CHAPTER 4

Semantic terrain representation

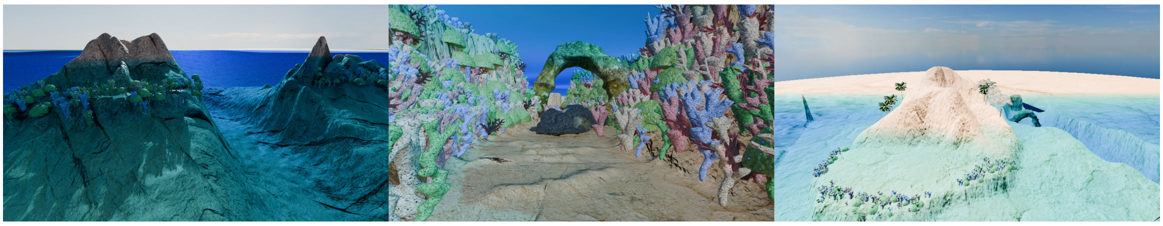

Figure 4.1: Our method can produce different scenes including coral reef islands and canyons at multiple scales using environmental objects to represent terrain features.

Abstract

This chapter introduces a novel method for procedural terrain generation, which leverages a sparse representation of environmental features to produce landscapes that are lightweight, plausible and adaptable to user desires. The method differs from traditional terrain generation approaches by emphasizing multi-scale user interaction and incorporating expert knowledge to model the evolution of terrain features over time. By representing terrain features as discrete entities, or "environmental objects", the method enables dynamic interaction between these entities and their surrounding environment, represented through continuous scalar and vector fields. The generation process is iterative and allows for user-guided modifications at any iteration, including the introduction of environmental events that can influence the terrain's evolution. The proposed approach is particularly flexible, capable of generating both terrestrial and underwater landscapes with a focus on large-scale plausibility and detailed, localized feature representation.

Contents

Back to summary

4.1 Introduction

Topographic maps are very useful tools for biologists, geologists or even oceanologists (which would call them "bathymetric charts"). These maps are displayed in 2D but provide 3D information about the altitude (or depth), but can also use symbology to represent the elements of different scale order need to be visible for interpreting correctly the context of a map. In cartography, map symbols are defined as geometric primitives such as points, polylines, polygons, and (more rarely) polyhedrons. Map symbols are important in order to extract as much information as possible while being an extreme simplification of the content of an environment, or an abstraction of the 3D nature of the terrain features.

This abstraction is useful to understand the relationship between the different features, which may enable to deduce physical rules in the evolution of a terrain through the process of observation. In fields where systemism is out of reach (earth science, biology, urbanism, ...), the deduction of rules through observation plays an important role in the understanding of phenomena and explication of situations. [A REDIRE VITE]

In such way, geologists can study the distribution of peaks in a mountain range, the location of soil types in an area (Figure 4.3), which in turn allow to deduce possible locations of karst networks (Figure 4.4), for example. Using the same abstraction tools, a biologist may interpret the effect of natural or artificial reefs on coastal erosion (Figure 4.2), or understand more clearly the interactions inside an ecosystem. Oceanologists may also deduce, using the same strategy, the formation of canyons and fans from old river systems (Figure 4.5). [A REDIRE]

The use of parametric lines representing coasts of islands or continents is heavily used as it is easy to understand the concept of interaction using curves. It also shows an easy way to present the evolution of this interaction with respect to time, as the continuous change of shape can be interpreted by our mind as a moving shape, providing a sense of velocity of change (Figure 4.2). While real coastlines are fractal, a representation using a continuous line is sufficient to be able to imagine the real scene. A complex representation would be, on the other hand, difficult to read, without any significant improvement on the possibility to understand the reality, harder to draw and edit, and much more subject to change with time and become obsolete.

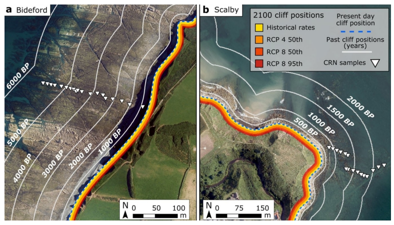

Figure 4.2: Evolution of coastlines at Bideford, UK and Scalby, UK over the last 6000 years and 2000 years respectively (BP = Before Present) (Shadrick et al., 2022)

Any element, like a coastline, then have an organic evolution over time, just like biologic elements as they can grow, shrink, and morph continuously. We see that one of the main differences of the evolution of organic elements, like the borders of a forest, and non-organic elements, like the borders of an island, reside in the time scale.

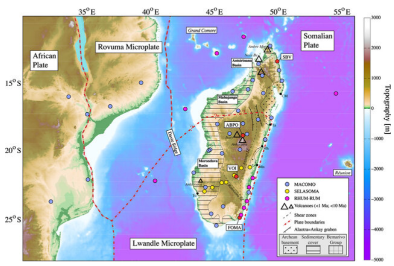

Figure 4.3: Sedimentary distribution over Madagascar island (Pratt et al., 2017)

Symbology can be used to represent at the same time large elements like regions with a specific type of soil, and very small and sparse elements like the peak of volcanos or buildings from Figure 4.3. We can find, on a single map, elements described as 0D (sparse points, like volcanos and buildings), 1D (curves like the plate boundaries and shear zones) and 2D (regions like the sedimentary cover and tectonic plates). The geometric shape used to describe an element may vary depending on the scale at which the topographist wishes to represent it, or depending on the work focus of the map. The observatories are presented as single points (0D) in an island-scale map, but could be found as regions (2D) on city-scale maps. Beaches may be represented as curves, points or regions if the objective is to present their distribution, position, or shapes.

A combination of multiple disciplines can be represented in a single topographic map to provide context about an area. We can find examples of representations with urbanism, geology, hydraulogy, and topography mixed together. Every discipline is linked with all others with relative strength. Urban regions are strongly affected by the geology and topography, while hydraulogy is directly impacted by topography. In this example, the karst geology is influenced by hydraulogy and geology, given that the presence of certain karst features are present in limestone areas with underground river networks. This also means that the presence of karst features can imply the presence (or a high probability of presence) of limestone and underground water flows, without the need to display them on a map.

Without its 3D representation, a karst expert may be able to visualize in 3D from Figure 4.4 a possible ground surface around dolines and lapies without additional information.

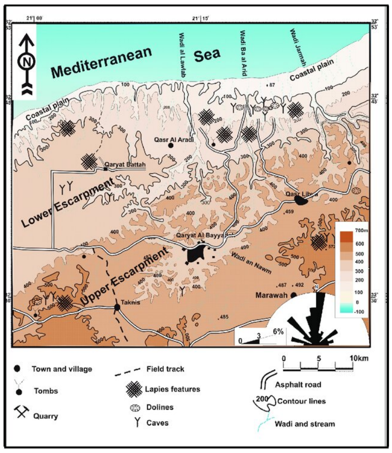

Figure 4.4: Geological features distribution of karstic landscape at Qasr Lybia, Libya (Amawy and Muftah, 2009)

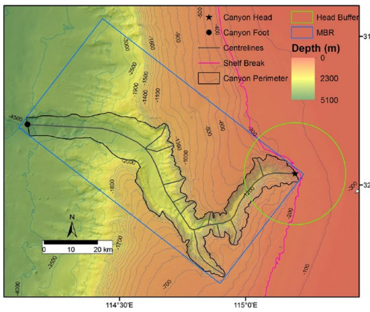

Figure 4.5: Localization of key parameters of Perth Submarine Canyon, Autralia (Huang et al., 2014)

The presence of symbols in a topographic map does not imply the presence of an element in the real world. We can commonly find indicative symbols like the head and foot of a canyon, or the area affected by an element, which have no real geometry. However, the presence of these symbols bring useful information. The head buffer displayed in Figure 4.5 add an explication on the deformation of the shelf break line. Looking at it in the other direction, the deformation on the shelf break with the canyon implies the presence of a canyon head.

Parallelly, terrain artists most of the time sketch the global shape of the terrain they will model beforehand, such that they can check, before the modeling part, that the consistency and plausibility of the terrain will be valid. Looking at a simplified map before starting the modeling step allows the designer to modify the overall shape of the terrain, at a large scale, before the 3D geometry comes into play, generating too much control points or vertices to be able to deal with. [TO CONTINUE]

As seen in the previous examples, by simplifying the surface details in maps or models we can concentrate more effectively on gaining a deep understanding of the underlying processes that shape the terrain. This focused understanding allows us to apply the insights we gain to a wide variety of terrain types. Essentially, this approach enables us to generalize our findings and apply them across diverse geographical landscapes, facilitating broader and more versatile applications of our knowledge.

In the use case of the generation of underwater environments, the elements to consider in a terrain vary greatly in size. Going from a single coral colony to a volcanic island, the use of topologic map enables biologists, geomorphologists, and hydrologists to study and understand the evolution of an area. The procedural construction of such landscape should fit the tools that the fields expert use in order to be coherent.

The question which led to our solution is the multi-scale user interaction: "Is it possible to provide an interaction mean for terrain generation allowing the user to interact with a small structure like a rock in the same manner as with a large structure like a mountain?". In this work, we want the user to be able to have a large scale representation of the terrain in order to generate a landscape that satisfies his needs while keeping the possibility to apply large modifications. In discussion with robotician users, we realise that we want to create a large landscape that can contain interesting configurations, select a smaller region that may have features in a disposition that fits its requirements and then refine again the given region. In the optic of generating a large scale terrain in which we could focus the generation effort in a certain region, we wished to be able to see a coarse representation that can be computed quickly.

Starting from an initial configuration or providing conditions on the desired output terrain, the algorithm we propose will let the different terrain features evolve as a multi-agent system in which the user can apply modifications or new constraints on the state of the environment. The resulting configuration is an environment conform to the constraints given by the user over the distribution of features present in the scene.

We aim for our terrain generation method to be versatile enough to handle both terrestrial and aquatic environments. This dual capability would allow the method to be applicable to landscapes above the water level, such as mountains and valleys, as well as to submerged terrains, such as oceanic canyons and coral landscapes.

Because many geographical terms and computer science terms are deceptive cognates, we will try to find middle ground in the naming of our introduced structures to avoid as much as possible any ambiguity between the research fields.

4.2 State of the art

Our work is highly ispired by the modeling of ecosystems, in which we consider abiological elements on the same level as biological organisms.

4.2.1 Ecosystem simulation

Modeling of ecosystem extremely important in many fields.

Simulations study how a population grows and spread or moves over time.

Two main philosophies can be extracted from the litterature: looking at population of species as continuous spaces, or studying each individual one by one.

From a large scale observation, where billions of individuals are indicernable from one another (like grass sproot, bacteria, whole population, ...), we consider the species as a continuum and borrow diffusion-reaction models from physics and chemistry [CITE TURING PATTERN].

On the other hand, when the number of individual becomes smaller or the density is low enough that we need to study individual one by one, the Individual-Based Models (IBM) become handy. This methods focus on the individuals as agents of a system, usually using sensing and interaction between agents to drive decisions, like spawning, growing and dying, in case of vegetation.

An hybrid method family, the ecological field models (Wu et al., 1985), proposed to describe how each individual modifies the state of its surrounding environment. These models represent the effect of each individual as spatial interferences, borrowing the field theory concept from physics. while itself being affected by the environment for decision making. Environment factors are intrinsicly includable in these models.

(Czaran, 1998)

Grid-based models

The grid-based models represent a density of a population in a discrete space. The interaction of each cell is usually limited to its direct neighbors.

This models can be generalized to what could be called "Tesselation models", or "Site-based neighborhood models" (Czaran, 1998). The grid is a specialization with each region being a unit square sharing adjacency with its direct neighbors. However, the use of Voronoi diagrams (or "Thiessen polygon" in the biological field) and its duality, the Delaunay triangulation, may be used. As such, each vertex in the Delaunay graph is a small area with uniform properties and the influence between two regions is defined using the graphs edges. The use of generalized topological maps furthermore dissociate the geometry to focus on the topological aspect of the ecosystem (Lemiere et al., 2023).

Considering small regions to simulate the evolution of the ecosystem enables the use of cellular automata theory.

Individual-based models

In opposition with the grid-based models, and in response to the increase of computer power, the research about ecosystem simulation leaned towards a new kind of model in which each individual of the system is represented independantly. The Individual-Based Models (IBM) sees each element of the system as an entity that spawn, grow, die, and possibly transport itself depending on its surroundings.

In order to simulate ecosystems with these models, the behaviour of single individuals with respect to the environment properties must be modeled. The main environmental factor being sunlight, the exact computation of shading for estimating the rate of growth of trees was expensive without the use of parallel computing and the use of GPU [CITE SOMETHING WITH GUERIN], and an approximation accepted is to use "distance-based models".

In the Fixed-Radius Neighborhood (FRN) method, the first appearance of a distance-based method, each tree has a radius inside which we consider that there is shade and competition of nutrients. It two trees' radii intersect, a competition let the strongest tree live while the weakest die.

As an improvement over FRN, the Zone of Influence (ZOI) method has been proposed to add more nuance in the previous method. Here, the radius around each tree represent a competition to resources. As such, the fraction of the radius contained in the intersection with another tree is used to define how the tree grows or even die, displaying a much smoother transition than the binary nature of FRN.

Introducing kernel functions instead of binary radii, representing a distance weight around each tree allows two radii to overlap without having to remove an entity, and even consider a different kernel for different pairs of species (see: a tree-tree kernel is highly different than a bush-tree kernel). Another advantage of the introduction of a kernel is to simulate multiple factors at once, like the importance of roots, the shade provide, the drainage of water, etc... (Berger and Hildenbrandt, 2000) This Field of Neighborhood (FON) method is a generalisation of the Fixed-Radius Neighborhood method since the later is simply the use of binary kernels for each tree.

Ecological Field models

The idea of using fields around each individual is not born from the Field of Neighborhood method. A third ideology, focused on the modeling of the effects caused by each individual on the ecological properties instead of the individual by themselves, rose from the mathematical field theory (Wu et al., 1985).

In this method, each individual vegetal element of the scene has a set of influence functions. The water, nutrients and light availability are influenced around each individual by kernel functions for the stem, the crown and the roots of each species. The terrain can then be seen as multiple continuous fields to represent each resource.

This type of implicit representation has recently resurfaced with the apparition of GPUs and high performance CPUs (see Signed Distance Fields in 3D modeling).

An analogy of Ecological Fields can be found in Grosbellet et al., *Environmental Objects for Authoring Procedural Scenes* (2016) . In this work, some objects in the 3D scene can influence the environmental properties such as shade, heat, humidity, or the possible presence of small elements like icicles, leaves, etc... In response to these changes, the geometry of the objects are affected by adding leaves, snow, icicles, grass, melted snow, and more, on them.

Our method, using an identical name, tend to generalize this idea of Environmental Objects by merging the idea of Ecological Fields.

4.2.2 Diffusion models

Reaction-diffusion models are fundamental tools in mathematical biology and ecology, providing insights into the complex patterns observed in natural ecosystems. These models combine chemical or biological reactions (reaction) with movement through space (diffusion) to describe processes such as population dynamics, infectious disease spread, and ecological invasions.

Initially developed to explain phenomena in chemistry and physics, reaction-diffusion models have been adapted to ecological contexts to simulate scenarios where organisms interact with each other and their environment over space and time. The application of these models in ecology helps in understanding how patterns of distribution and abundance arise, how they persist, and how they might change under various environmental pressures.

The concept of reaction-diffusion was initially introduced in the early 20th century by mathematician Alan Turing, whose 1952 paper, "The Chemical Basis of Morphogenesis," explored how non-uniform patterns emerge through mechanisms of chemical reaction and diffusion. Turing's theories laid the groundwork for understanding pattern formation in biological systems, including those seen in animal coats and developmental processes.

The adaptation of reaction-diffusion models to ecological contexts began in earnest several decades later as ecologists sought to explain spatial phenomena such as the spread of invasive species, the maintenance of species diversity, and the formation of vegetation patterns. These models provided a mathematical framework to simulate and predict the dynamics of complex biological systems influenced by both local interactions and spatial movement.

Over the years, the application of reaction-diffusion models has expanded with advancements in computational power and mathematical techniques, allowing for more detailed and accurate simulations of ecological processes. This historical progression not only reflects the increasing complexity and applicability of the models but also underscores their integral role in modern ecological research and management.

Ecological representation of spreading species uses diffusion models
Diffusion reaction with CA = Turing patterns
Diffusion with grid based
Diffusion with IBM
...

Mathematical model

Reaction-diffusion models describe the spatial and temporal evolution of populations, chemicals, or other ecological entities that interact (reaction) and spread (diffusion) through a medium. These models are widely used in ecosystem simulations to understand population dynamics, species dispersal, disease spread, and spatial pattern formation in ecological systems.

In the context of ecology, a reaction-diffusion model typically represents:

- Reaction: Local changes in population density due to birth, death, predation, or competition.
- Diffusion: Movement of individuals or species across space due to random dispersal or environmental factors.

These models help ecologists predict how populations evolve over time and across space, providing insights into species survival, biodiversity, and ecosystem stability.

The classical reaction-diffusion equation is expressed as:

$$\frac{\partial u(x,t)}{\partial t} = D\nabla^2 u(x,t) + R(u) \tag{4.1}$$

where $u(x,t)$ represents the population density (or concentration of a substance) at position x and at time t, D is the diffusion coefficient, determining the rate of spatial spread. $\nabla^2 u$ is the Laplacian operator, representing diffusion across space and $R(u)$ is the reaction term, which models growth, decay, or interactions between species.

Diffusion term

Diffusion is a fundamental process in ecological modeling that describes how individuals or substances spread across space due to random movement or environmental influences. In ecosystems, diffusion can model species dispersal, nutrient transport, disease spread, and invasive species expansion.

In mathematical models, diffusion is typically represented as a term in partial differential equations that governs how population densities change over time and space. While many ecological models assume isotropic diffusion (movement occurs equally in all directions), anisotropic diffusion provides a more realistic representation of species movement in structured or heterogeneous landscapes.

However, when diffusion is anisotropic, the diffusion coefficient varies depending on direction, and we must generalize the equation (Equation (4.1)) by introducing a diffusion tensor $\mathbf{D}$:

$$\frac{\partial u(x,t)}{\partial t} = \nabla \cdot (\mathbf{D}\nabla u(x,t)) + R(u) \tag{4.2}$$

The tensor $\mathbf{D}$ can now encompass the effect of relief and environmental factors like wind and currents in the spread of a species.

When diffusion is isotropic, $\mathbf{D} = D\mathbf{I}$, reducing the equation to the classic isotropic form.

Anisotropic diffusion is particularly useful in ecological modeling where movement rates are not uniform due to environmental structures or organism behavior. For example, wind influences the spread of pollen, seeds, and insect populations, leading to anisotropic diffusion. Similarly, marine organisms or pollutants diffuse anisotropically in ocean currents. Invasive species may spread preferentially along corridors such as rivers or roads, causing asymmetric diffusion. In ecosystems where microorganisms move in response to chemical gradients (chemotaxis), anisotropic diffusion better models movement patterns.

We may generalize the reaction-diffusion equation to account for both internal (density, species interaction, ...) and external (environmental factor, relief, seasonality, ...) influences on diffusion by writing:

$$\frac{\partial u}{\partial t} = \nabla \cdot (\mathbf{D}(*) \nabla u) + R(u) \tag{4.3}$$

where $\mathbf{D}(*)$ is a diffusion tensor dynamically defined by any relevant internal or external property of the system. For example:

A density-dependent diffusion can be expressed as $\mathbf{D}(*) = \mathbf{D}u^m$, allowing for crowding or dispersal-limited movement.

A seasonal dispersal effect can be incorporated as $\mathbf{D}(*) = \mathbf{D}(1 + Acos(\omega t))$, where dispersal varies cyclically over time.

A spatially varying diffusion may be defined as $\mathbf{D}(*) = \mathbf{D}f(x)$, such as the patchy diffusion formulation used in metapopulation models:

$$\mathbf{D}(*) = \mathbf{D} \begin{cases} D_1, & x \in \text{habitable patch} \\ D_2, & x \in \text{hostile patch} \end{cases} \tag{4.4}$$

*A*n anomalous diffusion (subdiffusive or superdiffusive) like the inclusion of Lévy flight term, incorporating random long distance jumps can be expressed as $\mathbf{D}(*) = \mathbf{D}u^m + \eta L(t)$.
This generalized form enables the integration of anisotropic effects, landscape heterogeneity, and adaptive movement behaviors within reaction-diffusion models, making it highly applicable to ecological and biological systems.

Reaction term
In physics and chemistry, the reaction $R(u)$ term describes how substances are produced or consumed in a system. Some common reaction formulations include:

*L*inear reaction terms (exponential growth with $\lambda > 0$ or decay $\lambda < 0$) as in the radioactive decay formulation:

$$R(u) = \lambda u$$

*S*econd-order reactions, usually found in chemical reactions between two molecules:

$$R(u, v) = \alpha u v$$

*T*uring pattern-forming systems proposed by Alan Turing to explain stripe and spot formations in animal coat patterns and chemical reactions

$$R(u, v) = \begin{cases} f(u, v) & \text{(Activator)} \\ g(u, v) & \text{(Inhibitor)} \end{cases}$$

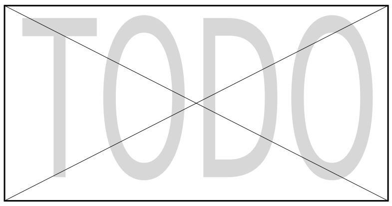

Figure 4.6: Turing patterns on animals

On the other hand, in ecology, the reaction term models population growth, competition, predation, and ecosystem interactions. Some of the most important ecological reaction terms are:

The logistic growth, that describes a single species population growth in a limited-resource environment, is defined with a growth rate r and a carrying capacity K:

$$R(u) = ru\left(1 - \frac{u}{K}\right)$$

The Lotka-Volterra competition model, that describe the trophic interactions between two or more species, for which the reaction-diffusion equations take into account each other species:

$$\frac{\partial u_i}{\partial t} = r_i u_i \left(1 - \frac{\sum_{j=1}^{N} a_{ij} u_j}{K_i}\right) \tag{4.5}$$

with N representing the number of species and a_{ij} the interaction coefficient (how strongly the species u_j inhibit the species u_i).
This formulation can also be rewritten in a vectorized form as the generalized Lotka-Volterra equation:

$$\mathbf{f} = \mathbf{r} - \mathbf{A}\mathbf{u} \tag{4.6}$$

where $\mathbf{f} = (f_1, f_2, ..., f_N)^\intercal$ represents the per capita growth rates of all species, $\mathbf{r} = (r_1, r_2, ..., r_N)^\intercal$ is the intrinsic growth rate vector, $\mathbf{u} = (u_1, u_2, ..., u_N)^\intercal$ is the species population vector and $\mathbf{A}$ is the interaction matrix, with elements a_{ij} describing the effect of species j on species i.

Since population growth depends on the per capita growth rate, the full population dynamics equation follows:

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{u} \circ \mathbf{f} \tag{4.7}$$

where $\circ$ represents the element-wise product, ensuring that each species' growth depends on both its population size and its per capita growth rate.

Convection

In vegetation ecosystem modeling, convection refers to the directed transport of biological or environmental variables under the influence of external forces such as wind, water flow, or slope. Unlike diffusion, which describes random movement, convection introduces a preferred direction of transport, significantly influencing ecosystem dynamics.

Mathematically, the convection-diffusion-reaction equation extends the classical reaction-diffusion model by adding a velocity field $\mathbf{v}$ that governs directed movement:

$$\frac{\partial u}{\partial t} + \mathbf{v} \cdot \nabla u = \nabla \cdot (\mathbf{D} \nabla u) + R(u) \tag{4.8}$$

Wind is a major dispersal mechanism for many plant species, affecting seed distribution, plant competition, and biodiversity. The dispersal efficiency of seeds and pollen depends on wind velocity, turbulence, and interactions with terrain features. In flat landscapes, wind disperses seeds relatively uniformly; however, in complex terrains, wind speed and direction are influenced by topography, vegetation cover, and atmospheric conditions. Wind channeled through valleys may enhance seed deposition in lowland areas, while mountain ridges and abrupt elevation changes can create wind shadows, reducing dispersal effectiveness.

The dispersal of seeds can be modeled using a convection-diffusion equation:

$$\frac{\partial S}{\partial t} + \mathbf{v} \cdot \nabla S = \nabla \cdot (\mathbf{D}_s \nabla S) - \lambda S \tag{4.9}$$

with $S(x,t)$ the seed density at position x and time t and λ a mortality rate due to environment constraints or predation.

A stochastic modelling of turbulent regime could be added to the velocity field with a stochastic noise term:

$$\frac{\partial S}{\partial t} + (\mathbf{v} + \sigma W(x,t)) \cdot \nabla S = \nabla \cdot (\mathbf{D}_s \nabla S) - \lambda S \tag{4.10}$$

On another hand, soil moisture distribution is essential for vegetation survival and ecosystem stability. The movement of water through soil is governed by surface runoff, subsurface infiltration, and capillary action, which are affected by slope, soil texture, and plant uptake rates. In hilly or mountainous regions, gravity-driven water flow dominates soil moisture distribution, creating hydrological gradients that shape vegetation structure.

The movement of soil moisture can be described by a modified form of the Richards equation:

$$\frac{\partial W}{\partial t} + \mathbf{v} \cdot \nabla W = \nabla \cdot (D_W \nabla W) - \beta B W \tag{4.11}$$

with $W(x,t)$ the water concentration at position x and time t, D_W the water diffusion coefficient, $\beta B W$ the plant water uptake (the amount of water absorbed or transpired, mainly by the roots of vegetation), and $\mathbf{v} = -g\nabla h$ a simplified velocity field provided by the slope.

Steeper slopes enhance runoff, reducing soil water retention and increasing drought susceptibility in these regions. Conversely, water accumulates in lowland areas, promoting denser vegetation and the formation of wetlands or riparian ecosystems. As a result, topography directly influences plant distribution, with drought-resistant species dominating slopes and water-demanding species thriving in valleys.

Fire is a key ecological disturbance in many vegetation ecosystems, influencing biomass turnover, species composition, and landscape structure. The spread of fire is influenced by wind velocity, fuel availability, and topography. Convection plays a crucial role, as rising hot air preheats vegetation ahead of the fire front, accelerating spread in the upslope direction.

Fire spread can be modeled as:

$$\frac{\partial F}{\partial t} + \mathbf{v} \cdot \nabla F = \nabla \cdot (\mathbf{D}_F \nabla F) + R(F,B) \tag{4.12}$$

with $F(x,t)$ the fire intensity, $\mathbf{v}$ the wind velocity field, $\mathbf{D}_F$ the diffusion coefficent and $R(F,B)$ the fuel consumption rate, which is dependant on biomass.

4.2.3 Computation of reaction-diffusion

The reaction-diffusion equations in complex landscapes cannot be solved analytically and require numerical methods for solving them. The

Advection (grid) model

Particle model

...

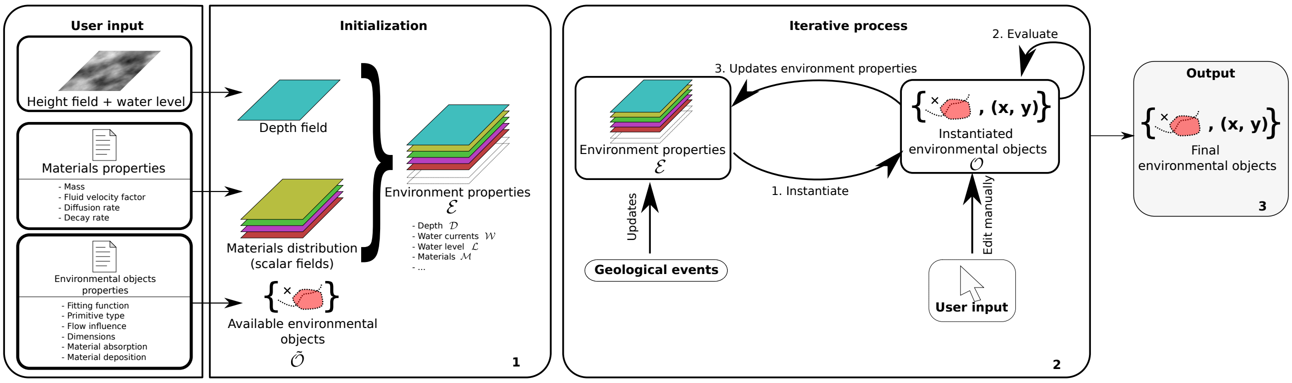

Figure 4.7: Overview of the pipeline of the method. The user provides as input an initial height field and sets the water level, as well as a definition of the environmental materials properties and environmental objects properties that will be used in the iterative process. These inputs are initialized as an initial set of environmental objects and scalar fields that represents the environmental attributes. In the iterative loop, new environmental objects are instantiated using the current state of the environment at their optimal position. The existing environmental objects in the terrain reevaluate their fitness function to grow or die and update the environmental attributes locally. At each iteration, geomorphic events can update the environmental attributes, while the user can interact directly with the environmental objects. The result of the whole process is a set of environmental objects which is a sparse representation of the features of the scene.

Decay

...

4.3 Description of the method

In this section, we present the pipeline and processes that underpin our method. Our approach introduces the concept of environmental objects, simplified terrain features that interact with their surroundings to simulate complex ecosystem dynamics. These environmental objects, which can represent natural features such as trees, rivers, or rocks, influence and are influenced by scalar fields like temperature, humidity, and elevation. The method is structured into several key phases: initialization, where the foundational elements of the terrain and environmental objects are set up; an iterative generation process, where these environmental objects are instantiated and interact with their environment; and finally, the production of a sparse representation of the scene's features. This section details each of these phases, explaining how the system dynamically adapts to user input and environmental changes.

4.3.1 Pipeline

Initialization

The generation of the terrain is initialized using an initial height field h and an initial water level $\mathcal{L}$. During this chapter we will include many features depending on altitude or depth, so we will use the shorthand notations $\mathcal{H} = h - \mathcal{L}$ and $\mathcal{D} = -\mathcal{H}$. The height field provides variation on the altitude, which can influence the generation process of the scene.

The list of available environmental objects $\widetilde{\mathcal{O}}$, representing the different features that can be present in the scene, are provided with their properties: type, size, generation rules and effects on the environmental attributes (Section 4.3.2). A target list of environmental objects $\hat{\mathcal{O}}$ can be defined to control the final result of the generation.

Finally, different environmental materials are defined with their properties such as diffusion speed, mass, decay rate and influence from the water currents. An initial environment configuration resulting from the initial height field $\mathcal{H}$ and water level $\mathcal{L}$, the environmental materials distribution $\mathcal{M}$ (represented as scalar fields $\mathcal{M} : \mathbb{R}^2 \mapsto \mathbb{R}$) and water currents $\mathcal{W}$ (as a vector field $\mathcal{W} : \mathbb{R}^2 \mapsto \mathbb{R}^2$) will be used by the environmental objects of the scene to simulate their growth and spawn at the most probable position. The environmental attributes are noted $\mathcal{E} = (\mathcal{D}, \mathcal{W}, \mathcal{L}, \mathcal{M})$ (Section 4.3.3).

The definition of environmental objects' properties and environmental attributes is done with field experts, providing the pertinent parameters required to model the evolution of the terrain features using expert knowledge (**??**).

The generation phase starts with an initial set of environmental objects present in the scene, which can optionally be pre-filled.

Generation process

Once the initialization phase is done, the generation begins. The generation process is incremental and its main loop is composed of two different steps: the instantiation of new environmental objects then the update of the environment. This loop is repeated until the user is satisfied with the look of his environment or following rules like a number of features threshold or a targeted list of environmental objects as described in the layout planner defined in Tutenel et al., *Rule-based layout solving and its application to procedural interior generation* (2009) .

Instantiation

At each iteration, new environmental objects can be created at their most fitting locations if possible. The generation rules provided in the initialization phase are used to find an optimal position from stochastic sampling (Section 4.4). All environmental objects are evaluating their state analytically using a fitness function and a skeleton fitting function provided as input (Section 4.4).

Environment update

Once the instantiation step is done, the environmental attributes are updated by each environmental object through environmental modifiers, which depose and absorb some of the environmental materials $\mathcal{M}$ (Section 4.5) while modifying the water currents $\mathcal{W}$ (Section 4.5.3) and the height field $\mathcal{H}$ around them. Finally, water currents and terrain slope displace environmental materials of the terrain until reaching a dynamic equilibrium in the environment at each iteration.

During the generation process, the user can alter directly the distribution and shapes of the environmental objects (Section 4.6.1) and perturb the generation process by planning geomorphic events that have impacts on the environmental attributes (Section 4.6.2).

Output

The output of our system is a set of environmental objects disposed in the plane. We do not provide the 3D representation of the environmental objects in this chapter, letting the user define the rendering method. The figures used in the chapter use a mix of implicit surfaces and triangular meshes.

4.3.2 Environmental objects

A geographical feature, also called object or entity, is defined as a discrete phenomenon located at or near the Earth's surface, relevant in geography and geographic information science (GIScience). It represents geographic information that can be depicted in maps, geographic information systems (GIS), and other forms of geographic media. This term includes both natural and human-made objects, ranging from tangible items like buildings or trees to intangible concepts like neighborhoods or savana. Features are distinct entities with defined boundaries, differentiating them from continuous geographic masses or processes occurring over time. They can be categorized as natural features, such as ecosystems, biomes, water bodies, and landforms, or artificial features, such as settlements, administrative regions, and engineered constructs. Geographic features are described by characteristics including identity, existence, classification, relationships with other features, location, attributes, and temporal aspects. Information about these features is stored in geographic databases using models like GIS datasets, which organize and represent these features in structured formats. As the term "feature" is overused in computer science, we will use the term environmental object in this work.

Each environmental object is shaped with a simple geometric shape called a "skeleton" that defines where it is located and how it fits into the environment. The skeleton can either be, as used in cartography, a point, a curve or a region. We will then refer to our environmental objects as point-based, curve-based or region-based, respectively. These environmental objects interact with the environment $\mathcal{E}$ by changing local conditions using the environmental modifiers. For example, the presence of a river might increase the moisture in the surrounding area, while a mountain might induce more rockiness in the soil composition. They can also absorb changes from the environment, such as a forest taking in humidity from the air. The placement of environmental object is determined by a fitness function ω, which evaluates how suitable a location is based on the environmental attributes. Once a suitable location is found, the skeleton fitting function optimizes the shape and position of the entity to fit as best as possible into the environment Γ.

4.3.3 Environmental attributes

In geography, a "field" refers to a continuous spatial phenomenon across a region where each point has a specific value of a variable, unlike discrete objects with distinct boundaries. Fields can be scalar, representing a single value at every point (like temperature or elevation), or vector, representing quantities with magnitude and direction (like wind velocity). Examples include topographic fields for elevation, climatic fields for temperature or precipitation, and magnetic fields for magnetic forces. Because of the ambiguous nature of the term "field" with mathematics and computer science, we will define the geographic fields as environmental attribute.

In an ecosystem simulation, each actor of the ecosystem has an impact on all other actors, which results in an exponentially growing computation effort as the number of elements of the terrain increase. We avoid this problem by considering the environmental attributes as a proxy to allow any environmental object to interact with any other one. Each of the environmental object have a local impact on the environmental attributes without knowledge of neighboring environmental objects. This modification of the environmental attributes are presented as the effect of environmental modifiers defined for each environmental object.

In this work, we have integrated vector environmental attributes (e.g., water currents $\mathcal{W}$) and scalar environmental attributes (e.g., altitude $\mathcal{H}$, water level $\mathcal{L}$, and various material properties $\mathcal{M}$) under the unified term "environment," denoted as $\mathcal{E} = (\mathcal{H}, \mathcal{W}, \mathcal{L}, \mathcal{M})$. The environmental materials represent abstract quantities such as the availability of sand, salt, moisture, or rocks at each point. It is important to emphasize that these materials are not to be visualized as physical layers stacked on the terrain surface. Instead, they should be understood as conceptual resource distributions that influence the environment and the behavior of environmental objects, rather than as something directly observable in the scene.

Environmental modifiers $\mathcal{E}^+$

The environment determine if a environmental object does belong at a certain position. When a environmental object is placed, its surrounding environmental attributes can be affected though environmental modifiers noted $\mathcal{E}^+ = (\mathcal{H}^+, \mathcal{W}^+, \emptyset, \mathcal{M}^+)$ defining a change of height $\mathcal{H}^+$, changes in the water currents $\mathcal{W}^+$ and environmental material alteration $\mathcal{M}^+$.

Environmental objects are subject to altitude conditions. However, to maintain a clear distinction between semantic modeling and 3D modeling, we do not compute the exact physical shape or detailed height field of each environmental object. Instead, we define a coarse height function, a parametric representation derived from the skeleton to provide a rough estimate of the changes in elevation around it. This simplified model of how the environmental object influences the surrounding terrain's altitude allows us to cheaply evaluate the potential presence of new entities that have altitude-dependent conditions without the need to perform expensive computations to generate a detailed height map of the entire terrain.

Altering the vector field of the water currents $\mathcal{W}$ is done by the composition of the effect $\mathcal{W}^+$ of each object at a position $\mathbf{p}$ as introduced in Wejchert and Haumann, *Animation aerodynamics* (1991) , while we use the formulation of Kelvinlets (Goes and James, 2017) in the computation of effect of each environmental object.

Each environmental object has intrinsic environmental materials that can be seen as "spreading" and "absorbed" around its skeleton over time. A coral reef may produce coral polyps and at the same time reduce the water currents. It grows thanks to the deposition of limestone from coral colonies. In our model, the colonies affect the environmental attributes through the deposition of environmental material $\mathcal{M}^+_{\text{limestone}}$, which in turn, is absorbed by the coral reef, without a direct exchange between the two environmental objects.

The alteration of a scalar environmental attribute is done by adding or removing some amount around the skeleton of the environmental object and diffusing it in the space, influenced by the water currents. We consider the system to be steady state, garantied by the introduction of a decay rate $k > 0$ in the computation of the diffusion and advection.

4.4 Placement of environmental objects in an environment

At each iteration of our algorithm, we want our environmental objects to be at plausible positions. We do not guaranty a temporal continuity between iterations as in Ecormier-Nocca et al., *Authoring consistent landscapes with flora and fauna* (2021) , so the objective is to add new environmental object in order to satisfy the users wishes, while conserving the plausibility of the scene. Rather than "forming" these environmental objects, our method "reveals" them, much like a paleontologist uncovers fossils during an excavation. A paleontologist does not dig randomly across the Earth to find fossils; instead, they analyze the geological context to identify the most likely locations. Similarly, our method observes the environment and estimates where certain elements are likely to exist. For example, in hydrology, if a river appears to originate from nowhere, it might suggest the presence of a karstic river system upstream. In urban planning, if many roads converge at a certain point, it is reasonable to expect that a city is located there. This approach ensures that Semantic Terrain Entities are revealed in positions that are contextually appropriate and coherent within the landscape.

We will follow the same intuition using a fitness function for each of the environmental object that may be spawn in the terrain. The fitness function defined $\omega : \mathcal{E} \mapsto \mathbb{R}$ provides a score indicating how well the environmental object may fit in this position. Evaluating this function at multiple position results in an approximation of the fitness map of the entity. Once the most probable position is found, we can find the most plausible shape of the environmental object using the skeleton fitting function Γ.

For this task, we place a new elements required at the most plausible position using the analysis of the fitness function of each environmental object. We know that each environmental object will modify the environment surrounding, which may make previously instantiated environmental objects unfitted. Knowing this,the goal is to add the new element at the position that will change the least the stability of the system.

Genetic algorithms or Depth First Search algorithms could be used to try many possibilities until a local or global minimum could be found, but this would require a large processing power. Naive genetic algorithms would place a environmental object at a certain position at each iteration and evaluate the stability of the environment, repreating this operation while varying slightly the position of the environmental objects or the type of environmental object instantiated at each iteration, resulting in way too much computation to stay interactive. The Depth First Seach algorithms would require to compute all the possible combinations of environmental objects and positions which, given the fact that we want a continuous position in order to work multi-scale, would require to compute an incredibly high amount of possible configurations in order to find a plausible situation, on average. We will work with an evolutionary algorithm to find a compromise between fast computation and a satisfying result.

Our placing algorithm is done in two steps: first, it identifies the global location where a environmental object best fits within the environment $\mathcal{E}$ using its fitness function ω, which evaluates the suitability of each point $\mathbf{p}$ based on the environmental attributes $\mathcal{E}_p$, including factors such as altitude $\mathcal{H}(\mathbf{p})$, water current velocity and direction $\mathcal{W}(\mathbf{p})$, water level $\mathcal{L}(\mathbf{p})$, and the availability of environmental material $\mathcal{M}(\mathbf{p})$. Second, once the most suitable location is identified, the algorithm determines the most plausible shape of the environmental object using the skeleton fitting function Γ.

To ease the reading the functions $\omega(\mathcal{E}(\mathbf{p}))$ and $\Gamma(\mathcal{E}(\mathbf{p}))$ with $\mathcal{E}(\mathbf{p})$ the environmental attributes at the point $\mathbf{p}$ of the terrain are simplified to $\omega(\mathbf{p})$ and $\Gamma(\mathbf{p})$ respectively.

4.4.1 Fitness function

"Darwinian fitness" refers to an organism's ability to survive and reproduce in its environment. It is a measure of how well-suited an organism is to its surroundings, and those with higher fitness are more likely to pass on their genes to the next generation. In a similar vein, the fitness function of a environmental object evaluates its suitability at a location in the terrain, extending the meaning to living features like forests and corals and non-living elements such as rivers, mountains, karsts, etc.

The fitness function is constructed by evaluating several environmental variables at a given location $\mathbf{p}$. These include altitude ($\mathcal{H}(\mathbf{p})$) and its gradient ($\nabla\mathcal{H}(\mathbf{p})$), the availability and gradient of various materials ($\mathcal{M}_i(\mathbf{p})$ and $\nabla\mathcal{M}_i(\mathbf{p})$), water current characteristics ($\mathcal{W}(\mathbf{p})$), and water level ($\mathcal{L}(\mathbf{p})$). Altitude and its gradient can influence the placement of objects like rivers, which may prefer lower elevations, or forests, which might thrive on slopes. Each material, such as limestone, sand, or clay, is considered separately, and their availability and gradients at a location are crucial for determining the suitability of different environmental objects, such as a coral reef requiring specific substrates. The velocity and direction of water currents are essential for placing aquatic features or determining where erosion might occur, while the proximity to water influences the likelihood of placing wetlands, lakes, or other hydrological features.

Different types of environmental objects require different criteria for their fitness evaluation. For example, a river might prioritize lower altitude and proximity to water currents, while a forest might prioritize higher altitude and specific material availability. The flexibility of the fitness function allows it to be customized for each environmental object, ensuring that the generated terrain remains coherent and realistic.

4.4.2 Skeleton fitting function

The seed point of a spawning environmental object is defined at the point $\mathbf{p}$ satisfying $\arg\max_{\mathbf{p}} \omega(\mathbf{p})$.

Point-based skeleton

The spawning position of a punctual environmental object is found at the local maxima of the skeleton fitting function from the seed point. While the skeleton fitting function doesn't have to be identical to the fitness function, we usually use $\Gamma = \omega$ for point-based environmental objects. If the two functions are different, the optimisation process simply follows the field's gradient $\nabla\Gamma$ until the local maxima is reached.

Curve-based skeleton

The skeleton of a curve-based environmental object is determined by the shape that fits the most given the environment it is added to. Using a modified version the Active Contours algorithm (Kass et al., 1988), we can minimize the energy E for the parametric curve C given $E(C) = E_{\text{internal}} + E_{\text{external}} + E_{\text{shape}} + E_{\text{gradient}}$ with

$$E_{\text{internal}} = \alpha_{\text{i}} \left(\int_C \left\| C'(s) \right\|^2 ds + \int_C \left\| C''(s) \right\|^2 ds \right) \tag{4.13}$$

$$E_{\text{external}} = \alpha_{\text{e}} \int_C \Gamma(C(s))\, ds \tag{4.14}$$

$$E_{\text{shape}} = \alpha_{\text{s}} \left(L - \int_C ds \right)^2 \tag{4.15}$$

$$E_{\text{gradient}} = \alpha_{\text{g}} \int_C \langle C'(s), \Delta\Gamma(C(s)) \rangle\, ds \tag{4.16}$$

In this configuration, E_{internal} induce a smooth continuity of the curve by reducing the spacing of each point while reducing the curvature. Another energy, E_{external} integrate the skeleton fitting function over the curve, often seen as an attractor of the points, that tries to descent the gradient to find local minima. At the same time, E_{shape} apply constraints on the curve shape, which, in this case, is to target a specific length L. As such, the curve search for an optimized shape given constraints in E_{shape} for minimizing E_{external}. We introduced a new term E_{gradient} in the energy computation that push the points of the curve in the direction of the slope of the skeleton fitting function, also providing an orientation for the curve.

If a steep coast can be found where the terrain slope is important near the water level, we can define $\Gamma = |\mathcal{H} + 1| \;/\; \|\Delta\mathcal{H}\|$, but no orientation is needed, thus we set $\alpha_g = 0$. In this case, the curve will follow a path at the water level and spread its extremities over areas with a steep slope. A river may also be symbolized as a parametric curve, but we need to add information about the direction and magnitude of the slope. As such, we can use $\Gamma = \mathcal{H}$ which forces the direction of the curve to fit with the terrain slope. The introduction of the gradient component provides also an orientation to shapes.

Internal energy

The internal energy, already introduced in Kass et al., *Snakes: Active contour models* (1988) is composed of two components imposing penalities on the local properties of the points of the curve. The first derivative forces the points along the curve to be evenly spaced by minimizing the curve tension, while the second derivative restrict it from forming sharp corners. As our aim is to represent natural elements, we rarely find sharp elements and thus, keep the original definition from the Snake formulation.

External energy

The external energy is also present in the original work. Using an external scalar field, each point of the curve is forced to follow the steepest slope of the field. The external field can be seen as an attractor for the curve.

Shape energy

We introduced the shape energy, an energy defined on the whole curve to apply constraints on its final shape. As many natural features have a given dimension, we may add a constraint on the lenght of the skeleton.

Gradient energy

In our application, having information about the orientation of environmental objects may be essential. For this purpose, we introduced a gradient energy component in the formulation. The equation impose that the direction of the curve at any point should be directed towards the gradient of the scalar field. As the external energy pushes points toward the lowest point of the scalar field, the gradient field restrict the gradient descent for the global curve into a specific way, which may feel more natural.

During the optimization process, the gradient of the scalar field is already evaluated by the external energy optimization, so the addition of the gradient component is almost free.

Region-based skeleton

Region-based environmental objects follows the same process than curve-based environmental objects to define their skeleton, at the exception of the gradient energy E_{gradient} which have null value on closed shapes.

The resulting energy E to minimize for a closed region whose borders are defined by the curve C can then be expressed as $E(C) = E_{\text{internal}} + E_{\text{external}} + E_{\text{shape}}$.

The internal energy is expressed identically as for curve-based environmental objects. The extenal energy however, has to be modified to take into account the interior of the region instead of only the borders. We will use the idea of the Chan-Vese algorithm, differentiating the energy value for the inside Ω and the borders C of the region (Chan and Vese, 2001; Getreuer, 2012).

By adding a factor for the inside λ_1 and a factor for the borders λ_2, we can add weight depending on the required optimization. We can then define the external energy component E_{external} as:

$$E_{\text{external}} = \lambda_1 \int_\Omega \Gamma(s)\, ds + \lambda_2 \int_C \Gamma(C(s))\, ds \tag{4.17}$$

We see that the definition of the energy formulation of the curve-based environmental objects is a specialization of the region-based, with $\lambda_1 = 0$ and $\lambda_2 = \alpha_e$

The use of external forces on the skeleton can be useful as some landscape features are easier to define by what they contain, while other by what they separate. For example, a lagoon is formed by coral reefs surrounding it, while a forest is formed by the climatic conditions inside it that are propice for trees. A lagoon may then be defined with $\lambda_1 = 0$ and $\Gamma_{\text{lagoon}} = -\mathcal{M}_{\text{coral limestone}}$ while a forest sets $\lambda_2 = 0$ and $\Gamma_{\text{forest}} = -\mathcal{M}_{\text{humidity}} + \mathcal{M}_{\text{shade}} + |\mathcal{M}_{temperature} - 10|^2$.

Finally, the shape constraint energy E_{shape} can target an area A, for example.

$$E_{\text{shape}} = \left(A - \int_\Omega ds\right)^2 \tag{4.18}$$

In our implementation, each environmental object's skeleton is a connected component as we define the boundaries by the connected curve C, but can be convex or concave. An infinite penality is added for the curve self-intersection.

At each iteration, environmental objects are interrogated to verify if they are still fitted to belong at their position. We first check that the fitness is above a given threshold $\omega > T_\omega$. If not, we remove the environmental object from the scene. On the other case, we can improve the shape of the skeleton by minimizing again their skeleton fitting function. As the number of environmental objects become important in the scene, the reajustment time for the skeletons becomes important. For environmental objects that have been present for multiple iterations, we consider less iterations and an increased convergence threshold up to a point where the features are static.

4.5 Environmental modifiers

Each environmental object once present in the scene has an impact on the environment $\mathcal{E}$ through their modifiers $\mathcal{E}^+$. We list three types of influence, which are the environmental material modifiers $\mathcal{M}^+$, the height modifiers $\mathcal{H}^+$ and the water modifiers $\mathcal{W}^+$. These modifiers are direct impact and can be computed as

$$\mathcal{E}^* = \mathcal{E} + \sum_{o \in \mathcal{O}} \mathcal{E}_o^+$$

4.5.1 Environmental material modifiers

The environment is composed of a scalar field for each of the possible material that can be found in the terrain. The scalar fields represents the availability of the material at any point, but not a height field. Each material is defined with a mass m, a fluid velocity factor ν, a diffusion rate D and finally a decay rate k.

Each environmental object in the terrain is a source and a sink of environmental materials. It is the main mean of communication between environmental objects as it allows them to interact with their surrounding environment. We define the amount of deposed material with $D_{\mathcal{M}}$ and $A_{\mathcal{M}}$ the amount of material deposed and absorbed by the environmental object and $\gamma(t) \in [0, 1]$ a factor related with the current state of the environmental object, which state that more material will be displaced when the environmental object is fully formed than when it was just spawn:

$$\int_0^t \gamma(t) \left(D_{\mathcal{M}} - A_{\mathcal{M}}\right) \, dt$$

The deposition and absorption around an environmental object is defined using the Gaussian kernel distance computation from the skeleton.

The scalar field for the material $\mathcal{M}$ is displaced by using a warp operator Φ, taking into account the water flow $\mathcal{W}$ and the terrain slope $\nabla\mathcal{H}$. We unified the warp with m the mass of the material and ν a influence factor of the fluid on the material:

$$\Phi(\mathbf{p}, t) = m\nabla\mathcal{H}(\mathbf{p}, t) + \nu\mathcal{W}(\mathbf{p}, t)$$

The environmental materials are also dispersed at a diffusion rate D, for which we can use the advection-diffusion-reaction equation to evaluate the distribution after a time t

$$\frac{\partial\mathcal{M}}{\partial t}\Phi\nabla\mathcal{M} = D\nabla^2\mathcal{M} - k\mathcal{M} \tag{4.19}$$

We solve Equation (4.19) numerically using Euler integration

$$\begin{aligned} \mathcal{M}(\mathbf{p}, t + dt) = \mathcal{M}(\mathbf{p}, t) &+ dt(D\nabla^2\mathcal{M}(\mathbf{p}, t) - k\mathcal{M}(\mathbf{p}, t) \\ &- \Phi(\mathbf{p}, t)\nabla\mathcal{M}(\mathbf{p}, t)) \end{aligned} \tag{4.20}$$

The introduction of the decay rate $k \in]0; 1]$ in the equation allows for the reach of a steady-state, where we can consider the simulation stable. As the user updates the state of the simulation manually, we observe the reach of this steady state before continuing the iterative steps.

4.5.2 Height modifiers

The computation of the height $\mathcal{H}(\mathbf{p})$ is done using the coarse height function of each environmental object affecting a point $\mathbf{p}$. We can then simplify the shape of a mountain to a cone, a reef as a curved cylinder or a coral boulder as a sphere. As we consider surface height and not surface volume, we use the top-view height field projection, which ignore overhangs and cavities.

The coarse height function is an implicit height field that is used for estimating the shape of the environmental object while providing a hint of the surface normal at each point. Using implicit surfaces allows us to evaluate the altitude and the slope at any point analytically, without requiring the computation of the whole field. The analytical solution induce the multi-scale aspect of the method.

We divide the coarse height functions in three categories:

- $\mathcal{G}$: environmental objects whose shape is defined from an absolute zero of the terrain,

- $\mathcal{A}$: environmental objects whose shape is defined using a notion of altitude, typical of coral-related elements as the depth from the water surface is prevalent,
- $\mathcal{F}$: environmental objects that are defined at the terrain surface. They represent objects that can be seen as bumps or carves from an aerial view.

We compute the depth change given for each category:

$$\mathcal{H}^+_{\mathcal{G}}(\mathbf{p}) = \max_{o \in \mathcal{G}} \mathcal{H}^+_o(\mathbf{p})$$
$$\mathcal{H}^+_{\mathcal{A}}(\mathbf{p}) = \min_{o \in \mathcal{A}} \mathcal{H}^+_o(\mathbf{p})$$
$$\mathcal{H}^+_{\mathcal{F}}(\mathbf{p}) = \sum_{o \in \mathcal{F}} \mathcal{H}^+_o(\mathbf{p})$$

The final altitude is computed as

$$\mathcal{H}^+ = \max(\mathcal{H}^+_{\mathcal{G}}, \mathcal{H}^+_{\mathcal{A}}) + \mathcal{H}^+_{\mathcal{F}} \tag{4.21}$$

4.5.3 Influence on water currents

We define our water currents as a vector field defined as

$$\mathcal{W}(\mathbf{p}) = \mathcal{W}_{\text{user}}(\mathbf{p}) + \mathcal{W}_{\text{simulation}}(\mathbf{p}) + \mathcal{W}^+_{\text{objects}}(\mathbf{p})$$

With $\mathcal{W}_{\text{user}}$ a user-defined vector field, $\mathcal{W}_{\text{simulation}}$ an analytical solution directly inspired by a wind flow simulation (Paris, Peytavie, Guérin, Argudo, and Galin, 2019), and $\mathcal{W}^+_{\text{objects}}$ the water flow alteration computed from the environmental objects. The component $\mathcal{W}_{\text{simulation}}$ is influenced by terrain surface level. Starting with an initial flow direction a, the vector field is adjusted by applying a warp influenced by the terrain gradient at various scales:

$$\mathcal{W}_{\text{simulation}}(\mathbf{p}) = \sum_{i=0}^{i=n} c_i \Phi_i \cdot v$$

Here, v is calculated as $v = a\,(1 + k_w\,|\mathcal{H}(\mathbf{p})|)$, where $|\mathcal{H}(\mathbf{p})|$ represents the distance to water level at point $\mathbf{p}$, and k_w is a scaling factor that accounts for Venturi effects. The term $\Phi_i \cdot v$ denotes the warping process at scale i, with c_i as the associated coefficient, defined as follows:

$$\Phi_i \cdot v = (1-\alpha)v + \alpha k_i \nabla\widetilde{\mathcal{H}_i}^{\perp}(\mathbf{p}) \qquad \alpha = \left\| \nabla\widetilde{\mathcal{H}_i}(\mathbf{p}) \right\|$$

In this formulation, k_i serves as the deviation coefficient, α represents the gradient of the smoothed terrain, and $\nabla\widetilde{\mathcal{H}_i}^{\perp}(\mathbf{p})$ is the vector orthogonal to the smoothed terrain. Consistent with the original paper, two scaling levels were employed ($n = 2$) using Gaussian kernels with radii of 200 m and 50 m. The weights were set to 0.8 and 0.2, with corresponding deviation coefficients $k_0 = 30$ and $k_1 = 5$.

$\mathcal{W}^+_{\text{objects}}$ is a deformation field defined as the accumulation of flow primitives (Wejchert and Haumann, 1991). Kelvinlets are applied on each environmental objects to deflect the water flow. We use the scale and grab formulations of the regularized Kelvinlets brushes (Goes and James, 2017), denoted as $s_\varepsilon(r)$ and $g_\varepsilon(r)$ respectively to simulate obstruction and diversion, are defined as

$$s_\varepsilon(r) = (2b - a)\left(\frac{1}{r_\varepsilon^3} + \frac{1}{2r_\varepsilon^5}\right)(sr)$$
$$g_\varepsilon(r) = \left[\frac{a-b)}{r_\varepsilon} I + \frac{b}{r_\varepsilon^3} rr^t + \frac{a\varepsilon^2}{2r_\varepsilon^3}\mathbf{I}\right]\mathbf{F}$$

with $a = \frac{1}{4\pi\mu}$ and $b = \frac{a}{4(1-v)}$ provided μ a shear modulus and v a Poisson ratio provided for each Kelvinlet, $r = \mathbf{p} - \mathbf{q}$ for $\mathbf{p}$ the evaluation position and $\mathbf{q}$ the center point of the Kelvinlet, $r_\varepsilon = \sqrt{\|r\|^2 + \varepsilon^2}$ the regularized distance, ε a radial scale for the deformation field, s a scaling factor and $\mathbf{F}$ the force vector of the grab operation. Deformations defined on curves use $\mathbf{q} = \mathbf{p}^*_C$ with $\mathbf{p}^*_C$ the closest point on the curve from the point $\mathbf{p}$ and $f = C'(\mathbf{p})$. We can then define $u_o(\mathbf{p}) = s_\varepsilon(\mathbf{q} - \mathbf{p}) + g_\varepsilon(\mathbf{p} - \mathbf{q})$. Finally, we can retrieve the velocity field from the objects:

$$\mathcal{W}^+_{\text{objects}}(\mathbf{p}) = \sum_{o \in \mathcal{O}} \lambda_o u_o(\mathbf{p})$$

4.5.4 Environment stability

Environmental objects and environmental materials are inspired by the ecological concept of biogeocoenosis, which describes the relationship between living organisms (biotic) and their non-living environment (abiotic). These interactions closely mirror the processes observed in natural ecosystems, where biotic and abiotic components continuously influence each other, leading to a balanced and stable environment. Gubanov and Degermendzhy [CITATION] describe biogeocoenosis as almost-closed systems, with many parallels possible with thermodynamics such as the concept of dynamic equilibrium. In a biogeocoenosis, the various processes, such as the spread of nutrients, the growth of vegetation, or the erosion of soil, tend toward a state of balance.

Similarly, in our method, the interaction between environmental objects and environmental materials is modeled using a reaction-diffusion-advection framework. This model captures how environmental materials are spread and absorbed within the terrain. Thanks to the decay rate of the environmental materials, the system progresses toward a dynamic equilibrium, where the distribution of environmental materials stabilizes. This stabilization reflects a state of balance where the rates of spreading and absorption and decay are equal, and the environmental attributes become coherent across the landscape. For example, a river might spread sediments downstream, which are gradually absorbed by the surrounding terrain, leading to a stable sediment distribution over time. Similarly, moisture emitted by a forest will eventually reach a balance with the surrounding soil and atmosphere, creating a stable, humid environment.

The only factor that can disrupt this equilibrium is a change in the environmental attributes. Changes such as an increase in temperature, a shift in wind patterns, or the introduction of a new environmental object can alter the balance, leading to a new phase of interaction and stabilization.

4.6 User interactions

The user can guide the generation process. The use of simple shapes as environmental objects facilitate the edition of the simulation, as we can interactively add, remove or modify environmental objects, or focus the generation process in a restricted area. Interaction with the environmental attributes is also provided as geomorphic events, that the user can invoke during the simulation. While the direct interactions on the environmental objects are instantaneous, as a the geomorphic events are active on a given duration.

4.6.1 Direct interactions with the environmental objects

The interactive nature of our simulation enables the user to modify the state of the terrain by manipulating directly the environmental objects of the scene. We assume the modifications applied between two iterations of the simulation.

Translating an environmental object is trivial, we simply requires to evaluate the state of the environmental objects at a translated position. The deformation of environmental objects can be applied on curve and region environmental objects by updating the control points of the skeleton and recomputing the resulting implicit surfaces. The evaluation positions used for region environmental objects are displaced by applying a cage deformation of the 2D shape using the Green coordinates of points in the shape. After the alteration of the region, evaluation points should be keeping a similar distribution than before, avoiding unexpected results during the interaction. By modifying an environmental object, the environmental attributes may change, which can result in the destruction of the now incompatible environment objects in the scene (Figure 4.8).

Figure 4.8: Starting from a coral colony developed around a canyon (*left*), the user edits the shape of the canyon, resulting in a different configuration of the scene, killing the corals that ends too deep in the water (*center*) and the development and growth of new corals at the previous location of the canyon (*right*).

As long as a non-zero fitness function is defined in the terrain, new environmental objects can be forced by the user at any point of the simulation.

Control over the region of the terrain that should be updated can be given by adjusting all fitness functions through a scalar field $\lambda : \mathbb{R}^2 \mapsto \mathbb{R}$ such that the fitness function $\omega(\mathbf{p})$ of any new environmental object is evaluated as $\omega^*(\mathbf{p}) = \lambda\mathbf{p}\omega(\mathbf{p})$. This is especially useful in the planning of robotic simulations as we can first generate the overall shape of our terrain and secondly focus the generation process around the areas that may be visited by the robot, avoiding useless simulations and computer power. Figure 4.12 shows an example of colonization of the coral polyps that we limited manually into an annulus.

Our water current simulation is modeled as a simple vector field. As such, the user is able to interact with it at any moment of the simulation, allowing for the death of sensible environmental objects while it will guide the simulation into a new landscape. By modifying the water currents, the user also modifies the transport rate of environmental materials at this position. The modification of currents is given as a stroke, a parametric curve C for which we evaluate $\Delta\mathcal{W}_{\text{user}}(\mathbf{p})$ just as for curved environment objects (Section 4.5.3).

4.6.2 Indirect interaction with environmental objects

A configuration file can define in advance the different geomorphic events that should be triggered during the simulation. This can be useful to generate landscapes that are close to some existing locations. Multiple geomorphic events can be triggered either as sudden or continuous environmental changes. These changes play a huge role in the morphology of landscapes. We define geomorphic events with a starting point and an ending point, such that at any time of the simulation we can compute the progress of the geomorphic event as $t_e \in [0,1]$.

Water level changes are important geomorphic events that shape the underwater landscapes. As previously submerged environmental objects get elevated above water level, flora and fauna terrain features dry and die. Deprived from the living part of the features, everything is more affected by terrestrial erosion. By updating the value of the depth $\mathcal{D}$ evaluated in the fitness functions, any environmental object that is sensible to the depth will be impacted automatically, that may be causing death (Figure 4.9). The modification of the water level is defined as

$$\mathcal{D}(\mathbf{p}) = \mathcal{D}_0(\mathbf{p}) + \sum_{e \in \text{events}} \Delta\mathcal{D}_e t_e$$

with $\Delta\mathcal{D}_e$ the amount of water rising or lowering during an geomorphic event. We assumed a linear evolution of the water level during an geomorphic event. This allows to evaluate the depth at any point in space and in time.

Figure 4.9: Lowering the water level by a few meters caused most of the coral objects to satisfy $\omega \leq 0$, causing their death. Since the water level (blue) decrease slowly, new coral objects spawn progressively at a lower altitude.

Subsidence and uplift are the main geomorphic events that create or destroy islands in the long term. These geomorphic events are simulated as a simple factor on the height field of the generated terrain (Figure 4.10). Subsidence is not always uniform in the terrain. As such, the user can provide a position $\mathbf{q}$ at which the subsidence is the strongest, the amount of subsidence applied $\Delta\mathcal{H}_e$ and a standard deviation σ for which we can then compute at any point in space and time of the simulation the height of the terrain

$$\mathcal{H}(\mathbf{p}) = \mathcal{H}_0(\mathbf{p}) \cdot \sum_{e \in \text{events}} G(\|\mathbf{p} - \mathbf{q}\|)\Delta\mathcal{H}_e t_e$$

with $G(x)$ the Gaussian function

$$G(x) = \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

Figure 4.10: Simulating subsidence on a part of the terrain (brown area) cause the depth value to change locally, resulting in the death of coral objects that find themselves too deep to survive. Here two subsidence geomorphic events are triggered in parallel.

Storms are factors of the geomorphology of coral reefs (Vila-Concejo and Kench, 2016; Oron et al., 2023) and coasts (Domínguez et al., 2005; Cowart et al., 2010). Due to the extreme wind and wave velocities coasts are highly eroded in a short time period and the more fragile corals near the water surface are broken, possibly causing breaches in the reefs and spreading polyps in the currents direction. While there are many factors at play to understand the apparition of storms and the hydrodynamics affecting it, we simplified the model of storms to the user as a single epicenter $\mathbf{q}$ with a wind velocity v_{wind} and a standard deviation σ representing the spread around the epicenter (Figure 4.11). The computation of water currents are then computed as

$$\mathcal{W}_{\text{user}}(\mathbf{p}) = \mathcal{W}^*_{\text{user}}(\mathbf{p}) + \sum_{e \in \text{events}} v_{\text{wind}} \frac{G(\|\mathbf{p} - \mathbf{q}\|}{G(0)}$$

In this case, we did not include the linear factor t_e as storms are usually conserving a constant force for the time of the few weeks or months of their occurrence.

with ΔT_e the change of heat during an geomorphic event, T_0 the temperature at the water surface, and c a very small factor.

The framework can easily be extended as the geomorphic event system stays similar for all geomorphic events. Including higher level simulations in the geomorphic event system can be added, such as the simulation of tectonic activity, the use of fluid dynamics for tsunami geomorphic events, the integration of human activity, ...

4.7 Results and discussion

Our method provides a way to generate scenes at different scales. We demonstrate this capacity with the generation of a large scene of an island (Figure 4.1) after what we focused the generation process in a canyon (Figure 4.13), then a small-scale visualization of coral colonies (Figure 4.12). In the examples, we rendered the environmental objects as a implicit tree or as individual meshes. The island, lagoons, reefs, canyons and sand ripples as implicit surfaces

A canyon scene can be generated using our method. The water flow is affected by the curve of the canyon such that the currents are oriented in the direction of the curve's tangent.In this example, we force the position of arches to be inside the canyon. The arches

Figure 4.11: The result of a storm localized on one side of the island (red area) modifies the result of the evaluation of environmental objects around its epicenter for a short period of time. Most of the coral objects died from the geomorphic event, except few environmental objects less sensible to water currents strength.

deposits a material "rock deposit", which is the main element of the fitness function of the Rock object. The "rock deposit" is slightly affected by water currents, but its mass make it highly affected by gravity. As such, rocks will spawn underneath arches. In reality, an arch is often created as part of a large coral boulder that sees the calcareous bottom part detached by the water currents, often resulting in an arch surrounded by big rocks and smaller rocks from the erosion of the first rocks. As such, we define an environmental object "Arch" with a fitness function $\omega_{arch}(\mathbf{p}) = 5 - d(canyon - \mathbf{p}) * \|\mathcal{W}(\mathbf{p})\|$, an environmental object "Rock" using $\omega_{rock}(\mathbf{p}) = \mathcal{M}_{rock\_deposit}(\mathbf{p})$ and Pebble using $\omega_{pebble}(\mathbf{p}) = \mathcal{M}_{smaller\_rock\_deposit}(\mathbf{p})$. Finally, sand ripples are simply described as curves appearing where there is a lot of sand available: $\omega_{ripple}(\mathbf{p}) = \mathcal{M}_{sand}(\mathbf{p})$. Following these simple rules, Figure 4.13 shows the emergence of details in the scene.

In this example we defined three different types of corals, coralA, coralB and coralC, to illustrate the possibility to model behaviours from the choice of fitness functions. Each of the coral types deposits a material "coral polyp" and "coral polyp A" ("coral polyp B" and "coral polyp C" respectively). By considering a fitness function that minimize the ratio $\frac{\text{coral polyp}}{\text{coral polyp A}}$, we can see an emergent behavior of the three types of coral fighting for the space colonization. Figure 4.12 shows the result of this simulation at three different interations. At the border between two colonies, none of the colonies make progression due to the amount of coral polyp specific from the other colony.

The proposed method aims to generate plausible landscapes using simplified versions of the evolution of an ecosystem and of the 3D representation. The biological realism of the result is highly correlated to the amount of simplification and assumptions, while the visual realism is completely dependent to the geometric functions used for the 3D modeling of the environmental objects. While proposing a flexible method that propose a generic approach for terrain generation, a close collaboration with fields experts and with graphists is needed to achieve optimal results.

Most simulation algorithm's quality depends on the size of the time step used, but with

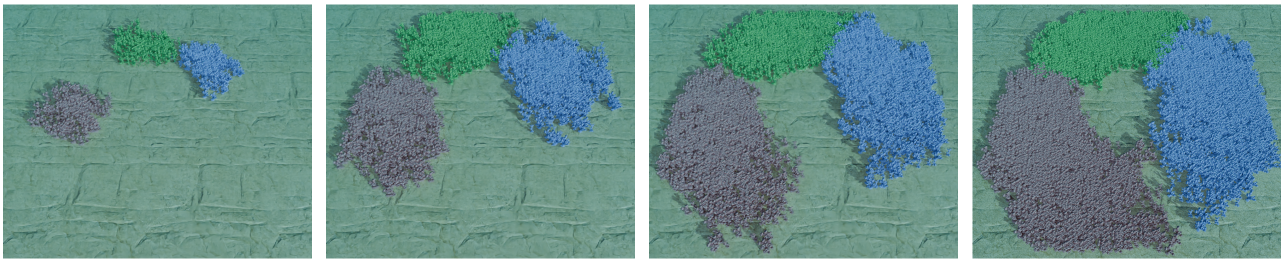

Figure 4.12: Three colonies of coral (red, blue, green) restricted to an annulus the middle section of the terrain fighting for the space.

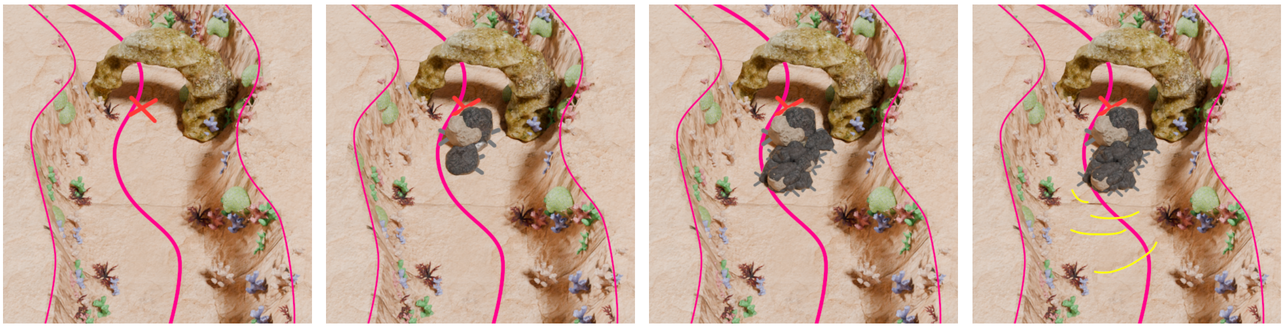

Figure 4.13: Evolution of a canyon scene at different iterations of the simulation. The apparition of an arch causes the spawning of rocks, pebbles, and finally some deposition of sand at the bottom of the canyon, spawning ripples.

the introduction of a decay rate in the environmental materials properties, we limit the influence of time steps by considering that steady-state are reachable. The material deposition and absorption on punctual environmental objects can be seen as a Dirac function δ centered at their position resulting in the advantage that material displacement function can use the definition of the diffusion equation instead of the advection-diffusion-reaction equation. This equation allowing us to evaluate the state of the material $\mathcal{M}$ without intermediate steps, but this is not applicable with curve- and region-based environmental objects.

4.8 Conclusion

We have proposed a method to generate terrains procedurally using sparse representations. This representation, the environmental objects, enables to introduce expert knowledge by the mean of the fitness functions that rule the environmental objects life cycle, but also to integrate the user in the loop during the generation process. We reduced the terrain resolution limitations by defining the environment objects as parametric features. Thanks to the sparse representation based on single points, curves and regions, we allow for direct manipulation of the environmental objects of the scene by the user which, thanks to the environment steady state consideration, also enables to include these interactions in the automatic simulation process. Integrating environmental properties in the fitness function of environmental objects allows the user to guide the generation through geomorphic events. Our method enables each environmental object of the scene to influence the environment locally, reducing the need of computations while also retrieving environmental attributes locally, which result in a parallelizable life-like simulation process. The genericity of the environment properties definitions should be sufficient for plausible generation of other landscape types as long as expert knowledge can be translated to environmental object's formalism.

Figure 4.14: A simple coral reef island is generated using an island, a lagoon, reefs coral polyps, beaches, trees and algae environmental objects. Trees appear on beaches and algae grow in the lagoon's sand.

We limited our work to the use of 2D scalar fields as they are more easily differentiable, interpretable and lighter than volumetric representations. However, future works include using 3D representations of the terrain and the environment to generate 3D terrains, including cavities, sub-terrestrial areas and the interior of coral structures.