Computation of β

$$\beta^e = \tan^{-1}\frac{\Delta U^e_{x_2}}{L_0 + \Delta U^e_{x_1}} = \tan^{-1}\frac{U^e_5 - U^e_2}{L_0 + U^e_4 - U^e_1}$$

First-order Taylor series expansion about $\left(\beta^e\right)^i_{n+1}$ **:**

$$\left(\beta^e\right)^{i+1}_{n+1} \approx \left(\beta^e\right)^i_{n+1} + \underbrace{\left(\frac{\partial \beta^e}{\partial \mathbf{U}^e}\right)^i_{n+1}\left(\delta \mathbf{U}^e\right)^{i+1}_n}_{\left(\delta\beta^e\right)^{i+1}_n}$$

Computation of β

$$\left(\delta\beta^e\right)_n^{i+1} = \left(\frac{\partial\beta^e}{\partial U_1^e}\right)_{n+1}^{i} \cdot \left(\delta U_1^e\right)_n^{i+1} + \left(\frac{\partial\beta^e}{\partial U_2^e}\right)_{n+1}^{i} \cdot \left(\delta U_2^e\right)_n^{i+1} + \cdots + \left(\frac{\partial\beta^e}{\partial U_6^e}\right)_{n+1}^{i} \cdot \left(\delta U_6^e\right)_n^{i+1}$$

$$\beta = \mathrm{ArcTan}\left[\frac{(U5 - U2)}{L0 + U4 - U1}\right];$$

```
FullSimplify[Grad[β, {U1, U2, U3, U4, U5, U6}]]
```

$$\left\{\underbrace{\frac{-U2+U5}{(L0-U1+U4)^2+(U2-U5)^2}}_{\frac{\partial\beta^e}{\partial U_1^e}},\ \underbrace{-\frac{L0-U1+U4}{(L0-U1+U4)^2+(U2-U5)^2}}_{\frac{\partial\beta^e}{\partial U_2^e}},\ \underbrace{0}_{\frac{\partial\beta^e}{\partial U_3^e}},\ \underbrace{\frac{U2-U5}{(L0-U1+U4)^2+(U2-U5)^2}}_{\frac{\partial\beta^e}{\partial U_4^e}},\ \underbrace{\frac{L0-U1+U4}{(L0-U1+U4)^2+(U2-U5)^2}}_{\frac{\partial\beta^e}{\partial U_5^e}},\ \underbrace{0}_{\frac{\partial\beta^e}{\partial U_6^e}}\right\}$$

Computation of β

- **Solve for last incremental displacement vector:**

$$\delta\mathbf{U}_n^{i+1} = \left[\mathbf{K_T}\left(\mathbf{U}_{n+1}^i\right)\right]^{-1} \cdot \boldsymbol{\Psi}\left(\mathbf{U}_{n+1}^i\right) = \left[\mathbf{K_T}\left(\mathbf{U}_{n+1}^i\right)\right]^{-1} \cdot \left\{\mathbf{P_f} - \mathbf{P_r}\left(\mathbf{U}_{n+1}^i\right)\right\}$$

- **Extract last incremental displacement vector for all elements:**

$$\left(\delta\mathbf{U}^e\right)_n^{i+1} = \overset{e}{\mathbf{E}}\left\{\delta\mathbf{U}_n^{i+1}\right\}$$

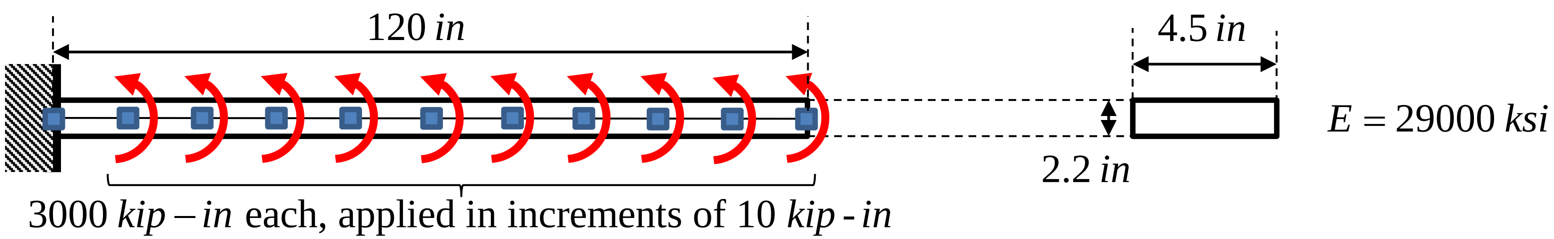
- **Find last incremental $\left(\delta\beta^e\right)_n^{i+1}$ for all elements:**

$$\left(\delta\beta^e\right)_n^{i+1} = \left(\frac{\partial\beta^e}{\partial U_1^e}\right)_{n+1}^i \cdot \left(\delta U_1^e\right)_n^{i+1} + \left(\frac{\partial\beta^e}{\partial U_2^e}\right)_{n+1}^i \cdot \left(\delta U_2^e\right)_n^{i+1} + \cdots + \left(\frac{\partial\beta^e}{\partial U_6^e}\right)_{n+1}^i \cdot \left(\delta U_6^e\right)_n^{i+1}$$

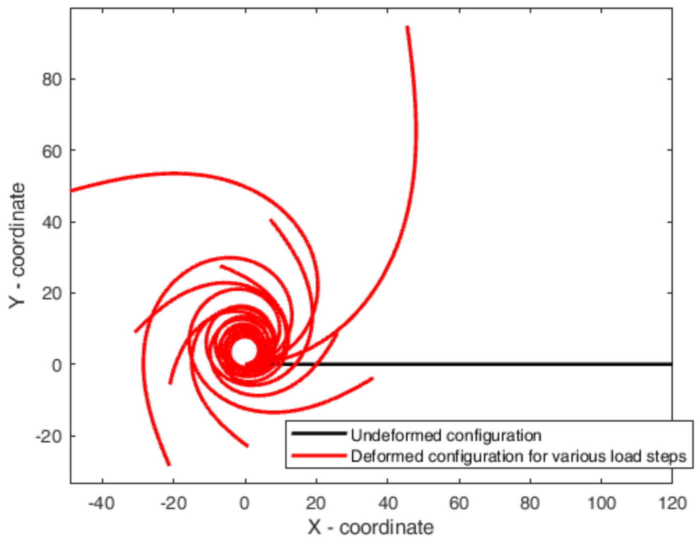
- **Update current $\left(\beta^e\right)_{n+1}^{i+1}$ for all elements:**

$$\left(\beta^e\right)_{n+1}^{i+1} \approx \left(\beta^e\right)_{n+1}^i + \left(\delta\beta^e\right)_n^{i+1}$$

Verification Example



Few selected load steps



Final load step