Week 2 Assignment

Task 1

$W \subseteq V \iff$ (1) $\vec{0} \in W$

(2) $\vec{x}+\vec{y} \in W$ whenever $\vec{x}, \vec{y} \in W$

V is vector space over a field F

(3) $\alpha \vec{x} \in W$ whenever $\alpha \in F$, $\vec{x} \in W$

①

$W_1 = \{(x, y, z) \in R^3 \mid x = 2y\}$

(1) $(0,0,0) \in W_1$

(2) $\vec{x}+\vec{y} \in W$ whenever $\vec{x}, \vec{y} \in W$

for two vector $\vec{u} = (x_0, y_0, z_0)$, $\vec{v} = (x_1, y_1, z_1) \in W_1$

$\vec{u}$, $\vec{v}$ should following $x = 2y$

$\Rightarrow$ $y_0 = \frac{1}{2}x_0$, $y_1 = \frac{1}{2}x_1$

$\vec{u} + \vec{v} = (x_0 + x_1, y_0 + y_1, z_0 + z_1)$

$\because$ $y_0 + y_1 = \frac{1}{2}x_0 + \frac{1}{2}x_1 = \frac{1}{2}(x_0 + x_1)$

$\therefore$ satisfied close under addition

(3) Let $\vec{x} = (x_0, y_0, z_0) \in W_1$

$\forall \alpha \in F$

$\because$ x_0, y_0 multiply same value α

$\therefore$ satisfied close under multiplication

W_1 is a subspace of R^3 #

Task 1

② $W_2 = \{(x,y,z) \in R^3 \mid y=0\}$

(1) $(0,0,0) \in W_2$

(2) $\because 0+0=0$ $\therefore$ satisfied close under addition

(3) $\forall \alpha \in F$ $\because \alpha \cdot 0 = 0$ $\therefore$ satisfied close under multiplication

W_2 is a subspace of R^3 #

③ $W_3 = \{(x,y,z) \in R^3 \mid x=2y \text{ and } z=2\}$

(1) $\because (0,0,0) \notin W_3$

$\therefore W_3$ is not a subspace of R^3 #

④ $W_4 = \{(x,y,z) \in R^3 \mid x=y^2\}$

(1) $(0,0,0) \in W_4$

(2) Let $\vec{u} = (x_0, y_0, z_0)$, $\vec{v} = (x_1, y_1, z_1) \in W_4$

$\vec{u}+\vec{v} = (x_0+x_1, y_0+y_1, z_0+z_1)$

$x_0 = y_0^2$

$x_1 = y_1^2$

$\vec{u}+\vec{v}$ not following $x=y^2$

$\because \underbrace{x_0+x_1 = y_0^2+y_1^2}_{x} \neq \underbrace{y_0+y_1}_{y}$ in $\vec{u}+\vec{v}$

W_4 is not a subspace of R^3 #

Task 2

$A^T = A \Leftrightarrow$ a symmetric matrix

$\because (A^T)^T = A, \quad (B+C)^T = B^T + C^T$

$(A + A^T)^T = A^T + A$

matrix addition has commutative propety

$\Rightarrow A + A^T = (A + A^T)^T$

$A + A^T$ is a symmetric matrix

Task 3.

(1) $-2x^2 + 3 = a(x^2 + 3x) + b(2x^2 + 4x - 1)$

$= (a + 2b)x^2 + (3a + 4b)x - b$

$\Rightarrow \begin{cases} a + 2b = -2 \\ 3a + 4b = 0 \\ -b = 3 \end{cases} \Rightarrow \begin{cases} a = 4 \\ b = -3 \end{cases} \in R \quad \therefore$ True

(2) $x^2 + 2x - 3 = a(-3x^2 + 2x + 1) + b(2x^2 - x - 1)$

$= (-3a + 2b)x^2 + (2a - b)x + (a - b)$

$\Rightarrow \begin{cases} -3a + 2b = 1 \\ 2a - b = 2 \\ a - b = -3 \end{cases} \Rightarrow \begin{cases} a = 5 \\ b = -8 \end{cases} \quad \therefore$ True

(3) $3x^2 + 4x + 1 = a(x^2 - 2x + 1) + b(-2x^2 - x + 1)$

$= (a - 2)x^2 + (-2a - b)x + (a + b)$

$\Rightarrow \begin{cases} a - 2b = 3 \cdots ① \\ -2a - b = 4 \cdots ② \\ a + b = 1 \cdots ③ \end{cases}$

①, ③ $\Rightarrow \begin{cases} a = \frac{5}{3} \\ b = \frac{2}{3} \end{cases}$ > No solution exists.

②, ③ $\Rightarrow \begin{cases} a = -5 \\ b = 6 \end{cases} \quad \therefore$ False

Task 4

① $(2,-1,1) = s(1,0,2) + t(-1,1,1)$

$(s-t,\ t,\ 2s+t)$

$\Rightarrow \begin{cases} s-t=2 \\ t=-1 \\ 2s+t=1 \end{cases} \Rightarrow \begin{cases} s=1 \\ t=-1 \end{cases} \Rightarrow$ Yes #

②

$(-1,2,1) = s(1,0,2) + t(-1,1,1)$

$= (s-t,\ t,\ 2s+t)$

$\Rightarrow \begin{cases} s-t=-1 \ \cdots ① \\ t=2 \ \cdots ② \\ 2s+t=1 \ \cdots ③ \end{cases}$ ①,② $\Rightarrow \begin{cases} s=1 \\ t=2 \end{cases}$ ②,③ $\Rightarrow \begin{cases} s=\frac{1}{2} \\ t=2 \end{cases}$ > No solution exists

$\Rightarrow$ No #

③ $(-1,1,1,2) = s(1,0,1,-1) + t(0,1,1,1)$

$= (s,\ t,\ s+t,\ -s+t)$

$\Rightarrow \begin{cases} s=-1 \ \cdots ① \\ t=1 \ \cdots ② \\ s+t=1 \ \cdots ③ \\ -s+t=2 \ \cdots ④ \end{cases}$ ①,② to ③ $\Rightarrow -1+1 \neq 1 \Rightarrow$ No solution exists

$\Rightarrow$ No #

Task 5.

Symmetric Matrix $M^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$M^2 = sM_1 + tM_2 + uM_3$

$= \begin{bmatrix} s & u \\ u & t \end{bmatrix}$

$\Rightarrow \begin{cases} a=s \\ b=u \\ c=t \end{cases} \Rightarrow$ Yes, $\{M_1, M_2, M_3\}$ is span of all 2X2 symmetric Matrices #