# MAC 239 Lista 2: Lógica de Primeira Ordem

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1. 
   - $\Gamma = \{p_1(c) \to \forall x(p_2(x))\}$
   - $\varphi = \forall x(p_1(c) \to p_2(x))$

$$T p_1(c) \to \forall x p_2(x)$$
$$F \forall x(p_1(c) \to p_2(x))$$
$$F p_1(c) \to p_2(b)$$
$$T p_1(c)$$
$$F p_2(b)$$

$$\begin{array}{cc} F p_1(c) & T\forall x p_2(x) \\ \times & T p_2(b) \\ & \times \end{array}$$

Como todos os ramos do Tableaux fecharam, podemos concluir que $\Gamma \vdash \varphi$.

2. 
   - $\Gamma = \{\forall x_1(\forall x_2(p_1(x_2) \to p_2(x_1)))\}$
   - $\varphi = \exists x_2 p_1(x_2) \to \forall x_1 p_2(x_1)$

$$T\forall x_1(\forall x_2(p_1(x_2) \to p_2(x_1)))$$
$$F\exists x_2 p_1(x_2) \to \forall x_1 p_2(x_1)$$
$$T\exists x_2 p_1(x_2)$$
$$F\forall x_1 p_2(x_1)$$
$$T p_1(c)$$
$$F p_2(b)$$
$$T\forall x_2(p_1(x_2) \to p_2(b))$$
$$T p_1(c) \to p_2(b)$$

$$\begin{array}{cc} F p_1(c) & T p_2(b) \\ \times & \times \end{array}$$

Como todos os ramos do Tableaux fecharam, podemos concluir que $\Gamma \vdash \varphi$.

3. - $\Gamma = \{\forall x_1 p(a, x_1, x_1),\ \forall x_1(\forall x_2(\forall x_3(p(x_1, x_2, x_3) \to p(f(x_1), x_2, f(x_3)))))\}$
   - $\varphi = p(f(a), a, f(a))$

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\dfrac{p(a,a,a) \vdash p(a,a,a)}{p(a,a,a), p(a,a,a) \to p(f(a),a,f(a)) \vdash p(a,a,a)}\,12
\quad
\dfrac{p(a,a,a) \to p(f(a),a,f(a)) \vdash p(a,a,a) \to p(f(a),a,f(a))}{p(a,a,a) \to p(f(a),a,f(a)), p(a,a,a) \vdash p(a,a,a) \to p(f(a),a,f(a))}\,12
}{p(a,a,a), p(a,a,a) \to p(f(a),a,f(a)) \vdash p(f(a),a,f(a))}\,8
}{p(a,a,a), \forall x_3(p(a,a,x_3) \to p(f(a),a,f(x_3))) \vdash p(f(a),a,f(a))}\,14
}{p(a,a,a), \forall x_2(\forall x_3(p(a,x_2,x_3) \to p(f(a),x_2,f(x_3)))) \vdash p(f(a),a,f(a))}\,14
}{p(a,a,a), \forall x_1(\forall x_2(\forall x_3(p(x_1,x_2,x_3) \to p(f(x_1),x_2,f(x_3))))) \vdash p(f(a),a,f(a))}\,14
}{\forall x_1 p(a,x_1,x_1), \forall x_1(\forall x_2(\forall x_3(p(x_1,x_2,x_3) \to p(f(x_1),x_2,f(x_3))))) \vdash p(f(a),a,f(a))}\,14
}{}
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Como partimos dos axiomas e apenas usando as regras do Cálculo de Sequentes, conseguimos chegar em $\Gamma \vdash \varphi$, podemos concluir o mesmo.

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\begin{array}{c}
T\forall x_1 p(a, x_1, x_1) \\
T\forall x_1(\forall x_2(\forall x_3(p(x_1, x_2, x_3) \to p(f(x_1), x_2, f(x_3))))) \\
Fp(f(a), a, f(a)) \\
Tp(a, a, a) \\
T\forall x_2(\forall x_3(p(a, x_2, x_3) \to p(f(a), x_2, f(x_3)))) \\
T\forall x_3(p(a, a, x_3) \to p(f(a), a, f(x_3))) \\
Tp(a, a, a) \to p(f(a), a, f(a)) \\
\begin{array}{cc}
/ & \backslash \\
Fp(a, a, a) & Tp(f(a), a, f(a)) \\
\times & \times
\end{array}
\end{array}
$$

Como todos os ramos do Tableaux fecharam, podemos concluir que $\Gamma \vdash \varphi$

4. 
- $\Gamma = \{\forall x_1((p(x_1) \to (\forall x_2(p(f(x_1, x_2)) \land p(f(x_2, x_1))))), p(a), p(c)\}$
- $\varphi = p(f(f(a, b), f(c, d)))$

$$T\forall x_1(p(x_1) \to (\forall x_2(p(f(x_1, x_2)) \land p(f(x_2, x_1)))))$$
$$Tp(a)$$
$$Tp(c)$$
$$Fp(f(a, b), f(c, d))$$
$$Tp(c_1) \to (\forall x_2(p(f(c_1, x_2)) \land p(f(x_2, c_1))))$$
$$\begin{array}{cc} Fp(c_1) & T\forall x_2(p(f(c_1, x_2)) \land p(f(x_2, c_1))) \\ & Tp(f(c_1, c_2)) \land p(f(c_2, c_1)) \\ & Tp(f(c_1, c_2)) \\ & Tp(f(c_2, c_1)) \end{array}$$

Independente do que for colocado no lugar de $c_1$ e $c_2$, o Tableaux nunca fechará $\to$ Não posso deduzir $\varphi$ de $\Gamma$.

5. 
- $\Gamma = \{\forall x_1(\forall x_3(p_1(x_1) \land (\exists x_2(p_2(x_1, x_2, x_3) \lor p_2(x_2, x_1, x_3)) \to p_1(x_3)))),$ $p_1(a), p_2(a, b, c), p_2(d, e, f), p_2(c, f, g)\}$
- $\varphi = p_1(g)$

$$T\forall x_1(\forall x_3(p_1(x_1) \land (\exists x_2(p_2(x_1, x_2, x_3) \lor p_2(x_2, x_1, x_3)) \to p_1(x_1))))$$
$$Tp_1(a)$$
$$Tp_2(a, b, c)$$
$$Tp_2(d, e, f)$$
$$Tp_2(c, f, g)$$
$$Fp_1(g)$$
$$T\forall x_3(p_1(g) \land (\exists x_2(p_2(g, x_2, x_3) \lor p_2(x_2, x_1, x_3)) \to p_1(g)))$$
$$Tp_1(g) \land (\exists x_2(p_2(g, x_2, c_1) \lor p_2(x_2, g, c_1)))$$
$$Tp_1(g)$$
$$\times$$

Como todos os ramos do Tableaux fecharam, podemos concluir que $\Gamma \vdash \varphi$.