SE167/SE267A Signal Processing & Spectral Analysis

Homework 1 (2022-01-4) **Due date: 2022-01-11**

Problem 1:

Consider a signal (as shown below)

$$x(t) = cos(2\pi f_1 t + \varphi_1) + 1.5cos(2\pi f_2 t + \varphi_2) + 0.7cos(2\pi f_3 t + \varphi_3)$$

Where $f_1 = 2.0\ Hz\ , \varphi_1 = 0.;\ \ f_2 = 2.1\ Hz\ , \varphi_2 = {}^{\pi}/_{6};\ and\ f_3 = 3.5\ Hz\ ,\ \varphi_3 = {}^{\pi}/_{4}$

a. Consider sampling rate as 100 Hz (100 points per sec), and generate 1024 points (Signal duration is 10.24 sec). Calculate the Fourier amplitude spectrum (only plot from 0.0 Hz to 6.0 Hz) and identify the dominant frequencies of the signal from Fourier amplitude spectrum.

b. Same sampling rate (100 Hz) and with duration of 10.24 sec. but add zero at the end of the data (add 1024 points of zero). Now the signal will have duration of 20.48 Hz.
Calculate the Fourier amplitude spectrum (only plot from 0.0 Hz to 6.0 Hz) and identify the dominant frequencies of the signal from Fourier amplitude spectrum.

c. Consider sampling rate as 10 Hz (10 points per sec). [0.1 sec/point <(1/(2 x 3.5)]. The duration of the signal is set 12.8 sec (A total of 128 points will be used for analysis). Calculate the Fourier amplitude spectrum (only plot from 0.0 Hz to 6.0 Hz) and identify the dominant frequencies of the signal from Fourier amplitude spectrum.

d. Same condition as (c), but with zero padding of 128 points at the end of the record (total data point is 256 points). The duration of the data is 25.6 sec.
Calculate the Fourier amplitude spectrum (only plot from 0.0 Hz to 6.0 Hz) and identify the dominant frequencies of the signal from Fourier amplitude spectrum.

[Use MATLAB code on FFT: y=fft(x,n)]

From these analyses, discuss the sampling rate, signal duration and effect of zero padding on the estimation of Fourier amplitude spectrum.

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SE 267A - HW1

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Problem 1.

Consider a signal made of 3 cosine functions.

$$x(t) = \cos(2\pi f_1 t + \phi_1) + 1.5\cos(2\pi f_2 t + \phi_2) + 0.7\cos(2\pi f_3 t + \phi_3)$$

where the frequencies of each cosine are

$$f_1 = 2.0\text{Hz},\ f_2 = 2.1\text{Hz}, f_3 = 3.5\text{Hz}$$

and the phases are

$$\phi_1 = 0.0\ ,\ \phi_2 = \frac{\pi}{6}\ ,\ \phi_3 = \frac{\pi}{4}$$

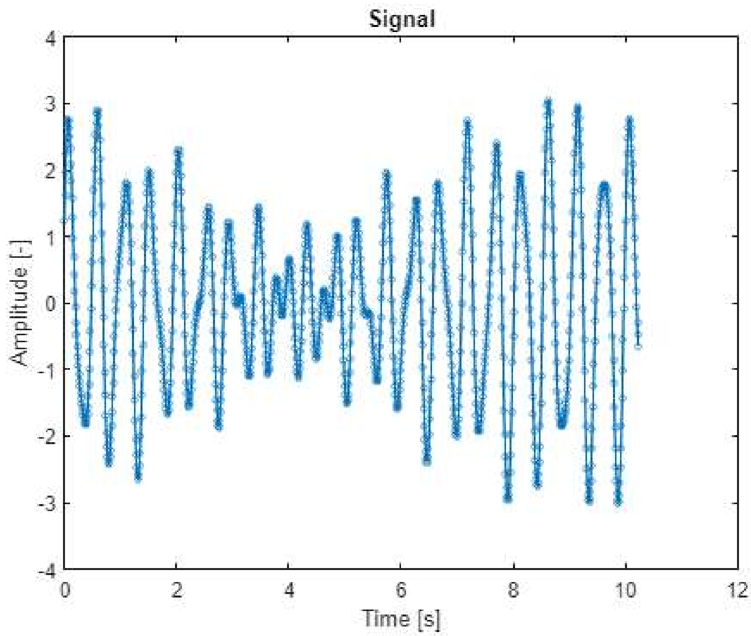
```
A = [1, 1.5, 0.7];
phase = [0, pi/6, pi/4];
freq = [2.0, 2.1, 3.5];
```

A.

Consider sampling rate as 100Hz and generate 1024 points. Calculate the fourier amplitude spectrum (from 0.0Hz to 6.0Hz) and indentify the dominante frequencies of the signal from the Fourier amplitude spectrum.

First plot the signal.

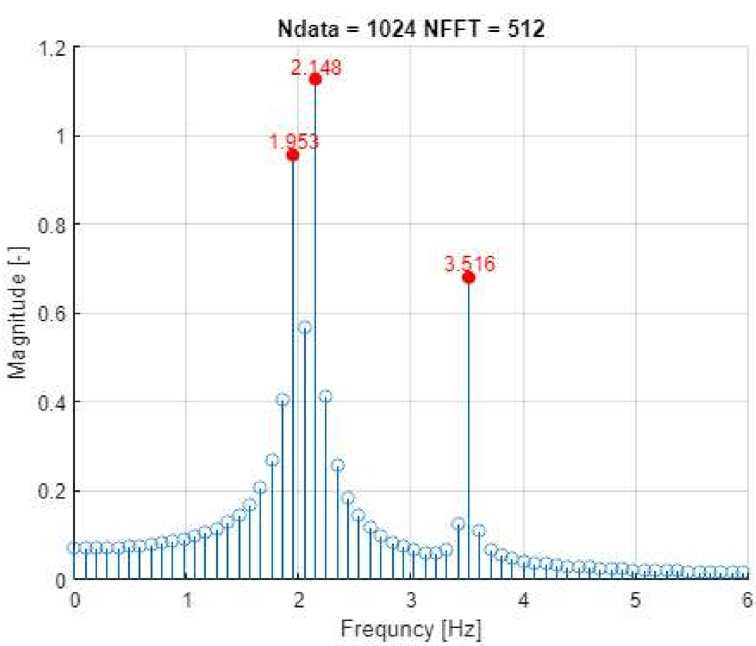
```
figure(); clf;
fs = 100; % Frequncy of data
dt = 1/fs; % Time increment
L = 1024; %length of data
n = 0:L -1;
t = n* dt;
x1 = A*sin(2*pi*freq'*t + phase');
plot(t,x1,'-o','MarkerSize',3);
ylabel("Amplitude [-]");
xlabel("Time [s]");
title("Signal")
```



Now to plot the fast fourier transform

```
figure(); clf; hold on;
x = x1;
N = 1024;
y = fft(x,N);
mag = abs(y)* 2/N;
f = (0:N-1)*fs/N;
id = (1:N/2); % First half

[pks,locs] = findpeaks(mag(id),f(id),'NPeaks',3,'MinPeakHeight',0.04);
stem(f(id), mag(id));
plot(locs, pks,'ro','MarkerFaceColor','r');
text(locs,pks,num2str(locs',4),'HorizontalAlignment','center','VerticalAlignment','bottom','Color','R');
xlim([0,6])
xlabel("Frequncy [Hz]");
ylabel("Magnitude [-]");
title("Ndata = 1024 NFFT = 512")
```

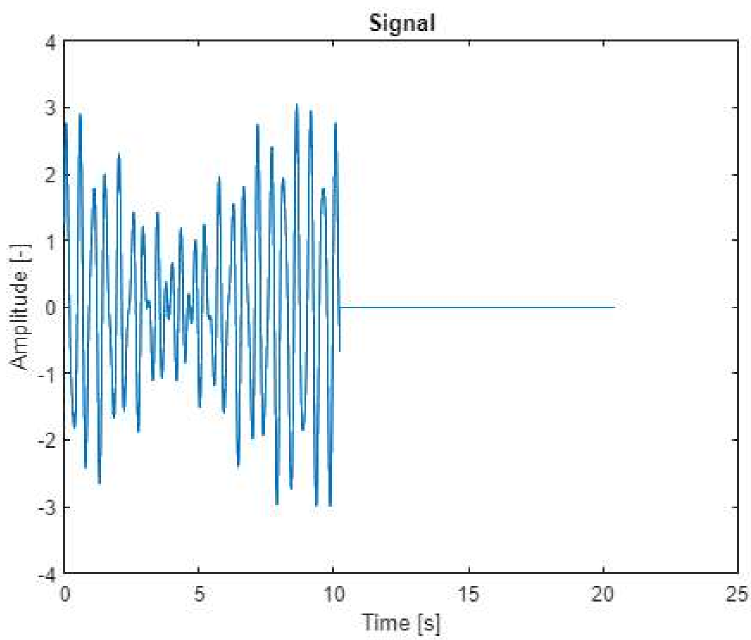


B.

Using the same sampling rate (100Hz) and with the duration of 10.24s, add 1024 zeros at the end of the signal. Repeat the analysis from above.

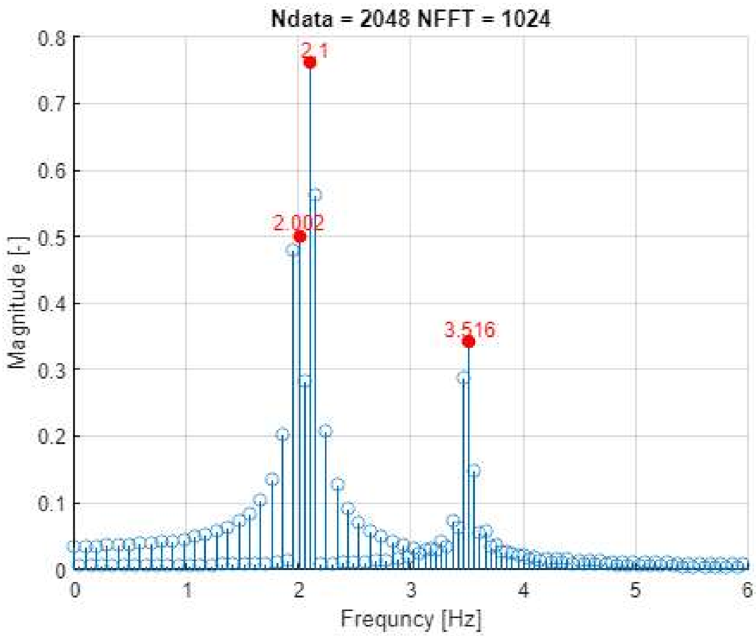
Plot the signal.

```
figure(); clf;
fs = 100;
dt = 1/fs;
L = 1024; %length of data
t = (0:L -1)* dt;
f = A*sin(2*pi*freq'*t + phase');
x2 = [f, zeros(1,1024)];
L = length(x2); %length of data
t = (0:L -1)* dt;
plot(t,x2)
ylabel("Amplitude [-]");
xlabel("Time [s]");
title("Signal")
```




```
figure(); clf; hold on;
x = x2;
N = L;
y = fft(x,N);
mag = abs(y)* 2/N;
f = (0:N-1)*fs/N;
id = (1:N/2); % First half

[pks,locs] = findpeaks(mag(id),f(id),'NPeaks',3,'MinPeakHeight',0.3);
stem(f(id), mag(id));
plot(locs, pks,'ro','MarkerFaceColor','r');
text(locs,pks,num2str(locs',4),'HorizontalAlignment','center','VerticalAlignment','bottom','Color','R');
xlim([0,6])
xlabel("Frequncy [Hz]");
ylabel("Magnitude [-]");
title("Ndata = 2048 NFFT = 1024")
grid on
```

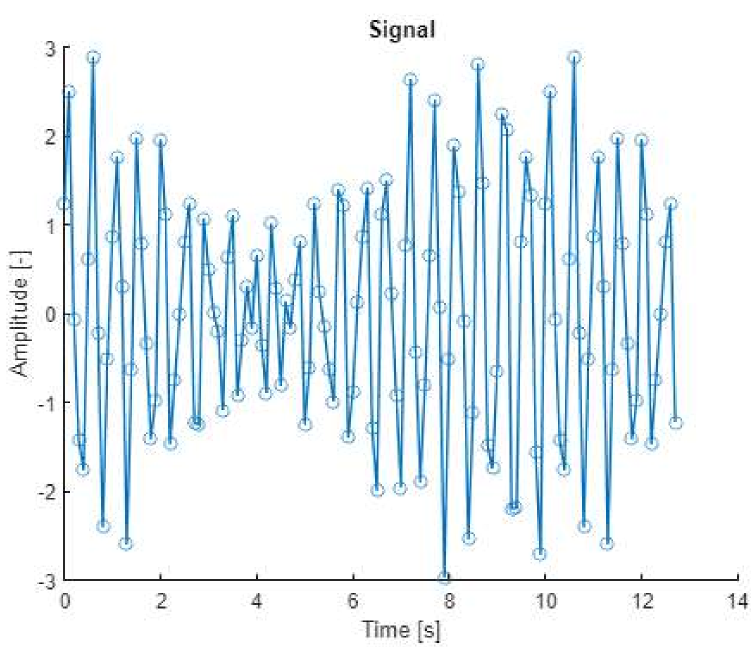


C.

Consider a sample rate of 10Hz. The duration of the signal is 12.8 second. There are 128 points for the analysis.

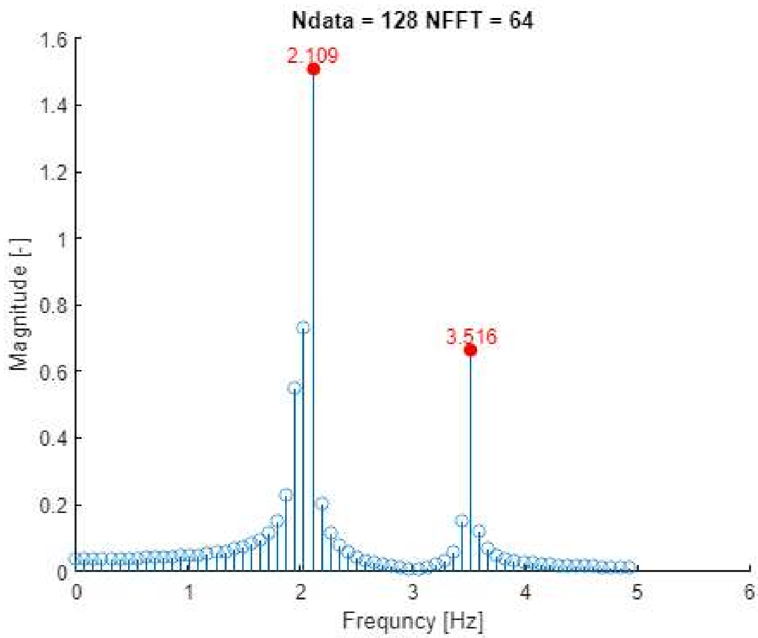
```
figure(); clf;
hold on;
fs = 10;
dt = 1/fs;
N = 128;
t = (0:N-1) * dt;
x3 = A*sin(2*pi*freq'*t + phase');
plot(t,x3,'-o');

ylabel("Amplitude [-]");
xlabel("Time [s]");
title("Signal")
```




```
figure(); clf; hold on;
x = x3;
```

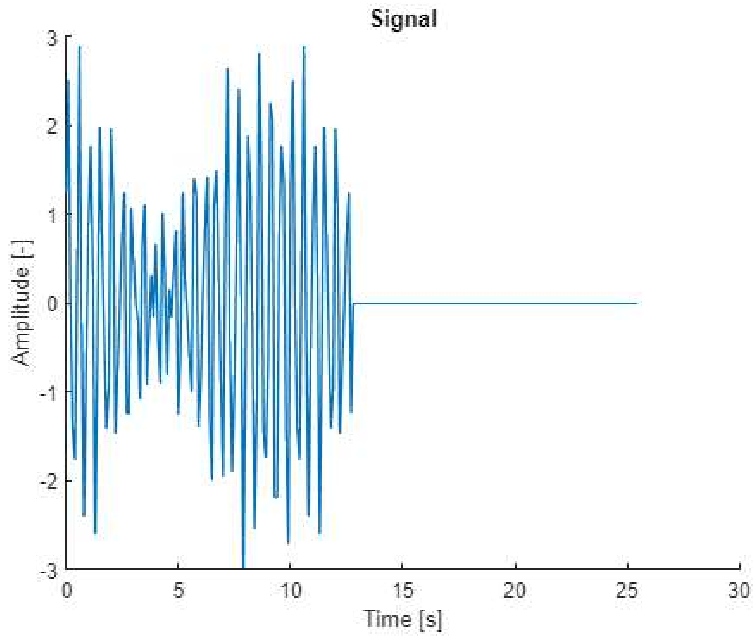
```
y = fft(x,N);
mag = abs(y)* 2/N;
f = (0:N-1)*fs/N;
id = (1:N/2); % First half
[pks,locs] = findpeaks(mag(id),f(id),'NPeaks',3,'MinPeakHeight',0.3);
stem(f(id), mag(id));
plot(locs, pks,'ro','MarkerFaceColor','r');
text(locs,pks,num2str(locs',4),'HorizontalAlignment','center','VerticalAlignment','bottom','Color','R');
xlim([0,6])
xlabel("Frequncy [Hz]");
ylabel("Magnitude [-]");
title("Ndata = 128 NFFT = 64")
```



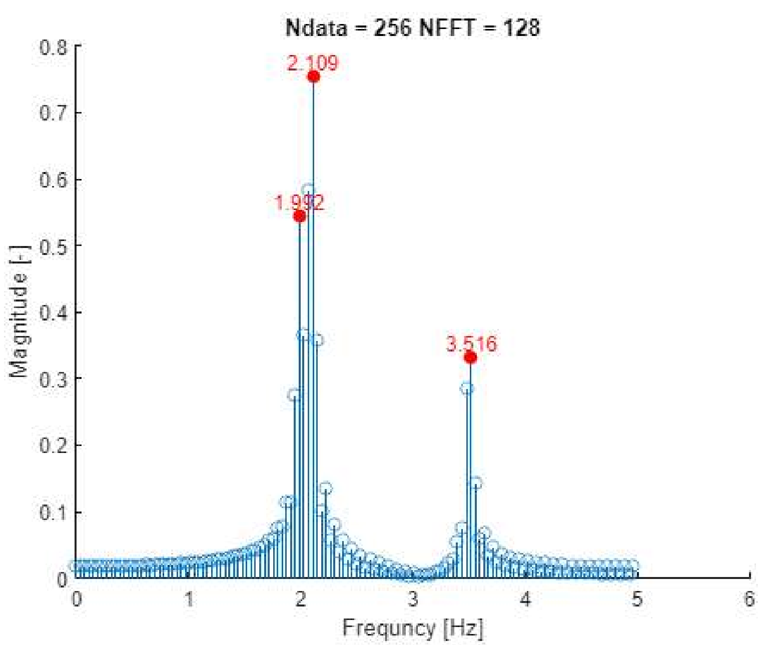
D.

Consider the same condition as part c, but with zero padding of 128 points at the end of the record. Repeat the same analysis.

```
figure(); clf; hold on;
fs = 10;
dt = 1/fs;
N = 128;
t = (0:N-1) * dt;
f = A*sin(2*pi*freq'*t + phase');
x4 = [f, zeros(1,128)];
L = length(x4); %length of data
t = (0:L -1)* dt;
plot(t,x4)
ylabel("Amplitude [-]");
xlabel("Time [s]");
title("Signal")
```

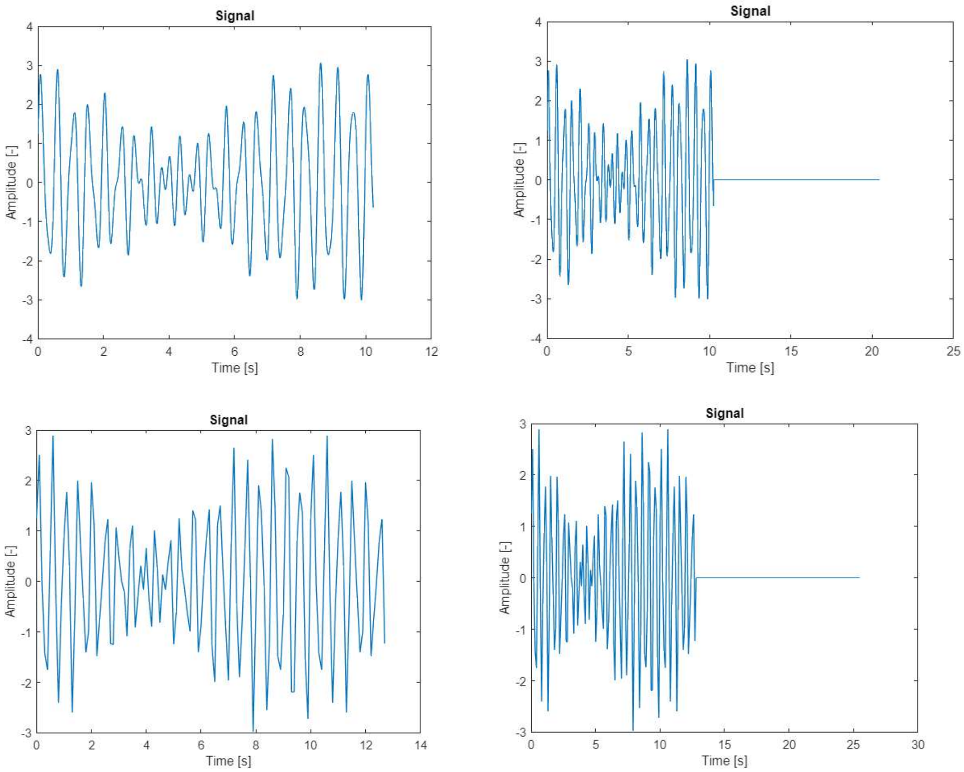



```
figure(); clf; hold on;
x = x4;
N = L;
y = fft(x,N);
mag = abs(y)* 2/N;
f = (0:N-1)*fs/N;
id = (1:N/2); % First half
[pks,locs] = findpeaks(mag(id),f(id),'NPeaks',3,'MinPeakHeight',0.3);
stem(f(id), mag(id));
plot(locs, pks,'ro','MarkerFaceColor','r');
text(locs,pks,num2str(locs',4),'HorizontalAlignment','center','VerticalAlignment','bottom','Color','R');
xlim([0,6])
xlabel("Frequncy [Hz]");
ylabel("Magnitude [-]");
title("Ndata = 256 NFFT = 128")
```



Discusssion

The signals are plotted below. The first two signals had sampling rates of 100Hz while the latter two had sampling rates of 50Hz. The main difference within each test was the extra padding added to the ends of test. The zero padding were the same length as the signal itself and the same sampling rate, effectively doubling the length of the signal but did not increase the information of the system.



The fast fourier transform are given below for the four cases. It can be seen that the faster sampling rate (allows for finer frequncy resolution in the frequency spectrum). The added padding increasing the peakiness of the spectrum; more of the signals are zero where they should be.