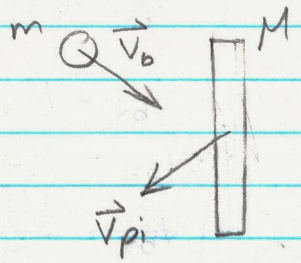
A ball with mass m, moving with velocity $\vec{v}_b$, strikes a paddle with mass M, moving with velocity $\vec{v}_{pi}$. The proportion of the ball's energy lost in the collision is L. What is the outgoing velocity $\vec{v}_o$?



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x-component

$$\begin{cases} m v_{bx} + M v_{pix} = m v_{ox} + M v_{pfx} & [1] \\ (1-L)\left(\frac{1}{2} m v_{bx}^2 + \frac{1}{2} M v_{pix}^2\right) = \frac{1}{2} m v_{ox}^2 + \frac{1}{2} M v_{pfx}^2 & [2] \end{cases}$$

[1] $$\frac{m}{M} v_{bx} + v_{pix} - \frac{m}{M} v_{ox} = v_{pfx}$$

[2] $$(1-L)\left(\frac{m}{M} v_{bx}^2 + v_{pix}^2\right) = \frac{m}{M} v_{ox}^2 + \left(\frac{m}{M} v_{bx} + v_{pix} - \frac{m}{M} v_{ox}\right)^2$$

$$(1-L)\left(\frac{m}{M} v_{bx}^2 + v_{pix}^2\right) = \frac{m}{M} v_{ox}^2 + \left(\frac{m}{M} v_{bx} + v_{pix}\right)^2 - 2\frac{m}{M}\left(\frac{m}{M} v_{bx} + v_{pix}\right) v_{ox} + \frac{m^2}{M^2} v_{ox}^2$$

$$\left(\frac{m}{M} + \frac{m^2}{M^2}\right) v_{ox}^2 - 2\frac{m}{M}\left(\frac{m}{M} v_{bx} + v_{pix}\right) v_{ox} + \left(\frac{m}{M} v_{bx} + v_{pix}\right)^2 - (1-L)\left(\frac{m}{M} v_{bx}^2 + v_{pix}^2\right) = 0$$

$$v_{ox} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $$a = \frac{m}{M}\left(1 + \frac{m}{M}\right)$$

$$b = -2\frac{m}{M}\left(\frac{m}{M} v_{bx} + v_{pix}\right)$$

$$c = \left(\frac{m}{M} v_{bx} + v_{pix}\right)^2 - (1-L)\left(\frac{m}{M} v_{bx}^2 + v_{pix}^2\right)$$

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$$\begin{cases} m v_{bx} + M v_{pix} = m v_{ox} + M v_{pfx} \\ v_{pfx} = (1 - L_x) v_{pix} \end{cases}$$

$$m v_{bx} + M v_{pix} = m v_{ox} + M(1 - L_x) v_{pix}$$

$$m v_{bx} + M L_x v_{pix} = m v_{ox}$$

$$\boxed{v_{bx} + \frac{M L_x}{m} v_{pix} = v_{ox}}$$