

Assignment Week 1

Task 1. Two vectors are parallel if there exists a scalar α such that $\vec{x} = \alpha \vec{y}$

①

$(-2, 1, 3)$ and $(4, 6, 1)$

$$\therefore \frac{-2}{4} = \frac{1}{2}; \frac{1}{6}$$

$$\Rightarrow \frac{1}{2} \neq \frac{1}{6}$$

\therefore not parallel #

②

$(1, 2)$ and $(-3, 6)$

$$\therefore \frac{1}{-3}; \frac{2}{6} = \frac{1}{3}$$

$$\Rightarrow \frac{1}{-3} = \frac{1}{3}$$

\therefore Parallel #

③

$(1, -2, 0, 1)$ and $(3, 0, 2, -5)$

$$\therefore \frac{1}{3} \neq \frac{1}{5}$$

\therefore not parallel #

④

$(10, 0, 2, -4, 8)$ and $(5, 0, 1, -2, -4)$

$$\therefore \frac{10}{5} = 2; \frac{0}{1} = 2; \frac{-4}{-2} = 2; \frac{8}{4} = 2$$

\therefore Parallel #

Task 2.

① $(2, 5, -1), (0, 4, 6)$ and $(-3, 7, 1)$

Let $(2, 5, -1)$ as reference point

$$\begin{aligned} \vec{v} &= (0, 4, 6) - (2, 5, -1) \\ &= (-2, 9, 7) \end{aligned} \quad \therefore \frac{-2}{-5} \neq \frac{9}{12}$$

$$\begin{aligned} \vec{w} &= (-3, 7, 1) - (2, 5, -1) \quad \therefore \vec{v}, \vec{w} \text{ not parallel} \\ &= (-5, 12, 2) \end{aligned}$$

$$\vec{n} = \vec{v} \times \vec{w} = \begin{vmatrix} 9 & 7 \\ 12 & 2 \end{vmatrix}, \begin{vmatrix} 7 & -2 \\ 2 & -5 \end{vmatrix}, \begin{vmatrix} -2 & 9 \\ -5 & 12 \end{vmatrix}$$

$$= (18 - 84, -35 + 4, -24 + 45)$$

$$= (-66, -31, 21)$$

$$\vec{z} \Rightarrow -66(x-2) + (-31)(y+5) + 21(z+1) = 0$$

$$\Rightarrow -66x + 132 - 31y - 155 + 21z + 21 = 0$$

$$\Rightarrow -66x - 31y + 21z - 22 = 0 \#$$

② $(1, 2, 1), (2, 4, 2)$ and $(-3, -6, -3)$

let $(1, 2, 1)$ as reference point

$$\vec{v} = (2, 4, 2) - (1, 2, 1)$$

$$= (1, 2, 1)$$

$$\vec{w} = (-3, -6, -3) - (1, 2, 1)$$

$$= (-4, -8, -4)$$

$$\therefore \frac{1}{-4} = \frac{2}{-8} = \frac{1}{4}, \exists k : k\vec{v} = \vec{w}$$

$\therefore \vec{v}, \vec{w}$ are parallel,

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\therefore the three points are collinear,
unique plane not exists.

Task 2

③ $(1, 1, 1), (2, 5, 2)$ and $(0, 0, 0)$

Let $(0, 0, 0)$ as reference point.

$$\vec{v} = (1, 1, 1)$$

$$\vec{w} = (2, 5, 2)$$

$$\vec{n} = (| \begin{matrix} 1 & 1 & 1 \\ 2 & 5 & 2 \end{matrix} |, | \begin{matrix} 1 & 1 & 1 \\ 2 & 5 & 2 \end{matrix} |, | \begin{matrix} 1 & 1 & 1 \\ 2 & 5 & 2 \end{matrix} |)$$

$$= (-3, 0, 3)$$

$\therefore \frac{1}{2} \neq \frac{1}{5} \therefore \vec{v}, \vec{w}$ not parallel

$$\vec{E} \Rightarrow -3x + 3z = 0 \#$$

Task 3

$$\textcircled{1} \quad D_{2 \times 5} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \#$$

② $f \in N \therefore$ coefficient of $f(x) \in N$

$$f(x) = 3.2x^2 + 2x + 0.5$$

$\therefore 3.2 \notin N, 0.5 \notin N$

$\therefore f(x) \notin \text{field } F$

Task 4.

①

false, $\deg(f+g)$ may less than or equal to n

② true $f(x) = 2x^2, g(x) = -2x^2 + x \Rightarrow f(x) + g(x) = x, \deg(f+g) = 1 \neq 2$

③

false, only if $a \neq 0$

$$\text{Let } V \in \mathbb{R}^2, \vec{x} = (1, 0), \vec{y} = (2, 3)$$

$$\text{if } a=0 \text{ then } a\vec{x} = a\vec{y} \Rightarrow 0 \cdot (1, 0) = 0 \cdot (2, 3)$$