Assignment 5

1) The initial parameters gave a χ^2 of 15267 for 2501 degrees of freedom. The χ^2 should approximate the number of degrees of freedom. It is not a good fit.

Using the new params, we get a χ^2 of 3272 for 2501 degrees of freedom. It is a much beter fit. (pset5_1.py)

2) Using Newton's method we got: (pset5_2.py)

$$H_0 = 67.34 \pm 3.93 \times 10^{-5}$$

$$\Omega_b h^2 = 2.25 \pm 1.14 \times 10^{-8}$$

$$\Omega_c h^2 = 1.19 \times 10^{-1} \pm 2.60 \times 10^{-8}$$

$$\tau = 1.27 \times 10^{-1} \pm 2.60 \times 10^{-8}$$

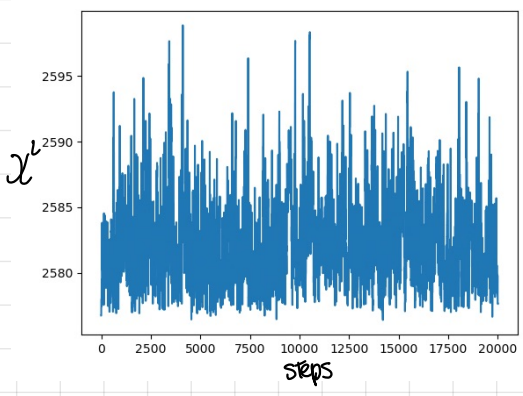
$$A_s = 2.43 \times 10^{-9} \pm 2.04 \times 10^{-10}$$

$$n_s = 9.72 \times 10^{-1} \pm 2.76 \times 10^{-8}$$

With a χ^2 of 2576

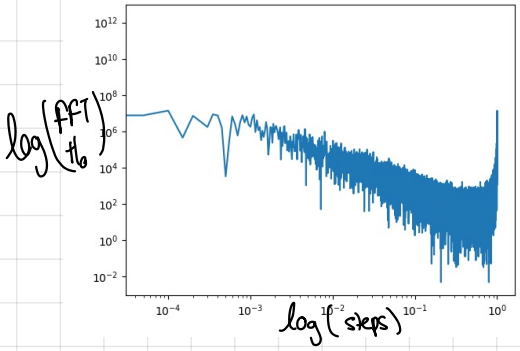
3) We compute the MCMC chain in pset5_3.py

We get a chain for χ^2:

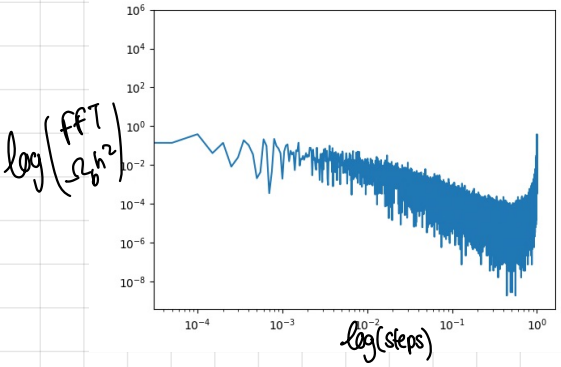


A chain on which we computed the fast fourier transform for :

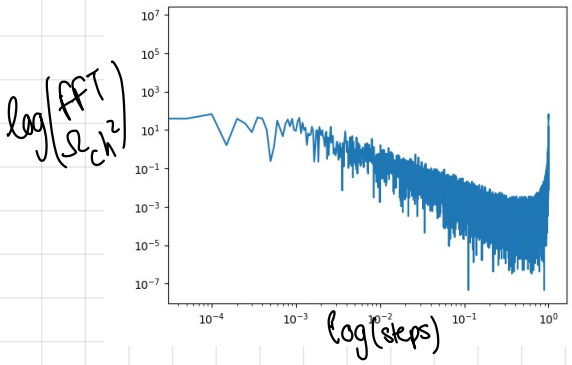
H_0 :



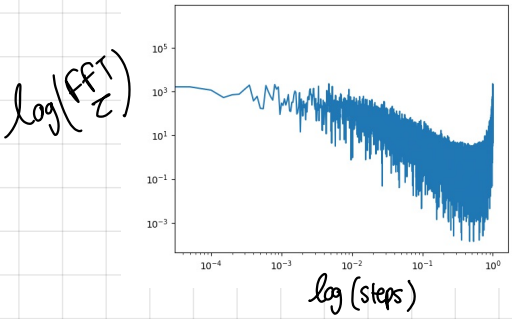
$\Omega_b h^2$:



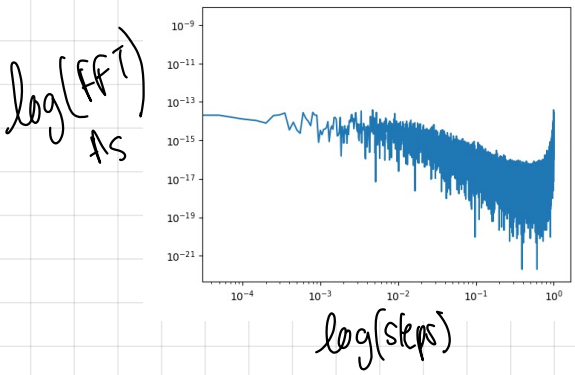
$\Omega_c h^2$:



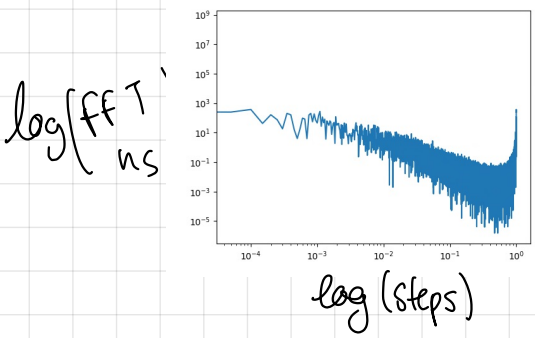
τ :



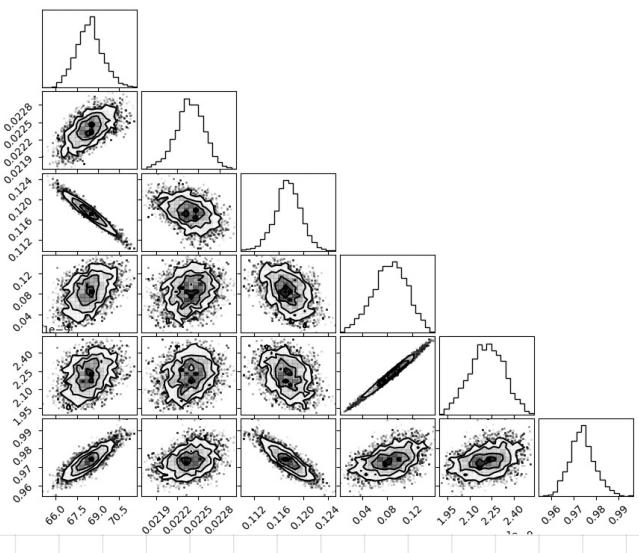
A_S :



n_s



we computed a corner plot based on the chain



Those graphs show a constant behavior (about the same distance from the mean) furthermore, the corner plot shows a rather correlated data.

4) In this last question, we compared, to our 3) chain that we cleaned using the importance sampling method, a chain with a fixed taw. We use as a tool for comparison the gelman rubin scatters method.

Question 1

[ ]:
```
import numpy as np
import camb
from matplotlib import pyplot as plt
import time


def get_spectrum(pars,lmax=3000):
    #print('pars are ',pars)
    H0=pars[0]
    ombh2=pars[1]
    omch2=pars[2]
    tau=pars[3]
    As=pars[4]
    ns=pars[5]
    pars=camb.CAMBparams()
    pars.set_cosmology(H0=H0,ombh2=ombh2,omch2=omch2,mnu=0.06,omk=0,tau=tau)
    pars.InitPower.set_params(As=As,ns=ns,r=0)
    pars.set_for_lmax(lmax,lens_potential_accuracy=0)
    results=camb.get_results(pars)
    powers=results.get_cmb_power_spectra(pars,CMB_unit='muK')
    cmb=powers['total']
    tt=cmb[:,0]    #you could return the full power spectrum here if you wanted␣
 ↪to do say EE
    return tt[2:]


plt.ion()

pars=np.asarray([60,0.02,0.1,0.05,2.00e-9,1.0]) #test chisq is  15267.
 ↪937968194292  for  2501  degrees of freedom.
pars=np.asarray([69, 0.022, 0.12,0.06, 2.1e-9, 0.95]) #q1 chisq is  3272.
 ↪203604462886  for  2501  degrees of freedom.
planck=np.loadtxt('Assignment5/COM_PowerSpect_CMB-TT-full_R3.01.txt',skiprows=1)
ell=planck[:,0]
spec=planck[:,1]
errs=0.5*(planck[:,2]+planck[:,3])
```

```
model=get_spectrum(pars)
model=model[:len(spec)]
resid=spec-model
chisq=np.sum( (resid/errs)**2)
print("chisq is ",chisq," for ",len(resid)-len(pars)," degrees of freedom.")
#read in a binned version of the Planck PS for plotting purposes
planck_binned=np.loadtxt('Assignment5/COM_PowerSpect_CMB-TT-binned_R3.01.
 ↪txt',skiprows=1)
errs_binned=0.5*(planck_binned[:,2]+planck_binned[:,3]);
plt.clf()
plt.plot(ell,model)
plt.errorbar(planck_binned[:,0],planck_binned[:,1],errs_binned,fmt='.')
plt.show()
```

Question 2

[ ]:
```
import numpy as np
from matplotlib import pyplot as plt
import camb

newtom_params_txt_file = "planck_fit_params.txt"

def get_spectrum(pars,lmax=3000):
    H0=pars[0]
    ombh2=pars[1]
    omch2=pars[2]
    tau=pars[3]
    As=pars[4]
    ns=pars[5]
    pars=camb.CAMBparams()
    pars.set_cosmology(H0=H0,ombh2=ombh2,omch2=omch2,mnu=0.06,omk=0,tau=tau)
    pars.InitPower.set_params(As=As,ns=ns,r=0)
    pars.set_for_lmax(lmax,lens_potential_accuracy=0)
    results=camb.get_results(pars)
    powers=results.get_cmb_power_spectra(pars,CMB_unit='muK')
    cmb=powers['total']
    tt=cmb[:,0]
    return tt[2:]
def num_derivs(fun,pars,dp,x):
    A=np.empty([len(x),len(pars)])
    for i in range(len(pars)):
        pp=pars.copy()
        pp[i]=pars[i]+dp[i]
        y_right=fun(pp)
        y_right=y_right[:len(spec)]
        pp[i]=pars[i]-dp[i]
        y_left=fun(pp)
```

```
        y_left=y_left[:len(spec)]
        A[:,i]=(y_right-y_left)/(2*dp[i])
    return A
def newton(fun,pars,dp,x,y,niter=3):
    errs=0.5*(planck[:,2]+planck[:,3])
    err_diag = np.asarray(errs**2)
    N = np.zeros((len(x),len(x)))
    np.fill_diagonal(N, err_diag)
    for i in range(niter):
        pred=fun(pars)
        pred=pred[:len(spec)]
        r=y-pred
        A=num_derivs(fun,pars,dp,x)
        lhs=A.T@np.linalg.inv(N)@A
        rhs=A.T@np.linalg.inv(N)@r
        step=np.linalg.inv(lhs)@rhs
        pars=pars+step
    return pars,np.linalg.inv(lhs)
pars=np.asarray([69, 0.022, 0.12,0.06, 2.1e-9, 0.95])
planck=np.loadtxt('Assignment5/COM_PowerSpect_CMB-TT-full_R3.01.txt',skiprows=1)
ell=planck[:,0] #x
spec=planck[:,1] #y

dx = 1e-8
dp = pars*dx
fitp,curve=newton(get_spectrum,pars,dp,ell,spec)

error_fitp=np.diag(np.linalg.cholesky(curve))
errs=0.5*(planck[:,2]+planck[:,3])
model=get_spectrum(fitp)
model=model[:len(spec)]
resid=spec-model
chisq=np.sum((resid/errs)**2)
print("chsiq is " + str(chisq))

f = open("Assignment5/planck_fit_params.txt", "a")
line = "Hubble constant is " + str(fitp[0]) + " ± " + str(error_fitp[0]) + "\n"
f.write(line)
line = "Baryon density is " + str(fitp[1]) + " ± " + str(error_fitp[1]) +"\n"
f.write(line)
line = "Dark matter density is " + str(fitp[2]) + " ± " + str(error_fitp[2])␣
 ↪+"\n"
f.write(line)
line = "Optical depth is " + str(fitp[3]) + " ± " + str(error_fitp[2])+ "\n"
f.write(line)
line = "Primordial amplitude of the spectrum is " + str(fitp[4]) + " ± " +␣
 ↪str(error_fitp[4])+ "\n"
```

```
f.write(line)
line = "Primordial tilt of the spectrum is " + str(fitp[5]) + " ± " +␣
 ↪str(error_fitp[5])
f.write(line)
```

- Hubble constant is 67.34446649082994 ± 3.929618194466233e-05
- Baryon density is 0.0224748518522885 ± 1.1372956877108872e-08
- Dark matter density is 0.11938840491314752 ± 2.604464866254484e-08
- Optical depth is 0.12650918521699278 ± 2.604464866254484e-08
- Primordial amplitude of the spectrum is 2.4307447362719345e-09 ± 2.0408733057146e-16
- Primordial tilt of the spectrum is 0.972377519638394 ± 2.7563004980729678e-08

Question 3:

[ ]:
```
import numpy as np
from matplotlib import pyplot as plt
import camb
import corner


def get_spectrum(pars,lmax=3000):
    H0=pars[0]
    ombh2=pars[1]
    omch2=pars[2]
    tau=pars[3]
    As=pars[4]
    ns=pars[5]
    pars=camb.CAMBparams()
    pars.set_cosmology(H0=H0,ombh2=ombh2,omch2=omch2,mnu=0.06,omk=0,tau=tau)
    pars.InitPower.set_params(As=As,ns=ns,r=0)
    pars.set_for_lmax(lmax,lens_potential_accuracy=0)
    results=camb.get_results(pars)
    powers=results.get_cmb_power_spectra(pars,CMB_unit='muK')
    cmb=powers['total']
    tt=cmb[:,0]
    return tt[2:]
def num_derivs(fun,pars,dp,x):
    A=np.empty([len(x),len(pars)])
    for i in range(len(pars)):
        pp=pars.copy()
        pp[i]=pars[i]+dp[i]
        y_right=fun(pp)
        y_right=y_right[:len(spec)]
        pp[i]=pars[i]-dp[i]
        y_left=fun(pp)
        y_left=y_left[:len(spec)]
        A[:,i]=(y_right-y_left)/(2*dp[i])
    return A
```

```
def newton(fun,pars,dp,x,y,niter=3):
    errs=0.5*(planck[:,2]+planck[:,3])
    err_diag = np.asarray(errs**2)
    N = np.zeros((len(x),len(x)))
    np.fill_diagonal(N, err_diag)
    for i in range(niter):
        pred=fun(pars)
        pred=pred[:len(spec)]
        r=y-pred
        A=num_derivs(fun,pars,dp,x)
        lhs=A.T@np.linalg.inv(N)@A
        rhs=A.T@np.linalg.inv(N)@r
        step=np.linalg.inv(lhs)@rhs
        pars=pars+step
    return pars,np.linalg.inv(lhs)
def run_chain(fun,pars,trial_step,data,nstep=20000,T=1):
    npar=len(pars)
    chain=np.zeros([nstep,npar])
    chisq=np.zeros(nstep)
    chain[0,:]=pars
    chi_cur=fun(pars,data)
    chisq[0]=chi_cur
    for i in range(1,nstep):
        pp=pars+get_step(trial_step)
        new_chisq=fun(pp,data)
        accept_prob=np.exp(-0.5*(new_chisq-chi_cur)/T)
        if np.random.rand(1)<accept_prob:
            pars=pp
            chi_cur=new_chisq
        chain[i,:]=pars
        chisq[i]=chi_cur
    return chain,chisq
def get_step(trial_step):
    if len(trial_step.shape)==1:
        return np.random.randn(len(trial_step))*trial_step
    else:
        L=np.linalg.cholesky(trial_step)
        return L@np.random.randn(trial_step.shape[0])
def process_chain(chain,chisq,T=1.0):
    dchi=chisq-np.min(chisq)
    #density in chain is exp(-0.5*chi^2/T), but
    #we wanted it to be exp(-0.5*chi^2)
    #so, we want to downweight by ratio, which is
    #exp(-0.5*chi^2*(1-1/T)).  We'll calculate the mean
    #and standard deviation of the chain, but will also
    #return the weights so you could calculate whatever you want
```

```
    wt=np.exp(-0.5*dchi*(1-1/T)) #the magic line that importance samples

    #calculate the weighted sum of the chain and the chain squared
    npar=chain.shape[1]
    tot=np.zeros(npar)
    totsqr=np.zeros(npar)
    for i in range(npar):
        tot[i]=np.sum(wt*chain[:,i])
        totsqr[i]=np.sum(wt*chain[:,i]**2)
    #divide by sum or weights
    mean=tot/np.sum(wt)
    meansqr=totsqr/np.sum(wt)

    #variance is <x^2>-<x>^2
    var=meansqr-mean**2
    return mean,np.sqrt(var),wt
def spectrum_chisq(pars,data):
    x=data['x']
    y=data['y']
    errs=data['errs']

    model=get_spectrum(pars)
    model=model[:len(spec)]
    resid=spec-model
    chisq=np.sum((resid/errs)**2)
    return chisq

pars=np.asarray([69, 0.022, 0.12,0.06, 2.1e-9, 0.95])
planck=np.loadtxt('Assignment5/COM_PowerSpect_CMB-TT-full_R3.01.txt',skiprows=1)
ell=planck[:,0] #x
spec=planck[:,1] #y

dx = 1e-8
dp = pars*dx
fitp,curve=newton(get_spectrum,pars,dp,ell,spec)

error_fitp=np.diag(np.linalg.cholesky(curve))
errs=0.5*(planck[:,2]+planck[:,3])
model=get_spectrum(fitp)
model=model[:len(spec)]
resid=spec-model
chisq=np.sum((resid/errs)**2)
print("chsiq is " + str(chisq))


data={}
data['x']=ell
```

```
data['y']=spec
data['errs']=errs

chain,chivec=run_chain(spectrum_chisq,fitp,curve,data)
mean,errs,wts=process_chain(chain,chivec)
print(errs)
steps = np.linspace(0,20000, 20000)
f = open("Assignment5/chain_4.txt", "a")
for line in chain:
    f.write(str(line) + "\n")
f.close()
f = open("Assignment5/chain_chi_4.txt", "a")
for line in chivec:
    f.write(str(line)+ "\n")
f.close()
f = open("Assignment5/pars_chain_4.txt", "a")
f.write(str(pars))
f.close()
f = open("Assignment5/errs_chain_4.txt", "a")
f.write(str(errs))
f.close()
plt.plot(steps, chivec)
corner.corner(chain)
for i in range (len(chain.T)):
    plt.loglog(np.linspace(0,1,len(chain.T[i])),np.abs(np.fft.fft(chain.
 ↪T[i]))**2)
plt.show()
```

Question 4:

[ ]:
```
import numpy as np
from matplotlib import pyplot as plt
import camb
import corner


def get_spectrum(pars,lmax=3000):
    H0=pars[0]
    ombh2=pars[1]
    omch2=pars[2]
    tau=pars[3]
    As=pars[4]
    ns=pars[5]
    pars=camb.CAMBparams()
    pars.set_cosmology(H0=H0,ombh2=ombh2,omch2=omch2,mnu=0.06,omk=0,tau=tau)
    pars.InitPower.set_params(As=As,ns=ns,r=0)
    pars.set_for_lmax(lmax,lens_potential_accuracy=0)
```

```
    results=camb.get_results(pars)
    powers=results.get_cmb_power_spectra(pars,CMB_unit='muK')
    cmb=powers['total']
    tt=cmb[:,0]
    return tt[2:]
def num_derivs(fun,pars,dp,x):
    A=np.empty([len(x),len(pars)])
    for i in range(len(pars)):
        pp=pars.copy()
        pp[i]=pars[i]+dp[i]
        y_right=fun(pp)
        y_right=y_right[:len(spec)]
        pp[i]=pars[i]-dp[i]
        y_left=fun(pp)
        y_left=y_left[:len(spec)]
        A[:,i]=(y_right-y_left)/(2*dp[i])
    return A
def newton(fun,pars,dp,x,y,niter=3):
    errs=0.5*(planck[:,2]+planck[:,3])
    err_diag = np.asarray(errs**2)
    N = np.zeros((len(x),len(x)))
    np.fill_diagonal(N, err_diag)
    for i in range(niter):
        pred=fun(pars)
        pred=pred[:len(spec)]
        r=y-pred
        A=num_derivs(fun,pars,dp,x)
        lhs=A.T@np.linalg.inv(N)@A
        rhs=A.T@np.linalg.inv(N)@r
        step=np.linalg.inv(lhs)@rhs
        pars=pars+step
    return pars,np.linalg.inv(lhs)
def run_chain(fun,pars,trial_step,data,nstep=20000,T=1):
    npar=len(pars)
    chain=np.zeros([nstep,npar])
    chisq=np.zeros(nstep)
    chain[0,:]=pars
    chi_cur=fun(pars,data)
    chisq[0]=chi_cur
    for i in range(1,nstep):
        pp = []
        for j in range(0,3):
            pp.append(pars[j]+get_step(trial_step)[j])
        pp.append(pars[3])
        for j in range(4,len(pars)):
            pp.append(pars[j]+get_step(trial_step)[j])
        new_chisq=fun(pp,data)
```

```
        new_chisq=fun(pp,data)
        accept_prob=np.exp(-0.5*(new_chisq-chi_cur)/T)
        if np.random.rand(1)<accept_prob:
            pars=pp
            chi_cur=new_chisq
        chain[i,:]=pars
        chisq[i]=chi_cur
    return chain,chisq
def get_step(trial_step):
    if len(trial_step.shape)==1:
        return np.random.randn(len(trial_step))*trial_step
    else:
        L=np.linalg.cholesky(trial_step)
        return L@np.random.randn(trial_step.shape[0])
def process_chain(chain,chisq,T=1.0):
    dchi=chisq-np.min(chisq)
    #density in chain is exp(-0.5*chi^2/T), but
    #we wanted it to be exp(-0.5*chi^2)
    #so, we want to downweight by ratio, which is
    #exp(-0.5*chi^2*(1-1/T)).  We'll calculate the mean
    #and standard deviation of the chain, but will also
    #return the weights so you could calculate whatever you want

    wt=np.exp(-0.5*dchi*(1-1/T)) #the magic line that importance samples

    #calculate the weighted sum of the chain and the chain squared
    npar=chain.shape[1]
    tot=np.zeros(npar)
    totsqr=np.zeros(npar)
    for i in range(npar):
        tot[i]=np.sum(wt*chain[:,i])
        totsqr[i]=np.sum(wt*chain[:,i]**2)
    #divide by sum or weights
    mean=tot/np.sum(wt)
    meansqr=totsqr/np.sum(wt)

    #variance is <x^2>-<x>^2
    var=meansqr-mean**2
    return mean,np.sqrt(var),wt
def spectrum_chisq(pars,data):
    x=data['x']
    y=data['y']
    errs=data['errs']

    model=get_spectrum(pars)
    model=model[:len(spec)]
    resid=spec-model
```

```
    chisq=np.sum((resid/errs)**2)
    return chisq

pars=np.asarray([69, 0.022, 0.12,0.0540, 2.1e-9, 0.95])
planck=np.loadtxt('Assignment5/COM_PowerSpect_CMB-TT-full_R3.01.txt',skiprows=1)
ell=planck[:,0] #x
spec=planck[:,1] #y

dx = 1e-8
dp = pars*dx
fitp,curve=newton(get_spectrum,pars,dp,ell,spec)

error_fitp=np.diag(np.linalg.cholesky(curve))
errs=0.5*(planck[:,2]+planck[:,3])
fitp[3] = pars[3]
model=get_spectrum(fitp)
model=model[:len(spec)]
resid=spec-model
chisq=np.sum((resid/errs)**2)
print("chsiq is " + str(chisq))


data={}
data['x']=ell
data['y']=spec
data['errs']=errs

chain,chivec=run_chain(spectrum_chisq,fitp,curve,data)
mean,errs,wts=process_chain(chain,chivec)
print(errs)
steps = np.linspace(0,20000, 20000)
f = open("Assignment5/fixed_chain_4.txt", "a")
for line in chain:
    f.write(str(line) + "\n")
f.close()
f = open("Assignment5/fixed_chain_chi_4.txt", "a")
for line in chivec:
    f.write(str(line)+ "\n")
f.close()
f = open("Assignment5/fixed_pars_chain_4.txt", "a")
f.write(str(pars))
f.close()
f = open("Assignment5/fixed_errs_chain_4.txt", "a")
f.write(str(errs))
f.close()
plt.plot(steps, chivec)
corner.corner(chain)
```

```
for i in range (len(chain.T)):
    plt.loglog(np.linspace(0,1,len(chain.T[i])),np.abs(np.fft.fft(chain.
 ↪T[i]))**2)
plt.show()
```

Code analysis

It analyses a chain and return chain plots, fft plots and saves the documents

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt

f = open("Assignment5/chain_3.txt", "r")
chain = []
lines = f.readlines()
d = open("Assignment5/chain_chi_3.txt", "r")
lines_d = d.readlines()
chain_chi = []
for i in range(len(lines_d)):
    chain_chi.append(float(lines_d[i]))

count_chain = 0
for r in range(0,len(lines),2):
    line1 = lines[r][1:].split(" ")
    line2 = lines[r+1][1:-2].split(" ")
    line1.append(line2[0])
    line1.append(line2[1])
    line = []
    line.append(chain_chi[count_chain])
    count_chain = count_chain + 1
    for j in line1:
        line = np.append(line,float(j))
    chain.append(np.array(line))
chain = np.array(chain)
steps = np.linspace(0,20000, 20000)

#for i in range(len(chain.T)):
    #plt.plot(steps, chain.T[i])
    #plt.loglog(np.linspace(0,1,len(chain.T[i])),np.abs(np.fft.fft(chain.
 ↪T[i]))**2)
re = open("Assignment5/planck chain.txt", "a")

for i in range(len(chain)):
    line = ""
    for j in range(len(chain[i])):
        line = line + " " + str(chain[i][j]) + " "
    line = line + str("\n")
    re.write(line)
```