Lista 4 - Álgebra Linear

Exercícios transcritos do livro Gilbert Strang - Álgebra Linear e suas aplicações

Exercício 1. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$. Verify that the trace equals the sum of the eigenvalues, and the determinant equals their product.

Exercício 5. Find the eigenvalues and the eigenvectors of

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

Check that $\lambda_1 + \lambda_2 + \lambda_3$ equals the trace and $\lambda_1\lambda_2\lambda_3$ equals the determinant.

Exercício 8. Show that the determinant equals the product of the eigenvalues by imagining that the characteristic polynomial is factored into

$$det(A - \lambda I) = (\lambda_1 - \lambda)(\lambda_2 - \lambda)\dots(\lambda_n - \lambda) \quad (16),$$

and making a clever choice of λ.

Exercício 9. Show that the trace equals the sum of the eigenvalues, in two steps. First, find the coefficient of $(-\lambda)^{n-1}$ on the right side of equation (16). Next, find all the terms in

$$det(A - \lambda I) = det \begin{bmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{bmatrix}$$

that involve $(-\lambda)^{n-1}$. They all come from the main diagonal! Find that coefficient of $(-\lambda^{n-1})$ and compare.

Exercício 11. ***The eigenvalues of*** A ***equal the eigenvalues of*** A^T. This is because $det(A - \lambda I)$ equals $det(A^T - \lambda I)$. That is true because ____. Show by an example that the eigenvectors of A and A^T are not the same.

Exercício 12. Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix} \qquad A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

Exercício 21. Compute the eigenvalues and eigenvectors of A and A^{-1}:

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} \text{ and } A^{-1} = \begin{bmatrix} -\frac{3}{4} & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$$

A^{-1} has the _____ eigenvectors as A. When A has eigenvalues λ_1 and λ_2, its inverse has eigenvalues ______.

Exercício 39. Challenge problem: *Is there a real 2 by 2 matrix* (other than I) *with* $A^3 = I$? Its eigenvalues must satisfy $\lambda^3 = I$. They can be $e^{2\pi i/3}$ and $e^{-2\pi i/3}$. What trace and determinant would this give? Contruct A.

Exercício 40. There are six 3 by 3 permutation matrices P. What numbers can be the *determinants* of P? What numbers can be *pivots*? What numbers can be the *trace* of P? What *four numbers* can be eigenvalues of P?