IEOR 221

Homework 8

Module 9 Options – Numerical Techniques (November 1, 2024)

For the problems below you may implement in your preferred programming language/environment (such as Python, Excel, C++) although Python is recommended. Present both your numerical answers and your code of implementation.

1) Price an European call option with spot stock price S_0 = 100.0, strike K = 100.0, time to maturity T = 1.0 year, risk free interest rate r = 6%, continuous dividend yield q = 6%, volatility σ = 35% using Monte Carlo (MC) simulation with

(1) plain MC (i.e., simulation without applying any variance reduction techniques)
(2) antithetic method by sampling paths generated from

$$S_{t+\Delta t} = S_t e^{\left(r-q-\frac{\sigma^2}{2}\right)\Delta t+\sigma\Delta W_t}, \; S'_{t+\Delta t} = S'_t e^{\left(r-q-\frac{\sigma^2}{2}\right)\Delta t-\sigma\Delta W_t}$$

(3) control variate method using S as control function by utilizing the fact

$$E_t\left[S_T\right] = S_t e^{(r-q)(T-t)}$$

Present both prices and error estimate. Use # of time step = 100, # of simulated stock path = 4000. Compare your answer with the BSM formula. Fix a seed for your random number generator if you can (e.g., set seed of random generator = 110124). Present your answers to 2 decimal places.
Due: **2 PM PDT Friday November 8, 2024**

2) Price the corresponding American call option with the same input as the previous problem using a 100 step CRR binomial tree. Also calculate delta, gamma, theta, vega and rho using the techniques discussed in the lecture. Present your answers to 4 decimal places.
Due: **2 PM PDT Friday November 15, 2024**

Note: depending on the choice of software platforms and seeds for the random number generator the MC results may vary slightly (the differences will getting smaller as the # of simulated path increases). On the other hand for a given binomial tree construction (CRR tree with 100 steps in our case) the results on a tree are unambiguous.