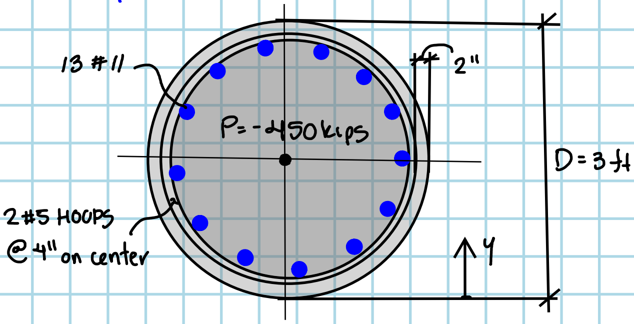
Moment curvature example

- Take this example only as reference and to help you debug your code. Don't take any value or parameter used here in your homework, use the ones specified in lecture and in the assignment



- Concrete section discretized in 20 equal thickness layers
- Steel divided according to the following table

Layer "i"	y_i (in)	A_{si} (in²)
1	4	3.09
2	11	4.70
3	18	4.70
4	25	4.70
5	32	3.09

→ Steel properties

$\rho_\ell = \dfrac{(13)(1.56\ in^2)}{\pi\,(3\times12)^2/4} = 2.00\,\%$

$f_{ye} = f_{yh} = 69$ ksi (Same yield stress for longitudinal and transverse reinforcement)

$E_s = 29000$ ksi (Young's modulus of steel)

$e_{sh} = 0.011$ (strain of onset hardening)

$e_{su} = 0.122$ (Uniform strain)

$f_{su} = 104$ ksi (ultimate strength

$P = 2.8$ (Power of the strain hardening region)

→ Concrete properties

$E_c = 3250$ ksi (Young's modulus of concrete)

$f'_c = 6.5$ ksi (compressive strength of concrete)

$\varepsilon'_c = -0.0027$

$\varepsilon'_{cu} = -0.006$

$f_{cr} = 0.458$ ksi (cracking strength of concrete)

$f'_{cc} = 9.05$ ksi (confined concrete comp. strength)

$\varepsilon'_{cc} = -0.0079$ (corresponding strain)

→ Parameters for M-φ

Tolerance = 0.01

P = -450 kips

$\varepsilon_o(1) = -1.17\times10^{-4}$ (strain at centroid)

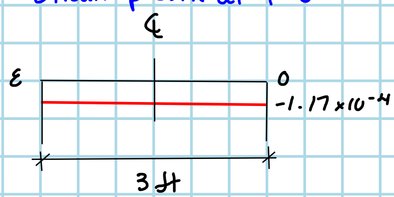
$\varphi_o(1) = 0$ (No curvature at start of loading)

$\Delta\varphi\cdot D = 2e_y/25$; $\Delta\varphi = 5.29\times10^{-6}$ 1/in

$\varepsilon_{s\,max} = -1.17\times10^{-4}$

$\varepsilon_{c\,min} = -1.17\times10^{-4}$

Strain profile at $\varphi = 0$



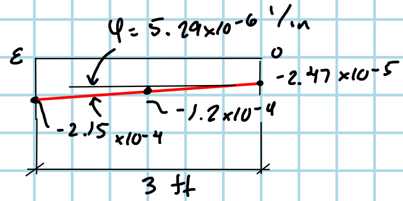
→ First step

$\Delta\varepsilon = -0.5\,\Delta\varphi = -2.6\times10^{-6}$ (initial guess)

$\varphi_o(2) = 5.29\times10^{-6}$ 1/in (curvature at current step)

$\varepsilon_o(2) = -1.2\times10^{-4}$ (Strain at centroid at current iteration)

Strain profile



* knowing the strain we can compute the stress at any concrete or steel fiber

Example: Concrete layer 4

- $y_c(4) = 6.3$ in
- $A_c(4) = 11.68$ in²
- $A_{cc}(4) = 37.56$ in²

- $\varepsilon_c(4) = -5.79\times10^{-5}$ (compression)
- $f_c(4) = -0.215$ ksi (unconfined concrete stress)
- $f_{cc}(4) = -0.188$ ksi (confined concrete stress)
- $F_c(4) = -9.58$ kips (Total force at layer 4)

Example: Steel layer 5

- $y_s(5) = 32$ in
- $A_s(5) = 3.09$ in^2

- $\varepsilon_s(5) = -1.93 \times 10^{-4}$ (strain at fiber 5, compression)
- $f_s(5) = -5.62$ ksi (stress at fiber 5, compression)
- * $f_{cc} = -0.627$ ksi
- $F_s(5) = A_s(5) \times (f_s(5) - f_{cc}^{*}) = -15.4$ kips (Total force at fiber 5)

* Confined concrete stress corresponding to the strain at fiber 5. Computed to avoid double counting (see Lecture 10).

* Evaluate equilibrium and obtaining normalized unbalanced load

$$F = \text{sum}(F_c) + \text{sum}(F_s) = -472 \text{ kips}$$

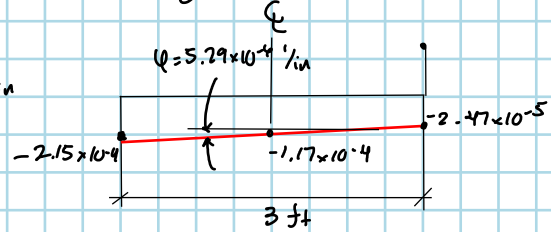
$$\text{error} = \frac{F - P}{A_{st} \cdot f_y} = -0.0157$$

* Iterate until reaching converges, i.e. $|\text{error}| < \text{tol}$. Update ε_o via brute_force or use any solver.

* Strain profile when convergence is reached

$\varepsilon_o = -1.17 \times 10^{-4}$

$\varphi_o = 5.29 \times 10^{-6}$ 1/in



* Compute es_max (Max tensile strain at steel fibers) and ec_min (Min compressive strain at concrete core)

$\varepsilon_{s\_max} = -4.31 \times 10^{-5}$

$\varepsilon_{c\_min} = -1.98 \times 10^{-4}$

→ Increase counter and go to next step. Where you'll increase the curvature.