Homework 2

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SE 211 Advance Structural Concrete

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Introduction

In this report, the properties of unconfined concrete are investigated. The key properties of the concrete is found using a compression test on a concrete cylinder which is used to model the stress-strain curve in the report. In part I, the concrete's behavior in the region before it reaches its maximum compressive strength is investigated. In the second part, the post peak response of concrete is investigated using theoretical equations for pre-peak and post-peak stress strain curves.

Part I – Modeling of the concrete compressive stress-strain response

For the first portion of the report, the compressive stress-strain data comes from a compression test on a 12 in. long by 6 in. diameter concrete cylinder recorded via an 8" gage length compressometer. The cylinder had been dried for over 28 days and had its aggregates sources in Southern California.

This portion of the report investigates the use of a stress and strain biases in correcting the measured stress-strain curve, estimating the modulus of elasticity of the concrete, the power coefficient for the pre-peak polynomial stress-strain curve, and the maximum post-peak displacement.

Question 1. Investigating the use of bias to correct stress-strain curve

A stress and strain bias or offset is selected in order to mitigate the errors from data collection during the test. For one, the stress does not start at zero as the compressometer needs to exert some stress in order to engage with the cylinder's face, since the cylinder is not snug against the compressometer when loaded. Secondly, once the needle of the compressometer engages with the cylinder, the needle itself requires some strain in order to stiffened and gather data. Thus, the initial readings are usually disregarded in ASTMs.

The stress of the measured curve was shifted upwards by the amount of the first stress datapoint, such that the following stresses were always positive. The strain was shifted towards the left by an amount that when a linear line is drawn from the near-linear portion of the beginning, the intercept of the line is zero.

In this report, the value of the stress bias was 32.1 ksi and the value of the strain bias was 0.00002. It should be noted that this affected the calculations in the rest of the report. A figure of the initial portions of the curve is shown in Figure 1.

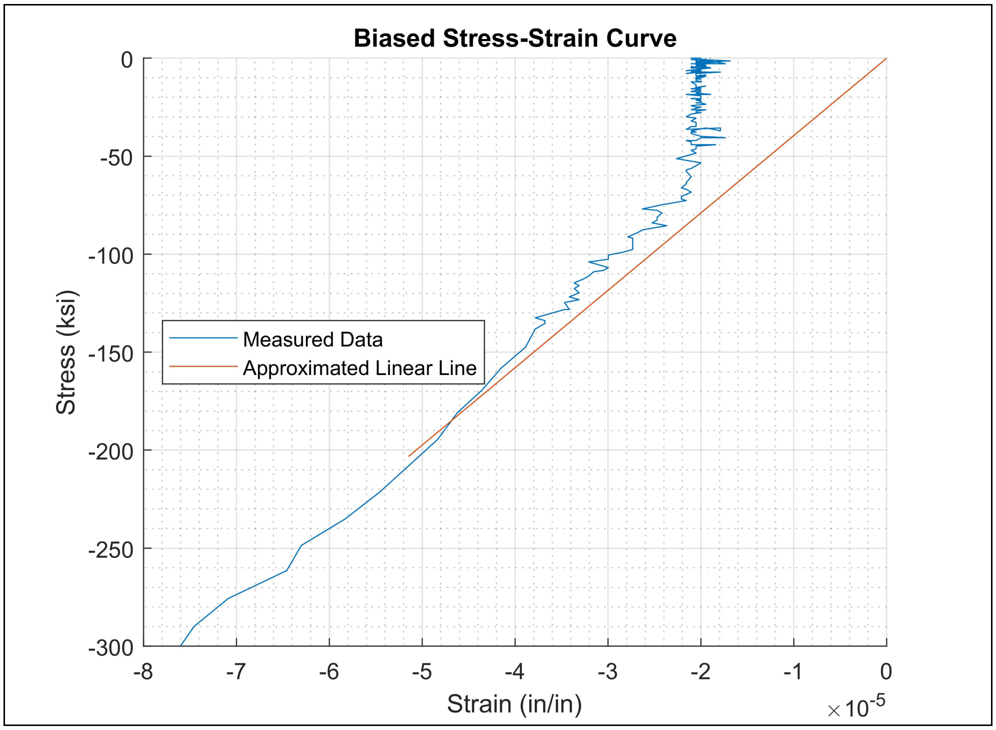

Figure 1 Biased Stress Strain Curve

Question 2. Identifying unconfined concrete stress-strain key parameters

Part (i) Determining modulus of elasticity and peak stress and strain

The peak point was found as the minimum of the data points and set as the ϵ_c', peak compressive strain, and f_c', peak compressive stress. It is noted that the actual peak strain can be set as a point between the unloading and reloading points at the peak since the actual peak can be found here if the concrete had not unloaded and since the stress remains constant for this region. In Figure 5, it is shown that the pre-peak curve is assumed to reach $\epsilon_c' = -0.00257$ and $f_c' = 4730$ ksi.

The modulus of concrete using ASTM C469 equation 3 is given as

$$E = \frac{S_2 - S_1}{\epsilon_2 - 0.000050}$$

where S_2 is the stress corresponding to 40% of the ultimate load, S_1 is the stress corresponding to a longitudinal strain ϵ_1, 50 millionths, and ϵ_2 is the longitudinal strain produced at S_2. In the analysis, a linear line was fit for the test data between ϵ_1 and ϵ_2 to calculate E. This would account for test data better as oppose to calculating E using only two data points. The modulus of elasticity was rounded to the nearest 50 ksi and was found to be 3200 ksi.

Part (ii) Compare modulus of elasticity with NCHRP 496 report

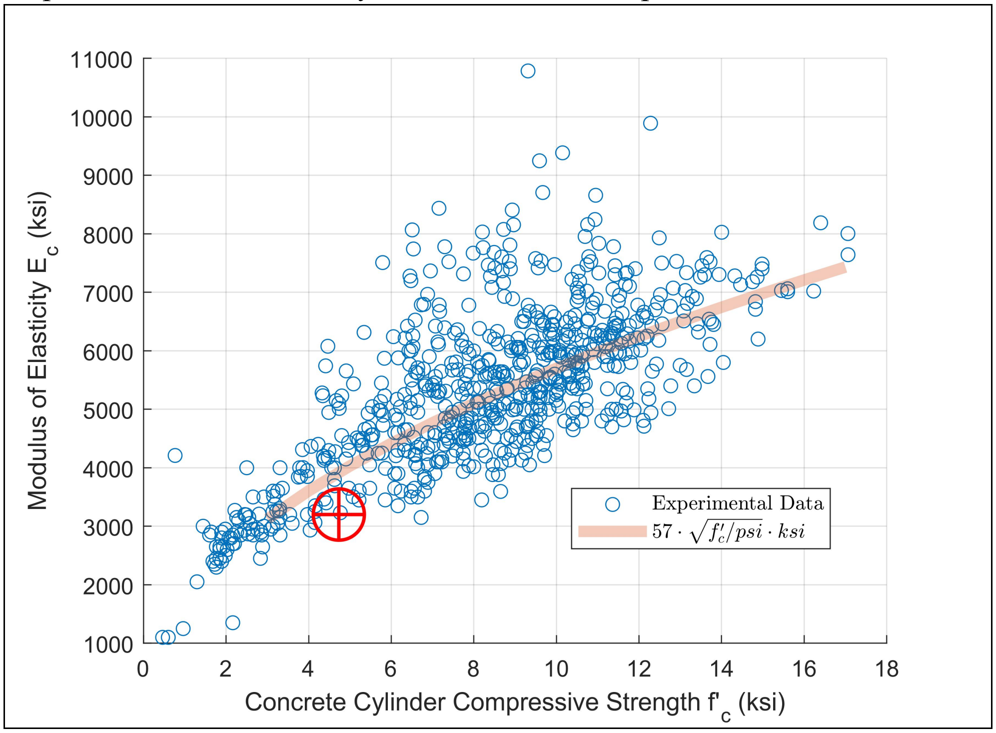

Figure 2 Experimental Test Data of Modulus of Elasticity to Compressive Strength

Figure 2 shows the modulus of elasticity for compressive strength using data from the National Cooperative Highway Research Program report 496, appendix E, provided by the instructor. The expected modulus of concrete given the compressive strength of concrete was also plotted using ACI 318-19 equation 19.2.2.1.b. This equation assumes a normal weight concrete density between 145-150 lb/ft^3 and is given as

$$E = 57 \cdot \sqrt{\frac{f_c'}{psi}} \cdot ksi$$

From the NCHRP report, there are many factors that influence the measurement of the modulus of elasticity. Factors in the laboratory include "moisture content and the loading conditions, such as top and bottom bearing plate sizes, loading rate, and specimen shape and size.". Other factors include the coarse aggregate content in the concrete and the properties of the aggregates. The modulus of elasticity as given by ACI 318-19 is given only as a function of the unit weight and the compressive strength which oversimplifies other properties of the concrete. This equation ignores the strength of the cement paste and the stiffness of the coarse aggregates. Other methods that account for the stiffness of the aggregate include the Comité Euro-International du Beton-Féderation Internatinale de la Précontratinte Model Code (7) given as

$$E_c = 3100\alpha_E \left(\frac{f_{cm}}{1.44}\right)^{\frac{1}{3}} \text{(ksi)}$$

$$E_c = 21500\alpha_E \left(\frac{f_{cm}}{10}\right)^{\frac{1}{3}} \text{(MPa)}$$

where α_E is 1.2 for basalt and dense limestone, 1.0 for quartz aggregate, 0.9 for limestone, and 0.7 for sandstone. E_c is the tangent modulus of elasticity at zero stress and at a concrete age of 28 days, and f_{cm} is the mean compressive strength of the concrete.

The specimen that was used in the test is lower than the expected modulus given by equation 19.2.2.1.b and by inspection, on the lower range of those with the same compressive strength. This is in part due to the aggregates that were used to make the concrete specimen. The aggregates used came from California which is known to have younger rocks which are softer. The higher range data points typically come from testing samples with harder aggregates, per description from instructor (no cited sources). Using the formula given above and solving for the α_E, the aggregate used in the concrete cylinder is assumed to be sandstone with $\alpha_E \approx 0.672$.

In figure 3, the ratio of the measured modulus of elasticity to compressive strength data to the expected modulus using the ACI 318-19 equation was plotted in a histogram with bins from 0.6-1.8 in intervals of 0.1. The histogram shows that there is a log normal distribution between the ratio of measured and predicted values with a little under 50% having a 1:1 ratio of measured to expected. The predicted modulus, using the ACI 318-19 equation, for the current test was 3,920 ksi, whereas the experiment had a modulus of 3,200 ksi. The ratio between the two is 0.816. The mean of the ratio between expected and measured modulus is 1.057, the median was 1.039, and the standard deviation was 0.197. The means that the data point falls just outside of one standard deviation of the mean. The ratios for each bin plotted as a cumulative distribution shows that the provided test data is on the lower end of the overall measured ratios, coming in <10 percentile. This means that the data collected from the test is not normal as predicted by ACI 318-19 or experimental data.

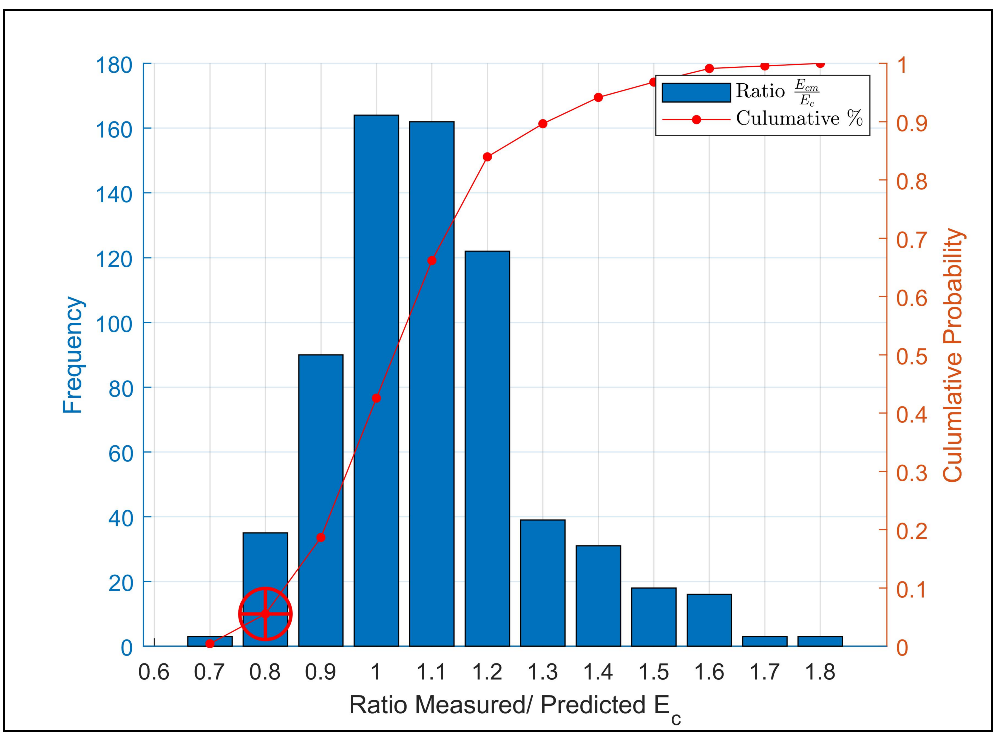

Figure 3 Ratio of Expected Modulus to Actual Modulus

Part (iii) Compute the power coefficient

The power coefficient n_E is used in modeling the stress-strain curve of concrete before it reaches its compressive strength in

$$f = -f_c'\left(1-\left(1-\frac{\epsilon_c}{\epsilon_c'}\right)^{n_E}\right)$$

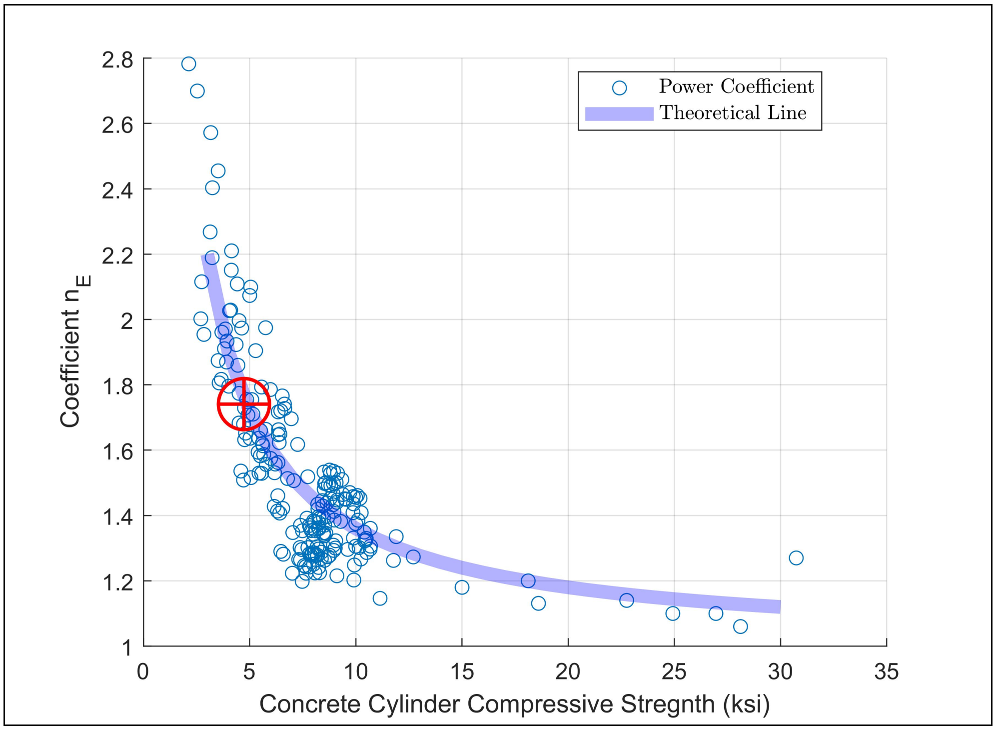

Figure 4 Experimental Test Data for Power Coefficient

Figure 4 shows the coefficient versus compressive strength of the concrete for various experiments performed at UC San Diego. The blue line is the theoretical line given by

$$n_E = 1 + \frac{3.6\text{ksi}}{f_c'}$$

which approximates the coefficient given the compressive strength of the concrete. In the data that was provided, the n_E factor was calculated as 1.74 which is plotted as the red circle in Figure 4. This is within ~1% from the theoretical $n_E = 1.76$ which means the theoretical equation predicts the correct power coefficient for the current test.

Part (iv) Comparing pre-peak stress-strain response of the test and theoretical response

The measured stress-strain curve is plotted up to $\epsilon_c' = -0.004$ as prescribed by the instructor. The theoretical stress-strain curve is plotted with the formula mentioned above, using $n_E = 1.74$. It is shown that the formula is a good approximation in small strains [-0.005, 0.] and at the peak strains [-0.0025, -0.002], though shows some variance in the strain range in between. This could be due to the nonlinear behavior of concrete affecting the constant power assumption. An optimization process can be used to find the best n_E value. Computed, but not shown, a n_E value between [1.9, 2.1] showed a better fit to the measurements. The formula is not a good fit for post peak response but is shown per instructor's request. The peak is shown to be at $\epsilon_c' = -0.00257$ and $f_c' = 4730$ ksi.

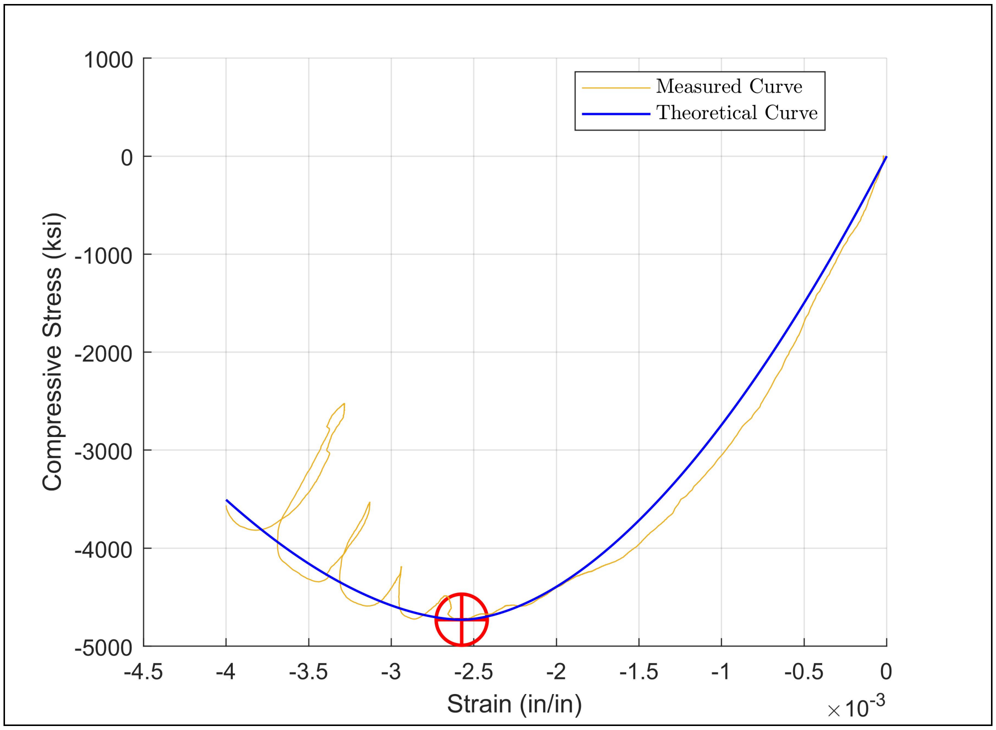

Figure 5Theoretical Curve to Measured Data

Part (v) Post peak stress-strain curve

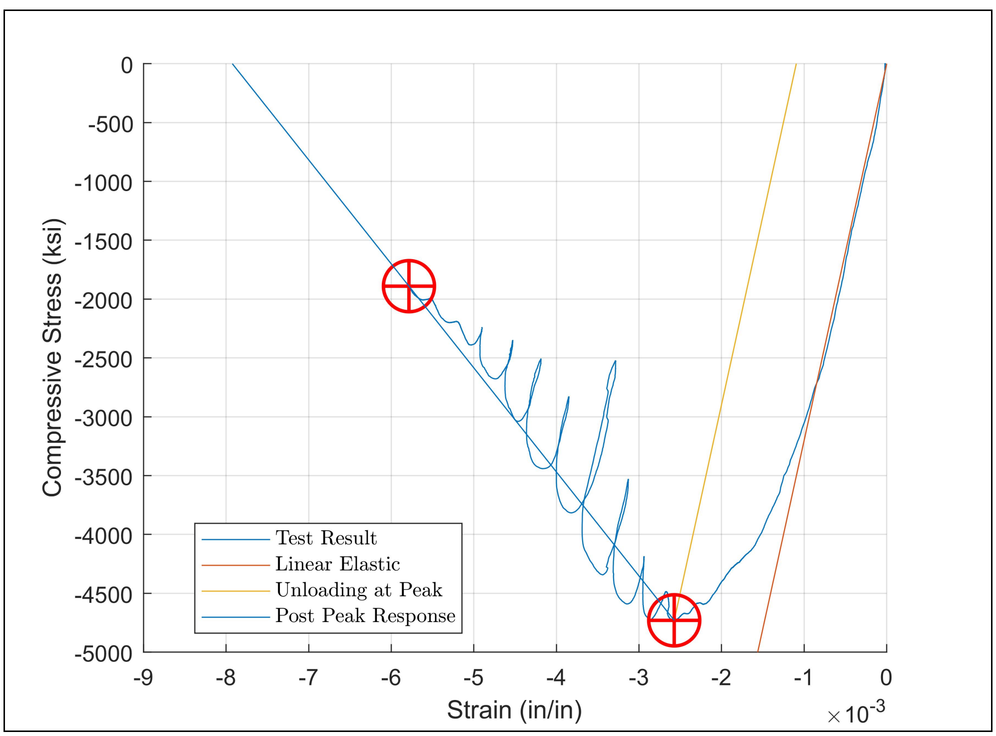

Figure 6 Post Peak Stress-Strain Curve

In Figure 6, the measured stress-strain curve is plotted to a post-peak stress value of $0.4 \cdot f_c'$. A linear-elastic line is drawn with a slope of $E_c = 3200$ksi. An unloading linear elastic line with the same slope is also plotted from the peak. Lastly a linear elastic line with a slope of $\eta \cdot E_c$ is shown from the peak representing the post-peak response of the concrete. Two points are also shown, the peak stress-strain point and the post-peak stress-strain point of $0.4 \cdot f_c'$.

During the post-peak stress-strain curve, unloading of the concrete can be seen to occur periodically. The slope of the unloading, when compared with the initial slope of the stress-strain curve, can be qualitatively described as parallel.

The η value relates the slope of the pre-peak and post-peak slopes and is calculated as the slope between (ϵ'_c, f_c') and $(\epsilon_{0.4f_c'}, f_c')$ divided by E_c, which was found to be $\eta = -0.276$. The post-peak response is thus approximated using the point-slope form by

$$f = \eta E_c(\epsilon_c' - \epsilon) - f_c'$$

where η is negative and f_c' is positive.

The ultimate post-peak strain, ϵ_{cu}' reached by the concrete, as approximated by this line, is calculated as

$$\epsilon_{cu}' = f_c' \left(\frac{1}{E_c} + \frac{1}{-\eta E_c}\right)$$

using simple geometry. In order to derive $\delta_{ppk,u}$, the ultimate post peak axial displacement at zero stress, you would have to multiply by the gage length, in this case 8 inches. From the measured data, $\epsilon_{cu}' = 0.00683$ and $\delta_{ppk,u} = 0.0546$ inches. It is noted that this will vary from other reports using the same data depending on the f_c' determined for the sample. As shown above, ϵ_{cu}' relies heavily on f_c'.

Part II – Practical Application – Axial response of a reinforced concrete column

In this part of the report, the axial response of a reinforce concrete column is investigated. The concrete column under consideration has a square cross section of 16” by 16” with the material properties as found in part I of this report. It has 4 #8 bars with a yielding stress of 69 ksi, and modulus of elasticity of 30,000 ksi. The axial compressive force and axial shortening were found for the column using different stress-strain curve in the pre-peak and post-peak regions.

Question 1 Assuming a Polynomial Stress-Strain Curve

The pre-peak response of the concrete is found using

$$f_c = -f_c'\left(1 - \left(1 - \frac{\epsilon_c}{\epsilon_c'}\right)^{n_E}\right)$$

and the post peak response is a linear line found

$$f_c = -\eta E_c(\epsilon - \epsilon_c') - f_c' \text{ and } f_c = E_c(\epsilon - \epsilon_c') - f_c'$$

where depending on which unloading path is chosen.

For the steel, the stress-strain response is given by

$$f_s = \mathrm{sign}(\epsilon) \cdot \min(|\epsilon| E_s, f_y)$$

Part (i) Analyzing the column with a single integration point

The assumed gauge length for this analysis was 120 inches.

In figure 7, the axial force versus axial shortening is given for the column, with the concrete and steel force-shortening relationship also given. The concrete is assumed to have reached failure, carrying 0 stress, at $\epsilon'_{cu} = -0.438$, using the η value calculated previously. The steel at this strain value still carries some stress and thus the total load capacity, axial force, does not go to 0 kips.

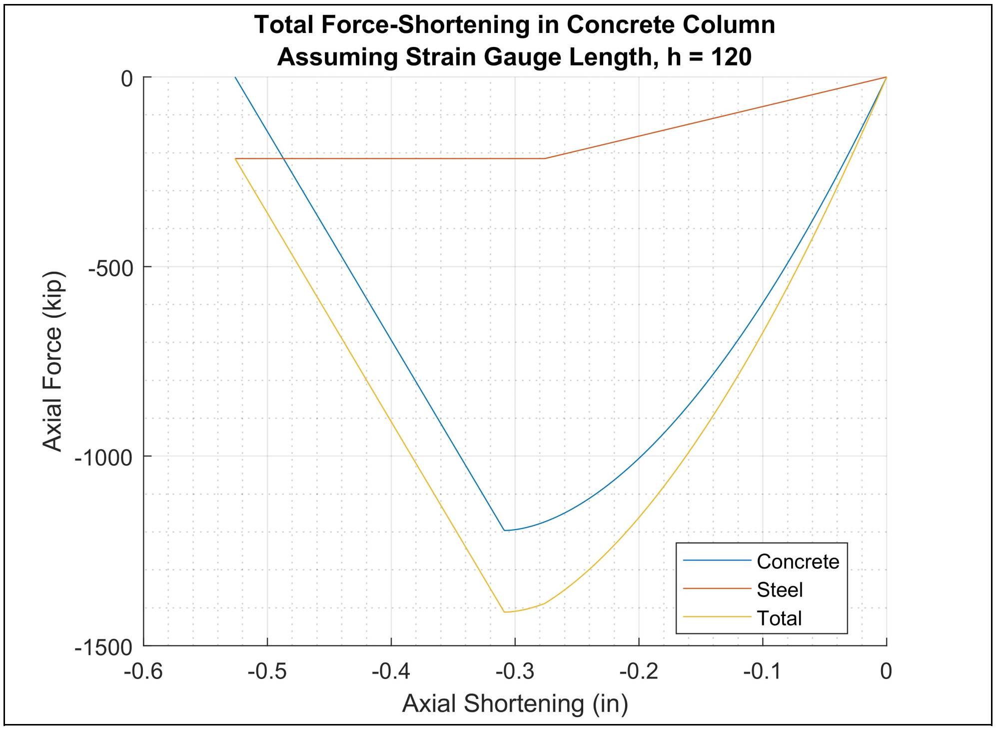

Figure 7 Axial-Shortening Curve for Gage Length =120

Part (ii) Analyzing the column with a three equal integration point

For this analysis, the concrete is broken into three sections, with heights of 40 inches as shown below. After reaching the peak, section B will develop a shear band which starts decreasing the stress in the column overall. This will result in both section A to start unloading elastically.

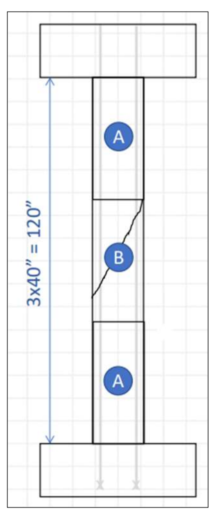

Figure 8 Diagram Showing Equal Gauge Length for Column Analysis

In Figure 9, the stress of each component in the column is plotted. For the concrete, the stress in both section A and B are assumed to both reach 0 ksi, as the stress is assumed to be the same stress throughout the unloading process. Section A reaches zero stress through unloading and section B reaching zero stress due to the development of the shear band. The stress in the steel ends up different since they follow the strain in the concrete they are bonded to.

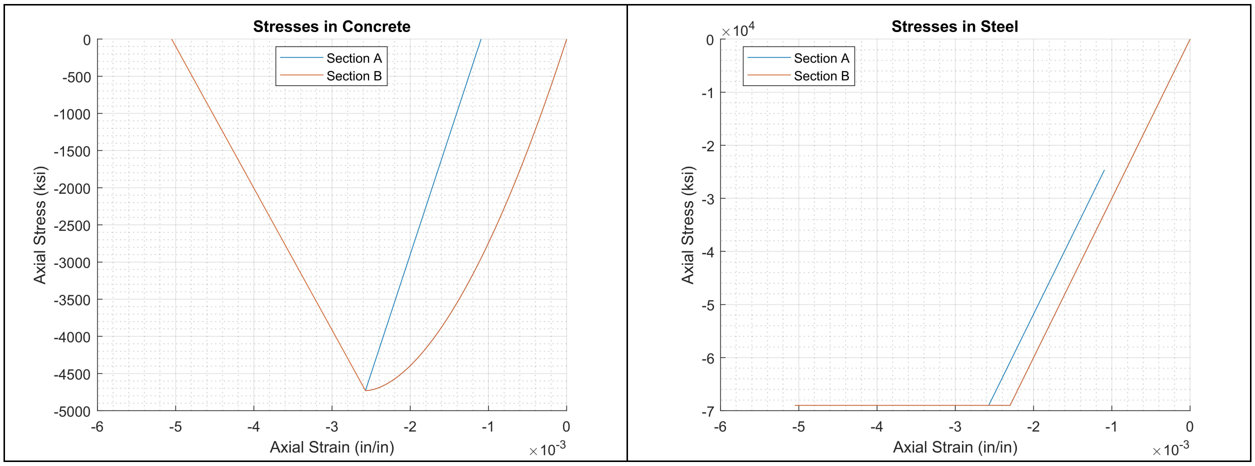

Figure 9 Stress-Strain Curve for Concrete and Steel

This leads to the difference in the end axial force when summing up the axial force-axial shortening curves for both sections. As is shown in Figure 10, the total axial force ends at force level capacity of the steel since the concrete is unable to carry any load. The axial force is found by multiplying the stress by the area of the material and the axial shortening is found by multiplying the strain by the gauge length, $h = 40$ inches.

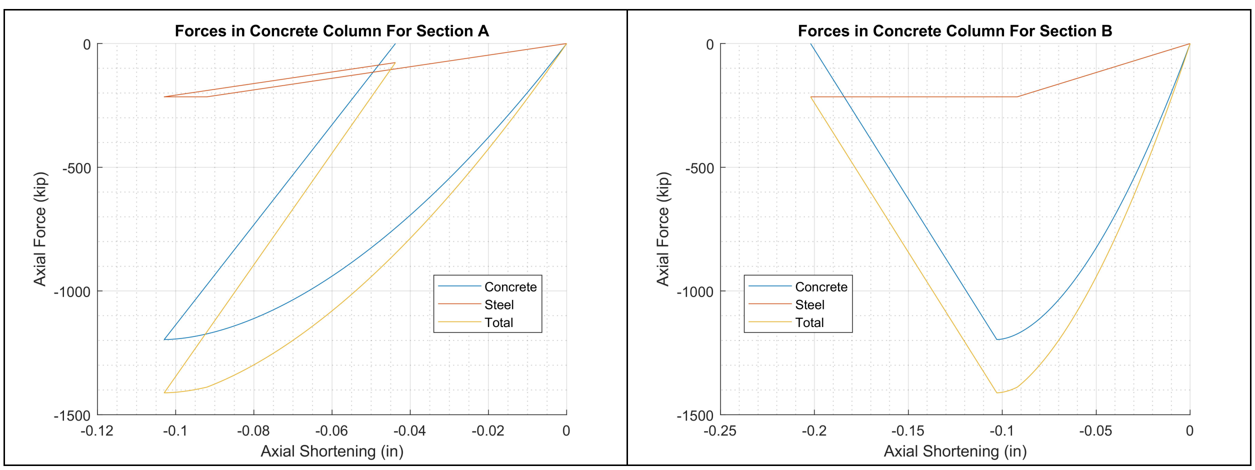

Figure 10 Axial-Shortening Curve for Section A and Section B

The combined response of the column is assumed to be the summation of the strains from 2 of section A and one of section B, given the axial force in section B. This is because the axial force for the entire column is assumed to be equal in A and B and section B should control the axial force in the entire column.

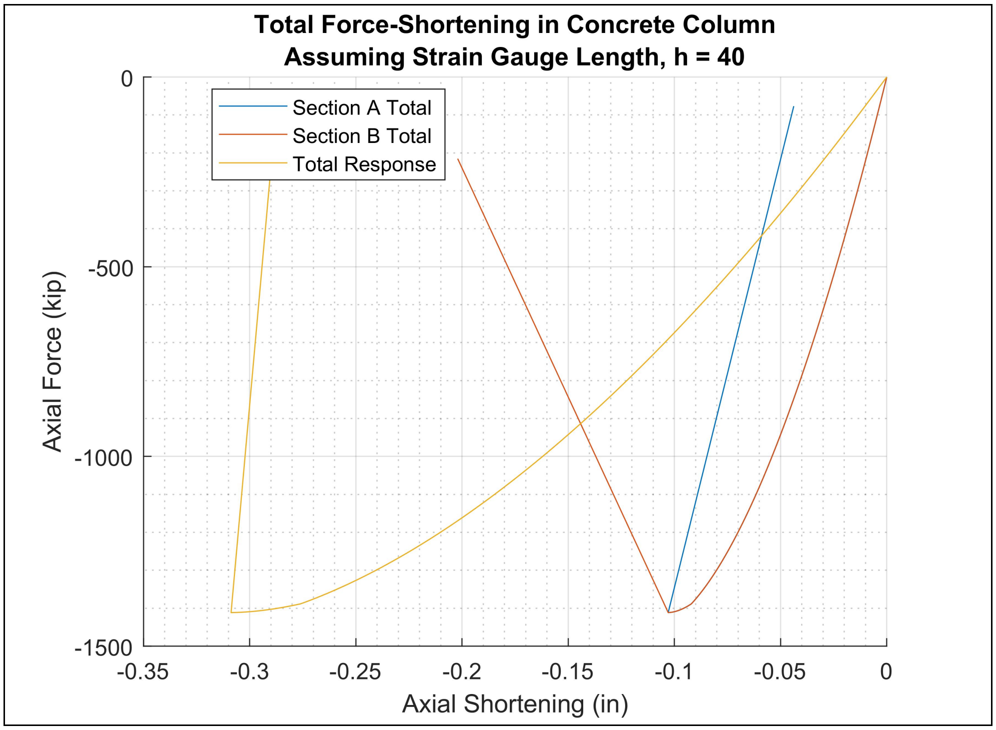

Figure 11 Axial-Shortening Curve for Gage Length =40

Question 2 Assuming Thorenfeldt's Stress-Strain Curve

The pre-peak and post-peak response of the concrete is found using Thorenfeldt's equation

$$f_c = -f_c' \frac{n\left(\frac{\epsilon}{\epsilon_c'}\right)}{n - 1 + \left(\frac{\epsilon}{\epsilon_c'}\right)^{n \cdot k}}$$

where

$$n = \frac{E_c}{E_c - \frac{f_c'}{-\epsilon_c'}} = \frac{n_E}{n_E - 1}$$

and

$$k = 0.67 + \frac{f_c'}{9000\text{psi}}$$

and the post peak unloading response is given as

$$f_c = E_c(\epsilon - \epsilon_c') - f_c'$$

For the steel, the stress-strain response is still given by

$$f_s = \text{sign}(\epsilon) \cdot \min\left(|\epsilon| E_s, f_y\right)$$

Part (i) Analyzing the column with a single integration point

For this analysis, the gauge length was assumed to be 120 inches. The stress-strain curve is plotted up to a compressive strain equal to -0.005 per instructor's request. The concrete, steel and total response is shown separately in Figure 12. The forces in the concrete have not assumed to reach zero stress and thus the end stress is much higher than those found in the first question. Now the carrying capacity of the column is assumed to have the capacity of the concrete and the steel.

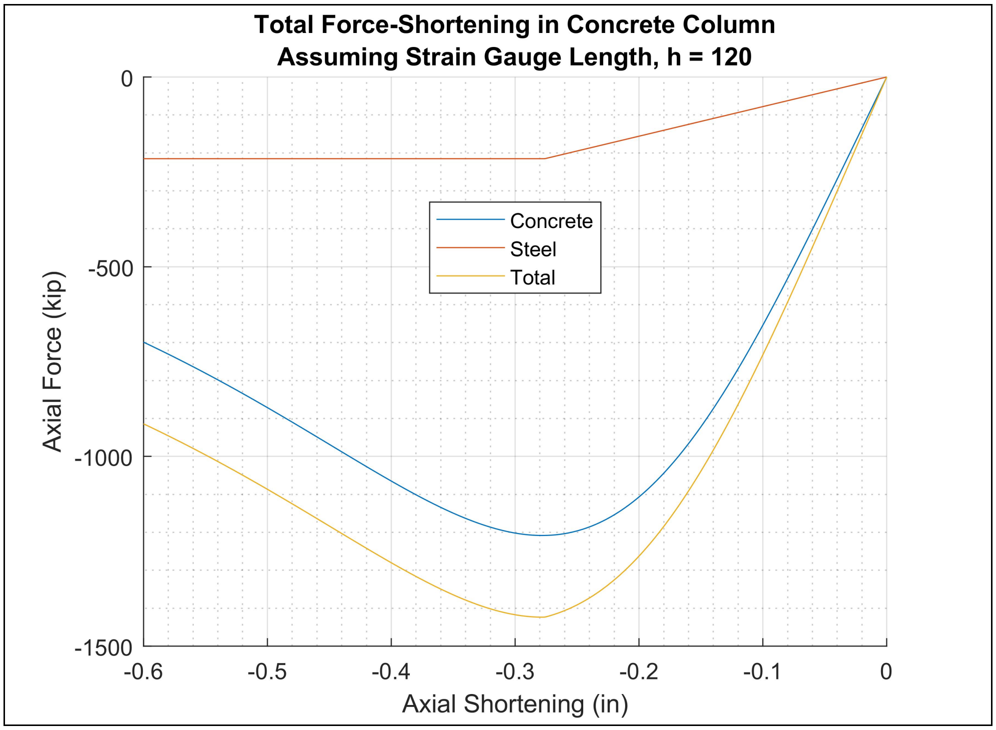

Figure 12 Axial-Shortening Curve for Gage Length =120

Part (ii) Analyzing the column with three equal integration point

The analysis in this portion reflects that of question 1, except the usage of Thorenfeldt's equation. The stress strain curve is shown in Figure 13, whereas mentioned previously, the stress does not reach zero ksi due to the stopping the analysis at -0.005 strain per the instructor. However, section A does still unload linear elastically once it reaches the peak stress-strain, in the process described previously. The stress in the steel also differs as their strain follows the strain of the concrete they're bonded to.

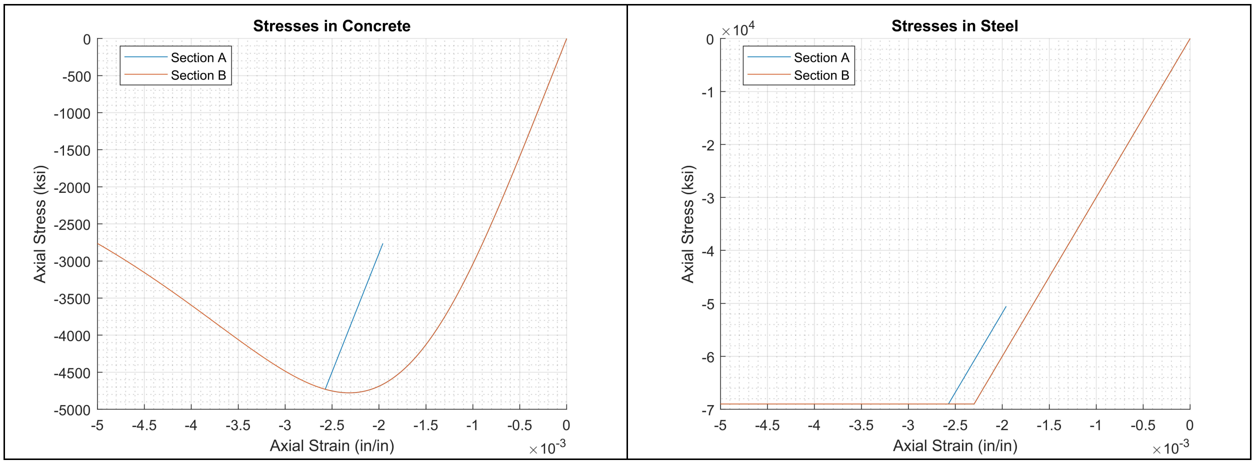

Figure 13 Stress-Strain Curve for Concrete and Steel

From the stress-strain curves, the axial force and axial shortening curves can be found using the process described previously. The stress of the concrete should reach the same stress levels as assumed but the total axial force is not going to be the same across the sections because of the behavior of the steel. The axial force carried by each section varies because the steel is carrying a different amount of force in each section.

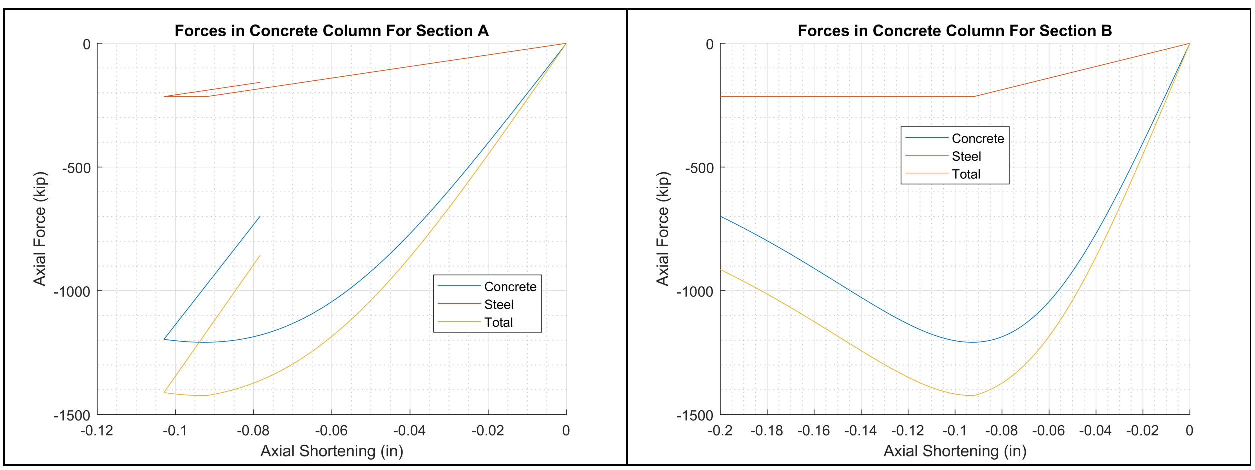

Figure 14 Axial-Shortening Curve for Section A and Section B

In the total response of the column, the axial shortening form both section A and the one section B is summed and plotted against the axial force of section B.

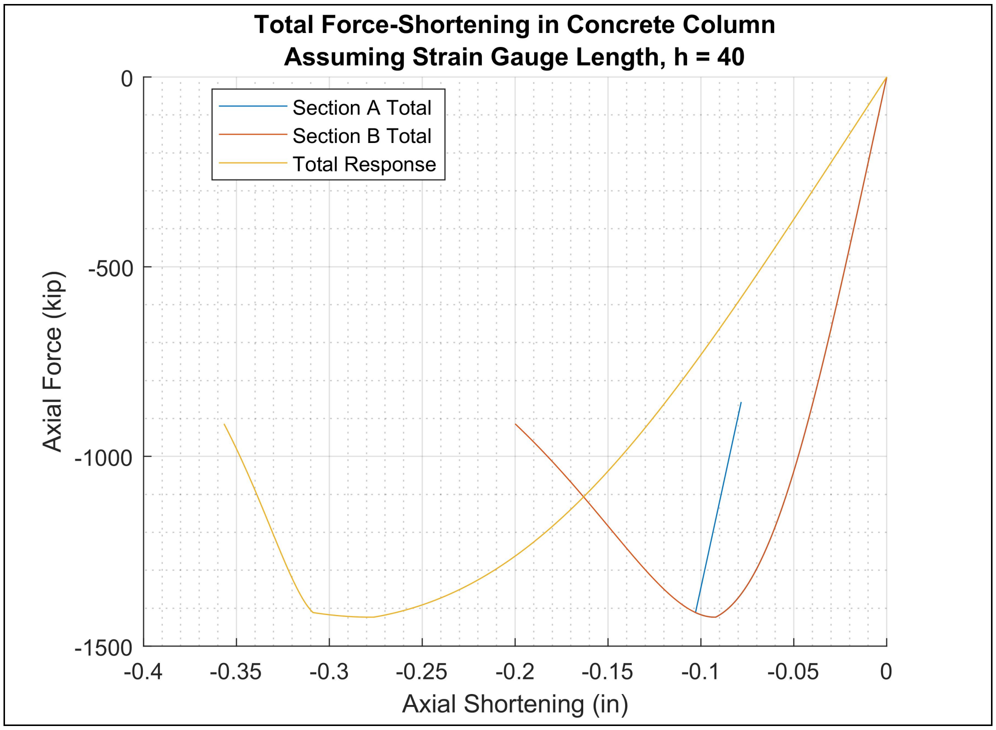

Figure 15 Axial-Shortening Curve for Gage Length =40

Question 3

Given a strainmeter that was placed significantly away from the shear band and measured an axial strain $= -0.002$, at failure, the axial stress in the section should be

$$f_c = E_c(\epsilon - \epsilon_c') - f_c'$$
$$f_c = 3200\big(-.002 - (-0.0257)\big) - 4.730$$
$$f_c = -0.00289 \text{ ksi}$$

$$f_s = E_s \cdot \epsilon$$

$$f_s = 3000 \text{ ksi} \cdot -0.002$$

$$f_s = 60 \text{ ksi}$$

Thus, the axial force in the column is taken as

$$F_c = f_c \cdot A_c + f_s \cdot A_s$$
$$F_c = 0.00289 \text{ ksi} \cdot 252.88 \text{ in}^2 + 60 \text{ ksi} \cdot 3.12 \text{ in}^2$$
$$F_c = -920.0 \text{kip}$$

In Figure 16, the response from all of the theoretical responses were plotted. The axial force for the column is shown to end up at different places as the assumed strain that the concrete ends on changes. For pre-peak stress-strain curve, it is shown that the different models predict the same response however, in the post peak region they differ significantly. This is majorly due to the assumed gage length and the assumed post-peak responses of the concrete.

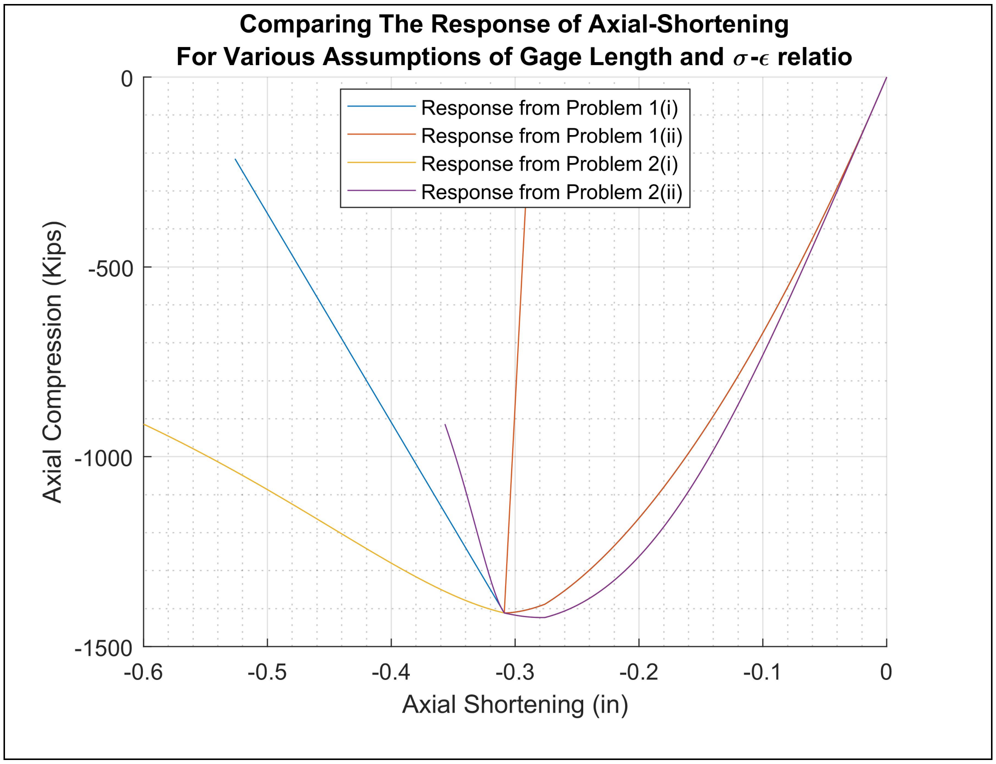

Figure 16 Combined Axial-Shortening Curve

Conclusion

In this report, the stress-strain behavior of concrete is analyzed using the results of a compression test on a concrete cylinder specimen. The key parameters of the pre-peak, stress-strain curve was derived using theoretical and standard formula were compared with experimental data.

ASTM C469 was used to compute the modulus of elasticity for the experiment and a modulus of 3200 ksi was found. This compared with other experimental data and the ACI 318-19 equation for modulus of elasticity given a compressive strength was lower than expected. One of the main reasons for this is the equation provided by ACI 318-19 only accounts for the unit weight and compressive strength of the mixture but not the stiffness of the coarse aggregate used in the concrete, which has been found to be another critical factor in determining the modulus. The aggregates used in concrete plays an important role in determining the strength as shown by this experiment. The test result was lower than those predicted because the aggregates used in southern California are typically softer than those in other regions. One way to account for the stiffness of the aggregate is to use the code from the Comité Euro-International du Beton-Féderation Internatinale de la Précontratinte which uses an α_E factor that takes this into account in another formula.

The theoretical power coefficient used in modeling the pre-peak stress-strain curve formula was a good estimate for the experimentally found power coefficient. The pre-peak stress-strain curve can be well approximated by a quadratic function if test data is not available.

The post peak response is experimentally shown to unload and reload multiple times during the test. For this report, the behavior is simplified to a straight line. With the post-peak stress-strain curve, the line was assumed to be between 0.4f’c and the peak with the slope as a ratio of the initial modulus of elasticity.

When the ultimate deformation is calculated, the strain gauge used to measure the post-peak response needs to be considered. This is because, the strain gauge may be too long to fully represent the stress-strain relationship between different portions of the concrete. There may be a portion of the concrete that cracks and will increase in strain as it continues to be loaded, however there are also other portions of the concrete that will unload as there's less compressive stress in the column overall. It is important to consider the gauge length when calculating the shortening of the concrete based on its strain.

In the second portion of the report, different formulas were used to assume the behavior in the pre-peak and post-peak stress-strain curve and axial-force to axial shortening curves of a concrete column. The assumption of the gauge length is shown to be important as the response varied significantly between the two scenarios. With the equal integration points, axial shortening is shown to be 45.0% less for part 1 and 40.5% less for part 2 than the respective single integration point calculations. This is due to better approximation of the behavior of the concrete and steel for different sections of the concrete. A longer gauge length will not properly account for the various stress-strain relationships of the concrete and steel throughout the column.

Reference

ACI Committee 318. (1995). *Building code requirements for structural concrete: (ACI 318-95)*. Farmington Hills, MI: American Concrete Institute

ASTM International. *C469/C469M-14 Standard Test Method for Static Modulus of Elasticity and Poisson's Ratio of Concrete in Compression*. West Conshohocken, PA; ASTM International, 2014. doi: https://doi.org/10.1520/C0469_C0469M-14d

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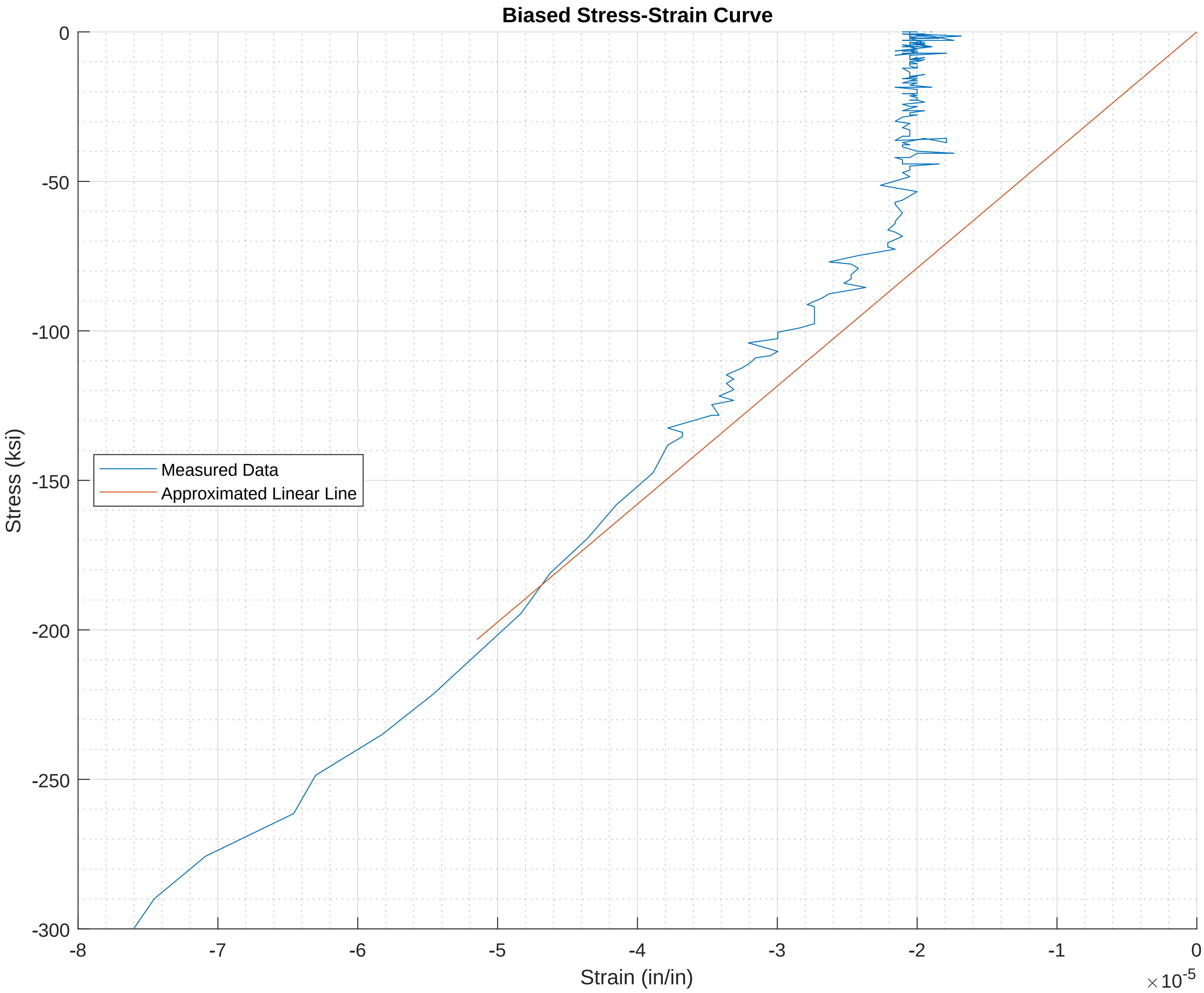
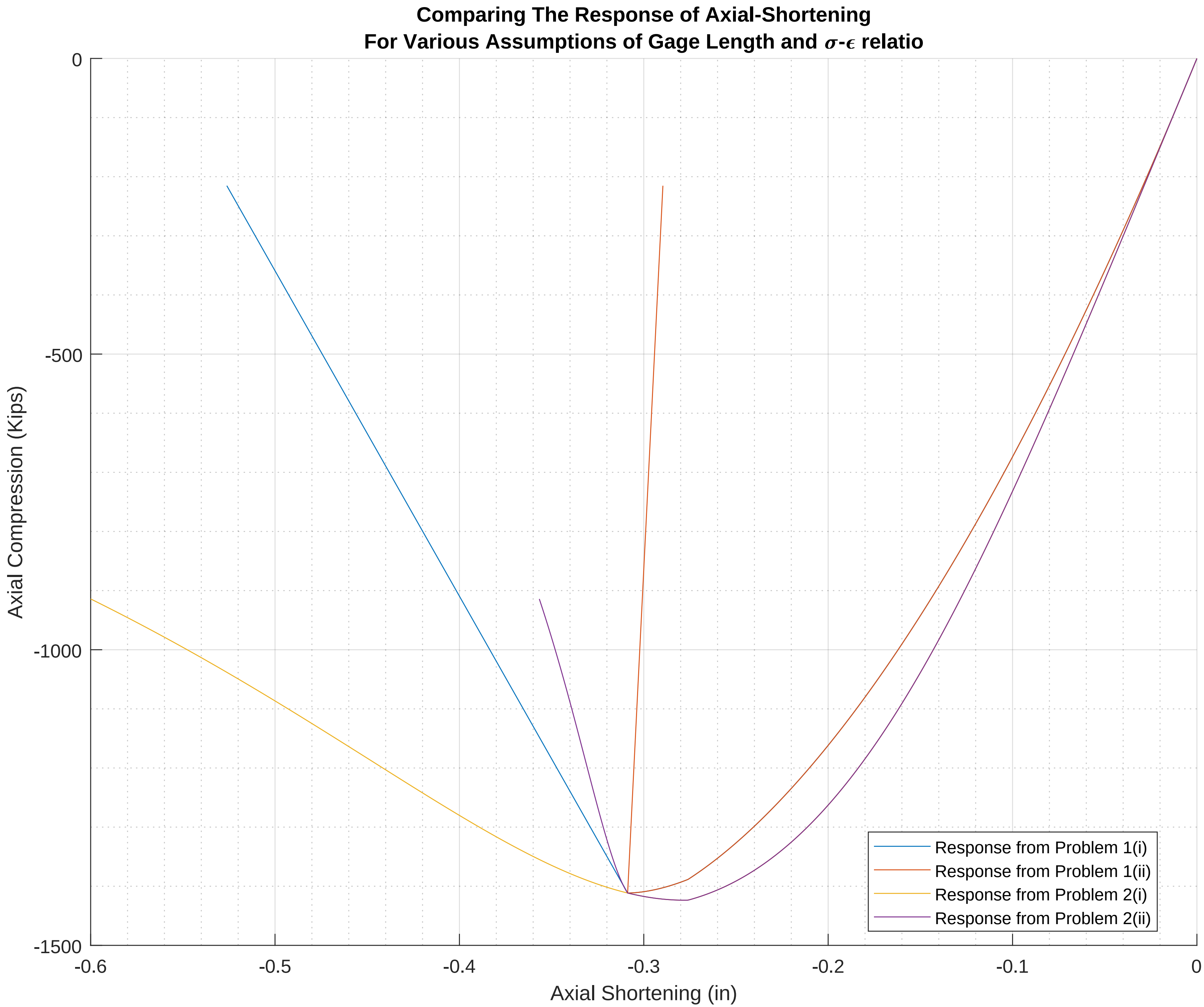
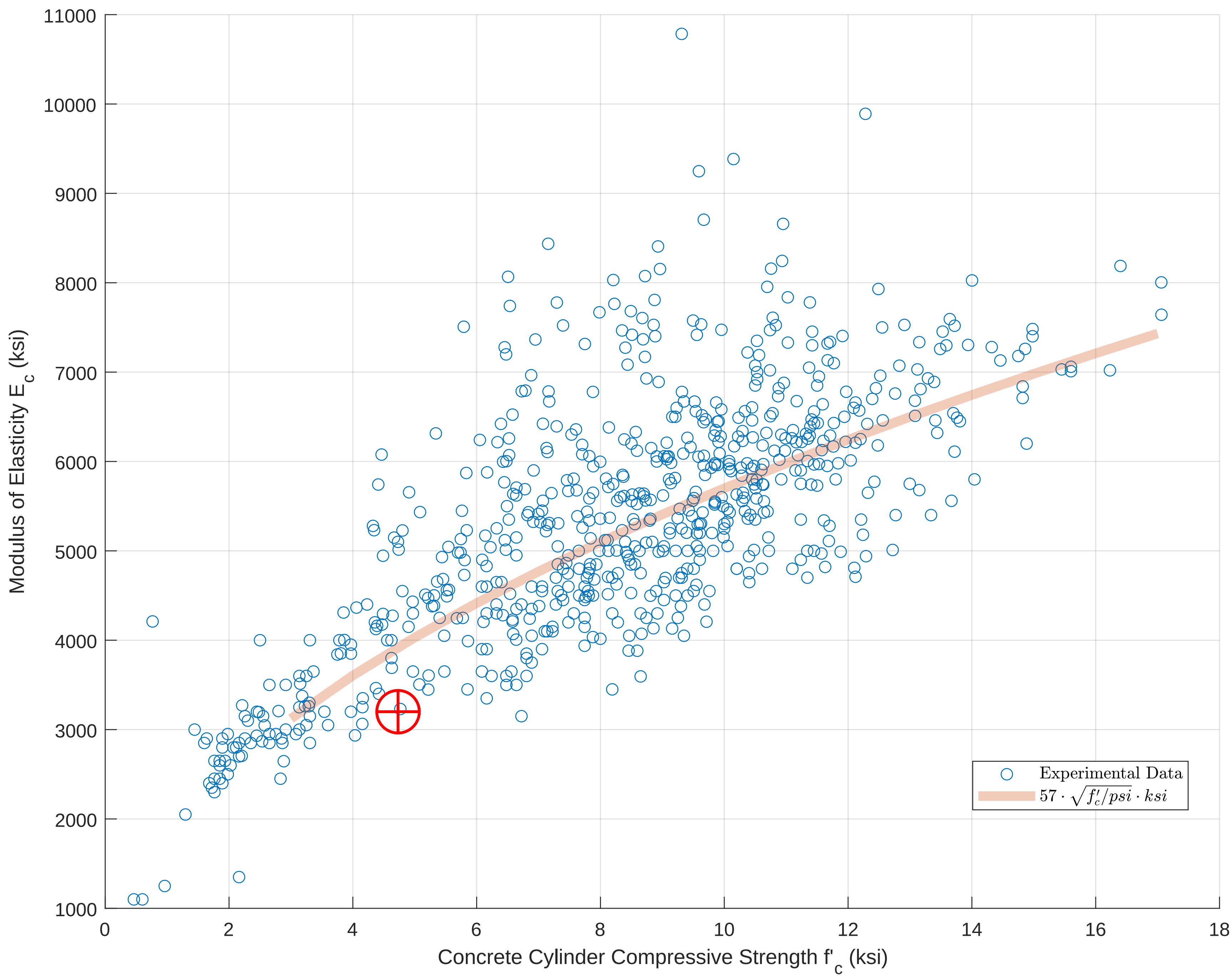
Appendix A. Figures

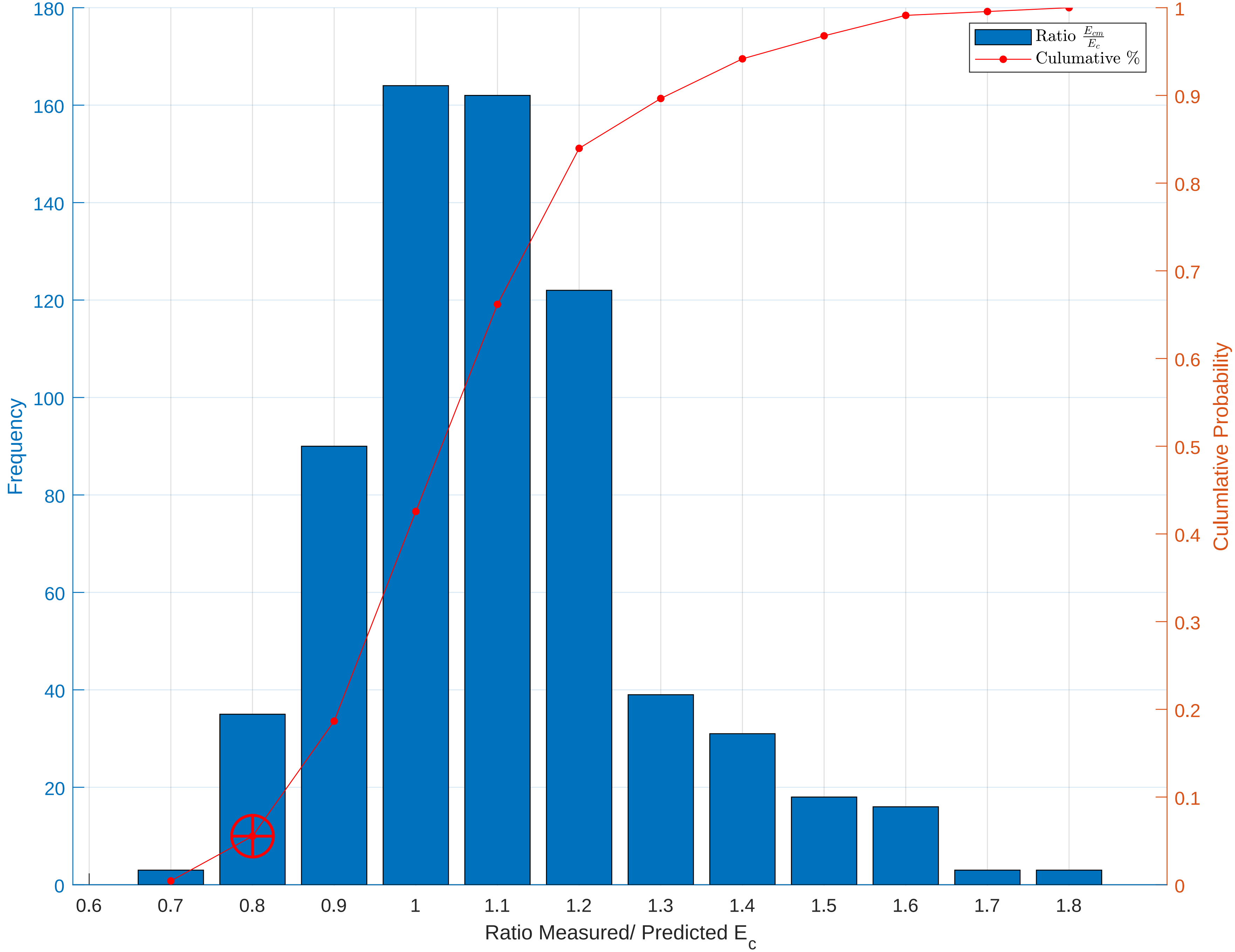


Biased Stress-Strain Curve
Stress (ksi)
Strain (in/in)
$\times 10^{-5}$
0
-50
-100
-150
-200
-250
-300
-8
-7
-6
-5
-4
-3
-2
-1
0
Measured Data
Approximated Linear Line

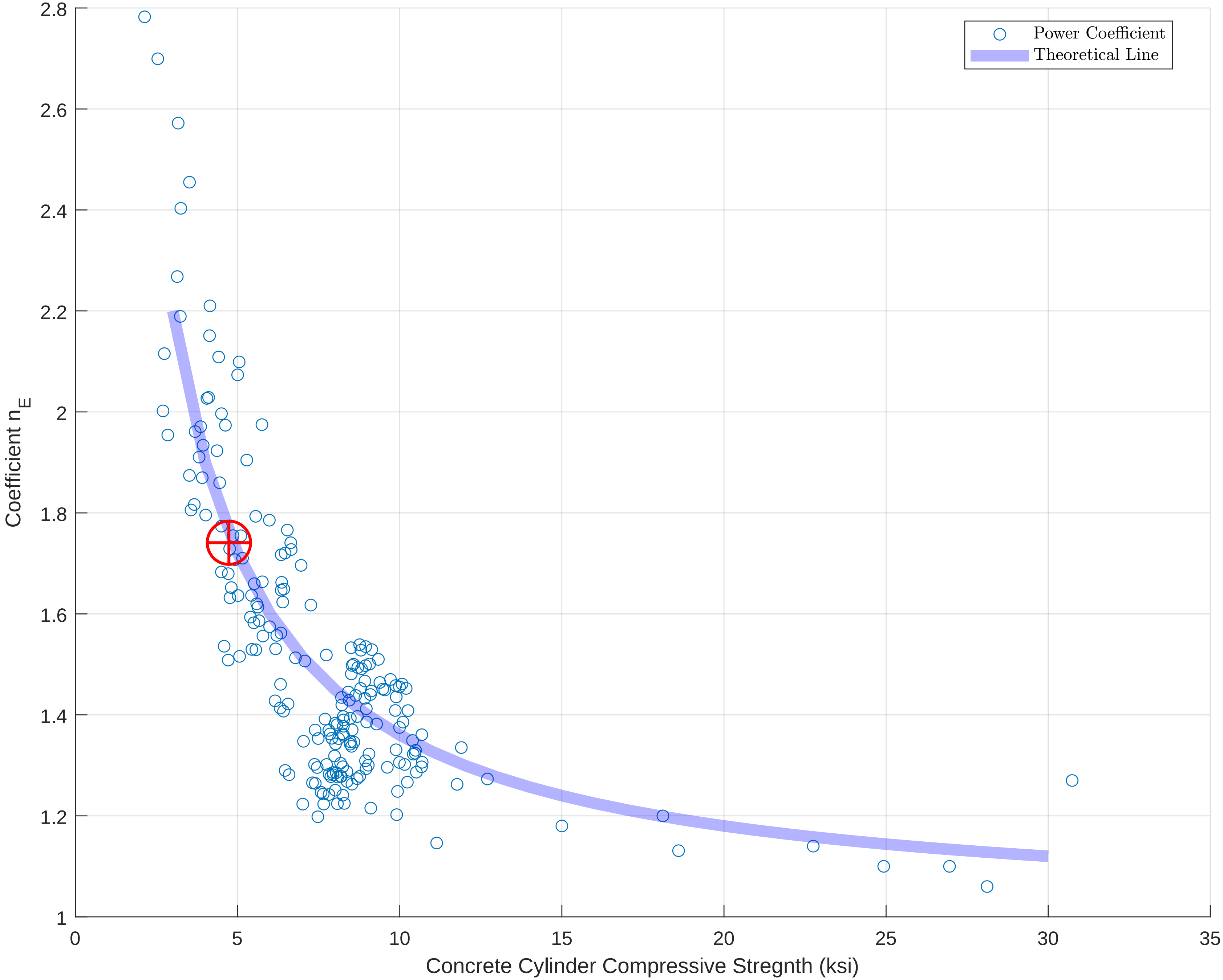

Comparing The Response of Axial-Shortening
For Various Assumptions of Gage Length and σ-ϵ relatio
Axial Compression (Kips)
0
-500
-1000
-1500
Axial Shortening (in)
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
Response from Problem 1(i)
Response from Problem 1(ii)
Response from Problem 2(i)
Response from Problem 2(ii)



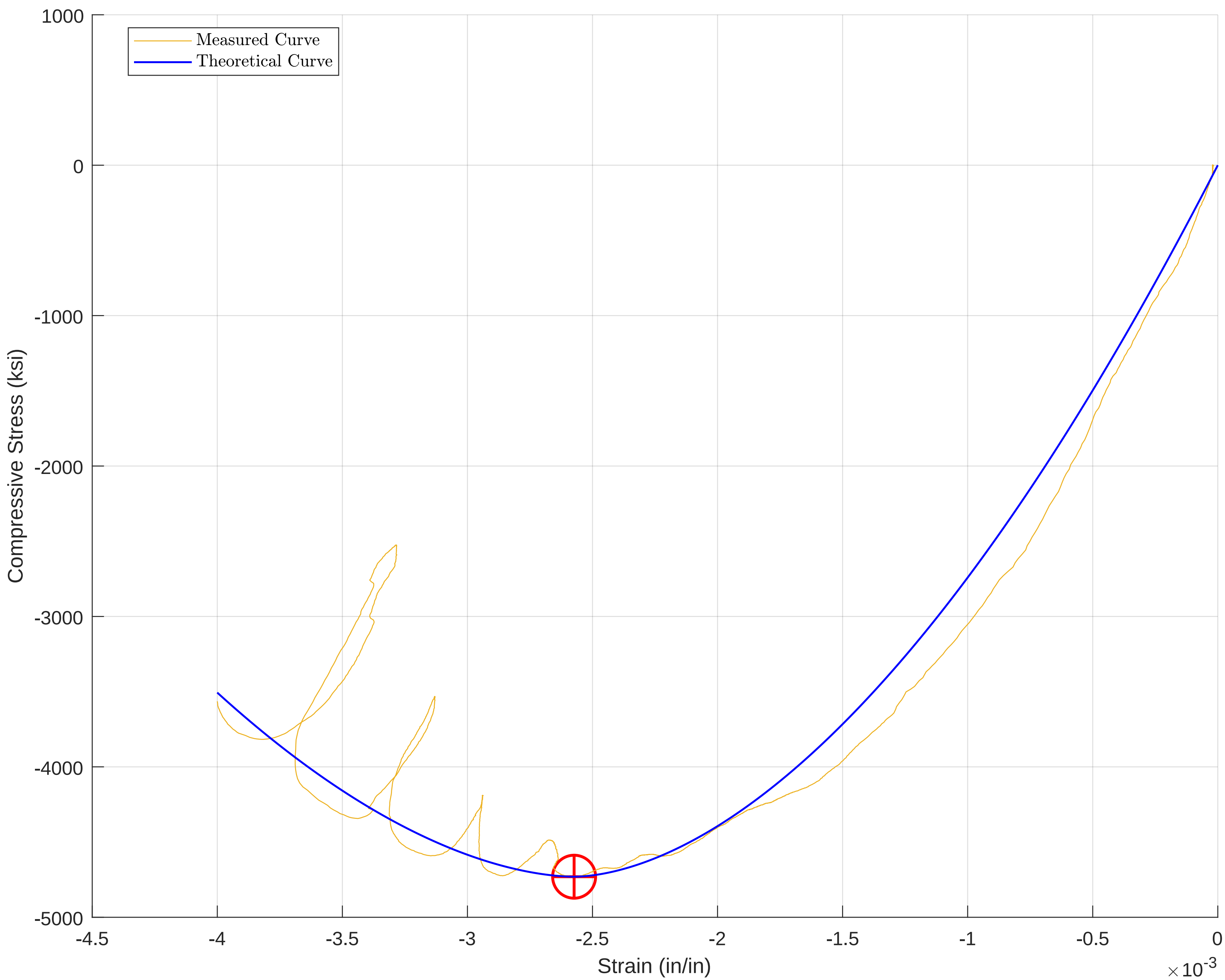
Modulus of Elasticity E_c (ksi)
Concrete Cylinder Compressive Strength f'_c (ksi)
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
2
4
6
8
10
12
14
16
18
Experimental Data
$57 \cdot \sqrt{f'_c/psi} \cdot ksi$



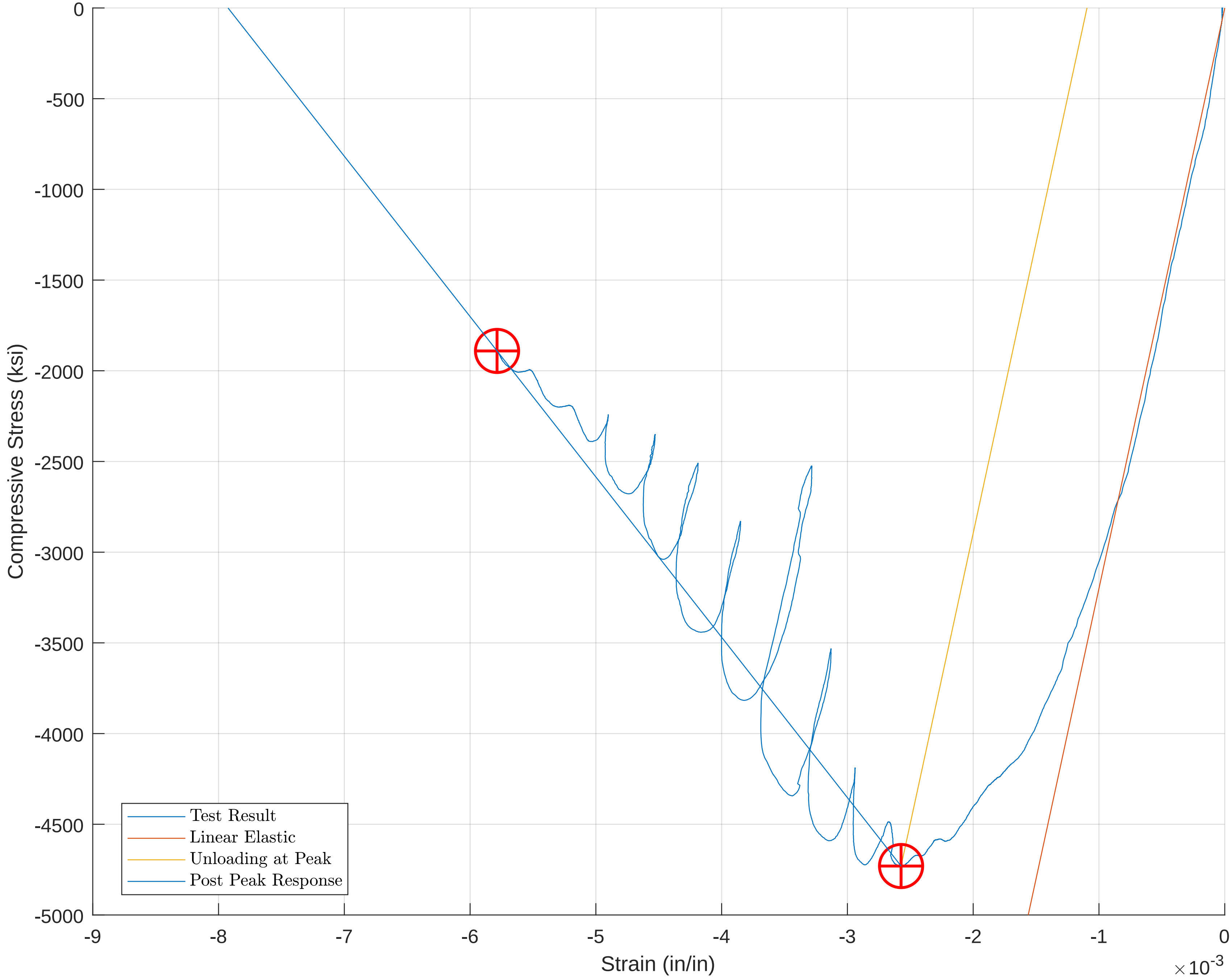
Ratio $\frac{E_{cm}}{E_c}$
Culumative %
Frequency
Culumlative Probability
Ratio Measured/ Predicted E_c
0
20
40
60
80
100
120
140
160
180
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



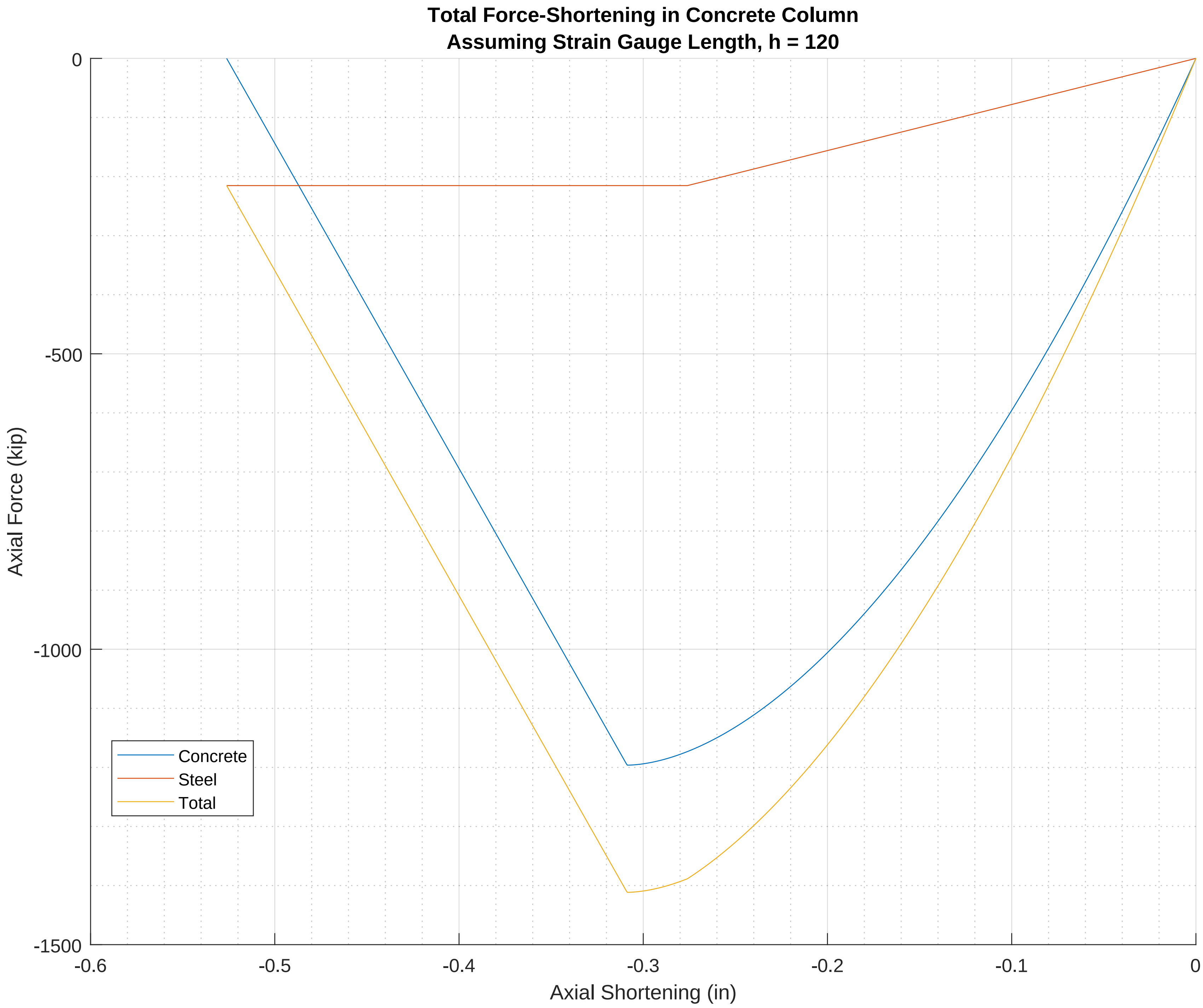
Power Coefficient
Theoretical Line
Coefficient n_E
Concrete Cylinder Compressive Stregnth (ksi)
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0
5
10
15
20
25
30
35



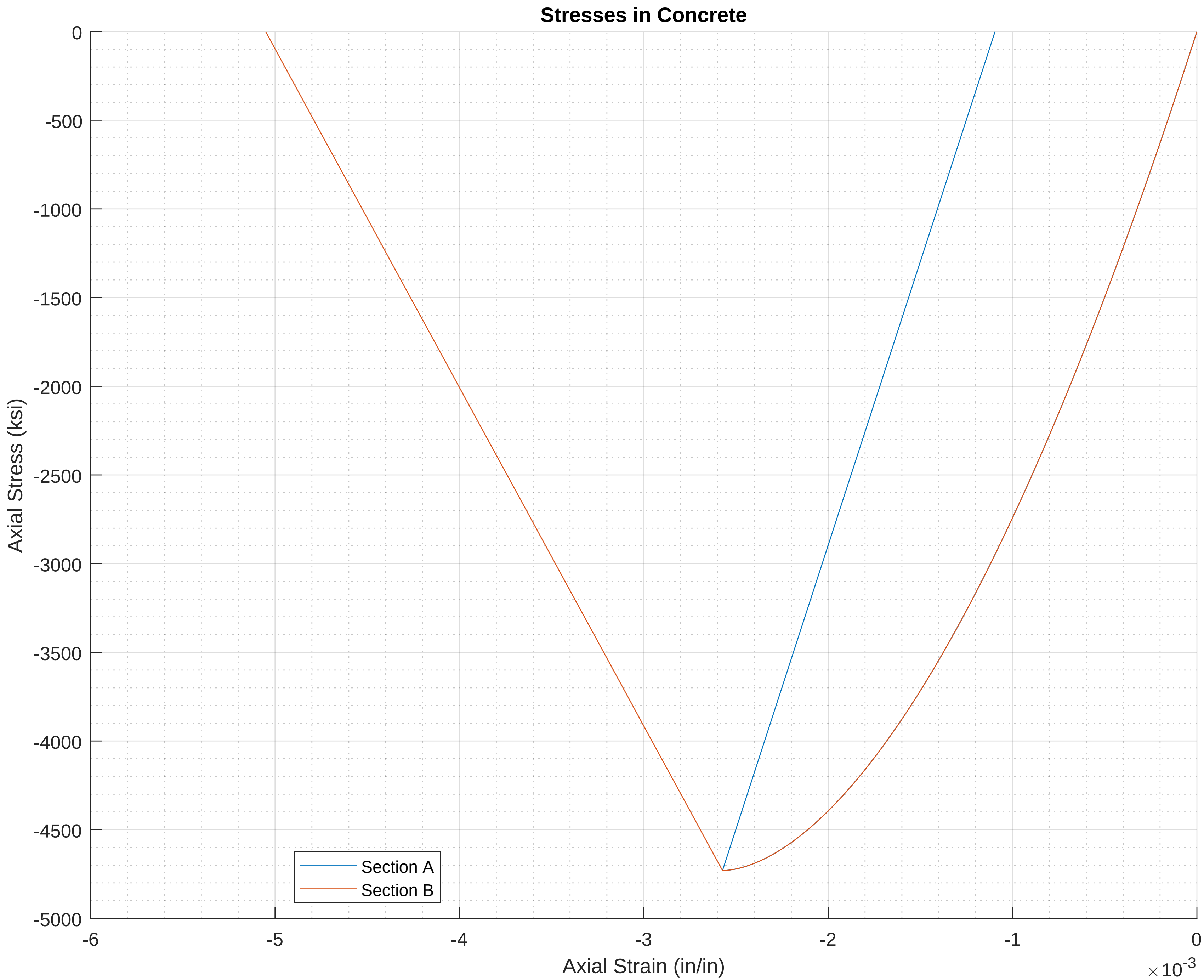
Measured Curve
Theoretical Curve
Compressive Stress (ksi)
1000
0
-1000
-2000
-3000
-4000
-5000
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
Strain (in/in)
×10^-3

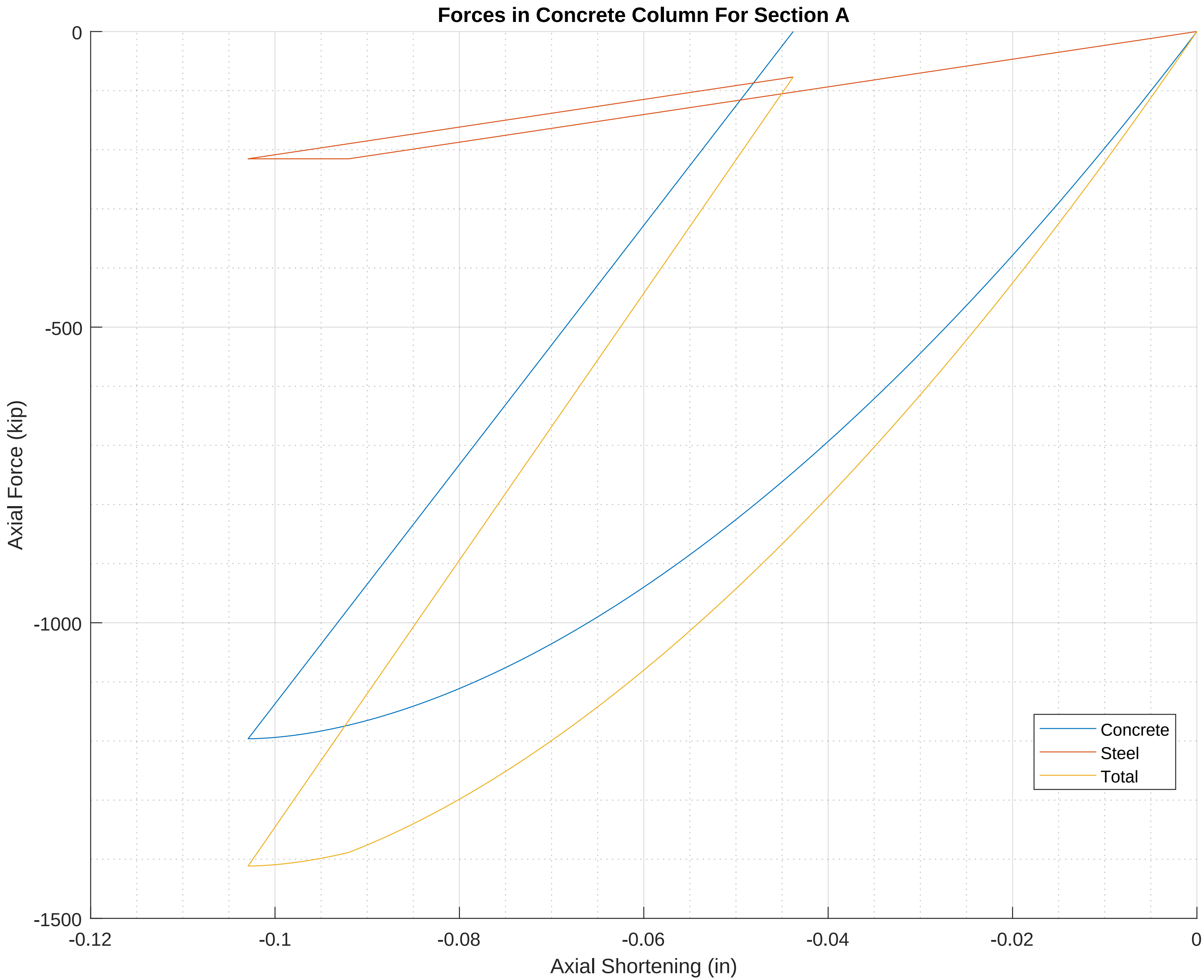


0
-500
-1000
-1500
-2000
-2500
-3000
-3500
-4000
-4500
-5000
Compressive Stress (ksi)
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
Strain (in/in)
$\times 10^{-3}$
Test Result
Linear Elastic
Unloading at Peak
Post Peak Response

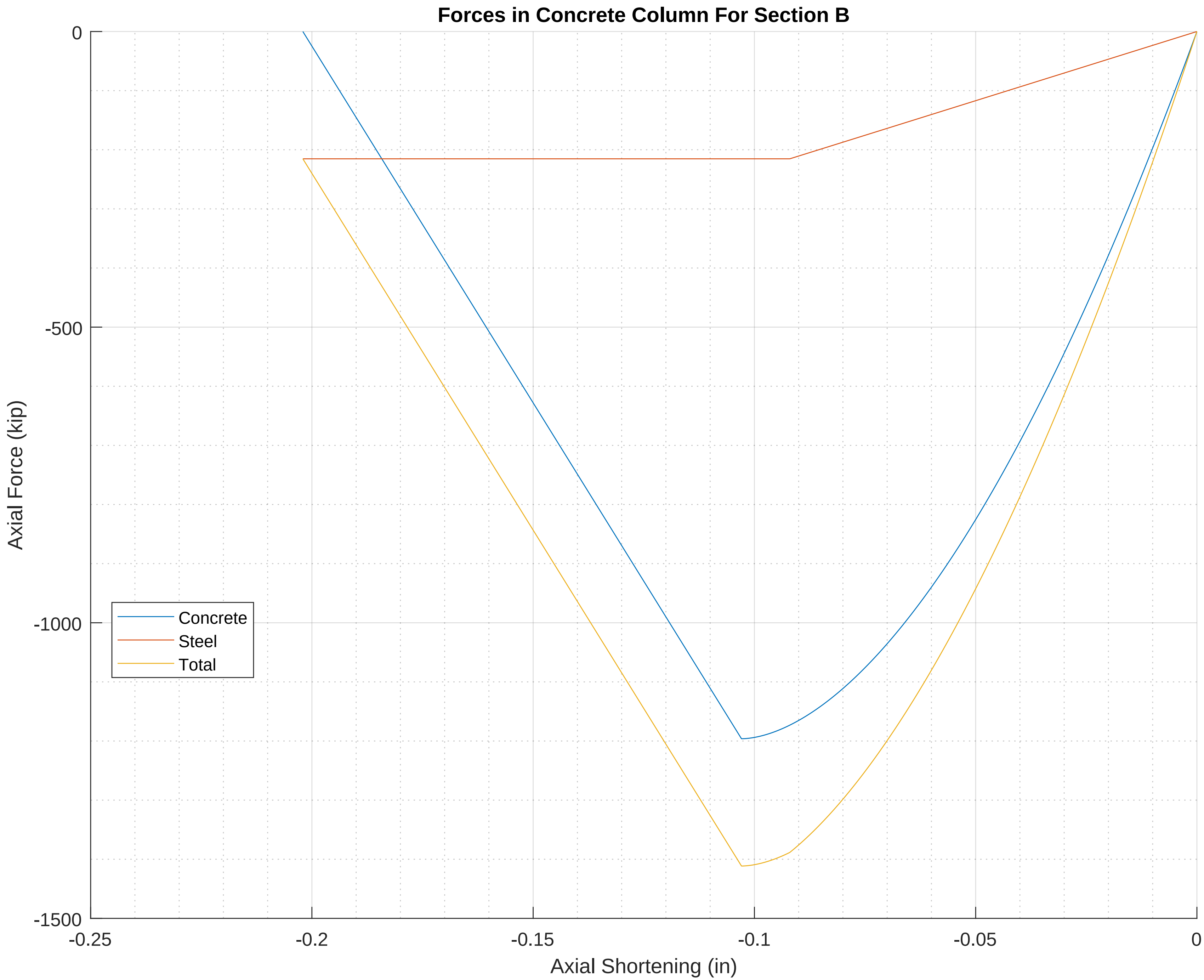


Total Force-Shortening in Concrete Column
Assuming Strain Gauge Length, h = 120
0
-500
-1000
-1500
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
Axial Force (kip)
Axial Shortening (in)
Concrete
Steel
Total

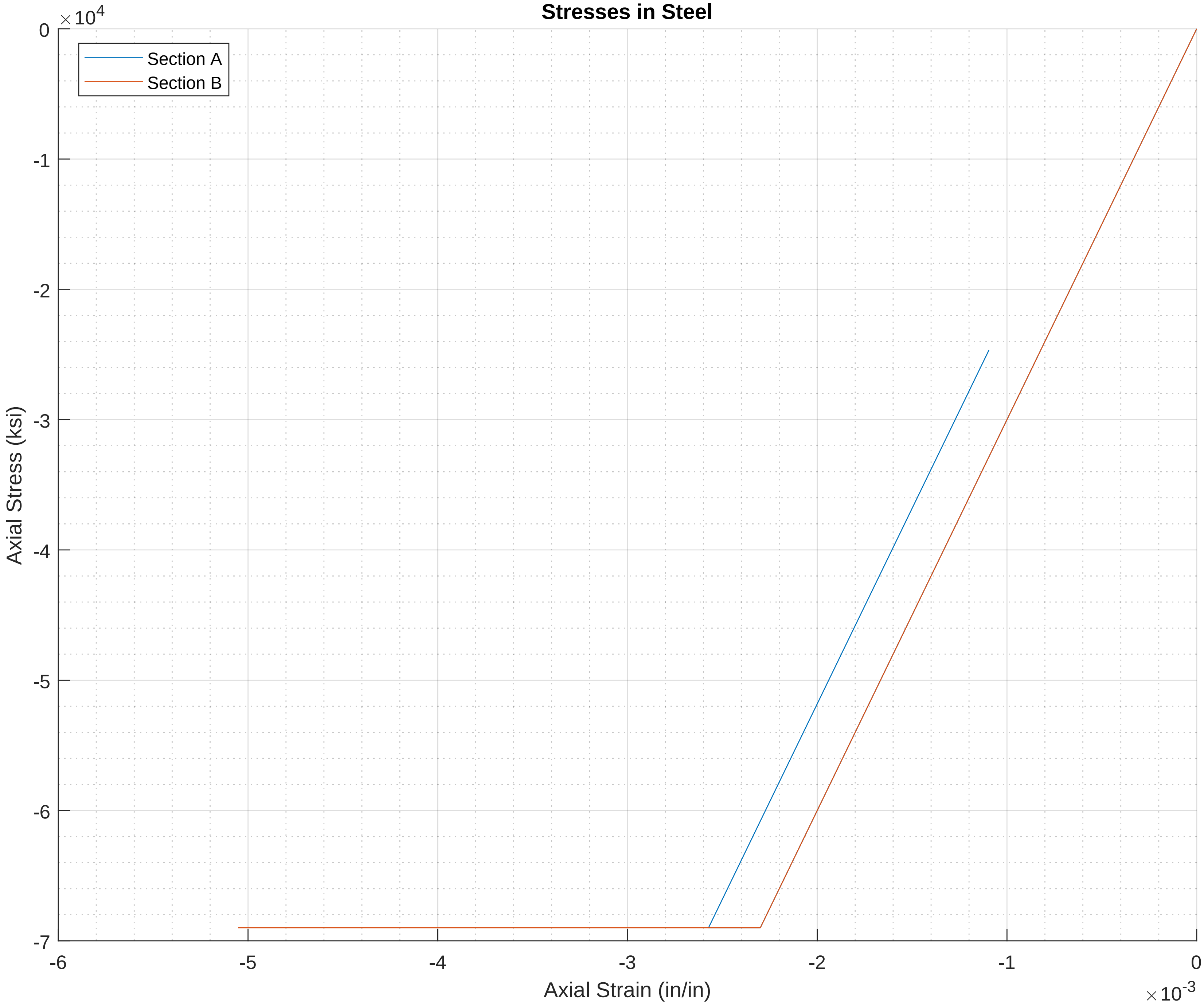

Stresses in Concrete
Axial Stress (ksi)
Axial Strain (in/in)
Section A
Section B
0
-500
-1000
-1500
-2000
-2500
-3000
-3500
-4000
-4500
-5000
-6
-5
-4
-3
-2
-1
0
×10^-3



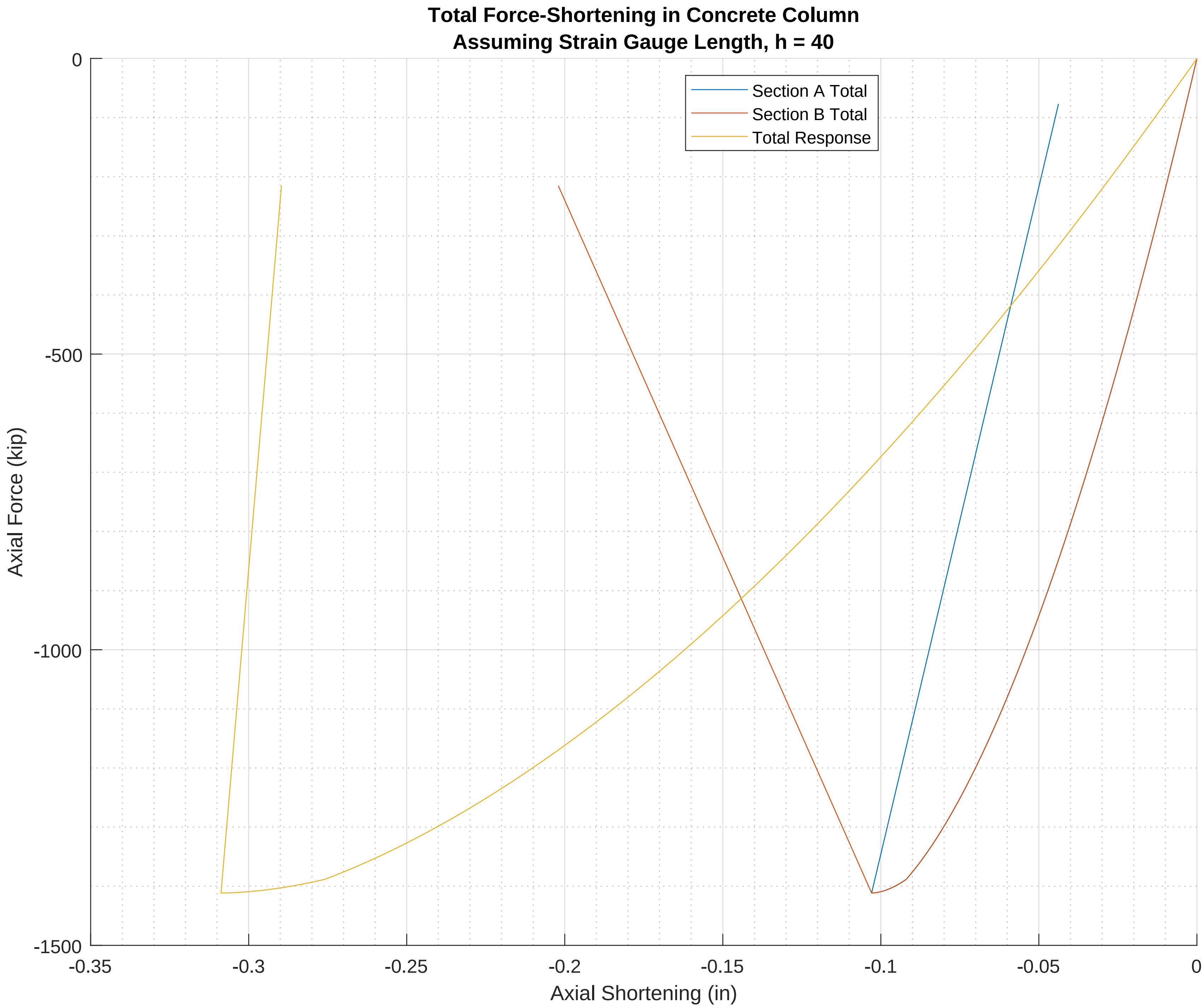
Forces in Concrete Column For Section A
0
-500
-1000
-1500
-0.12
-0.1
-0.08
-0.06
-0.04
-0.02
0
Axial Force (kip)
Axial Shortening (in)
Concrete
Steel
Total

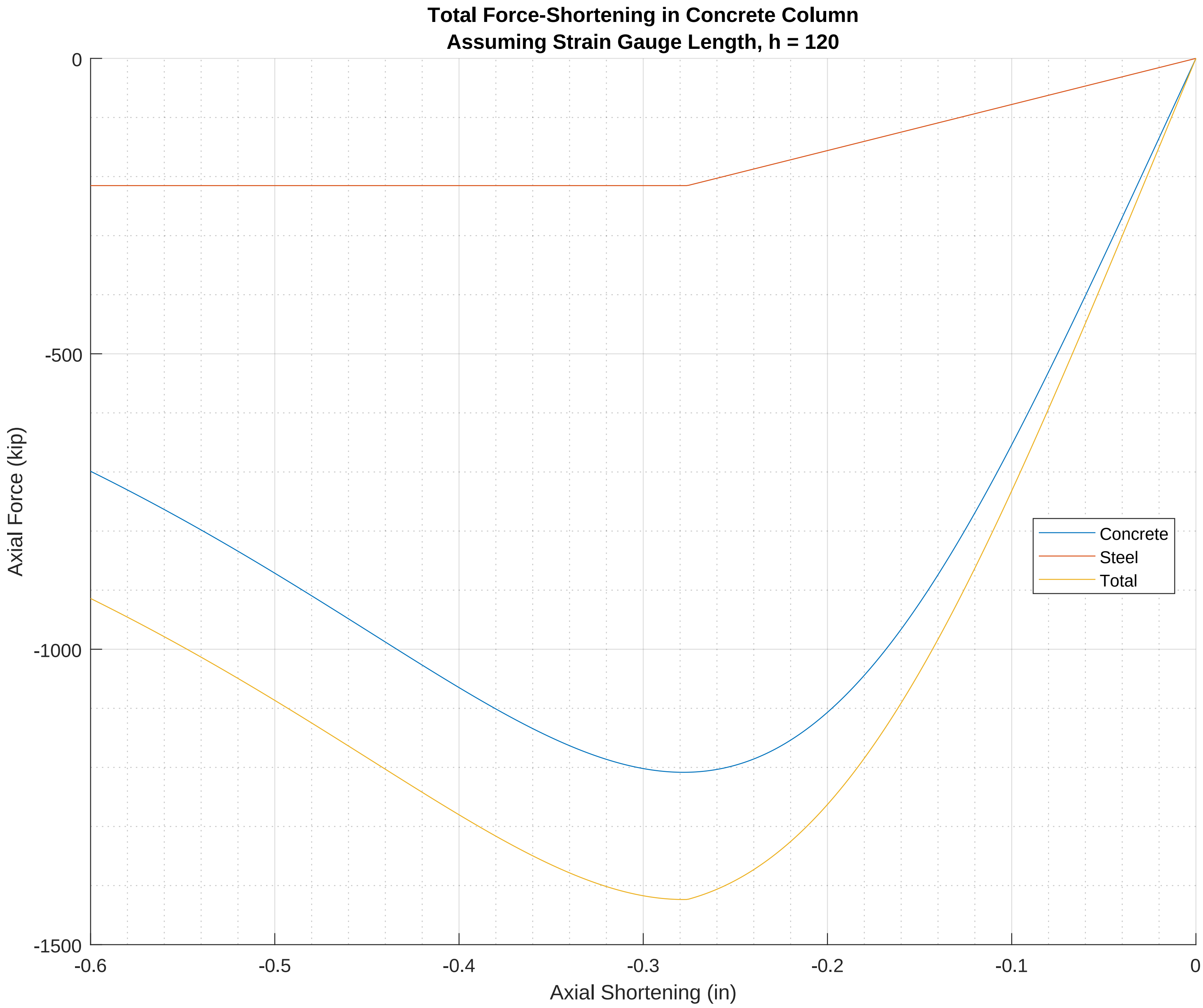


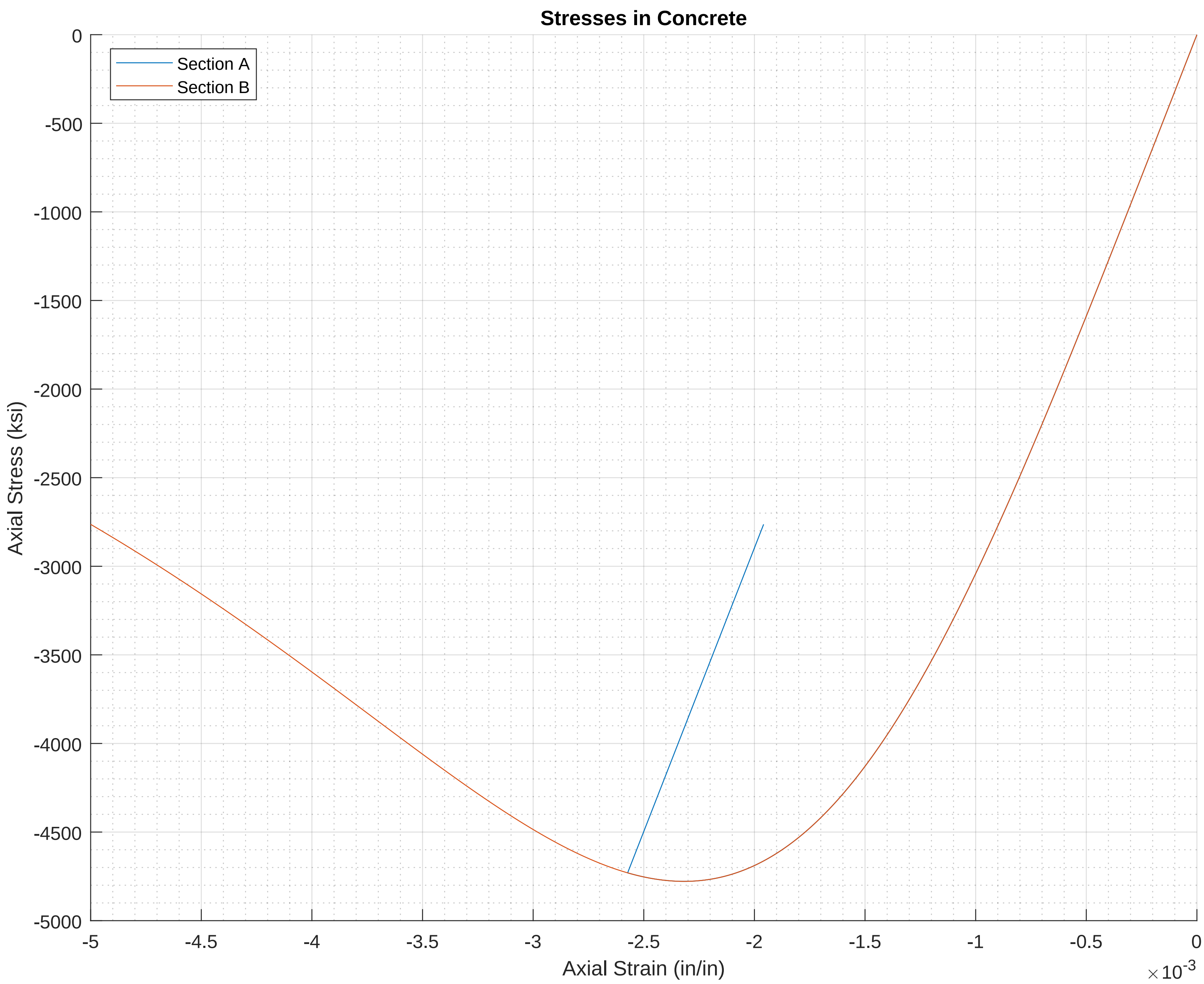
Forces in Concrete Column For Section B
Axial Force (kip)
Axial Shortening (in)
0
-500
-1000
-1500
-0.25
-0.2
-0.15
-0.1
-0.05
0
Concrete
Steel
Total

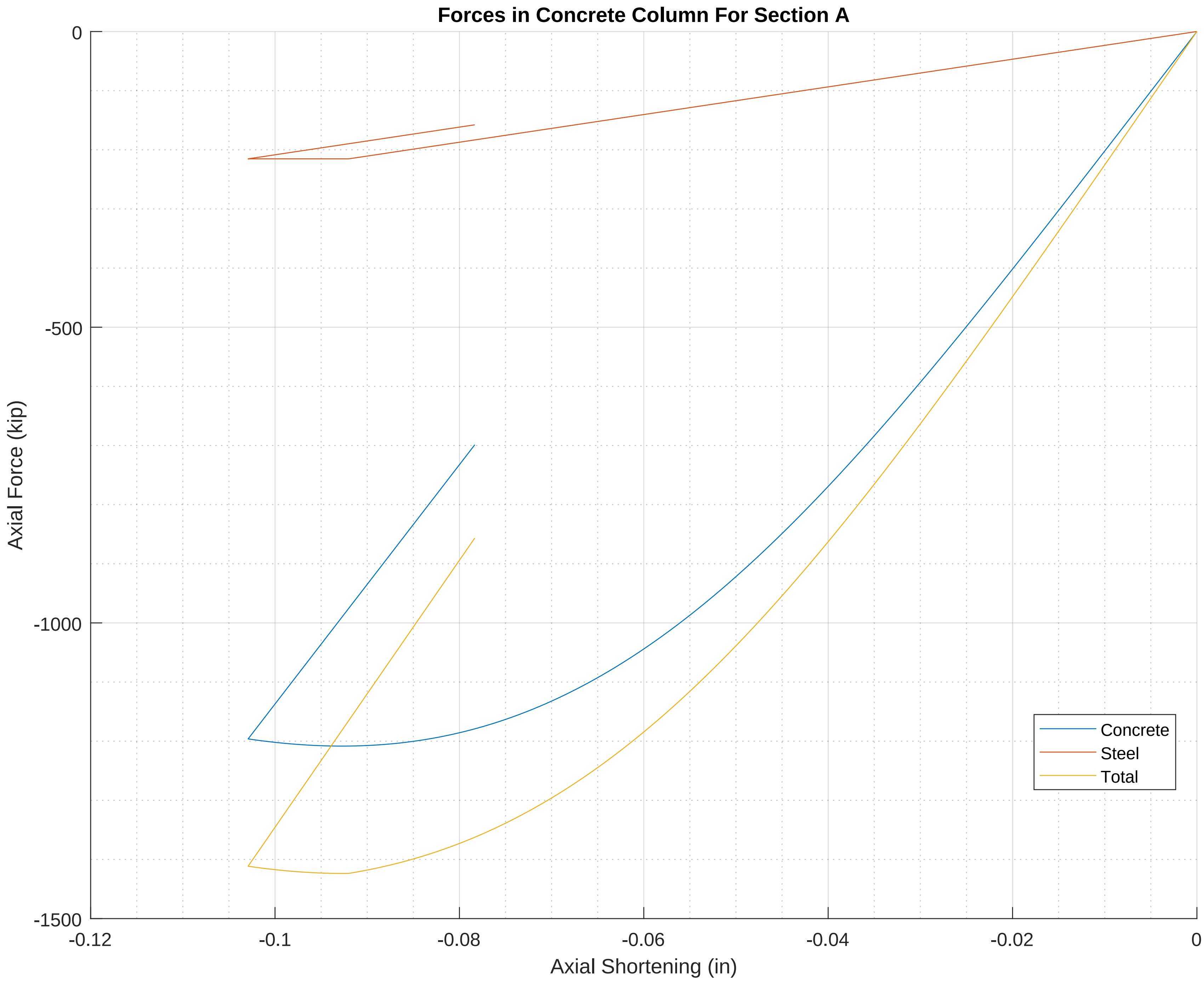


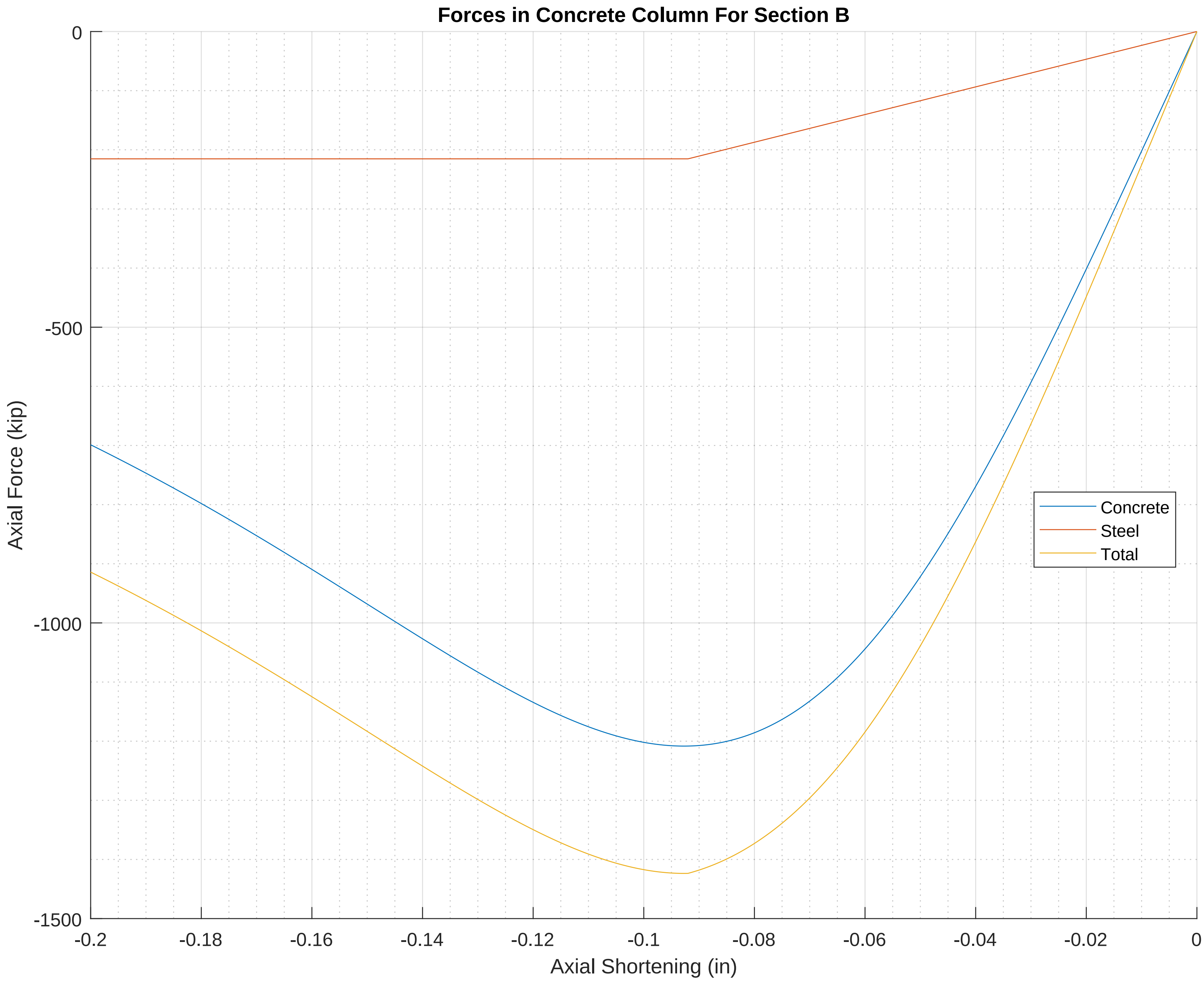
Stresses in Steel
×10^4
0
-1
-2
-3
-4
-5
-6
-7
Section A
Section B
Axial Stress (ksi)
-6
-5
-4
-3
-2
-1
0
Axial Strain (in/in)
×10^-3


Total Force-Shortening in Concrete Column
Assuming Strain Gauge Length, h = 40
Section A Total
Section B Total
Total Response
0
-500
-1000
-1500
Axial Force (kip)
-0.35
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
Axial Shortening (in)

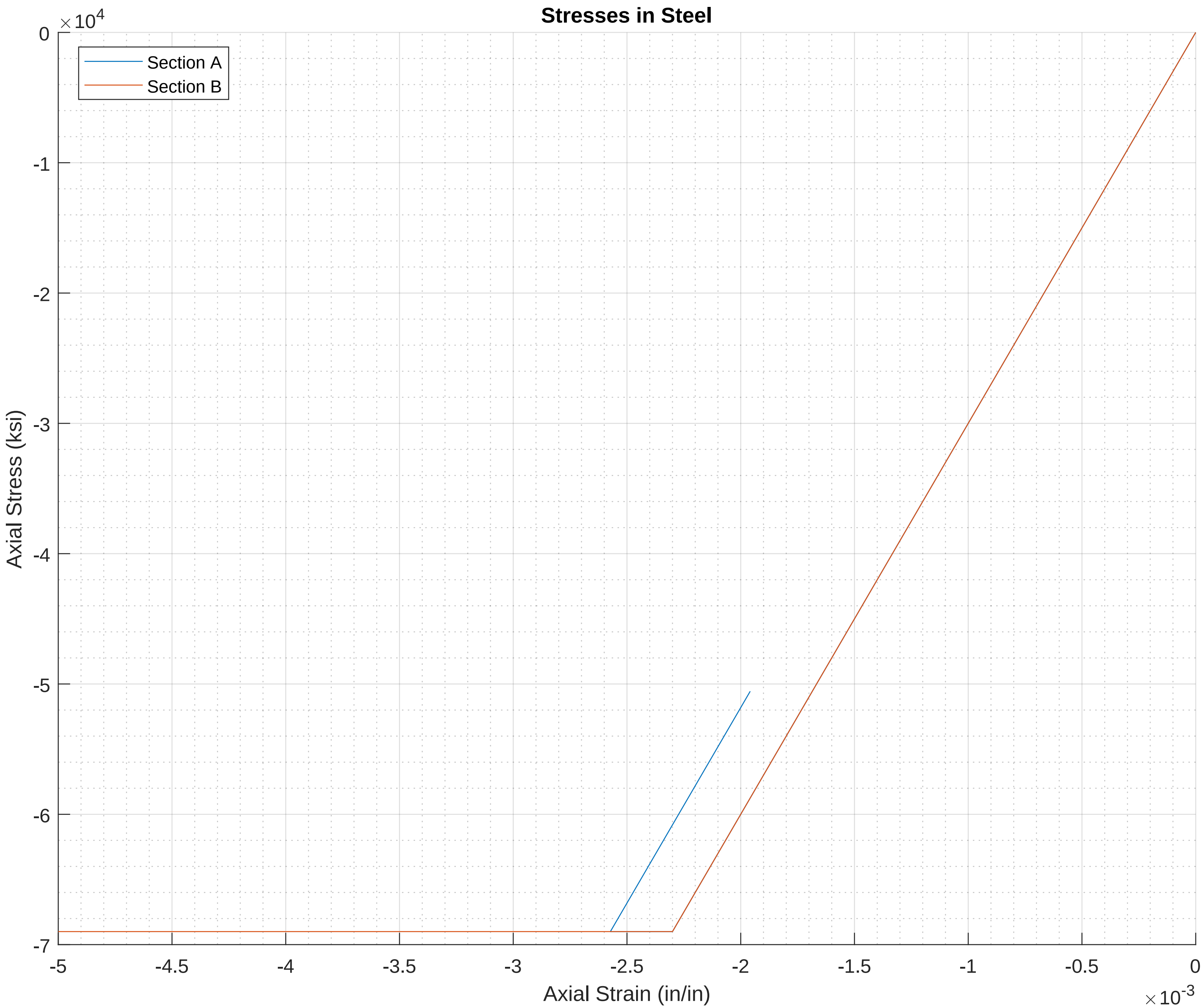

Total Force-Shortening in Concrete Column
Assuming Strain Gauge Length, h = 120
0
-500
-1000
-1500
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
Axial Shortening (in)
Axial Force (kip)
Concrete
Steel
Total

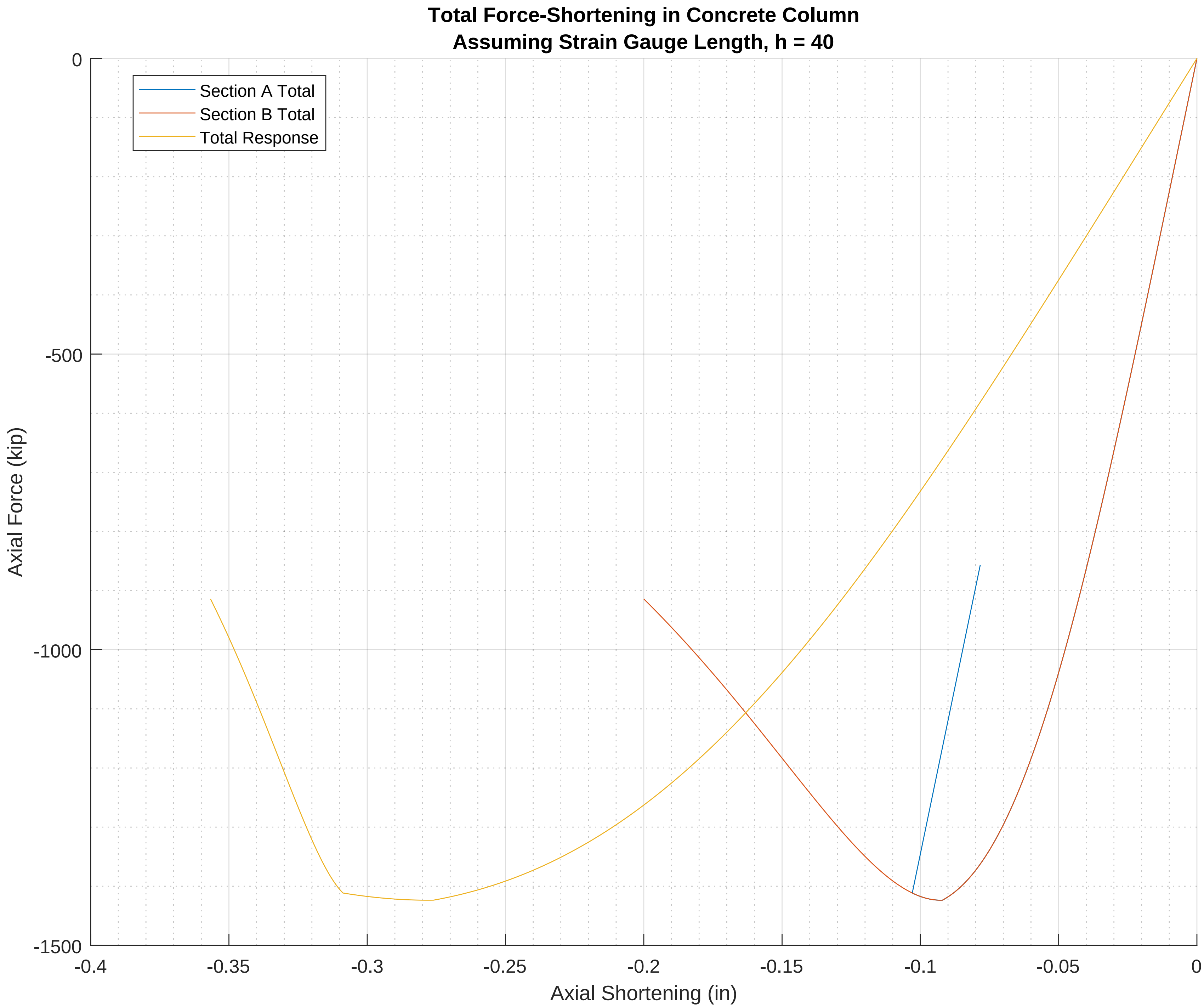

Stresses in Concrete
Section A
Section B
Axial Stress (ksi)
Axial Strain (in/in)
$\times 10^{-3}$
0
-500
-1000
-1500
-2000
-2500
-3000
-3500
-4000
-4500
-5000
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0


Forces in Concrete Column For Section A
Axial Force (kip)
Axial Shortening (in)
0
-500
-1000
-1500
-0.12
-0.1
-0.08
-0.06
-0.04
-0.02
0
Concrete
Steel
Total



Forces in Concrete Column For Section B
0
-500
-1000
-1500
Axial Force (kip)
-0.2
-0.18
-0.16
-0.14
-0.12
-0.1
-0.08
-0.06
-0.04
-0.02
0
Axial Shortening (in)
Concrete
Steel
Total


Stresses in Steel
×10^4
0
-1
-2
-3
-4
-5
-6
-7
Section A
Section B
Axial Stress (ksi)
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
Axial Strain (in/in)
×10^-3


Total Force-Shortening in Concrete Column
Assuming Strain Gauge Length, h = 40
Section A Total
Section B Total
Total Response
0
-500
-1000
-1500
-0.4
-0.35
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
Axial Shortening (in)
Axial Force (kip)

Appendix B. MATLAB Code

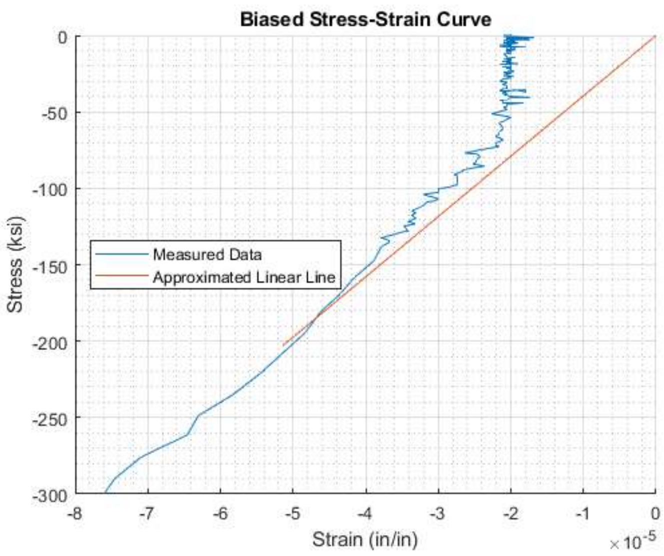
```
clc; clear;
[strain, stress] = get_data();
stress_bias = -stress(1) % set bias as the first stress point
```

```
stress_bias = 32.0573
```

```
Ec = polyfit(strain(250:260),stress(250:260),1); Ec(1)% Find linear line slope
```

```
ans = 3.9481e+06
```

```
strain_bias = -2 * 10^-5;

stress = stress + stress_bias;
strain = strain + strain_bias;

close all; figure; hold on;
plot(strain,stress,"DisplayName","Measured Data");
plot(linspace(0,strain(250)), Ec(1)*linspace(0,strain(250)),"DisplayName","Approximated Linear Line")

title("Biased Stress-Strain Curve");xlabel("Strain (in/in)"); ylabel("Stress (ksi)")
grid on; grid minor; axis([-0.00008 0 -300 0]); legend("Location","West");
print_figure(1)
```




```
[min_stress, min_id] = min(stress)
```

```
min_stress = -4.7302e+03
min_id = 1045
```

```
fc_prime = min_stress
```

```
fc_prime = -4.7302e+03
```

```
ec_prime = strain(min_id)
```

```
ec_prime = -0.0026
```

```
[~, start_id] = min(abs(-5*10^-5 - strain(1:min_id)))
```

```
start_id = 250
```

```
[~, end_id] = min(abs(0.4*fc_prime - stress(1:min_id)))
```

```
end_id = 339
```

```
coef = polyfit(strain(start_id:end_id),stress(start_id:end_id),1);
Ec_test = round(coef(1)/5,-4) *5 % Rounding the elastic modulus
```

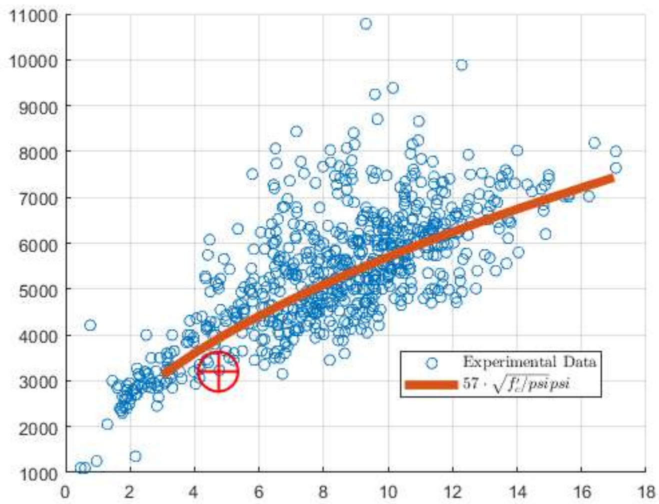
```
Ec_test = 3200000
```

```
Ec_test/1000 % Ksi
```

```
ans = 3200
```

```
close all; hold on;
[fc, E_c] = get_Modulus_data();
scatter(fc,E_c,"Displayname","Experimental Data");
Ec = @(fc) 57*sqrt(fc*1000); range = 3:17;
plt = plot(range,Ec(range), 'LineWidth',5,"DisplayName","$57 \cdot \sqrt{f'_c/psi}psi$"); plt.Color(4) = 0.3;
```

```
plot_point(-fc_prime/1000, Ec_test/1000)
```




```
figure();close all; grid on; hold on; xlabel("Ratio Measured/ Predicted E_c");
% Histogram
expected_Ec = Ec(fc);
ratio_expected = E_c./expected_Ec;
mean(ratio_expected)
```

```
ans = 1.0570
```

```
median(ratio_expected)
```

```
ans = 1.0387
```

```
std(ratio_expected)
```

```
ans = 0.1966
```

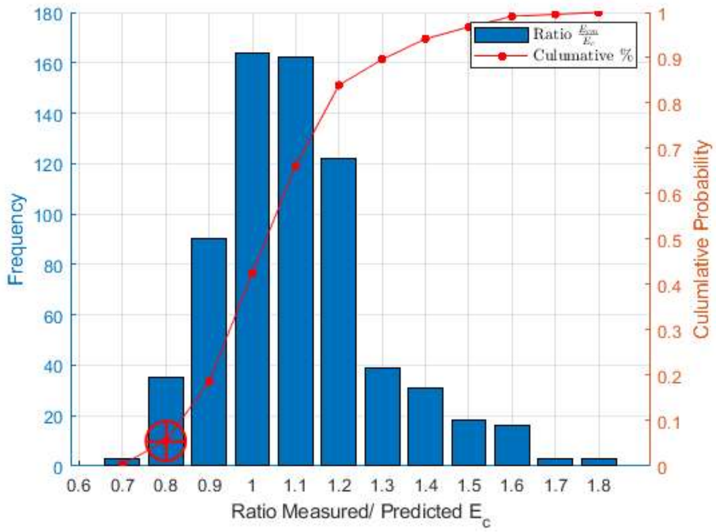
```
range = 0.6:0.1:1.8;
[counts, bins]= histcounts(ratio_expected,range);
bins = bins(2:end);
yyaxis left; ylabel("Frequency");
bar(bins, counts,"DisplayName","Ratio $\frac{E_{cm}}{E_c}$");

% CDF
yyaxis right; ylabel("Culumlative Probability")
cdf = cumsum(counts) / sum(counts);
plot(bins, cdf,'r',"Marker",".", "MarkerSize",12,"DisplayName","Culumative \%");
xticks(range);
Ec = @(fc) 57000*sqrt(fc);
Ec(-fc_prime)
```

```
ans = 3.9203e+06
```

```
test_ratio = Ec_test/Ec(-fc_prime)
```

```
test_ratio = 0.8163
```

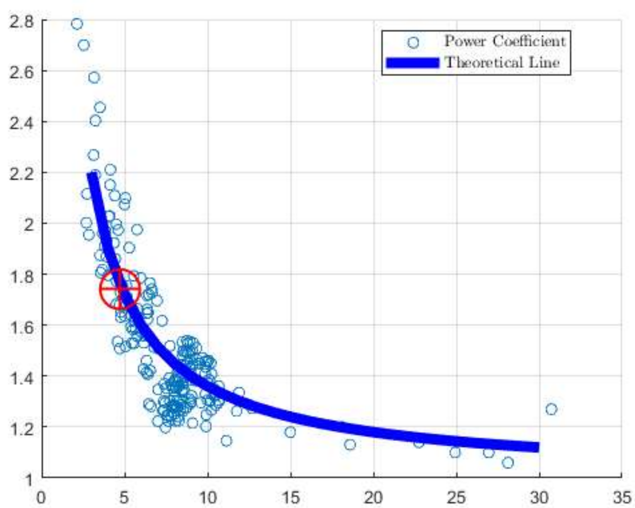
```
discretize(test_ratio, range);
plot_point(range(ans), cdf(ans-1));
print_figure(3)
```



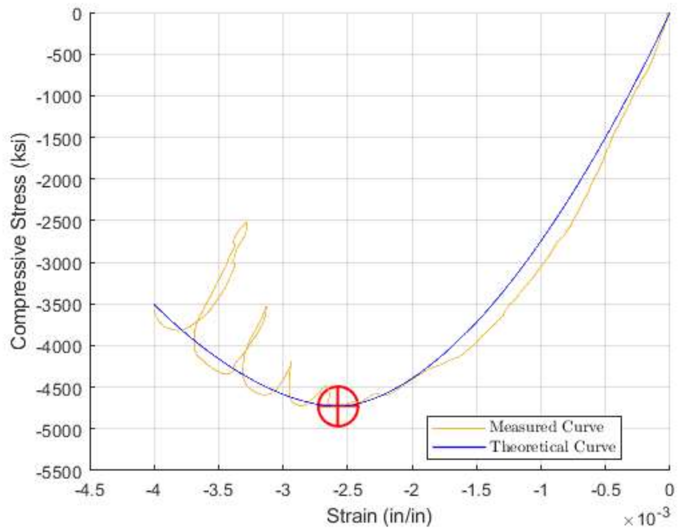

```
close all; hold on;
n_test = Ec_test/(fc_prime/ec_prime)
```

```
n_test = 1.7409
```

```
[fc, nE] = get_nE_data();
scatter(fc,nE,'DisplayName','Power Coefficient');
range = 3:30;
nE = @(f) 1+3.6./(f); % fc is in ksi
pl = plot(range,nE(range),'b','linewidth',6,'DisplayName','Theoretical Line');
plot_point(-fc_prime/1000, n_test);
```




```
close all; hold on;
xlabel("Strain (in/in)"); ylabel("Compressive Stress (ksi)");
plot_point(strain(min_id),fc_prime)
[~, end_id] = min(abs(-4*10^-3 - strain));

plot(strain(1:end_id), stress(1:end_id),"DisplayName","Measured Curve");
nE = 1 - 3.6/fc_prime*1000;
range = linspace(strain(end_id),0,end_id);
fc = @(ec) real(fc_prime*(1-(1-abs(ec./ec_prime)).^nE)); % fc is in ksi
plot(range,fc(range),'Color',[0, 0, 1],'linewidth',1,'DisplayName','Theoretical Curve');
print_figure(5); ylim([-5500 0])
```




```
close all; hold on;  axis([-9*10^-3 0 -5000 0]);
[~, end_id] = min(abs(0.4*fc_prime - stress(min_id:end))); end_id = end_id + min_id% Finding where stress = 0.4*f'c after the peak
```

```
end_id = 3513
```

```
eta = (stress(end_id) - fc_prime)/(strain(end_id)-ec_prime)/ Ec_test % Find the eta value
```

```
eta = -0.2762
```

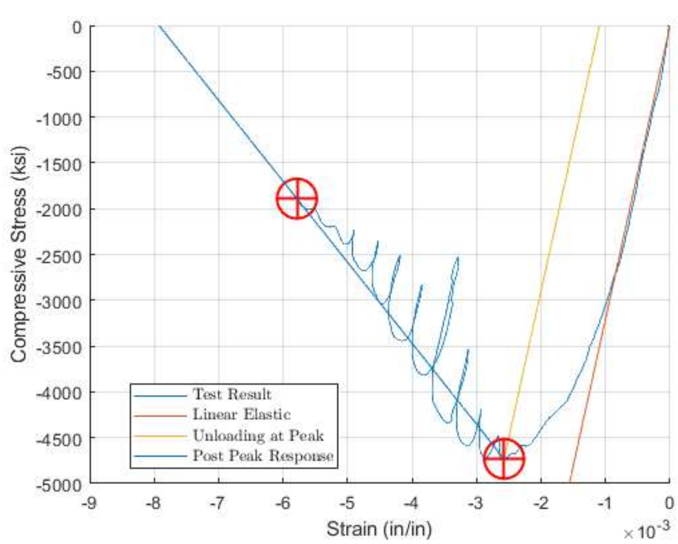
```
linear_elastic = @(ec, e, f) Ec_test*(ec-e) +f; % Function for linear line
post_peak = @(ec) eta*Ec_test*(ec-ec_prime) +fc_prime; % Function for post peak

linear_elastic_range = linspace(strain(min_id),0,100); % Linear elastic straings
post_peak_range = linspace(strain(min_id),ec_prime - fc_prime/eta/Ec_test,100); % Post Preak Range

plot(strain(1:end_id), stress(1:end_id),"DisplayName","Test Result"); % Test data
plot(linear_elastic_range,linear_elastic(linear_elastic_range,0,0),"DisplayName","Linear Elastic"); % Linear Elastic
plot(linear_elastic_range,linear_elastic(linear_elastic_range,ec_prime,fc_prime),"DisplayName","Unloading at Peak"); %Unloading
plot_point(strain(min_id),fc_prime); plot_point(strain(end_id),stress(end_id)); % Big Red Points

plot(post_peak_range,post_peak(post_peak_range),"DisplayName","Post Peak Response");
xlabel("Strain (in/in)"); ylabel("Compressive Stress (ksi)")
e_cu = fc_prime * (1/eta/Ec_test + 1/-Ec_test)
```

```
delta_ppk = 0.0068
```

```
delta_ppk = e_cu * 8
```

```
ans = 0.0546
```

```
print_figure(6)
```

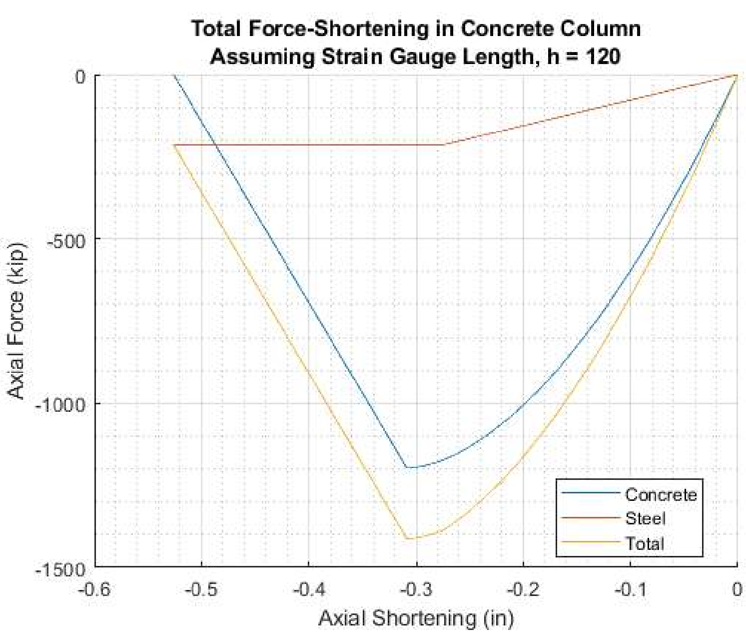



```
function [strain, stress] = get_data()
    opts = spreadsheetImportOptions("NumVariables", 2);
```

```matlab
    opts.Sheet = "Cylinder test data"; % Specify sheet and range
    opts.DataRange = "A6:B8491";
    opts.VariableNames = ["Strain", "Stress"]; % Specify column names and types
    opts.VariableTypes = ["double", "double"];
    tbl = readtable("C:\Users\Louis Lin\Workspace\Academic\UCSD\SE 211\Homework\HW 2\data\SE211 HW2 2021 Students.xlsx", opts, "UseExcel",
    strain = tbl.Strain; stress = tbl.Stress;
end

function [fc, Ec] = get_Modulus_data()
    opts = spreadsheetImportOptions("NumVariables", 2);
    opts.Sheet = "Ec (NCHRP 496)";
    opts.DataRange = "B10:C698";
    opts.VariableNames = ["fc1", "Ec"];
    opts.VariableTypes = ["double", "double"];
    tbl = readtable("C:\Users\Louis Lin\Workspace\Academic\UCSD\SE 211\Homework\HW 2\data\SE211 HW2 2021 Students.xlsx", opts, "UseExcel",
    fc = tbl.fc1; Ec = tbl.Ec;
end

function [fc, nE] = get_nE_data()
    opts = spreadsheetImportOptions("NumVariables", 2);
    opts.Sheet = "nE";
    opts.DataRange = "C4:D212";
    opts.VariableNames = ["fc1", "nE"];
    opts.VariableTypes = ["double", "double"];
    tbl = readtable("C:\Users\Louis Lin\Workspace\Academic\UCSD\SE 211\Homework\HW 2\data\SE211 HW2 2021 Students.xlsx", opts, "UseExcel",
    fc = tbl.fc1; nE = tbl.nE;
end

function plot_point(x,y)
    scatter(x, y,500,'red','+','LineWidth',1.5,'HandleVisibility',"off");
    scatter(x, y,500,'red','o','LineWidth',1.5,'HandleVisibility',"off");
    legend('Interpreter',"latex",'Location',"best"); grid on;
end

function print_figure(no)
    % Saves the figures in a consistent manner
    orient(gcf,'landscape');
    folder = '..\figures\';
    name = 'Figure' +string(no);
    print(folder+name,'-dpdf','-fillpage','-PMicrosoft Print to PDF','-r600','-painters')
    print(folder+name,'-djpeg','-PMicrosoft Print to PDF','-r600','-painters')
end
```

```
clear; clc;

As = 4*0.78; % in^2
Ac = 16*16- As; % in^2
fy = 69000; % psi
Es = 30000000; % psi

Ec = 3200000; % psi from Part I
ec_prime = -0.002573; % in/in from Part I
fc_prime = 4730.237; % psi from Part I
nE = 1.761; % From Part I
```

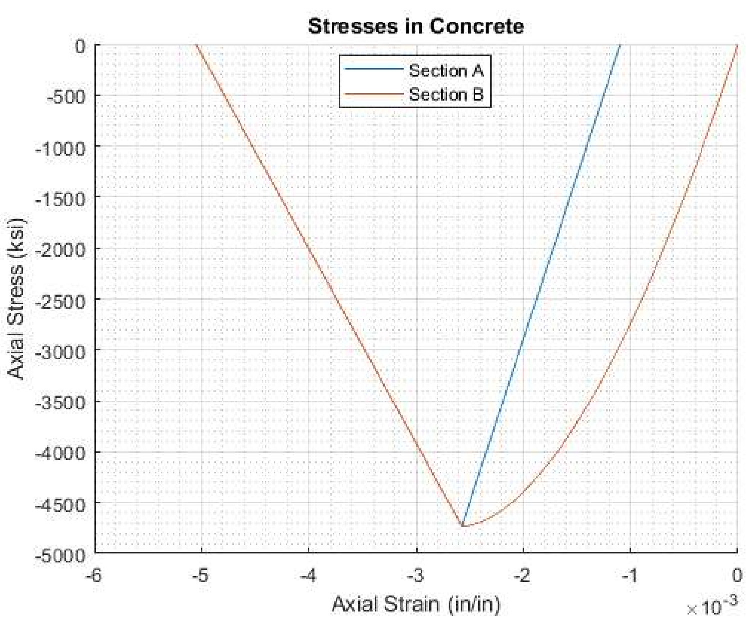
```
% With a single integration point
h = 120; % in
eta = 1/(1+0.04/(abs(fc_prime)/Ec*h));
max_ec = -fc_prime/eta/Ec + ec_prime;

% Concrete
pre_pk_concrete = @(e) -fc_prime*(1-(1-e/ec_prime).^nE);
post_pk_concrete = @(e) -eta*Ec*(e-ec_prime) - fc_prime;
f_concrete = @(e) (e <= ec_prime).*post_pk_concrete(e) + ~(e <= ec_prime).*pre_pk_concrete(e);
% Steel
f_steel = @(e) sign(e).*min(Es*abs(e),fy);

% Compute axial compressive force vs axial displacement
range = linspace(max_ec,0,1000);
concrete_axial_compression = Ac*f_concrete(range)/1000;
steel_axial_compression = As*f_steel(range)/1000;
Total_shortening1 = range*h;
Total_force1 = concrete_axial_compression + steel_axial_compression;

% Plotting
title = ["Total Force-Shortening in Concrete Column","Assuming Strain Gauge Length, h = 120"];
x = [Total_shortening1;Total_shortening1;Total_shortening1];
y = [concrete_axial_compression; steel_axial_compression; Total_force1];
names = ["Concrete"; "Steel"; "Total"];
print_plots(x, y, title, names, true)
print_figure("7")
```



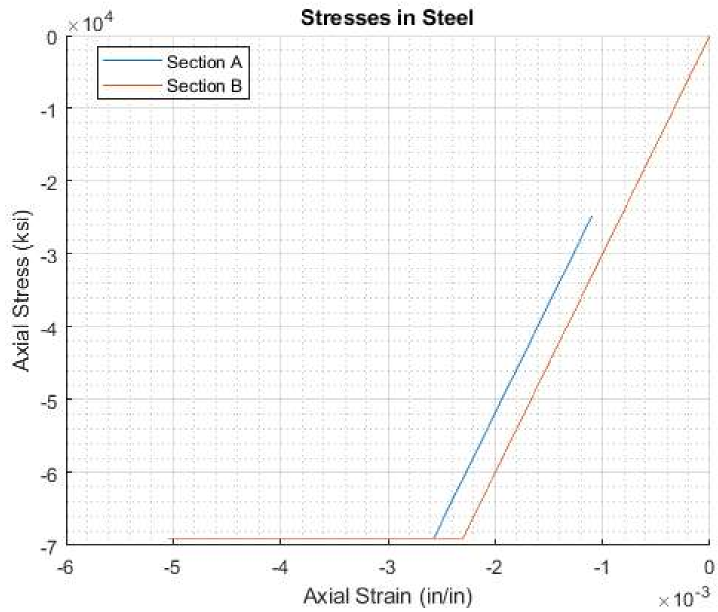
Part (ii) Using 3 Equal Integration Points

```
% With 3 equal Integration points
h = 40;
eta = 1/(1+0.04/(fc_prime/Ec*h)); %
max_ec = -fc_prime/eta/Ec + ec_prime;
```

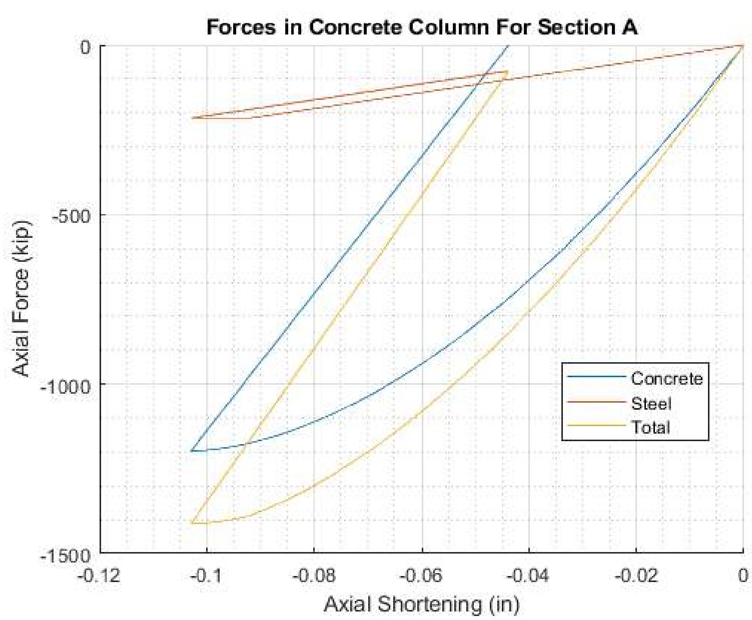
```
% Range for Analysis
pre_pk_range = linspace(0,ec_prime,500);
post_pk_range_A = linspace(ec_prime,ec_prime+fc_prime/Ec,50);
post_pk_range_B = linspace(ec_prime,max_ec,50);

% Concrete Stress-Strain Curve
pre_pk_concrete = @(e) -fc_prime*(1-(1-e/ec_prime).^nE); % Pre-peak formula
post_pk_concrete_A = @(e) Ec*(e-ec_prime) - fc_prime; % Post-peak formula
post_pk_concrete_B = @(e) -eta*Ec*(e-ec_prime) - fc_prime; % Post-peak formula

% Steel Stress-Strain Curve
f_steel_A = @(e) Es*(e -ec_prime) - fy;
f_steel_B = @(e) sign(e).*min(Es*abs(e),fy);

% Strain Ranges
Section_A_strain = [pre_pk_range, post_pk_range_A];
Section_B_strain = [pre_pk_range, post_pk_range_B];

% Stress Values
concrete_A_stress = [pre_pk_concrete(pre_pk_range), post_pk_concrete_A(post_pk_range_A)];
concrete_B_stress = [pre_pk_concrete(pre_pk_range), post_pk_concrete_B(post_pk_range_B)];
steel_A_stress = [f_steel_B(pre_pk_range), f_steel_A(post_pk_range_A)];
steel_B_stress = f_steel_B(Section_B_strain);

% Force Values
concrete_A_force = Ac/1000*concrete_A_stress; % kips
concrete_B_force = Ac/1000*concrete_B_stress; % kips
steel_A_force = As/1000* steel_A_stress; % kips
steel_B_force = As/1000* steel_B_stress; % kips

% Section compressive force
Section_A_force = concrete_A_force + steel_A_force;
Section_B_force = concrete_B_force + steel_B_force;

% Section shortening
Section_A_shorten = Section_A_strain * h;
Section_B_shorten = Section_B_strain * h;

% Total Column
Total_shortening2 = 2*Section_A_shorten + Section_B_shorten;
Total_force2 = Section_B_force;

% Plotting
title = "Stresses in Concrete";
x = [Section_A_strain;Section_B_strain];
y = [concrete_A_stress; concrete_B_stress];
names = ["Section A"; "Section B"];
print_plots(x, y, title, names, false)
print_figure("8_concrete")
```



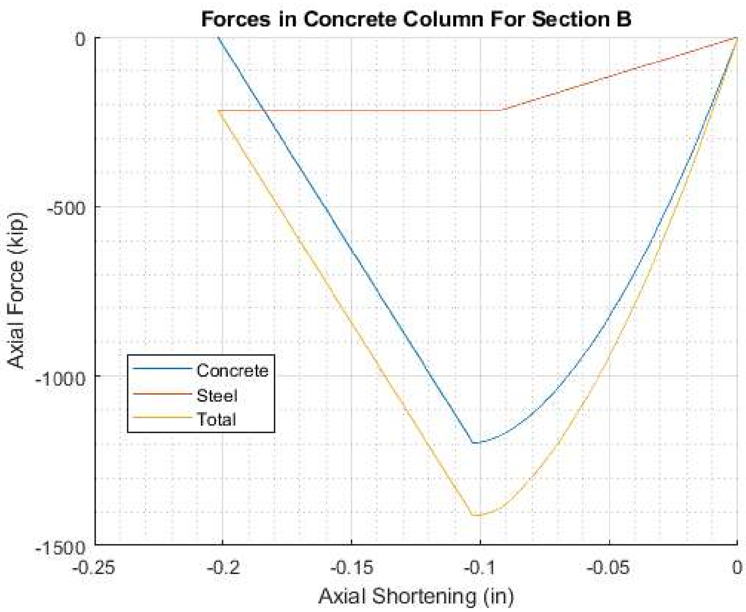
```
title = "Stresses in Steel";
x = [Section_A_strain;Section_B_strain];
y = [steel_A_stress; steel_B_stress];
names = ["Section A"; "Section B"];
print_plots(x, y, title, names, false)
print_figure("8_steel")
```



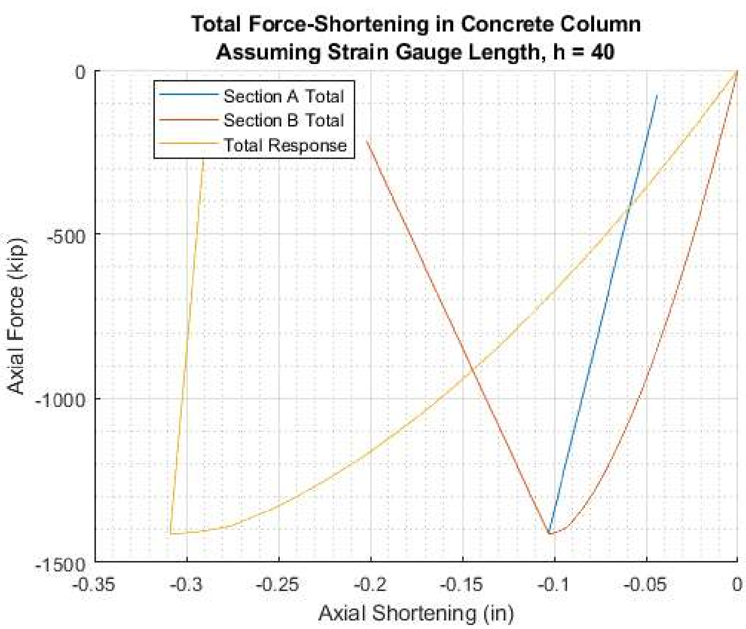

```
title = "Forces in Concrete Column For Section A";
x = [Section_A_shorten;Section_A_shorten;Section_A_shorten];
y = [concrete_A_force; steel_A_force; Section_A_force];
names = ["Concrete"; "Steel"; "Total"];
print_plots(x, y, title, names, true)
print_figure("8_forceA")
```




```
title = "Forces in Concrete Column For Section B";
x = [Section_B_shorten;Section_B_shorten;Section_B_shorten];
y = [concrete_B_force; steel_B_force; Section_B_force];
names = ["Concrete"; "Steel"; "Total"];
print_plots(x, y, title, names, true); ylim([-1500, 0])
print_figure("8_forceB")
```




```
title = ["Total Force-Shortening in Concrete Column","Assuming Strain Gauge Length, h = 40"];
x = [Section_A_shorten;Section_B_shorten;Total_shortening2];
y = [Section_A_force; Section_B_force; Total_force2];
names = ["Section A Total"; "Section B Total"; "Total Response"];
print_plots(x, y, title, names, true)
print_figure("8_total")
```



Question 2

Using Thorenfeldt's equation to model the entire stress-strain curve, pre and post peak responses.

Part (i) Using Single Integration Point

```
% With a single integration point
h = 120; % in
max_ec = -0.005;

% Thorenfeldt Eq. Parameters
n= nE/(nE-1)
```

```
n = 2.3141
```

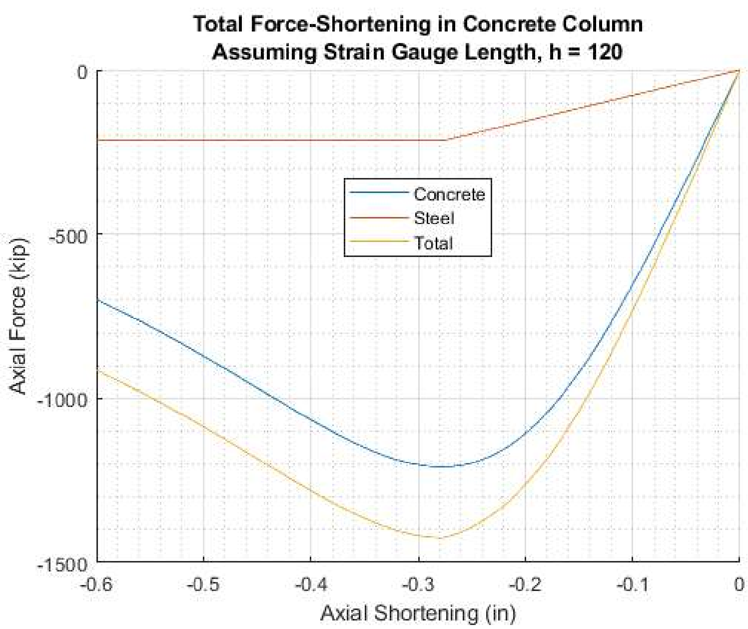
```
n = Ec / (Ec + fc_prime/ec_prime)
```

```
n = 2.3502
```

```
k = 0.67  + fc_prime/9000
```

```
k = 1.1956
```

```
Thorenfeldt = @(e) -fc_prime*(n*(e/ec_prime))./(n-1+(e/ec_prime).^(n*k)); % Concrete
f_steel = @(e) sign(e).*min(Es*abs(e),fy); % Steel

% Compute axial compressive force vs axial displacement
range = linspace(0,max_ec,500);

% Stress values
concrete_stress = Thorenfeldt(range);
steel_stress = f_steel(range);

% Compression force
concrete_axial_compression = Ac/1000*concrete_stress;
steel_axial_compression = As/1000*steel_stress;
Total_force3 = concrete_axial_compression + steel_axial_compression;

% Shortening
Total_shortening3 = h * range;

% Plotting
title = ["Total Force-Shortening in Concrete Column","Assuming Strain Gauge Length, h = 120"];
x = [Total_shortening3;Total_shortening3;Total_shortening3];
y = [concrete_axial_compression; steel_axial_compression; Total_force3];
names = ["Concrete"; "Steel"; "Total"];
print_plots(x, y, title, names, true)
print_figure("9")
```



Part (ii) Using Three Equal Integration Point

```
% With 3 equal Integration points
h = 40;
max_ec = -0.005;

% Thorenfeldt Eq. Parameters
n = Ec / (Ec + fc_prime/ec_prime)
```

```
n = 2.3502
```

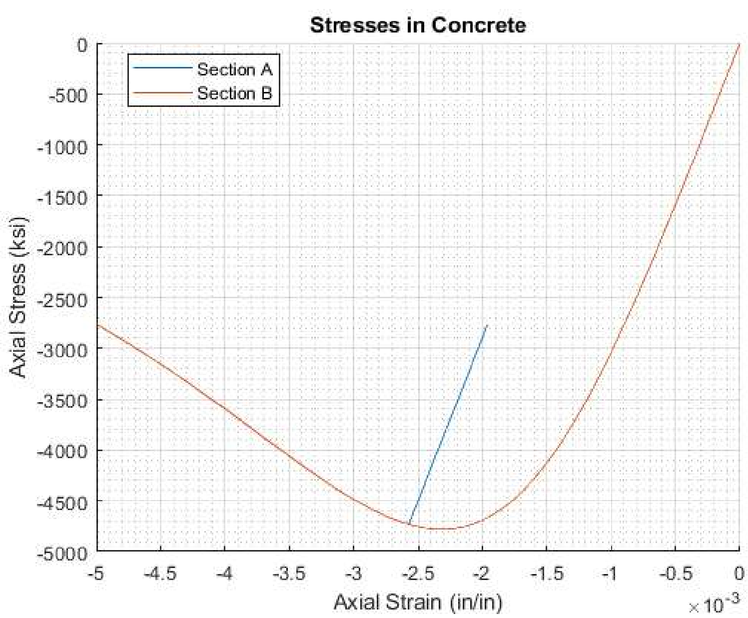
```
k = 0.67  + fc_prime/9000
```

```
k = 1.1956
```

```
% Concrete Stress-Strain Curve
Thorenfeldt = @(e) -fc_prime*(n*(e/ec_prime))./(n-1+(e/ec_prime).^(n*k)); % Concrete
post_pk_concrete_A = @(e) Ec*(e-ec_prime) - fc_prime; % Post-peak formula

% Steel Stress-Strain Curve
```

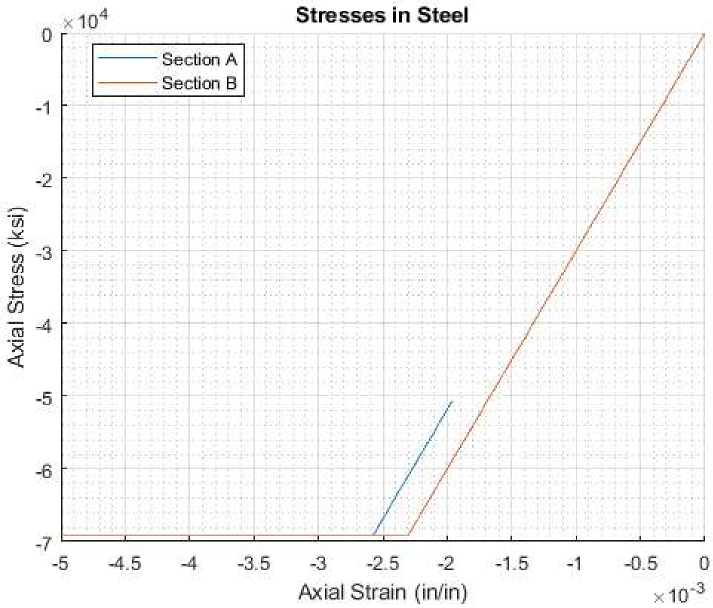
```
f_steel_A = @(e) Es*(e -ec_prime) - fy;
f_steel_B = @(e) sign(e).*min(Es*abs(e),fy);

% Range for Analysis
pre_pk_range = linspace(0,ec_prime,500);
post_pk_range_A = linspace(ec_prime,(fc_prime + Thorenfeldt(max_ec))/Ec + ec_prime,100);
post_pk_range_B = linspace(ec_prime,max_ec,100);

% Strain Ranges
Section_A_strain = [pre_pk_range, post_pk_range_A];
Section_B_strain = [pre_pk_range, post_pk_range_B];
Combined_strain = [3*pre_pk_range , 2*post_pk_range_A + post_pk_range_B];

% Stress Values
concrete_A_stress = [Thorenfeldt(pre_pk_range), post_pk_concrete_A(post_pk_range_A)];
concrete_B_stress = Thorenfeldt(Section_B_strain);
steel_A_stress = [f_steel_B(pre_pk_range), f_steel_A(post_pk_range_A)];
steel_B_stress = f_steel_B(Section_B_strain);

% Force Values
concrete_A_force = Ac/1000*concrete_A_stress; % kips
concrete_B_force = Ac/1000*concrete_B_stress; % kips
steel_A_force =    As/1000*steel_A_stress; % kips
steel_B_force =    As/1000*steel_B_stress; % kips

% Section compressive force
Section_A_force = concrete_A_force + steel_A_force;
Section_B_force = concrete_B_force + steel_B_force;

% Section shortening
Section_A_shorten = Section_A_strain* h; % *h
Section_B_shorten = Section_B_strain*h; % *h

% Total Responses
Total_force4 = Section_B_force;
Total_shortening4 = 2*Section_A_shorten + Section_B_shorten;
```

```
title = "Stresses in Concrete";
x = [Section_A_strain;Section_B_strain];
y = [concrete_A_stress; concrete_B_stress];
names = ["Section A"; "Section B"];
print_plots(x, y, title, names, false)
print_figure("9_concrete")
```

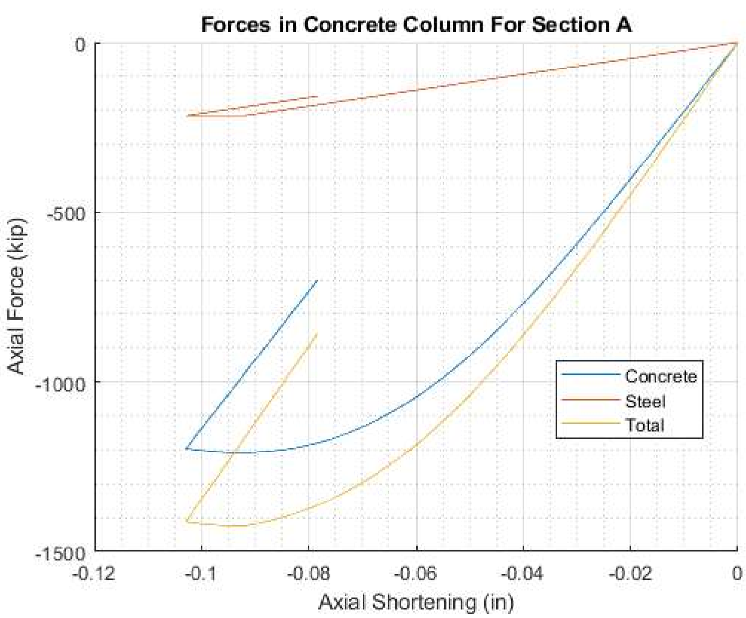



```
title = "Stresses in Steel";
x = [Section_A_strain;Section_B_strain];
y = [steel_A_stress; steel_B_stress];
names = ["Section A"; "Section B"];
print_plots(x, y, title, names, false)
```

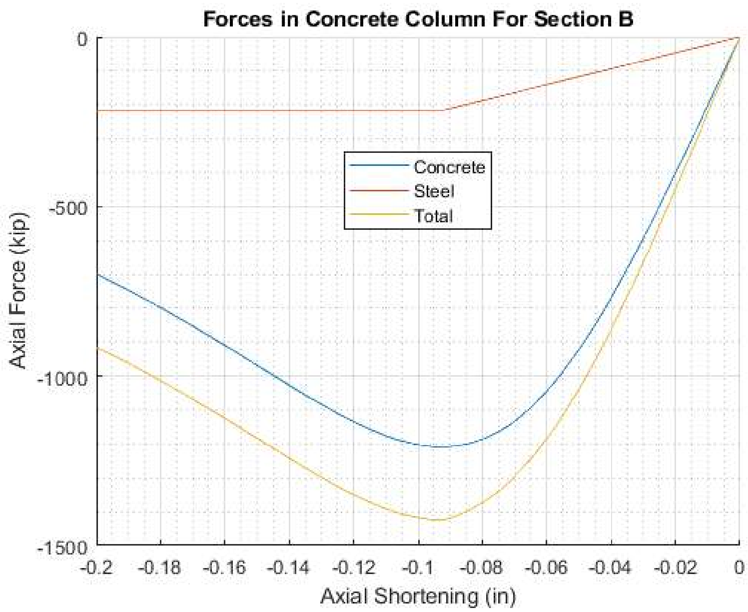
```
print_figure("9_steel")
```



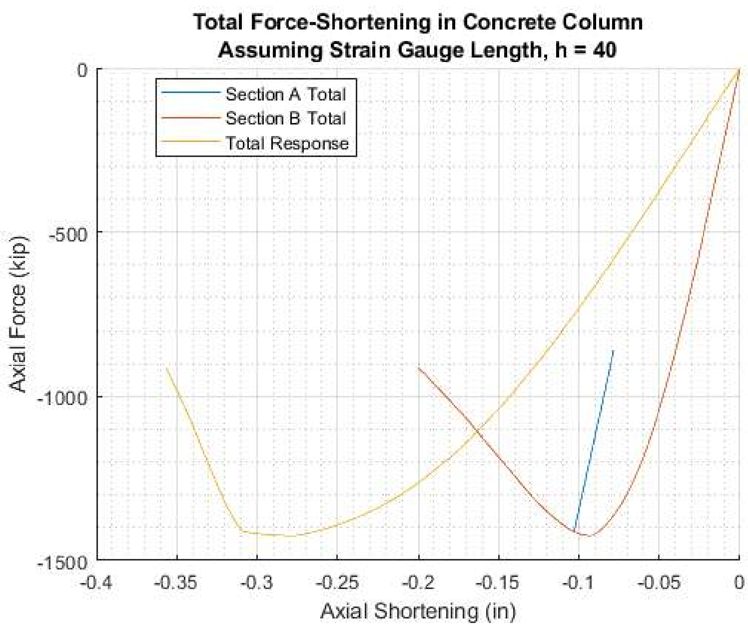

```
title = "Forces in Concrete Column For Section A";
x = [Section_A_shorten;Section_A_shorten;Section_A_shorten];
y = [concrete_A_force; steel_A_force; Section_A_force];
names = ["Concrete"; "Steel"; "Total"];
print_plots(x, y, title, names, true)
print_figure("9_forceA")
```



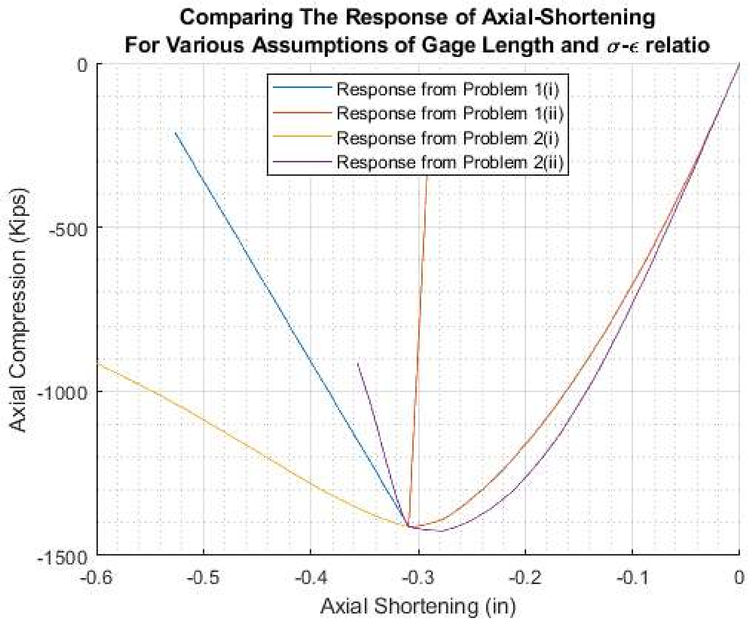

```
title = "Forces in Concrete Column For Section B";
x = [Section_B_shorten;Section_B_shorten;Section_B_shorten];
y = [concrete_B_force; steel_B_force; Section_B_force];
names = ["Concrete"; "Steel"; "Total"];
print_plots(x, y, title, names, true)
print_figure("9_forceB")
```




```
title = ["Total Force-Shortening in Concrete Column","Assuming Strain Gauge Length, h = 40"];
x = [Section_A_shorten;Section_B_shorten;Total_shortening4];
y = [Section_A_force; Section_B_force; Total_force4];
names = ["Section A Total"; "Section B Total"; "Total Response"];
print_plots(x, y, title, names, true)
print_figure("9_total")
```




```
close all; figure; axis on; grid minor; grid on; hold on; legend('Location',"Best"); clear title;
title(["Comparing The Response of Axial-Shortening","For Various Assumptions of Gage Length and \sigma-\epsilon relatio"]);
ylabel("Axial Compression (Kips)"); xlabel('Axial Shortening (in)');
plot(Total_shortening1,Total_force1,"DisplayName","Response from Problem 1(i)");
plot(Total_shortening2,Total_force2,"DisplayName","Response from Problem 1(ii)");
plot(Total_shortening3,Total_force3,"DisplayName","Response from Problem 2(i)");
plot(Total_shortening4,Total_force4,"DisplayName","Response from Problem 2(ii)");
print_figure("10")
```




```
function print_figure(no)
    % Saves the figures in a consistent manner
    orient(gcf,'landscape');
    folder = '..\figures\';
    name = 'Figure' +string(no);
    print(folder+name,'-dpdf','-fillpage','-PMicrosoft Print to PDF','-r600','-painters')
    print(folder+name,'-djpeg','-PMicrosoft Print to PDF','-r600','-painters')
end

function print_plots(x, y, title_, name, plot_forces)
    figure(); hold on; grid on; grid minor;
    legend('Location',"Best"); title(title_);
    for j = 1:size(x,1)
        plot(x(j,:), y(j,:), "DisplayName",name(j))
    end
    if plot_forces
        ylabel("Axial Force (kip)"); xlabel('Axial Shortening (in)');
%         ylim([-1600 0])
    else
        ylabel("Axial Stress (ksi)"); xlabel('Axial Strain (in/in)');
%         ylim([-5000 0])
    end
end
```