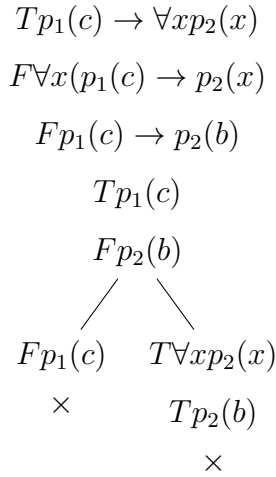


# MAC 239 Lista 2: Lógica de Primeira Ordem

Lourenço Henrique Moinheiro Martins Sborz Bogo, NUSP=11208005

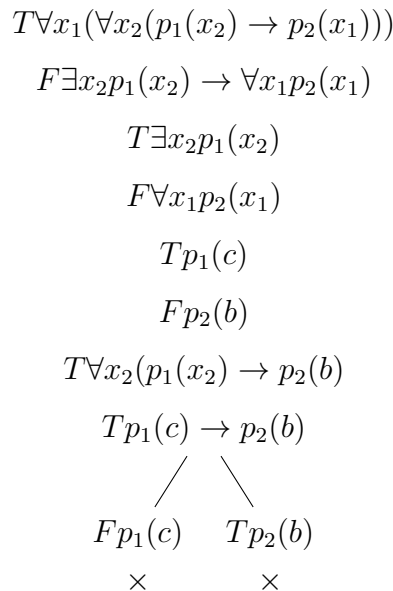
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1.
  - $\Gamma = \{p_1(c) \rightarrow \forall x(p_2(x))\}$
  - $\varphi = \forall x(p_1(c) \rightarrow p_2(x))$



Como todos os ramos do Tableaux fecharam, podemos concluir que  $\Gamma \vdash \varphi$ .

2.
  - $\Gamma = \{\forall x_1(\forall x_2(p_1(x_2) \rightarrow p_2(x_1)))\}$
  - $\varphi = \exists x_2 p_1(x_2) \rightarrow \forall x_1 p_2(x_1)$



Como todos os ramos do Tableaux fecharam, podemos concluir que  $\Gamma \vdash \varphi$ .

3. •  $\Gamma = \{\forall x_1 p(a, x_1, x_1), \forall x_1 (\forall x_2 (\forall x_3 (p(x_1, x_2, x_3) \rightarrow p(f(x_1), x_2, f(x_3)))))\}$
- $\varphi = p(f(a), a, f(a))$

$$\begin{array}{c}
\frac{p(a, a, a) \vdash p(a, a, a)}{p(a, a, a), p(a, a, a) \rightarrow p(f(a), a, f(a)) \vdash p(a, a, a)}^{12} \quad \frac{p(a, a, a) \rightarrow p(f(a), a, f(a)) \vdash p(a, a, a) \rightarrow p(f(a), a, f(a))}{p(a, a, a) \rightarrow p(f(a), a, f(a)), p(a, a, a) \vdash p(a, a, a) \rightarrow p(f(a), a, f(a))}^{12} \\
\hline
\frac{p(a, a, a), p(a, a, a) \rightarrow p(f(a), a, f(a)) \vdash p(f(a), a, f(a))}{p(a, a, a), \forall x_3 (p(a, a, x_3) \rightarrow p(f(a), a, f(x_3))) \vdash p(f(a), a, f(a))}^{14} \\
\hline
\frac{p(a, a, a), \forall x_3 (p(a, a, x_3) \rightarrow p(f(a), a, f(x_3))) \vdash p(f(a), a, f(a))}{p(a, a, a), \forall x_2 (\forall x_3 (p(a, x_2, x_3) \rightarrow p(f(a), x_2, f(x_3)))) \vdash p(f(a), a, f(a))}^{14} \\
\hline
\frac{p(a, a, a), \forall x_2 (\forall x_3 (p(a, x_2, x_3) \rightarrow p(f(a), x_2, f(x_3)))) \vdash p(f(a), a, f(a))}{p(a, a, a), \forall x_1 (\forall x_2 (\forall x_3 (p(x_1, x_2, x_3) \rightarrow p(f(x_1), x_2, f(x_3))))) \vdash p(f(a), a, f(a))}^{14} \\
\hline
\frac{p(a, a, a), \forall x_1 (\forall x_2 (\forall x_3 (p(x_1, x_2, x_3) \rightarrow p(f(x_1), x_2, f(x_3))))) \vdash p(f(a), a, f(a))}{\forall x_1 p(a, x_1, x_1), \forall x_1 (\forall x_2 (\forall x_3 (p(x_1, x_2, x_3) \rightarrow p(f(x_1), x_2, f(x_3))))) \vdash p(f(a), a, f(a))}^{14}
\end{array}$$

Como partimos dos axiomas e apenas usando as regras do Cálculo de Sequentes, conseguimos chegar em  $\Gamma \vdash \varphi$ , podemos concluir o mesmo.

$$\begin{array}{c}
T\forall x_1 p(a, x_1, x_1) \\
T\forall x_1 (\forall x_2 (\forall x_3 (p(x_1, x_2, x_3) \rightarrow p(f(x_1), x_2, f(x_3))))) \\
Fp(f(a), a, f(a)) \\
Tp(a, a, a) \\
T\forall x_2 (\forall x_3 (p(a, x_2, x_3) \rightarrow p(f(a), x_2, f(x_3)))) \\
T\forall x_3 (p(a, a, x_3) \rightarrow p(f(a), a, f(x_3))) \\
Tp(a, a, a) \rightarrow p(f(a), a, f(a)) \\
\swarrow \quad \searrow \\
Fp(a, a, a) \quad Tp(f(a), a, f(a)) \\
\times \quad \times
\end{array}$$

Como todos os ramos do Tableaux fecharam, podemos concluir que  $\Gamma \vdash \varphi$

4. •  $\Gamma = \{\forall x_1((p(x_1) \rightarrow (\forall x_2(p(f(x_1, x_2)) \wedge p(f(x_2, x_1))))) , p(a), p(c)\}$   
 •  $\varphi = p(f(f(a, b), f(c, d)))$

$$\begin{array}{c}
 T\forall x_1(p(x_1) \rightarrow (\forall x_2(p(f(x_1, x_2)) \wedge p(f(x_2, x_1))))) \\
 Tp(a) \\
 Tp(c) \\
 Fp(f(a, b), f(c, d)) \\
 Tp(c_1) \rightarrow (\forall x_2(p(f(c_1, x_2)) \wedge p(f(x_2, c_1)))) \\
 \swarrow \quad \searrow \\
 Fp(c_1) \quad T\forall x_2(p(f(c_1, x_2)) \wedge p(f(x_2, c_1))) \\
 \quad \quad \quad Tp(f(c_1, c_2)) \wedge p(f(c_2, c_1)) \\
 \quad \quad \quad Tp(f(c_1, c_2)) \\
 \quad \quad \quad Tp(f(c_2, c_1))
 \end{array}$$

Independente do que for colocado no lugar de  $c_1$  e  $c_2$ , o Tableaux nunca fechará  $\rightarrow$  Não posso deduzir  $\varphi$  de  $\Gamma$ .

5. •  $\Gamma = \{\forall x_1(\forall x_3(p_1(x_1) \wedge (\exists x_2(p_2(x_1, x_2, x_3) \vee p_2(x_2, x_1, x_3)) \rightarrow p_1(x_3)))) ,$   
 $p_1(a), p_2(a, b, c), p_2(d, e, f), p_2(c, f, g)\}$   
 •  $\varphi = p_1(g)$

$$\begin{array}{c}
 T\forall x_1(\forall x_3(p_1(x_1) \wedge (\exists x_2(p_2(x_1, x_2, x_3) \vee p_2(x_2, x_1, x_3)) \rightarrow p_1(x_3)))) \\
 Tp_1(a) \\
 Tp_2(a, b, c) \\
 Tp_2(d, e, f) \\
 Tp_2(c, f, g) \\
 Fp_1(g) \\
 T\forall x_3(p_1(g) \wedge (\exists x_2(p_2(g, x_2, x_3) \vee p_2(x_2, g, x_3)) \rightarrow p_1(g))) \\
 Tp_1(g) \wedge (\exists x_2(p_2(g, x_2, c_1) \vee p_2(x_2, g, c_1))) \\
 Tp_1(g) \\
 \times
 \end{array}$$

Como todos os ramos do Tableaux fecharam, podemos concluir que  $\Gamma \vdash \varphi$ .