# Energy savings for UAV flight in unsteady gusting conditions through trajectory optimization

Lou Grimaud

Illinois Institute of Technology

July 2014

# Table of Contents

# Table of Contents

# Introduction and motivations

- The goal is to find optimal energy extraction strategies in unsteady flows.
  - UAV energy saving
  - Vertical axis wind turbine
  - Helicopter rotor blade
- How do vehicle dynamics relate to unsteady aerodynamic effects
- When do these unsteady effects start to become significant

# Table of Contents

# Different types of soaring[1]

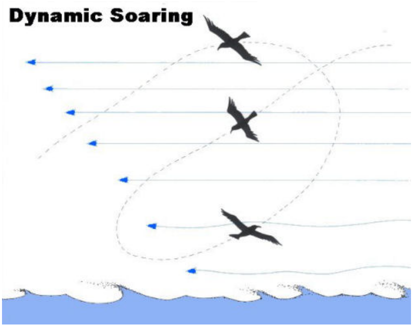



Figure : Dynamic soaring maneuver

**Spatial wind gradients**

- Thermal updrafts
- Horizontal wind gradients

**Temporal wind gradients**

- Natural turbulences
- Building and natural feature wake

[1] Rayleigh, *The soaring of birds*, Nature, 1883

# Defining the energy extraction problem

What is an "optimal trajectory"?

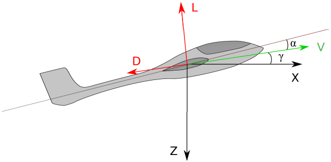
- Maximum energy at the end of the cycle
- Maximizing the energy gain at each instant of the cycle
- *Minimize the energy input needed for sustainable flight*

**The neutral energy loop**

Finding the minimal wind gust amplitude that allows to maintain altitude and speed over a gust.

# Aircraft model[2]

$$m\ddot{x} = -L' \cdot sin(\gamma) + D' \cdot cos(\gamma)$$
$$m\ddot{z} = L' \cdot cos(\gamma) - D' \cdot sin(\gamma) - m \cdot g$$




Lissaman's non-dimensional variables

- Velocities with $V^*$ the optimal glide speed
- Time with $T = \frac{V^*}{g}$
- Lift and drag coefficients $L = \frac{C_l}{C_l^*}$ $D = \frac{C_d}{C_d^*}$
- Dynamic pressure $Q = \frac{L'}{MgL} = \frac{\rho V^2 C_l^*}{2Mg}$
  $Q = (U - U_g)^2 + (W - W_g)^2$

$$\frac{dU}{dT} = -LQ \cdot sin(\gamma) + DQ \cdot cos(\gamma)$$
$$\frac{dW}{dT} = LQ \cdot cos(\gamma) - DQ \cdot sin(\gamma) - 1$$

[2] Lissaman P and Patel C. Neutral energy cycles for a vehicle in sinusoidal and turbulent vertical gusts. 45th AIAA meeting,863, 2007

# Quasi-steady lift and drag model

- NACA0009 characteristic

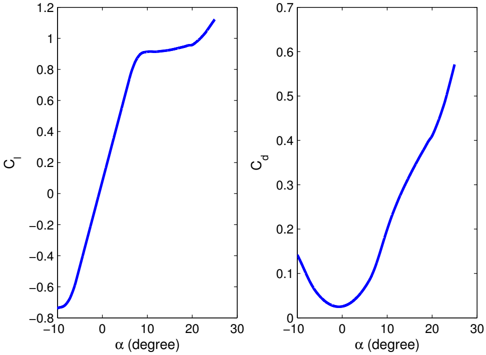



Figure : Simplified lift and drag for the NACA0009 airfoil

- Lissaman's quadratic drag

$$D = \frac{1+L^2}{2G^*}$$

# Wind profiles

We define three different wind profiles:

- Vertical wind gust:

$$W_g = W_a \cdot \sin(2\pi \frac{T}{T_g})$$
$$U_g = 0$$

- Horizontal wind gust:

$$W_g = 0$$
$$U_g = W_a \cdot \cos(2\pi \frac{T}{T_g})$$

- Combined wind gust:

$$W_g = W_a \cdot \sin(2\pi \frac{T}{T_g})$$
$$U_g = W_a \cdot \cos(2\pi \frac{T}{T_g} + \varphi)$$

# Optimization algorithm - a minimization problem

- State vector

$$T_i = \frac{i}{N} T_g \quad x = \begin{bmatrix} \dots \\ X_i \\ Z_i \\ U_i \\ W_i \\ L_i/\alpha_i \\ \dots \\ W_a \end{bmatrix} \quad i \in [0, N]$$

- Cost function: $W_a$
- Constraints: equations of motion, limits on the range of angles of attack allowed, neutral energy loop conditions.

# Constraints formulation

- Equations of motion (with Simpson's discreet integral)

$$y_i = \begin{bmatrix} X_i \\ Z_i \\ U_i \\ W_i \end{bmatrix} \quad \dot{y_i} = \begin{bmatrix} U_i \\ W_i \\ -L_i Q_i \cdot sin(\gamma_i) + D_i Q_i \cdot cos(\gamma_i) \\ L_i Q_i \cdot cos(\gamma_i) - D_i Q_i \cdot sin(\gamma_i) - 1 \end{bmatrix}$$

$$0 = y_{i+1} - y_i - \frac{1}{6}(\dot{y_i} + 4\dot{y_m} + \dot{y_{i+1}})\delta t \quad \forall i \in [0, N-1]$$

- Neutral energy loop

$$\begin{aligned} Z_1 &= Z_N & L_1 &= L_N \\ W_1 &= W_N & U_1 &= U_N \end{aligned}$$

$$\begin{aligned} W_2 - W_1 &= W_N - W_{N-1} \\ U_2 - U_1 &= U_N - U_{N-1} \end{aligned}$$

- Limits in range of $C_l$

$$\begin{aligned} L_{\min} &\leq L_i \leq L_{\max} \\ \alpha_{\max} &\leq \alpha_i \leq \alpha_{\max} \end{aligned}$$

# Comparison with Lissaman's results - Quadratic model

Lissaman found a gust amplitude $W_a = 0.128$ for this case

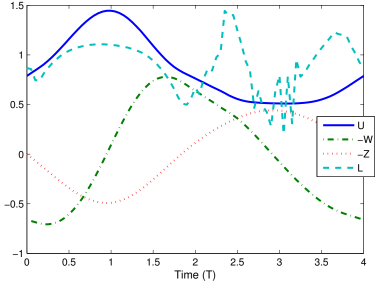



Figure : Optimization results for a $4T$ long vertical gust $W_a = 0.129$

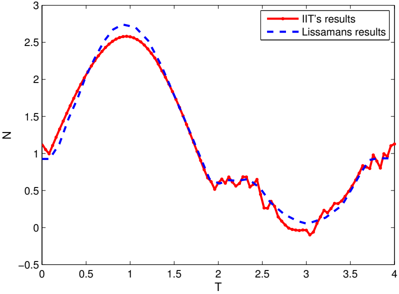



Figure : Comparison with Lissaman's non-dimensional normal force $N$ for a $4T$ long vertical gust

# Quasi-steady lift to drag model - NACA009

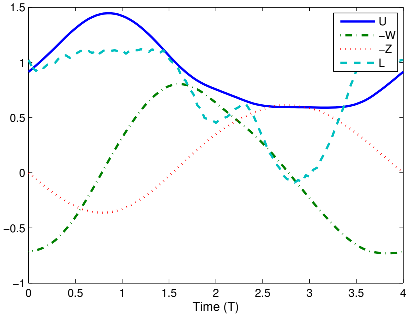



Figure : $4T$ long vertical gust for the NACA0009 airfoil, $W_a = 0.205$

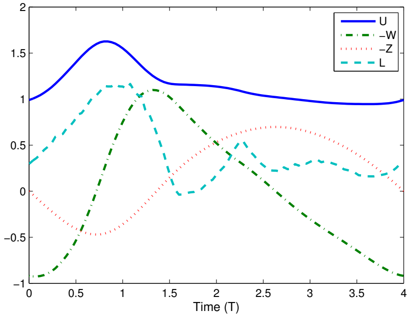



Figure : $4T$ long combined gust for the NACA0009 airfoil, $W_a = 0.516$

# Gust duration ($Tg$) dependency

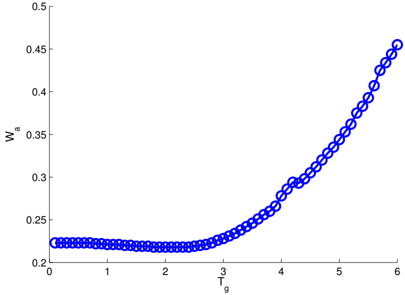



Figure : Influence of gust duration on the minimum gust amplitude for vertical gusts

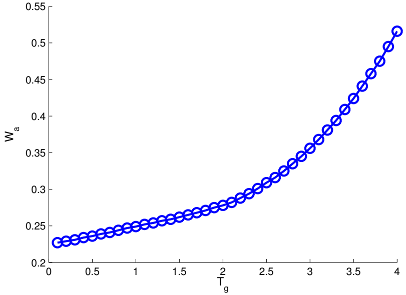



Figure : Influence of gust duration on the minimum gust amplitude for combined gusts

# Difference between short and long gusts

We can see that there is tipping point around $T_g = 2.5$

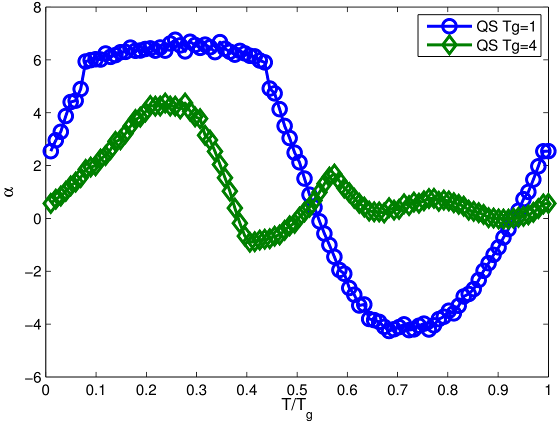



Figure : Difference between short and long gust angle of attack profile for combined gusts

# Angle of attack limitation

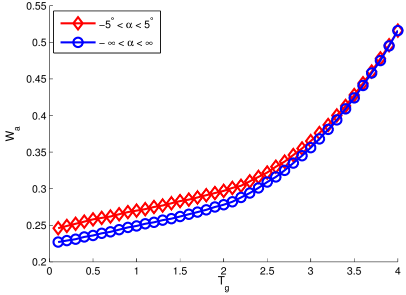



Figure : Difference in performance for combined wind gusts if no high angle of attack are allowed

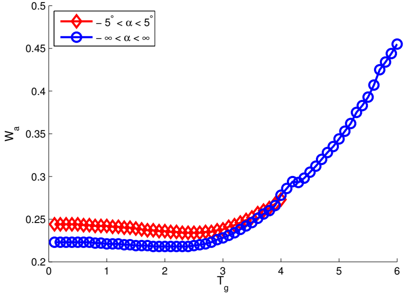



Figure : Difference in performance for vertical wind gusts if no high angle of attack are allowed

# Phase influence

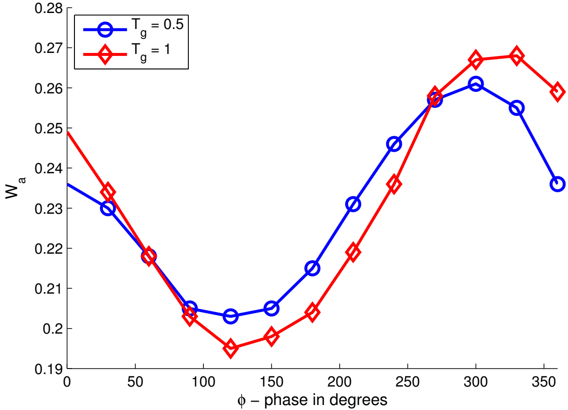



Figure : Influence of the phase between the components of the combined gust

# Table of Contents

# Pitching mechanism and experimental conditions

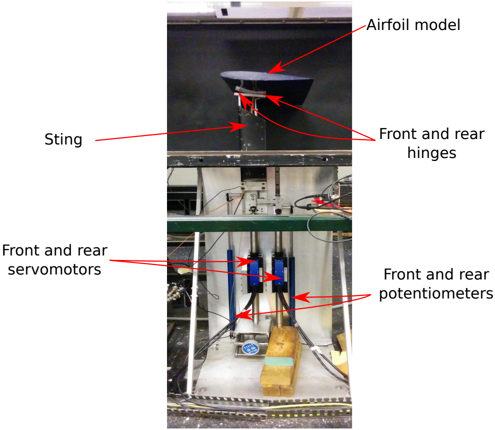



Figure : Airfoil model inside the wind tunnel

Experimental conditions

- Free stream velocity: 3 m/s
- Airfoil: NACA0009
- Reynolds number 50000

Controller and data acquisition

- Angle of attack controlled by simulink® and two servomotors
- Servos position measured by two linear potentiometers
- Piezoelectric force balance (NANO17) to measure the forces on the airfoil

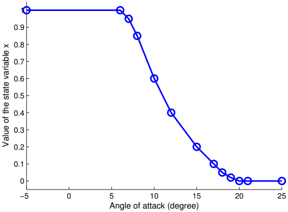
# The GK model concept

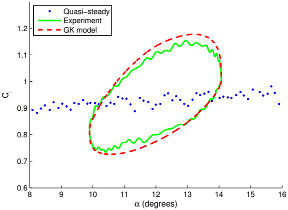
The Goman and Khrabrov model[3]

$$C_l = f(\alpha, x)$$
$$C_d = g(\alpha, x)$$
$$\tau_1 \frac{dx}{dt} + x = x_0(\alpha - \tau_2 \dot{\alpha})$$

- **Lift and drag model**
- **Non-linear state map**
- **Time constants $\tau_1$ and $\tau_2$**

$$C_l = 2\pi\alpha(0.6x + 0.4) + C_{l0}$$
$$C_d = \frac{((2-x)C_l)^2}{G_{\max}} + C_{d0}$$




[3] Goman M and Khrabrov A. *Journal of Aircraft*, 31(5):1109 âĂŞ 1115,1994. - a non-linear state space model

# Quasi-steady map and state variable

$$C_l = 2\pi\alpha(0.6x + 0.4) + C_{l0}$$
$$C_d = \frac{((2-x)C_l)^2}{G_{\max}} + C_{d0}$$

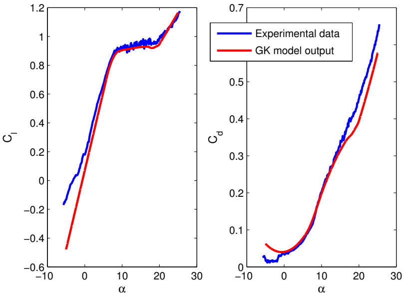



Figure : Lift and drag coefficient in the quasi-steady case

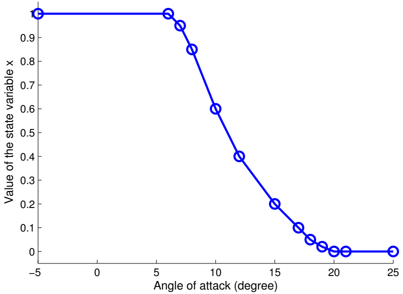



Figure : Quasi-steady profile for the state variable $x$

# Time constant determination

Periodic sinusoidal pitching at different frequencies $k = \pi \frac{cf}{u}$

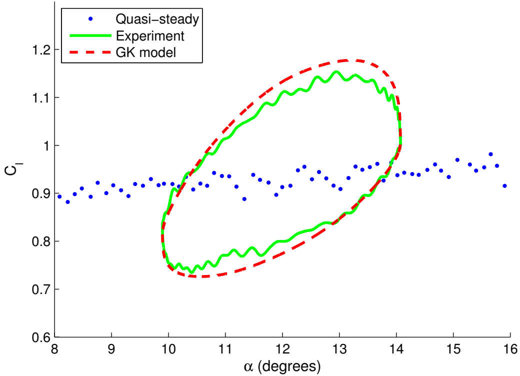



Figure : Comparison of experimental lift coefficient and model prediction after tunning of the time constant at $k = 0.128$

We find $\tau_1 = \mathbf{3.1t}^+$ and $\tau_2 = \mathbf{4.29t}^+$ (with $t^+ = \frac{c}{u}$)

# Comparison with periodic measurements - Lift

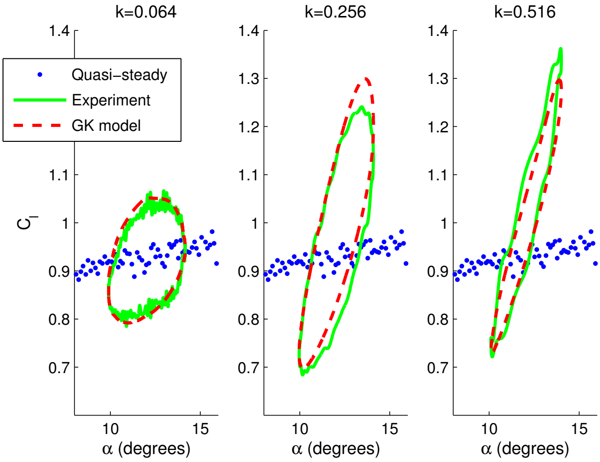



Figure : Lift measurement and prediction during sinusoidal pitching around 12 degree

# Comparison with periodic measurements - Drag

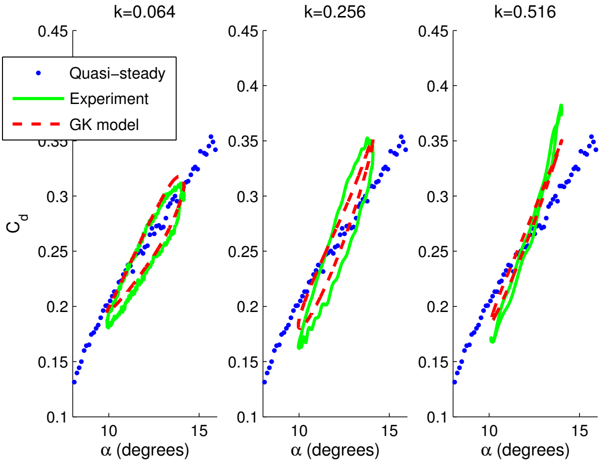



Figure : drag measurement and prediction during sinusoidal pitching around 12 degree

# Pseudo-random case - Lift

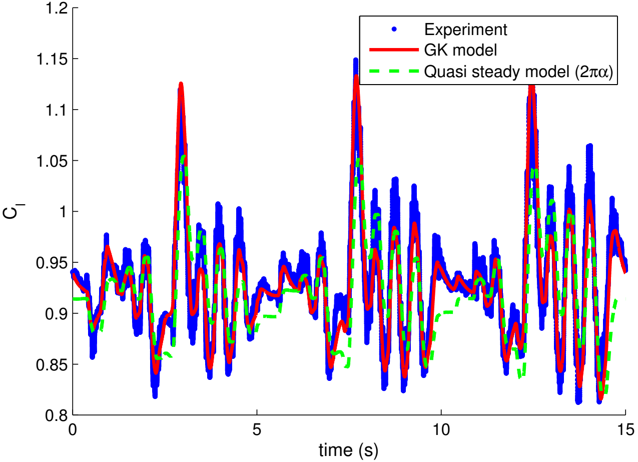



Figure : Unsteady effects of random pitching on the lift

# Pseudo-random case - Drag

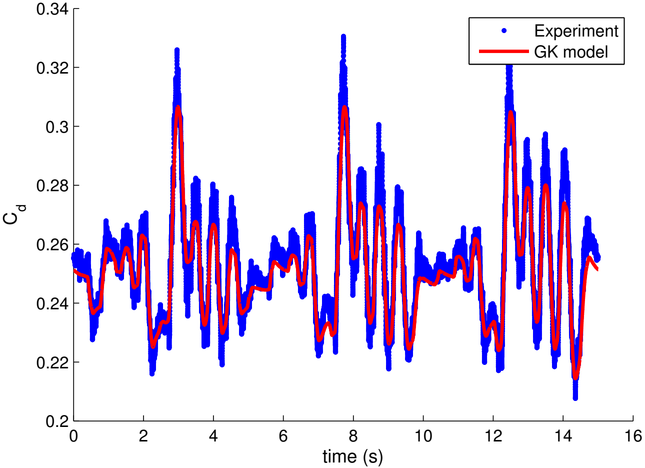



Figure : Unsteady effects of random pitching on the drag

# Table of Contents

# Froude number equivalence

Gliding vehicle time scale

$$T = \frac{g}{V*}$$

Airfoil time scale

$$t^{+} = \frac{c}{u}$$

For a vehicle flying at $V^{*}$

$$Fr = \frac{T}{t+} = \frac{V^{*2}}{g \cdot c}$$

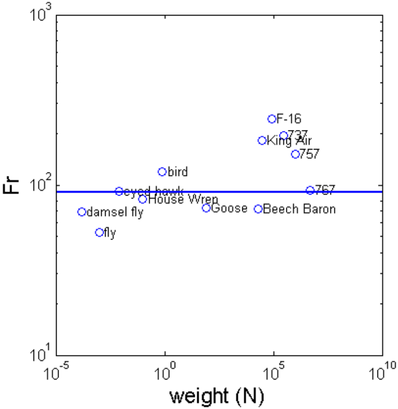



Figure : T to t+ ratio for various flying objects

# Difference with the quasi-steady model optimization

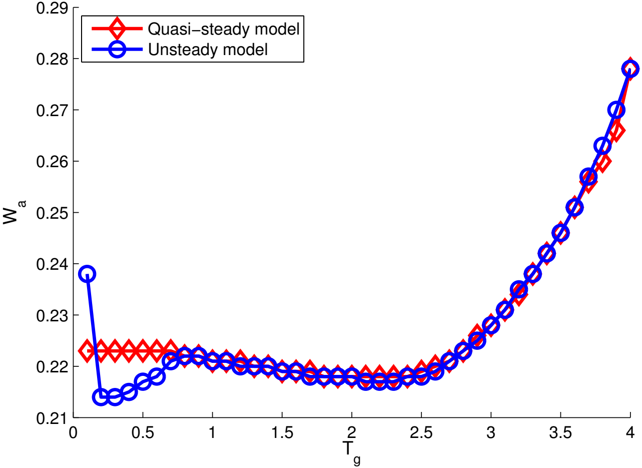



Figure : Performance difference between quasi-steady and unsteady model for vertical gusts

# Difference with the quasi-steady model optimization

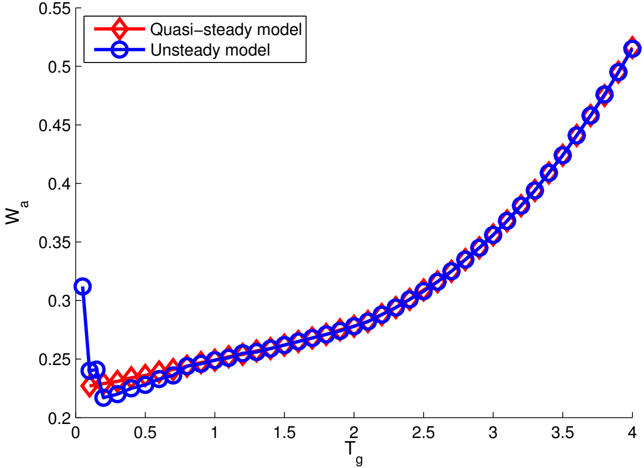



Figure : Performance difference between quasi-steady and unsteady model for combined gusts

# A closer look at $T_g \in [0.2, 0.5]$ ($k \in [0.05, 0.175]$)

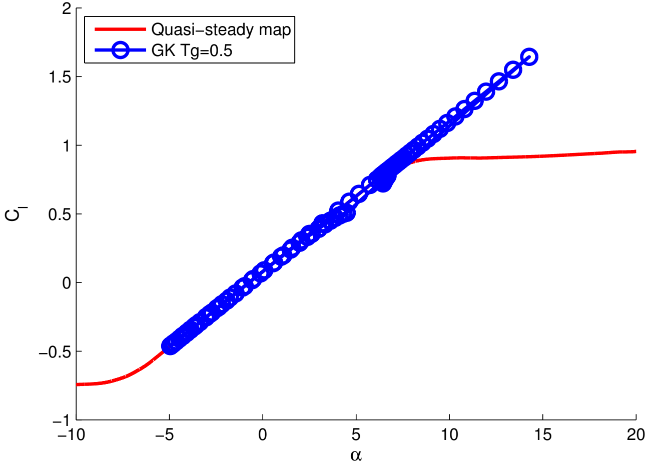



Figure : Lift coefficient versus angle of attack for 0.5T long vertical wind gusts with the unsteady model

# A closer look at good performing short gusts

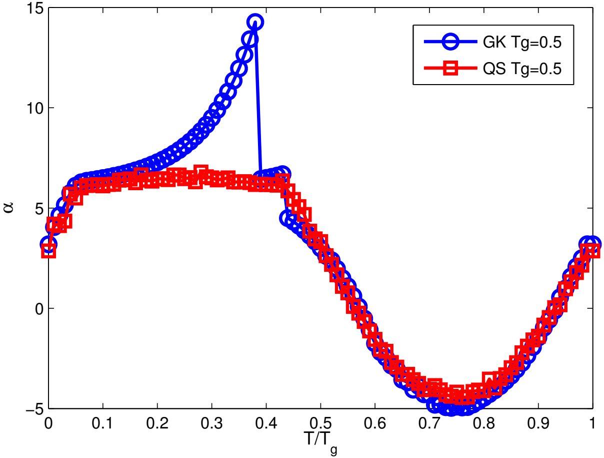



Figure : Angle of attack for short vertical gusts with the quasi-steady (QS) and unsteady (GK) model

# Difference around $T_g \leq 0.2$ ($k \geq 0.175$)

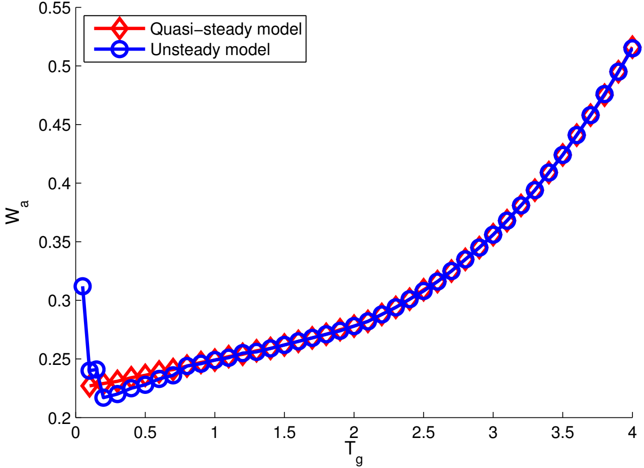



Figure : Performance difference between quasi-steady and unsteady model for combined gusts

# Difference around $T_g \leq 0.2$

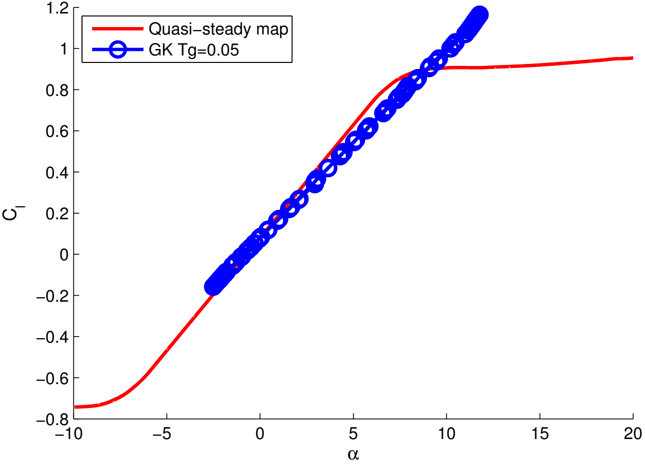



Figure : Lift coefficient versus angle of attack for a 0.05T long combined gust

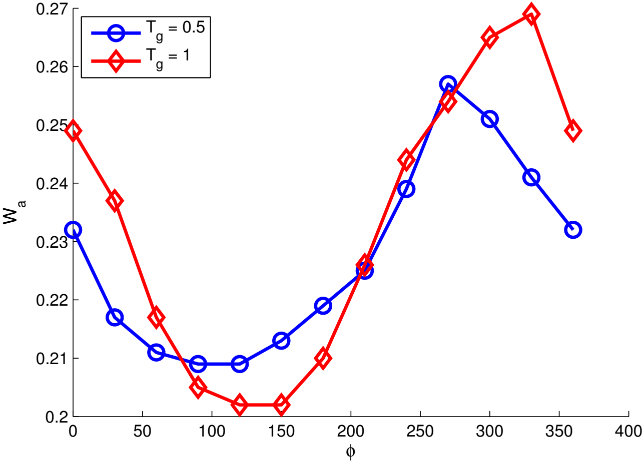



Figure : Influence of the phase between the components of the combined gust in the unsteady model case

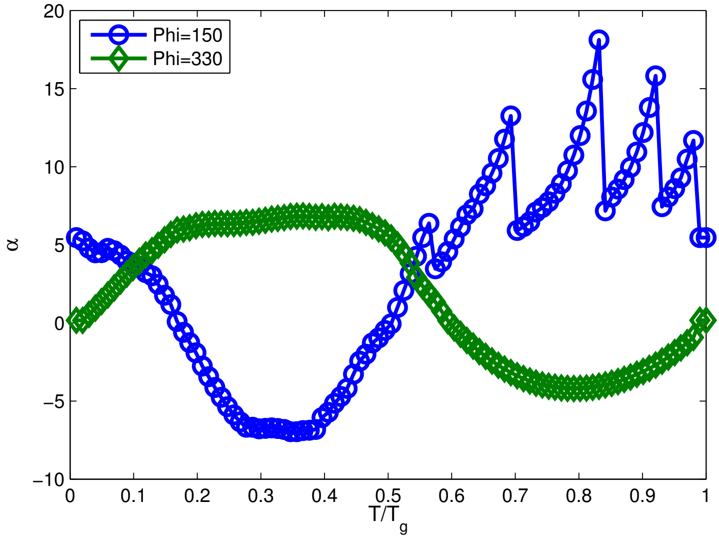



Figure : Angle of attack profile for different phase angle when $T_g = 1$

# Table of Contents

**GK model predictions:**

- Accurate prediction of lift and *drag* for arbitrary pitch motion
- The drag coefficient shares the same state variable as the lift
- The model is fast enough to be used for optimization algorithms

**Trajectory optimization**

- Neutral energy flight is possible through combinations of vertical and horizontal gusts
- Vehicle and airfoil time scale are related through the Froude number
- Unsteady aerodynamic effects are seen for gusts shorter than $0.7T$
- The unsteady effects are beneficial for $T_g \in [0.2, 0.7]$ as they let the airfoil achieve higher $C_l$ values
- The unsteady effects are detrimental for $T_g$ smaller than $0.2$, where the "angle of attack lag" becomes significant

# Possible improvements

**GK model prediction:**

- Extending the GK model to a plunging and surging flow
- Devise a more rigorous way to obtain the time constants
- Implement a model for the moment coefficient
- Link the state variable and the flow configuration

**Trajectory optimization**

- Further investigations should be performed for very short gusts
- The effects of surging and plunging are not considered
- Introduce a 3rd degree of freedom to account for the moment of inertia