Secret Hitler Statistics

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1 p-values

Let X_i be the number of i-player games in which they have a given role (say, Liberal) and let X be the total number of games in which they have that role. That is,

$$X = X_5 + X_6 + X_7 + X_8 + X_9 + X_{10}$$

We want to find the p-value for their observed affiliations. That is, given some observed number of games x and an expected number of games μ with a given role, we want to calculate $p = P(|X - \mu| \geq |x - \mu|)$.

Begin by calculating the probability of each number of games $P(X = x)$. Let n_i be the number of i-player games that the person participated in and let $n = \sum_{i=5}^{10} n_i$. Then

$$\begin{aligned} P(X = x) &= \sum_{x_5=0}^{n_5} \cdots \sum_{x_9=0}^{n_9} P(X_5 = x_5 \cap \cdots \cap X_9 = x_9 \cap X_{10} = x - x_5 - x_6 - \cdots - x_9) \\ &= \sum_{x_5=0}^{n_5} \cdots \sum_{x_9=0}^{n_9} P(X_5 = x_5) \ldots P(X_9 = x_9) P(X_{10} = x - x_5 - x_6 - \cdots - x_9) \end{aligned}$$

where

$$P(X_i = x_i) = \binom{n_i}{x_i} p_i^{x_i} (1 - p_i)^{n_i - x_i}$$

and p_i is the probability of being assigned the role in question in an i-player game. If there are k players with that role, then $p_i = \frac{k}{i}$.

Once all the probabilities have been calculated, the expected value can be calculated.

$$\begin{aligned} \mu &= E[X] \\ &= \sum_{x=0}^{n} x P(X = x) \end{aligned}$$

And the p-value can be calculated using the expected value and the probabilities.

$$\begin{aligned} p &= P(|X - \mu| \geq |x - \mu|) \\ &= P(X \geq \mu + |x - \mu|) + P(X \leq \mu - |x - \mu|) \\ &= \sum_{k=\lceil \mu + |x-\mu| \rceil}^{n} P(X = k) + \sum_{k=0}^{\lfloor \mu - |x-\mu| \rfloor} P(X = k) \end{aligned}$$