Modified Mander's uniaxial stress-strain relationship for concrete (no loading rate/creep considered)

1. Notation:

D_s = hoop diameter (measured to the center of the hoop)
E_c = short-term loading concrete elastic modulus at time (positive)
E_{sec} = f'_c / ε'_c, unconfined concrete secant modulus (positive)
E_{secc} = $f'_{cc} / \varepsilon'_{cc}$, confined concrete secant modulus (positive)
E_{cu} = unconfined concrete secant unloading modulus (negative)
E_{ccu} = confined concrete secant unloading modulus (negative)
f_c = concrete stress (tension is positive)
f'_c = unconfined concrete cylinder compressive strength magnitude (positive)
f'_{cc} = confined concrete compressive strength magnitude (positive)
f'_{cu} = unconfined concrete compressive stress at crushing (negative)
f'_{ccu} = confined concrete compressive stress at crushing (negative)
f'_t = direct concrete cracking strength
f'_l = passive confining stress provided by the lateral reinforcement
f'_{le} = effective confining stress
f_{yh} = yield strength of confining reinforcement
G_L = *gage length*
h_x= length of confined rectangular concrete core (measured to the center of the hoop)
h_y= width of confined rectangular concrete core (measured to the center of the hoop)
K_c = confinement coefficient (i.e. $\tan^2\theta$)
K_e = confinement efficiency coefficient
s = vertical hoop o.c. spacing
s' = clear vertical spacing between hoops or hoop-sets
w' = clear horizontal spacing between tied longitudinal bars

ε_c = concrete strain (positive is tensile)
ε'_t = concrete cracking strain (positive)
ε'_c = strain at the unconfined compressive strength f'_c (negative)
ε'_{cc} = strain at f'_{cc} (negative)
ε_{cu} = unconfined concrete ultimate compressive strain (negative)
ε_{ccu} = confined concrete ultimate compressive strain (negative)

λ_c = gage length ratio for unconfined concrete

ρ_s = volumetric confinement ratio

2. Modulus of Elasticity of Concrete in ACI 363 and ACI 318

Unless E_c is given, use

$$E_c = 40{,}000 psi \sqrt{\frac{f_c'}{psi}} + 1{,}000{,}000\text{psi} \tag{1a}$$

or that in ACI 318

$$E_c = 57{,}000 psi \sqrt{\frac{f_c'}{psi}} \tag{1b}$$

3. Tensile stress-strain relationship (Vecchio and Collins) accounting for tension-stiffening,

Before cracking, that is, while $0 \le \varepsilon_c \le \varepsilon_t'$,

$$f_c = \varepsilon_c E_c \tag{2}$$

Tension stiffening after cracking, that is, while $\varepsilon_c > \varepsilon_t'$

$$f_c = \frac{0.7 f_t'}{1 + \sqrt{500\varepsilon_c}} \tag{3}$$

where,

$$f_t' = 300 psi\, Ln\left(1 + \frac{f_c'}{1800 psi}\right) \tag{4}$$

with the tensile strain at cracking defined as,

$$\varepsilon_t' = \frac{f_t'}{E_c} \tag{5}$$

4. Modified Mander's compressive stress-strain relationship for unconfined concrete:

Elastic and Hardening range,

$$f_c = -\left(1-\left(1-\frac{\varepsilon_c}{\varepsilon_c'}\right)^{n_E}\right)f_c' \qquad while\ \varepsilon_c' \le \varepsilon_c \le 0$$

Softening range,

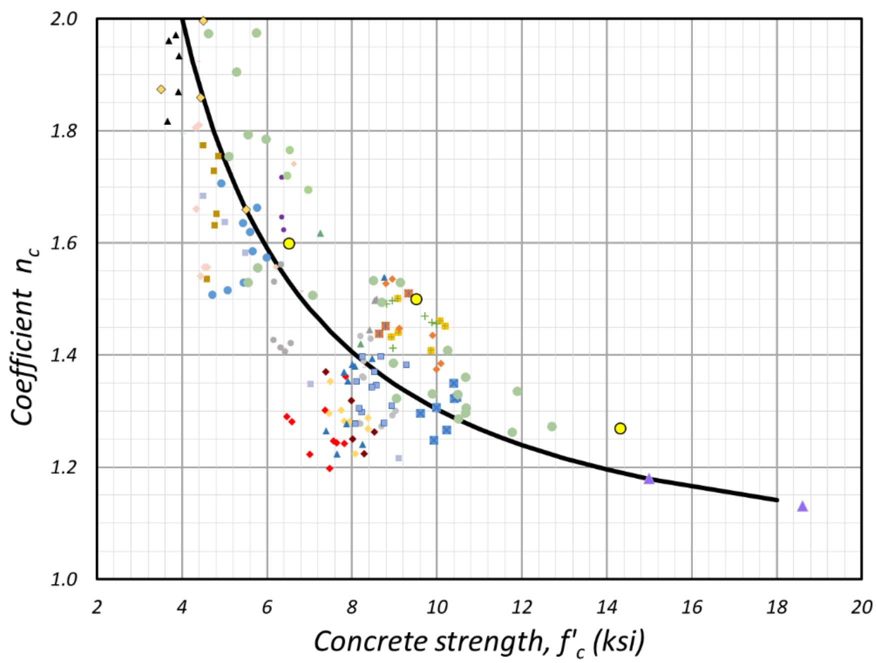
$$f_c = -\frac{\left(1+\frac{1}{\lambda_c}\left(\frac{\varepsilon_c}{\varepsilon_c'}-1\right)\right)}{\left(r_c - 1 + \left(1+\frac{1}{\lambda_c}\left(\frac{\varepsilon_c}{\varepsilon_c'}-1\right)\right)^{r_c}\right)} \quad r_c f_c' \qquad while\ \varepsilon_{cu} \le \varepsilon_c \le \varepsilon_c' \tag{6a}$$

and after crushing,

$$f_c = 0 \quad while\ \varepsilon_c \le \varepsilon_{cu} \tag{6c}$$

where,

$$n_E = 1 + \left(\frac{3{,}600 psi}{f_c'}\right)$$



Unless ε_c' is measured use,

$$\varepsilon_c' = -n_E \frac{f_c'}{E_c} \tag{7}$$

$$r_c = \frac{1}{1 - 1/n_E} \tag{9}$$

$$\varepsilon_{cu}' = \varepsilon_c' + \lambda_c \left(-0.005 - \varepsilon_c'\right) + \left(1 - \lambda_c\right)\left(\varepsilon_c - \varepsilon_c'\right)\frac{E_{cu}}{E_c} \tag{10}$$

$$\lambda_c = 1 \qquad \textit{elastic and hardening range, i.e. while } \varepsilon_c' \le \varepsilon_c \le 0 \tag{11}$$

$$\lambda_c = \frac{16''}{l_{p1}} \qquad \text{softening range, i.e. while } \varepsilon_{cu} \le \varepsilon_c \le \varepsilon_c' \tag{12}$$

$$E_{cu} = \frac{f_{cu}' - f_c'}{\varepsilon_{cu} - \varepsilon_c'} \tag{13}$$

and,

$$f'_{cu} = -\frac{\left(1+\frac{1}{\lambda_c}\left(\frac{\varepsilon_{cu}}{\varepsilon'_c}-1\right)\right)}{\left(r_c - 1 + \left(1+\frac{1}{\lambda_c}\left(\frac{\varepsilon_{cu}}{\varepsilon'_c}-1\right)\right)^{r_c}\right)} \quad r_c\, f'_c \tag{14}$$

The third term in Eq. 10 is the strain caused by the elastic rebound of the unconfined concrete away from the localization region.

5. Modified Mander's compressive stress-strain relationship for confined concrete

5.1. Formulation

$$f_c = -\frac{\left(1+\frac{1}{\lambda_c}\left(\frac{\varepsilon_c}{\varepsilon'_{cc}}-1\right)\right)}{\left(r_{cc} - 1 + \left(1+\frac{1}{\lambda_c}\left(\frac{\varepsilon_c}{\varepsilon'_{cc}}-1\right)\right)^{r_{cc}}\right)} \quad r_{cc}\, f'_{cc} \tag{15a}$$

Note: λ_c shall be calculated similar to unconfined concrete, i.e. Eq. 11 and Eq 12. However, with the respective confined concrete strain limits, ε'_{ccu} and ε'_{cc}.

and after crushing,

$$f_c = 0 \quad \textit{while } \varepsilon_c \le \varepsilon_{ccu} \tag{15b}$$

where,

$$f'_{cc} = f'_c + K_c f'_{le} \tag{16}$$

$$\varepsilon'_{cc} = \varepsilon'_c\left(1+20\frac{f'_{le}}{f'_c}\right) \tag{17}$$

$$K_c = 4.1 \tag{18}$$

$$r_{cc} = \frac{1}{1-\frac{E_{secc}}{E_c}} \tag{19}$$

$$E_{secc} = -\frac{f'_{cc}}{\varepsilon'_{cc}} \tag{20}$$

$$\varepsilon'_{ccu} = -0.02 \tag{21}$$

$$E_{ccu} = \frac{f'_{ccu} - f'_{cc}}{\varepsilon_{ccu} - \varepsilon'_{cc}} \tag{22}$$

and,

$$f'_{ccu} = -\frac{\left(1 + \frac{1}{\lambda_c}\left(\frac{\varepsilon_{ccu}}{\varepsilon'_{cc}} - 1\right)\right)}{\left(r_{cc} - 1 + \left(1 + \frac{1}{\lambda_c}\left(\frac{\varepsilon_{ccu}}{\varepsilon'_{cc}} - 1\right)\right)^{r_{cc}}\right)} \quad r_{cc}\, f'_{cc} \tag{23}$$

5.2. Effective confining stress

For concrete confined with circular hoops

$$f'_{le} = K_e f'_l \tag{24}$$

where

$$K_e = \frac{A_e}{A_{cc}} = \frac{\frac{\pi D_s^2}{4}\left(1 - \frac{s'}{2D_s}\right)^2}{\frac{\pi D_s^2}{4} - A_{st}} \approx \frac{\frac{\pi D_s^2}{4}\left(1 - \frac{s'}{2D_s}\right)^2}{\frac{\pi D_s^2}{4}} = \left(1 - \frac{s'}{2D_s}\right)^2 \tag{25}$$

and

$$2 f_{yh} A_b = f'_l s D_s \quad \rightarrow \quad f'_l = 2\frac{f_{yh} A_b}{s D_s} \tag{26}$$

but

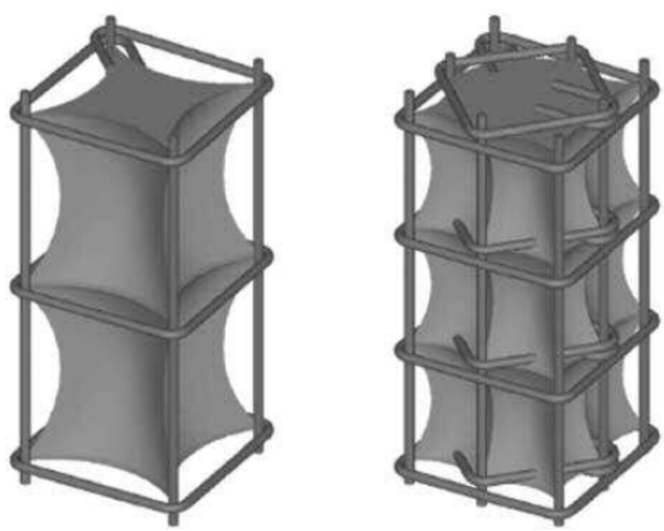
$$\rho_s = \frac{Vol.\ hoop}{Vol.\ concrete} = \frac{\pi A_b D_s}{\pi s\left(\frac{D_s}{2}\right)^2} = \frac{4A_b}{sD_s} \quad \rightarrow \quad A_b = \frac{\rho_s s D_s}{4} \tag{27}$$

(27) in (26) results in,

$$f_l' = \frac{1}{2} f_{yh} \rho_s \tag{28}$$

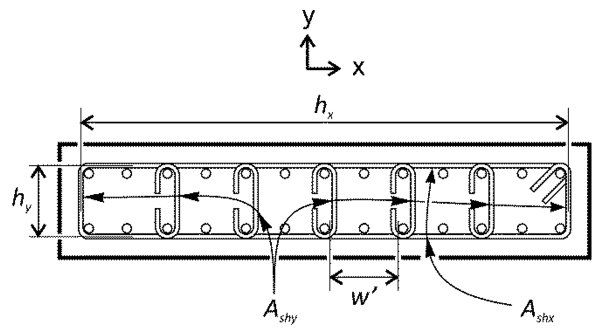
For concrete confined with rectangular or polygonal hoops. Like in columns confined with circular hoops, the effective confining pressure is also defined as:

$$f_{le}' = K_e f_l' \tag{29}$$

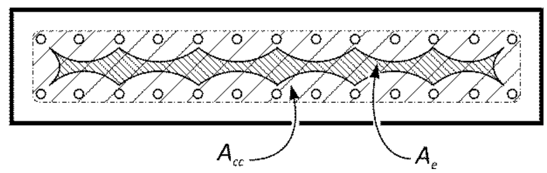


From Paultre et al.

However, the efficiency coefficient must account for vertical and horizontal arching since such arching develops in nodes in columns confined with rectangular and square hoops. Assuming vertical and horizontal arching with a slope at 45 degrees from the nodes formed by the intersection of a longitudinal bar and two orthogonal hoops or cross ties, then the area of effectively confined concrete core A_e can be approximated as:



Hoop vertical spacing: ***s***
Clear hoop spacing: ***s'***



$$A_e = \left[h_x h_y - \frac{\sum \left(w_i'\right)^2}{6} \right] \left(1 - \frac{s'}{2h_x}\right) \left(1 - \frac{s'}{2h_y}\right) \tag{30}$$

and the effectiveness coefficient becomes,

$$k_e = \frac{A_e}{A_{cc}} = \frac{\left[h_x h_y - \frac{\sum \left(w_i'\right)^2}{6} \right] \left(1 - \frac{s'}{2h_x}\right) \left(1 - \frac{s'}{2h_y}\right)}{h_x h_y - A_{st}} \approx \left[1 - \frac{\sum \left(w_i'\right)^2}{6 h_x h_y} \right] \left(1 - \frac{s'}{2h_x}\right) \left(1 - \frac{s'}{2h_y}\right) \tag{31}$$

Now, in rectangular columns the confining stresses from the hoop reinforcement detailed in the two orthogonal directions can differ. The confining pressure f_l' is a function of the confining pressures in each of the two orthogonal directions f_{lx}' and f_{ly}',

$$f_l' = max\left(\sqrt{f_{l1}' \, f_{l2}'}, 0.04 f_{l2}'\right) \text{ where } f_{l2}' = max\left(f_{lx}', f_{ly}'\right) \tag{32}$$

where

$$f_{lx}' = \frac{A_{shx}}{h_y s} f_{yh} = \rho_x f_{yh} \tag{33}$$

and

$$f_{ly}' = \frac{A_{shy}}{h_x s} f_{yh} = \rho_y f_{yh} \tag{34}$$

Finally, the volumetric confining ratio ρ_s (not used here for anything, but used in other models) is the sum of the two geometric ratios,

$$\rho_s = \rho_x + \rho_y \tag{35}$$