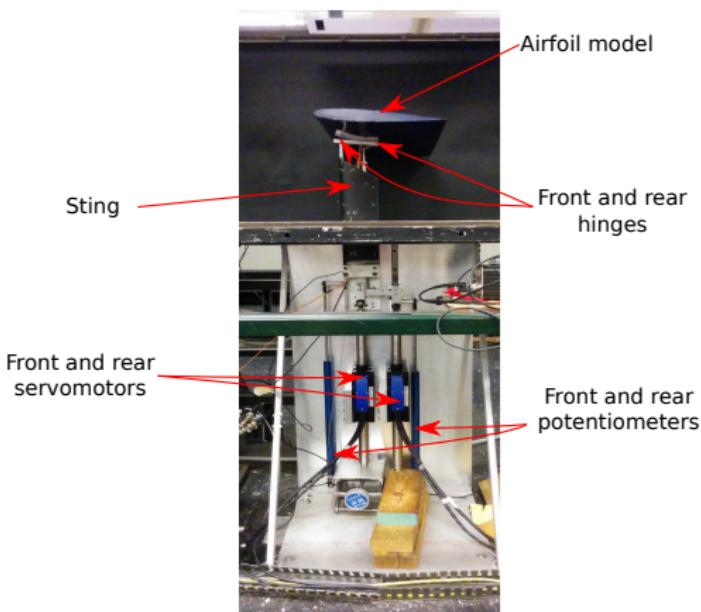


Figure : Airfoil model inside the wind tunnel

## Experimental conditions

- ▶ Free stream velocity: 3 m/s
- ▶ Airfoil: NACA0009
- ▶ Reynolds number 50000

## Controller and data acquisition

- ▶ Angle of attack controlled by simulink® and two servomotors
- ▶ Servos position measured by two linear potentiometers
- ▶ Piezoelectric force balance (NANO17) to measure the forces on the airfoil

# The GK model concept

The Goman and Khrabrov model<sup>2</sup> - a non-linear state space model

$$C_l = f(\alpha, x)$$

$$C_d = g(\alpha, x)$$

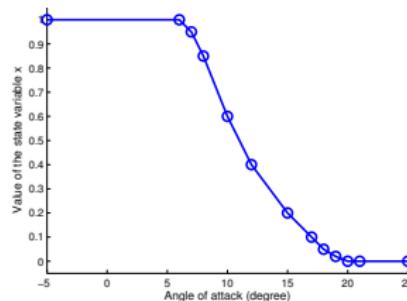
$$\tau_1 \frac{dx}{dt} + x = x_0(\alpha - \tau_2 \dot{\alpha})$$

- ▶ Lift and drag model

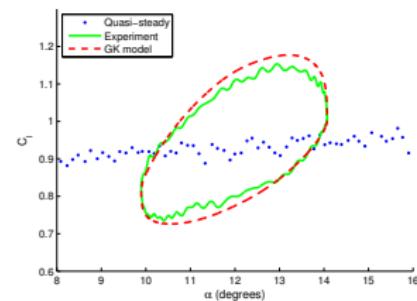
$$C_l = 2\pi\alpha(0.6x + 0.4) + C_{l0}$$

$$C_d = \frac{((2-x)C_l)^2}{G_{\max}} + C_{d0}$$

- ▶ Non-linear state map



- ▶ Time constants  $\tau_1$  and  $\tau_2$

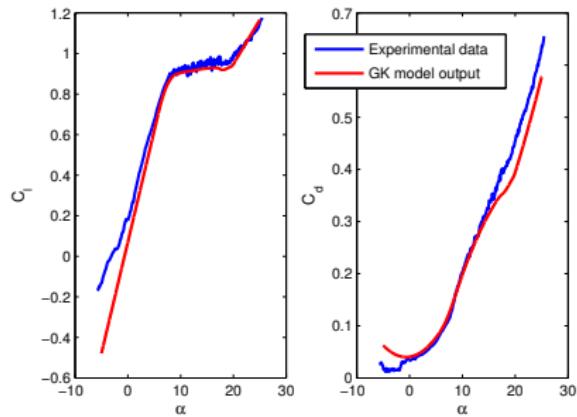


<sup>2</sup>Goman M and Khrabrov A. *Journal of Aircraft*, 31(5):1109–1115, 1994.

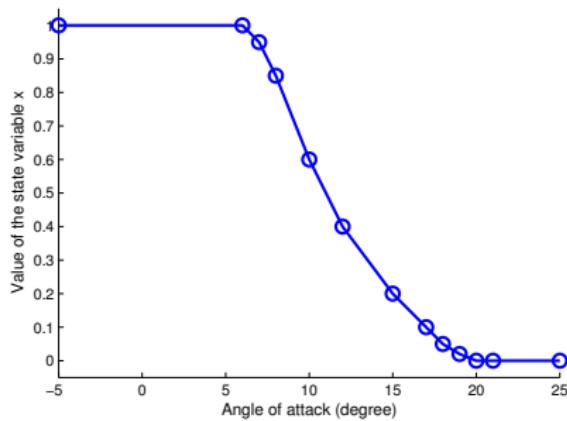
# Quasi-steady map and state variable

$$C_l = 2\pi\alpha(0.6x + 0.4) + C_{l0}$$

$$C_d = \frac{((2-x)C_l)^2}{G_{\max}} + C_{d0}$$



**Figure :** Lift and drag coefficient in the quasi-steady case



**Figure :** Quasi-steady profile for the state variable  $x$

# Time constant determination

Periodic sinusoidal pitching at different frequencies  $k = \pi \frac{cf}{u}$

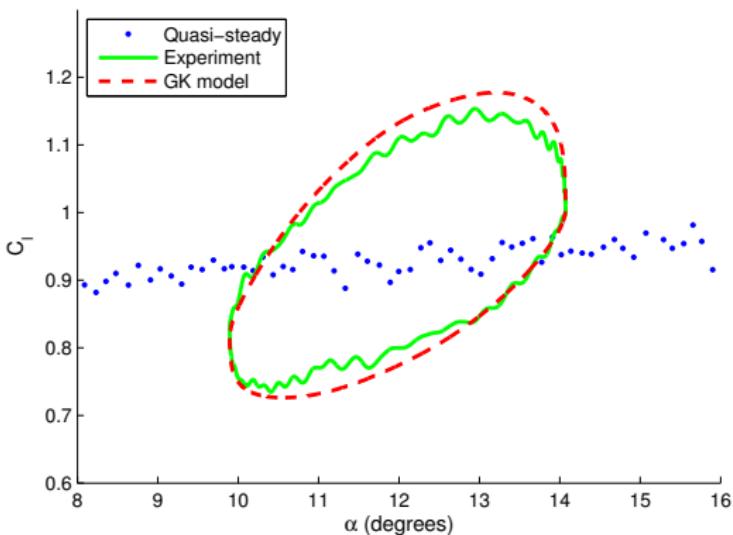


Figure : Comparison of experimental lift coefficient and model prediction after tuning of the time constant at  $k = 0.128$

We find  $\tau_1 = 3.1t^+$  and  $\tau_2 = 4.29t^+$  (with  $t^+ = \frac{c}{u}$ )

# Comparison with periodic measurements - Lift

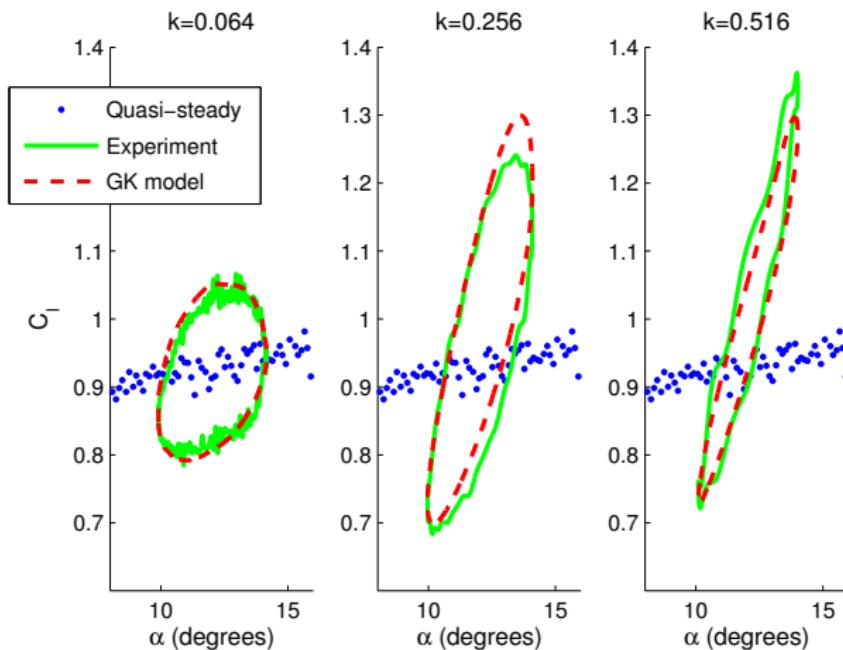


Figure : Lift measurement and prediction during sinusoidal pitching around 12 degree

# Comparison with periodic measurements - Drag

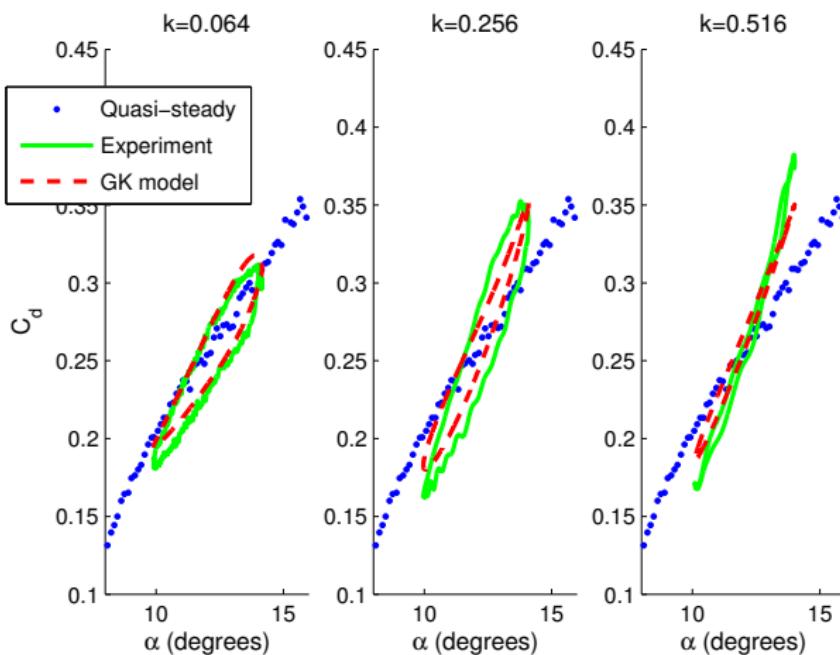


Figure : drag measurement and prediction during sinusoidal pitching around 12 degree

# Pseudo-random case - Lift

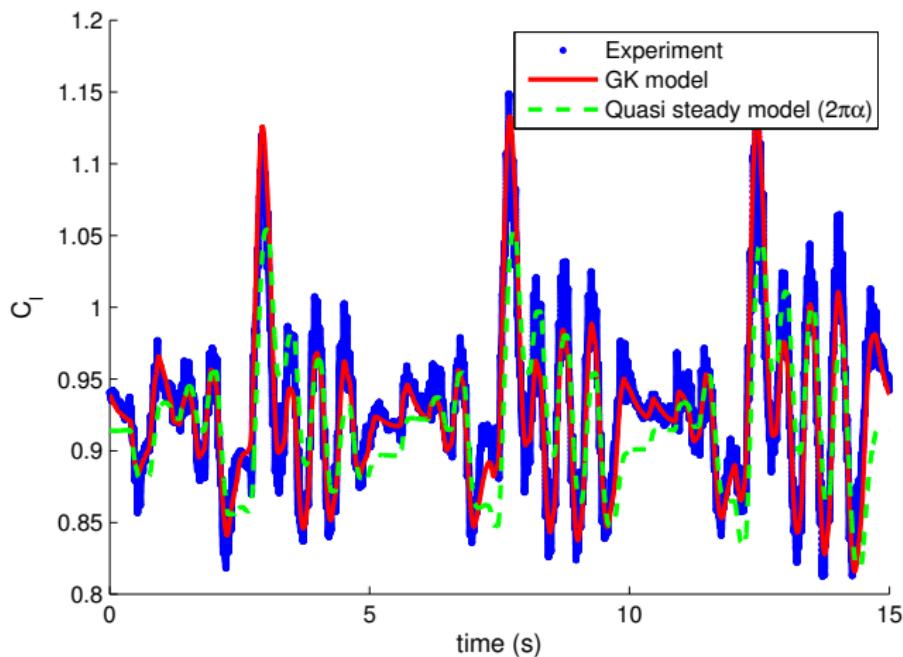


Figure : Unsteady effects of random pitching on the lift

# Pseudo-random case - Drag

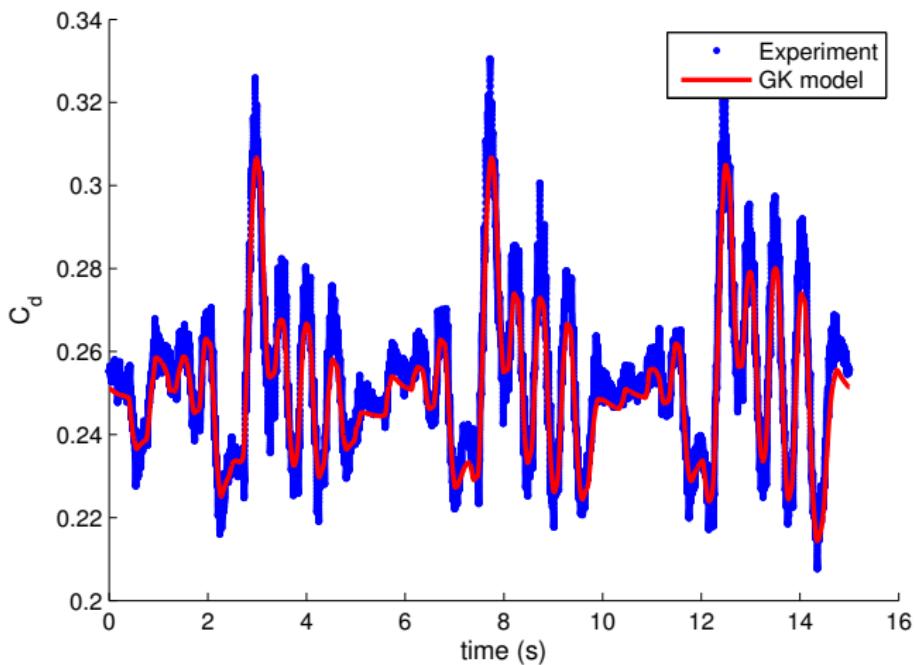


Figure : Unsteady effects of random pitching on the drag

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# Froude number equivalence

Gliding vehicle time scale

$$T = \frac{g}{V^*}$$

Airfoil time scale

$$t^+ = \frac{c}{u}$$

For a vehicle flying at  $V^*$

$$Fr = \frac{T}{t^+} = \frac{V^{*2}}{g \cdot c}$$

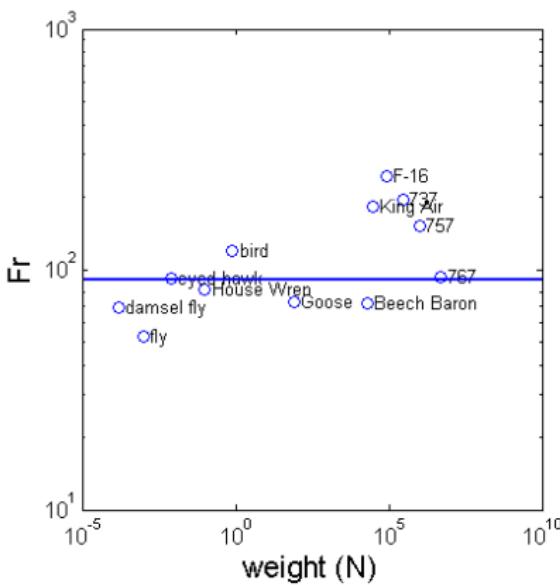


Figure : T to t+ ratio for various flying objects

# Difference with the quasi-steady model optimization

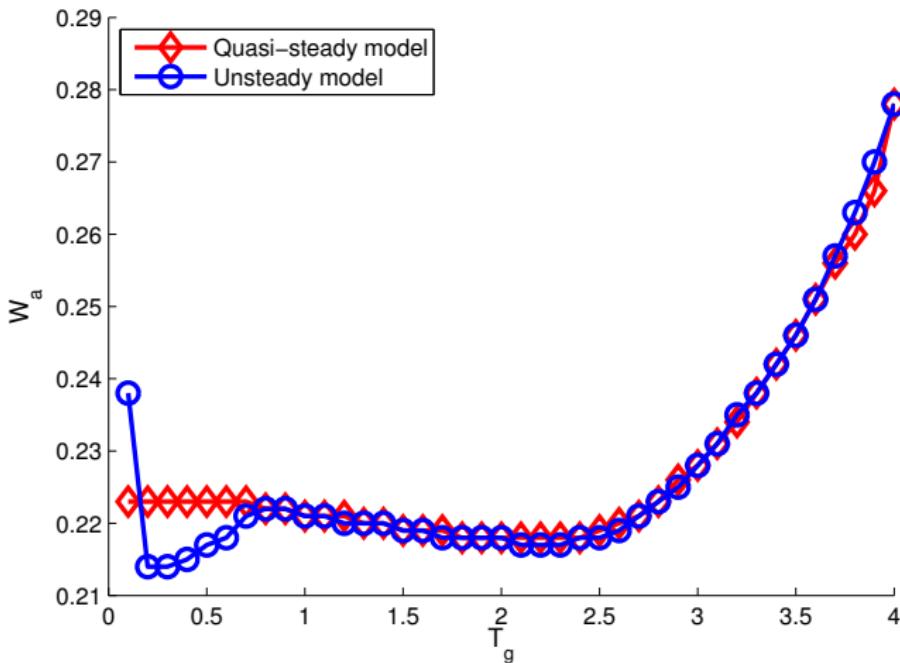


Figure : Performance difference between quasi-steady and unsteady model for vertical gusts

# Difference with the quasi-steady model optimization

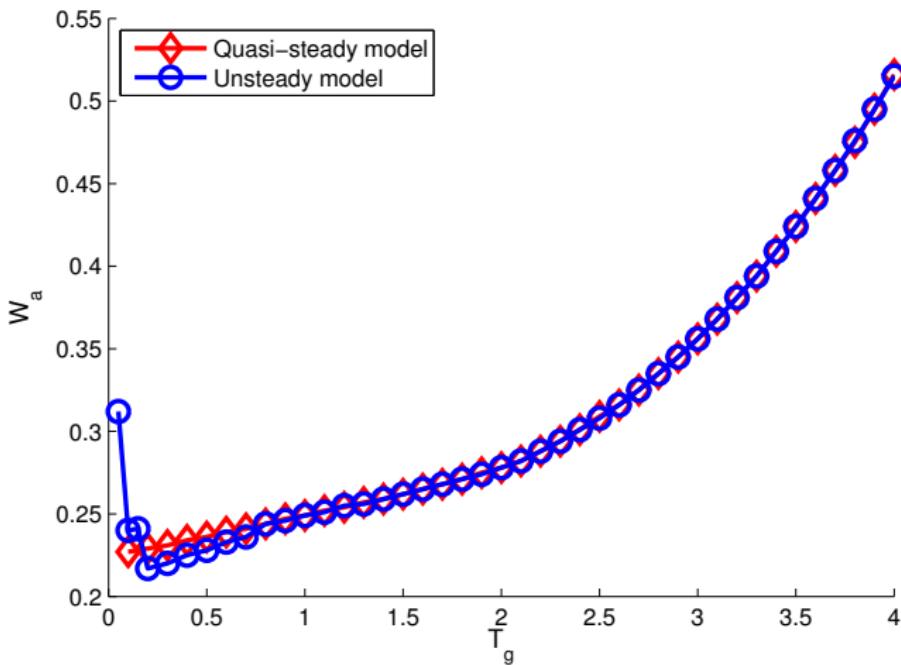


Figure : Performance difference between quasi-steady and unsteady model for combined gusts

# A closer look at $T_g \in [0.2, 0.5]$ ( $k \in [0.05, 0.175]$ )

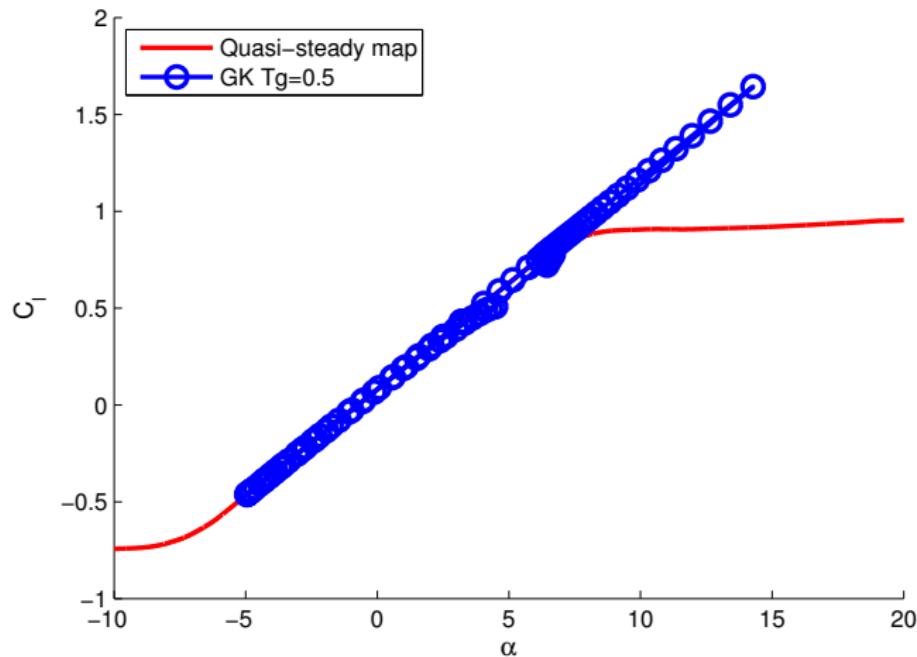
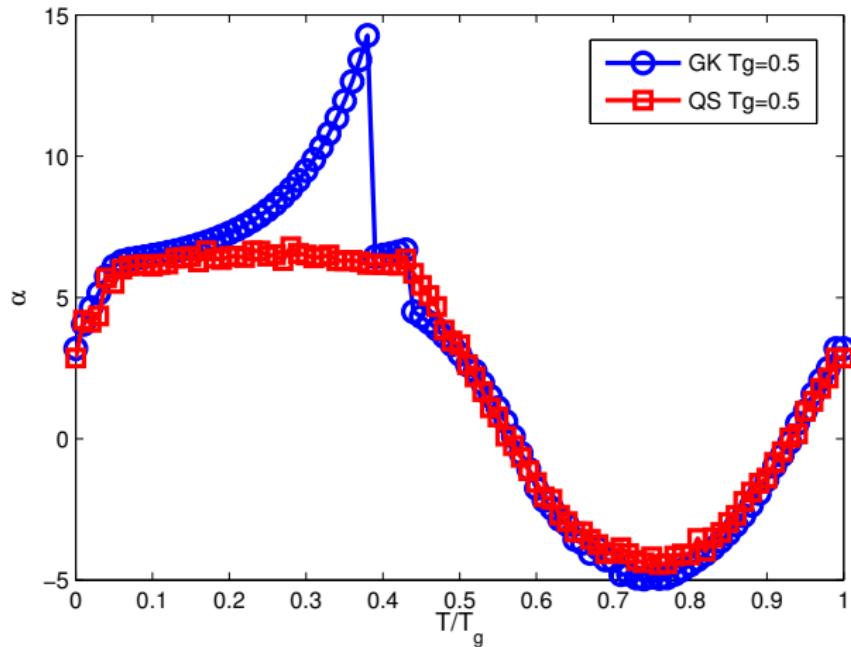


Figure : Lift coefficient versus angle of attack for 0.5T long vertical wind gusts with the unsteady model

# A closer look at good performing short gusts



**Figure :** Angle of attack for short vertical gusts with the quasi-steady (QS) and unsteady (GK) model

# Difference around $T_g \leq 0.2$ ( $k \geq 0.175$ )

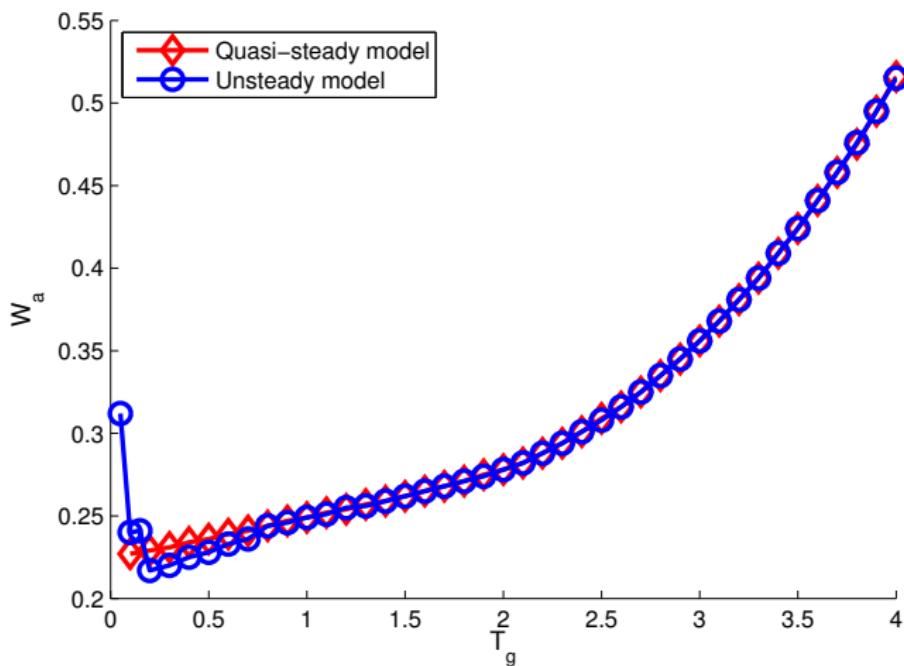


Figure : Performance difference between quasi-steady and unsteady model for combined gusts

# Difference around $T_g \leq 0.2$

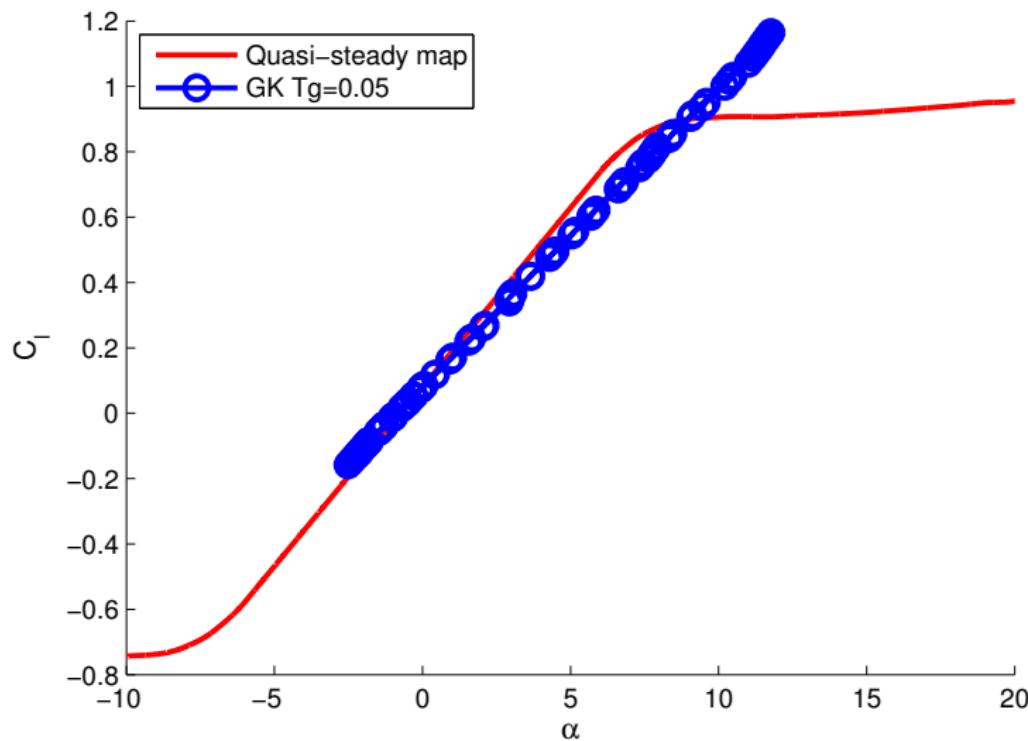


Figure : Lift coefficient versus angle of attack for a  $0.05T$  long combined

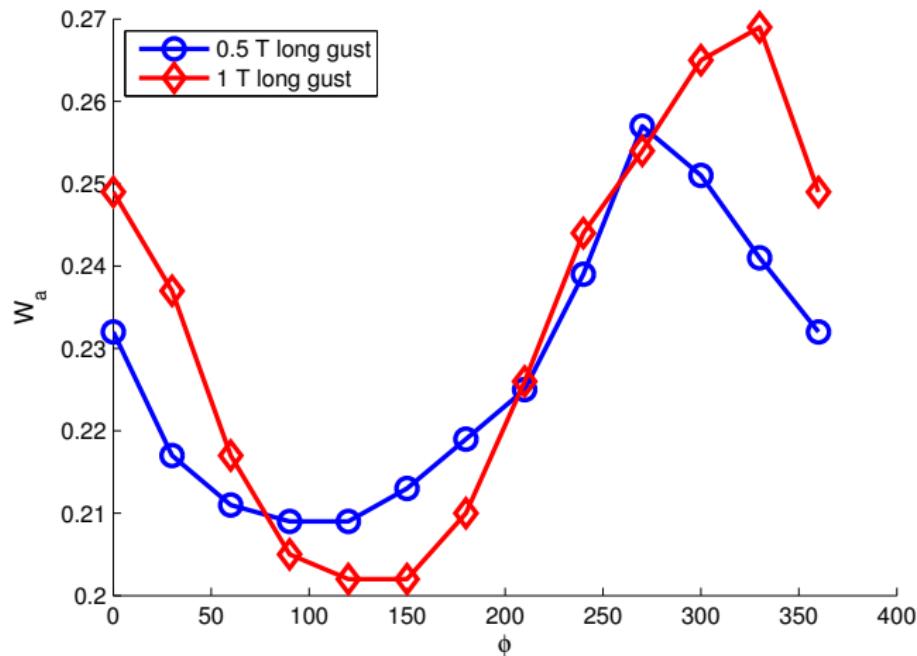


Figure : Influence of the phase between the components of the combined gust in the unsteady model case

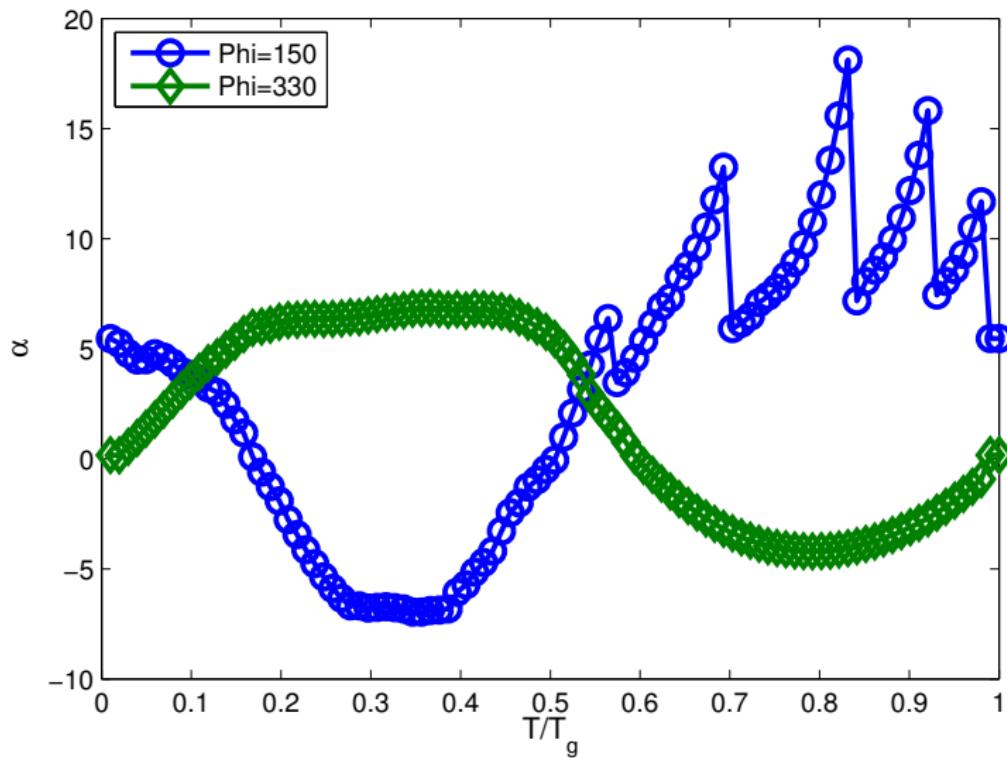


Figure : Angle of attack profile for different phase angle when  $T_g = 1$

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## GK model prediction:

- ▶ Accurate prediction of lift and *drag* for arbitrary pitch motion
- ▶ The drag coefficient shares the same state variable as the lift
- ▶ The model is fast enough to be used for optimization algorithms

## Trajectory optimization

- ▶ Neutral energy flight is possible through combinations of vertical and horizontal gusts
- ▶ Vehicle and airfoil time scale are related through the Froude number
- ▶ Unsteady aerodynamic effects are seen for gusts shorter than  $0.7T$
- ▶ The unsteady effects are beneficial for  $T_g \in [0.2, 0.7]$  as they let the airfoil achieve higher  $C_l$  values
- ▶ The unsteady effects are detrimental for  $T_g$  smaller than 0.2, where the “angle of attack lag” becomes significant

# Possible improvements

## GK model prediction:

- ▶ Extending the GK model to a plunging and surging flow
- ▶ Devise a more rigorous way to obtain the time constants
- ▶ Implement a model for the moment coefficient
- ▶ Link the state variable and the flow configuration

## Trajectory optimization

- ▶ Further investigations should be performed for very short gusts
- ▶ The effects of surging and plunging are not considered
- ▶ Introduce a 3rd degree of freedom to account for the moment of inertia