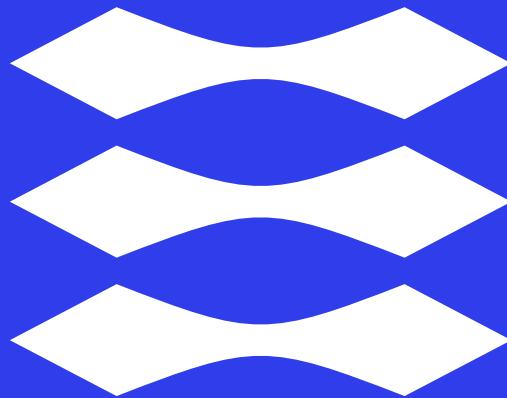


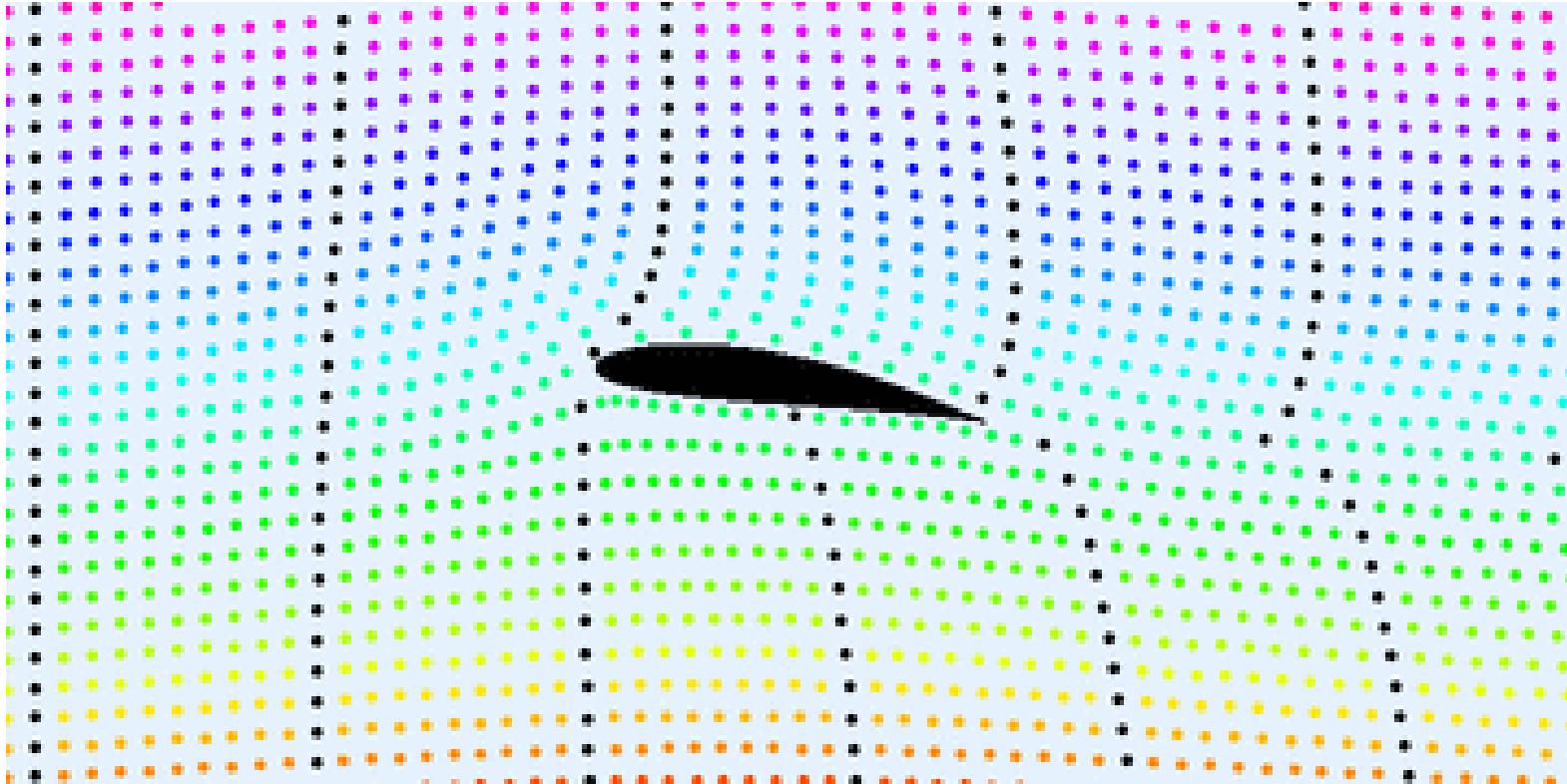
**DTU**



SRP på DTU Vind Energi 2024

# Vinge Aerodynamik

# Fluid Mekanik og Aerodynamik



[https://en.wikipedia.org/wiki/Lift\\_\(force\)#/media/File:Karman\\_trefftz.gif](https://en.wikipedia.org/wiki/Lift_(force)#/media/File:Karman_trefftz.gif)

Robert Mikkelsen

DTU Vind og Energi Systemer

Sektion for Aero- og Fluid Dynamik



GE's Haliade-X wind turbine

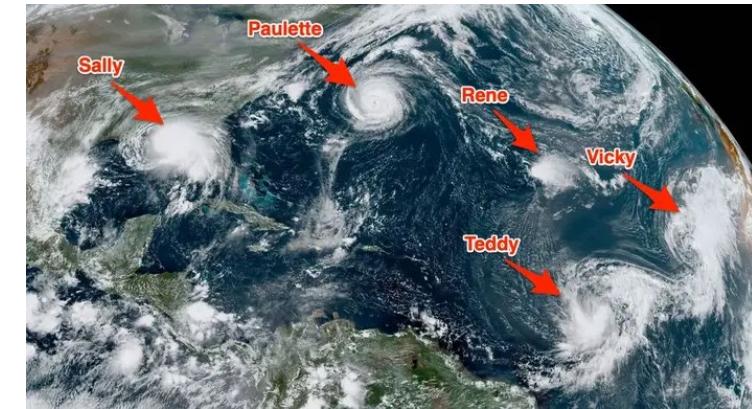
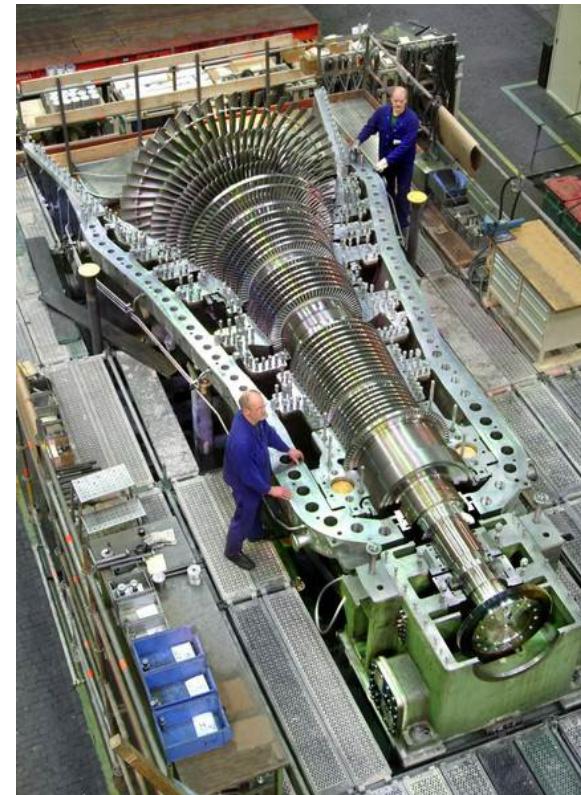
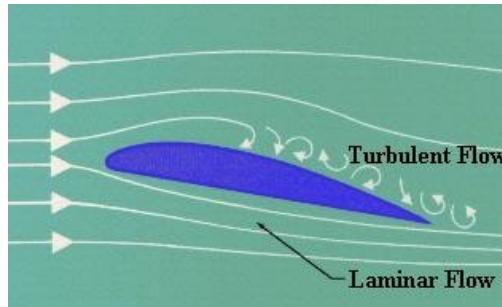
# Aerodynamik

**Studie af luften bevægelse omkring solide legemer**

- Luft hastighed  $V[m/s]$
- Trykfordelinger på legemer  $p[N/m^2, Pa]$
- De samlede kræfter og momenter  $F[N], M[Nm]$

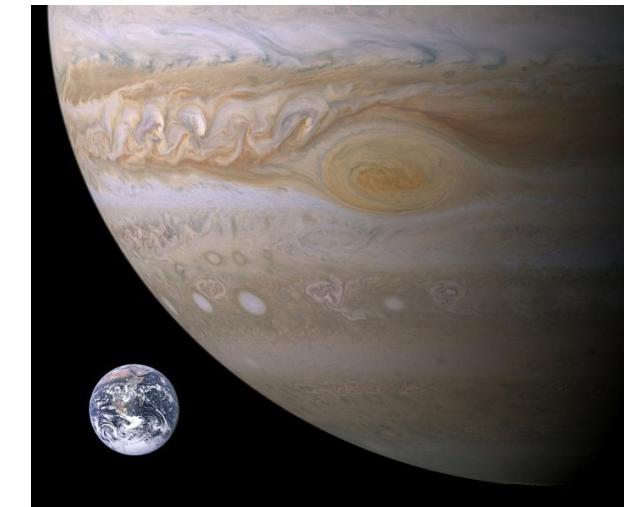
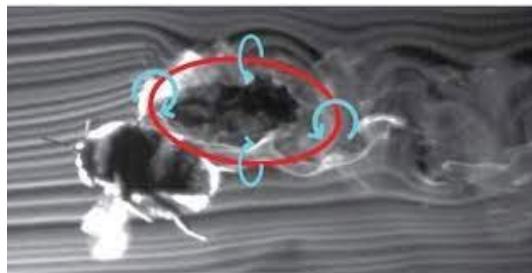
**Strømlinede eller "bluff" bodies/legemer**

# Fluid Mekanik – fysik og ingeniør disipliner...



NOAA GOES-East

- Aerodynamik
- Hydrodynamik
- Oceanografi
- Meteorologi
- Geologi
- Bio-fluid mekanik
- Strømninger med forbrænding
- Turbomaskineri
- Smøringsmekanik
- Andre interne strømninger, rør, procesanlæg, indeklima
- Visse teoretisk interessante strømninger
  - Solen, kaos, hvirvel dynamik,...

[http://da.wikipedia.org/wiki/Billede:Jupiter-Earth-Spot\\_comparison.jpg](http://da.wikipedia.org/wiki/Billede:Jupiter-Earth-Spot_comparison.jpg)

# Først lidt Fluid Mekanik... Hvad er Tryk?

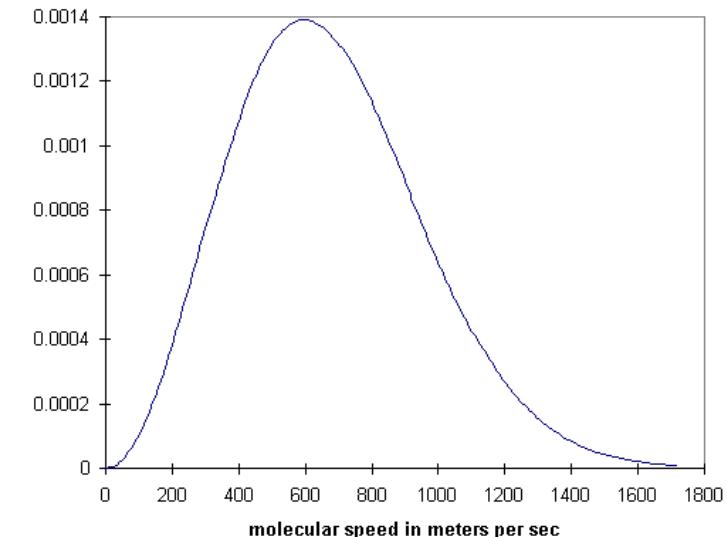
- Skalar, ingen retning, isotropisk [N/m<sup>2</sup>, Pa]
- Virker normal til alle overflader

## Luft:

- Molekulær hastighed  $V_m \approx 5-600 \text{ m/s}$
- Middelfaststand  $10^{-10} \text{ m}$
- Kompressibelt

## Vand:

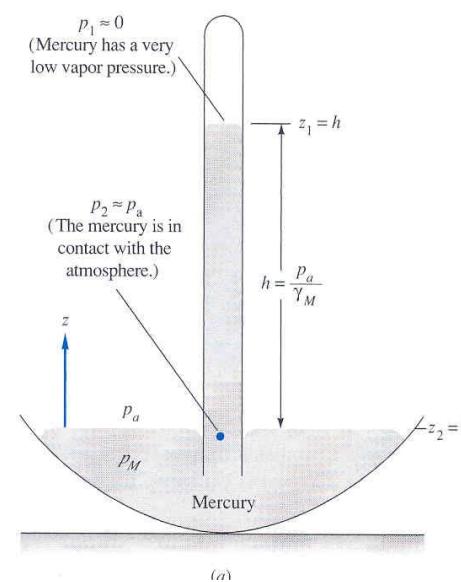
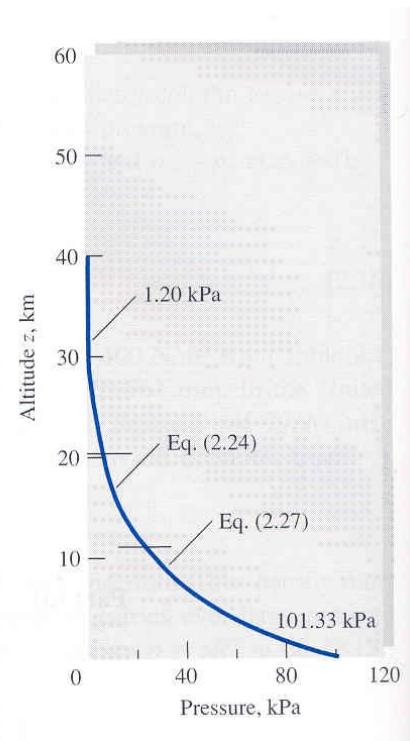
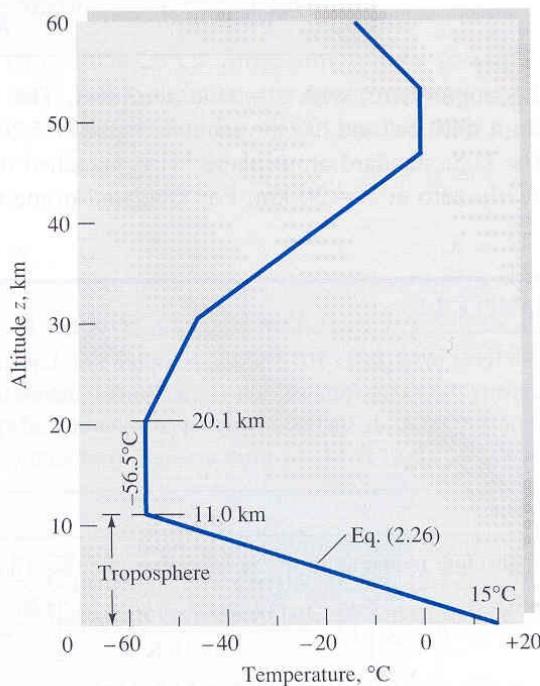
- Næsten inkompressibelt
- $\rho = 1000 \text{ kg/m}^3$  ved  $T = 4^\circ\text{C}$ , Havvand:  $\rho = 1025 \text{ kg/m}^3$



## Tryk termer:

- Statisk, hydrostatisk, overtryk(gage)
- Dynamisk, stagnations

# Atmosfæren



Barometer



**Ideal gas ligning:**  $p = \rho R T$

$$R = \frac{R_u}{M} = \frac{8314,3 \text{ J/(kgmol} \cdot \text{K)}}{28,98 \text{ kg/kgmol}}$$

$R$ : gas konstant for luft

$R_u$ : universal gas konstant

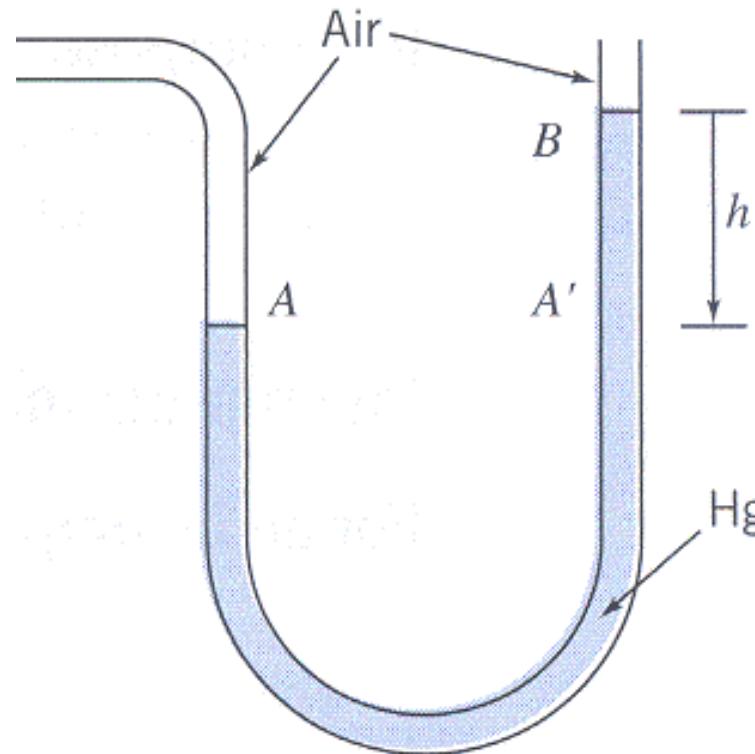
$M$ : molær masse af luft

**Isoterm:**  $RT=konstant$

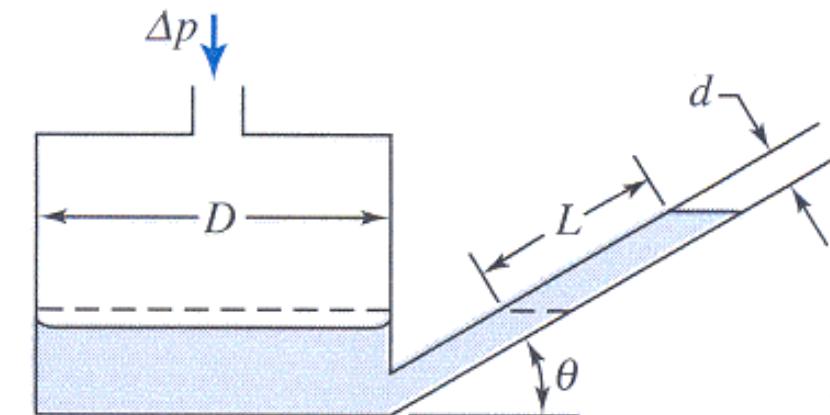
$$RT = K = \frac{p}{\rho} \Rightarrow p_1 V_1 = p_2 V_2 = K$$

# Måling af tryk – U-rørs manometer

$$\Delta p = p - p_0 = \rho g h$$



$$\Delta p = p - p_0 \sim \rho g L \sin\theta$$



# Tryk definitioner

**Statisk tryk,  $p_s$ :** Det tryk en fluid partikel oplever – måles på siden af en væg

$$p_s$$

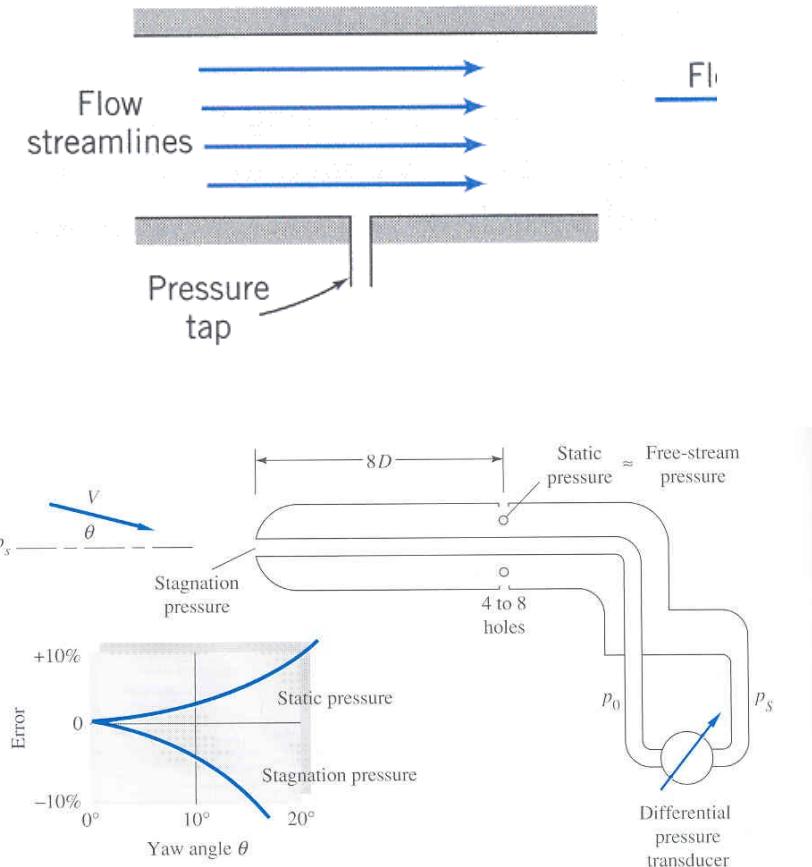
**Stagnations tryk  $p_o$ :** Tryk når fluid partikel decelereres til  $V=0$ , måles i front af probe

$$p_o = p_s + \frac{1}{2} \rho V_1^2 \Leftrightarrow p_o - p_s = \frac{1}{2} \rho V_1^2 = \Delta p \Leftrightarrow$$

$$V = \sqrt{\frac{2\Delta p}{\rho}}$$

**Dynamisk tryk  $p_{dyn}$ :** Vind tryk – forskel mellem stagnations- og statisk tryk

$$p_{dyn} = \frac{1}{2} \rho V^2$$



# Stationær strømning langs strømlinje - Bernoulli

**Bernoulli's ligning, stationær, inkompressible**

$$\left( \frac{p_2}{\rho} - \frac{p_1}{\rho} \right) + \frac{1}{2} \left( V_2^2 - V_1^2 \right) + g(z_2 - z_1) = 0$$

**...eller...**

$$\frac{p}{\rho} + \frac{1}{2} V^2 + gz = konst.$$

**Begrænsninger:**

- **Inviskos,  $\mu=0$**
- **Gælder langs strømlinje**
- **Stationær,  $d/dt=0$**
- **Inkompressible,  $\rho=konstant$**

# Aerodynamik

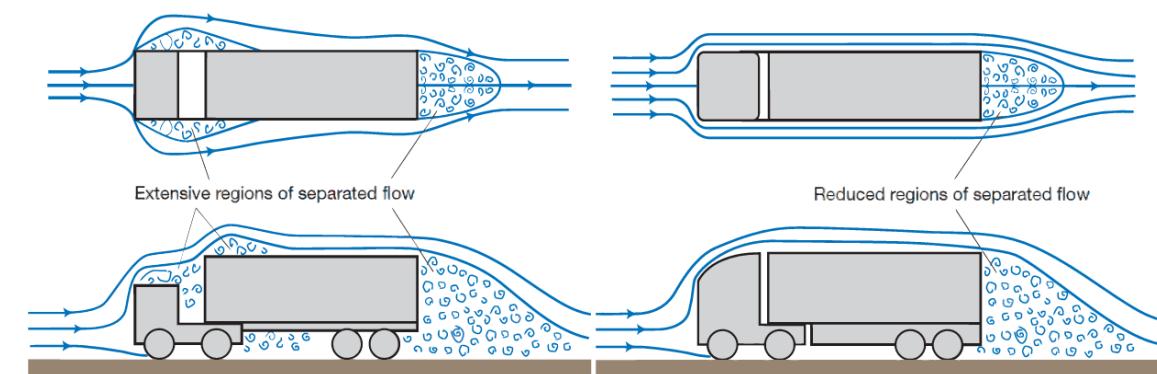
- **Lift kræfter dominerer – Vinger**

- Fly
- Vindmøller
- Propeller
- Racerbiler – downforce



- **Drag kræfter dominerer - "Bluff bodies"**

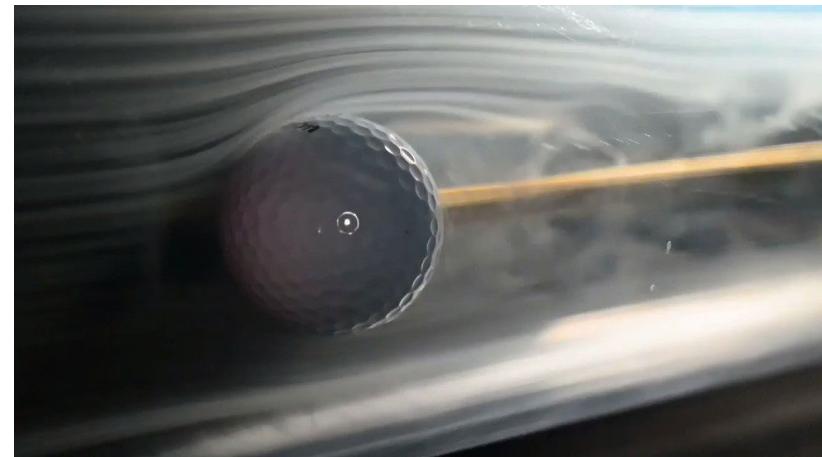
- Transport fartøjer: Biler, lastbiler, skibe, tog, cykel, motorcykel, etc.
- Bygnings strukturer: Huse, broer, etc.
- Mindre strukturer, del-strukturer monteret på fartøjer, bygninger, ...
- etc.



# Drag domineret strømning

## Generelt:

- Ofte separeret strømning
- Meget svært at beregne
- Eksperimentelle målinger nødvendige (vind tunnel)



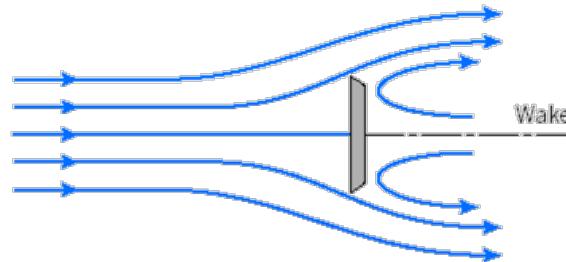
## Modstand – Drag force:

$$D [N] = C_D \cdot \frac{1}{2} \rho V^2 A \quad \Leftrightarrow \quad C_D = \frac{D}{\frac{1}{2} \rho V^2 A}$$

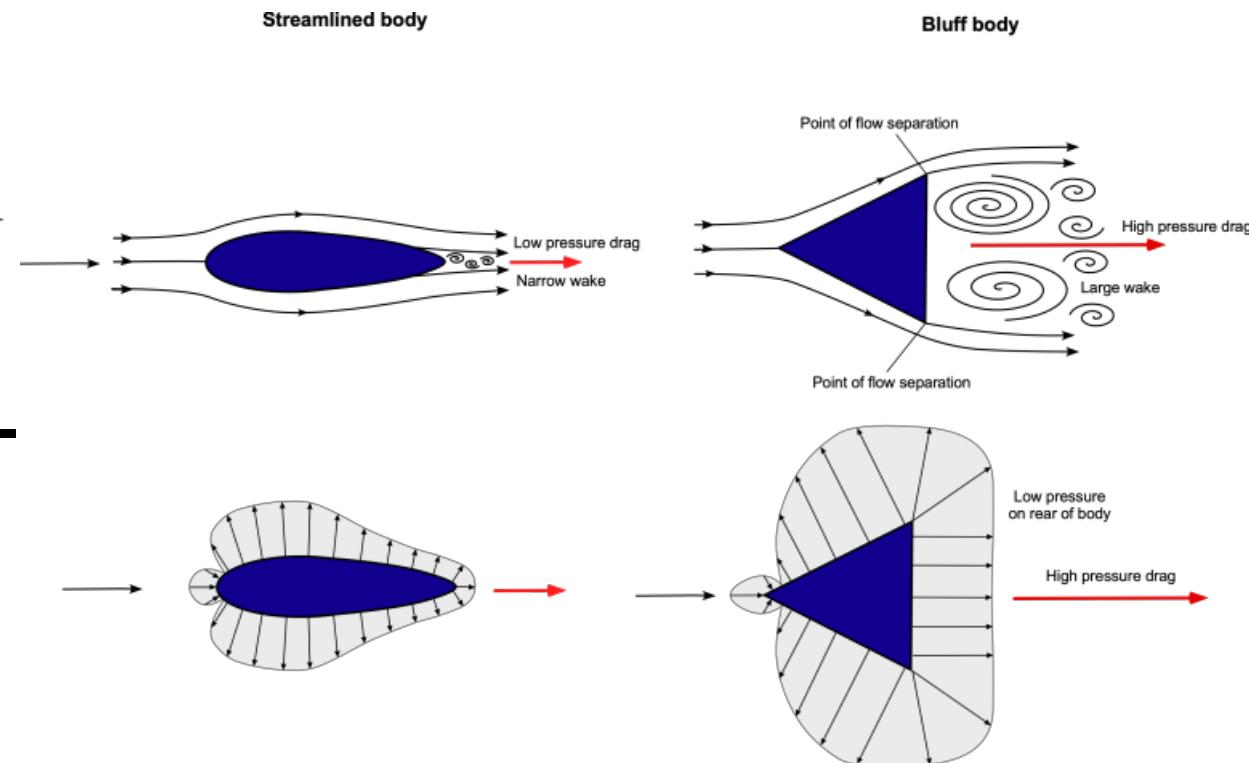


# Tryk modstand – pressure drag

Tryk modstand hidrører fra trykfordeling  
på legeme



$$F_D = \int_{\text{surface}} p dA$$

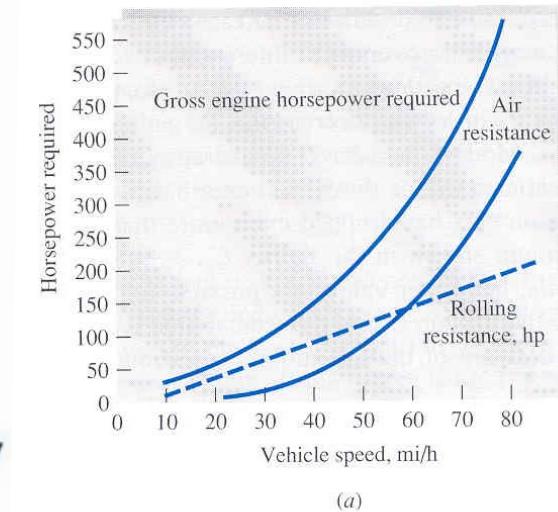
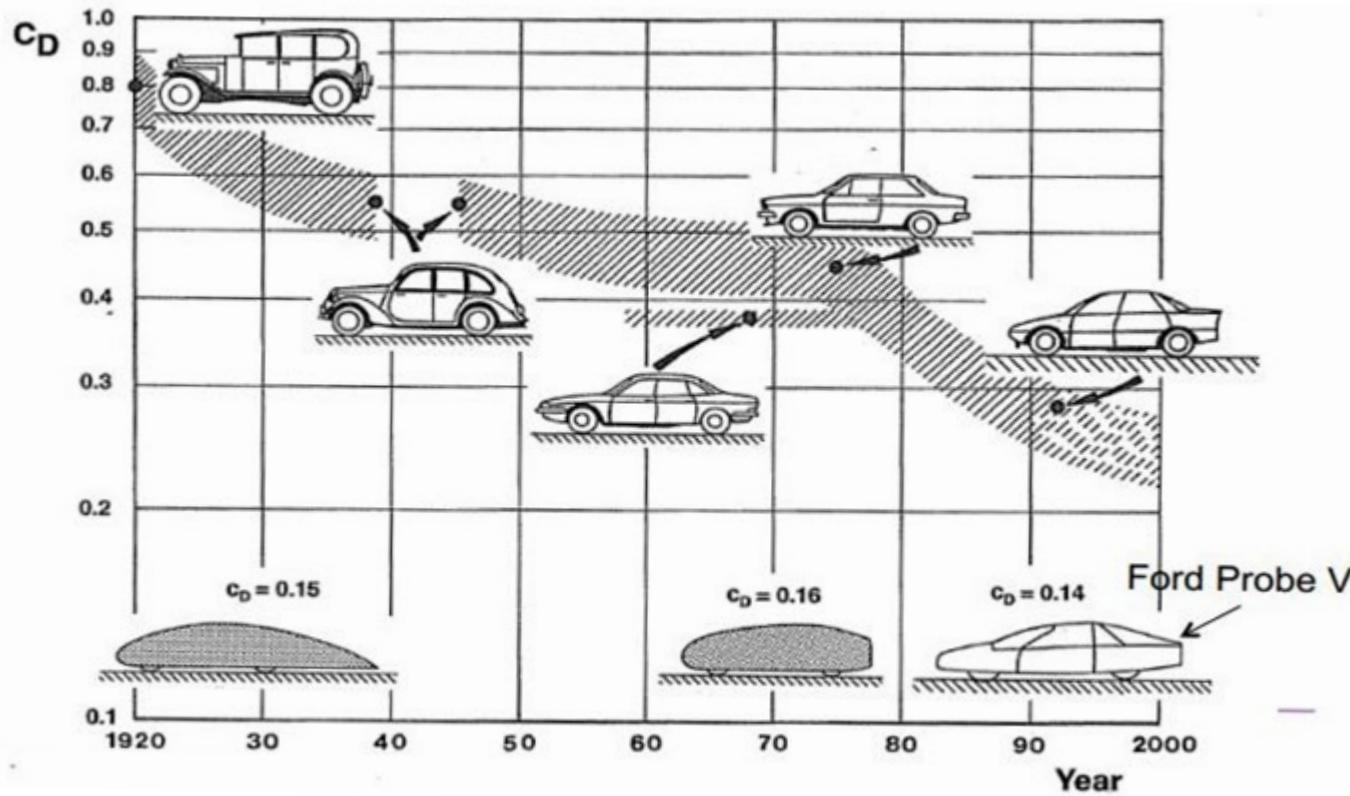


Generelt udtryk for modstands kraft D -  
drag

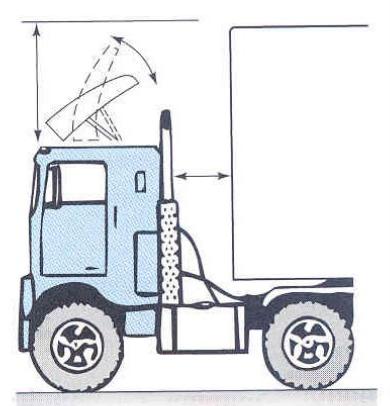
$$D [\text{N}] = C_D \cdot \frac{1}{2} \rho V^2 A \quad \Leftrightarrow \quad C_D = \frac{D}{\frac{1}{2} \rho V^2 A}$$

Drag koefficienter,  $C_D$ , findes ofte  
eksperimentelt (vind tunnel)

# Modstand på kørertøjer



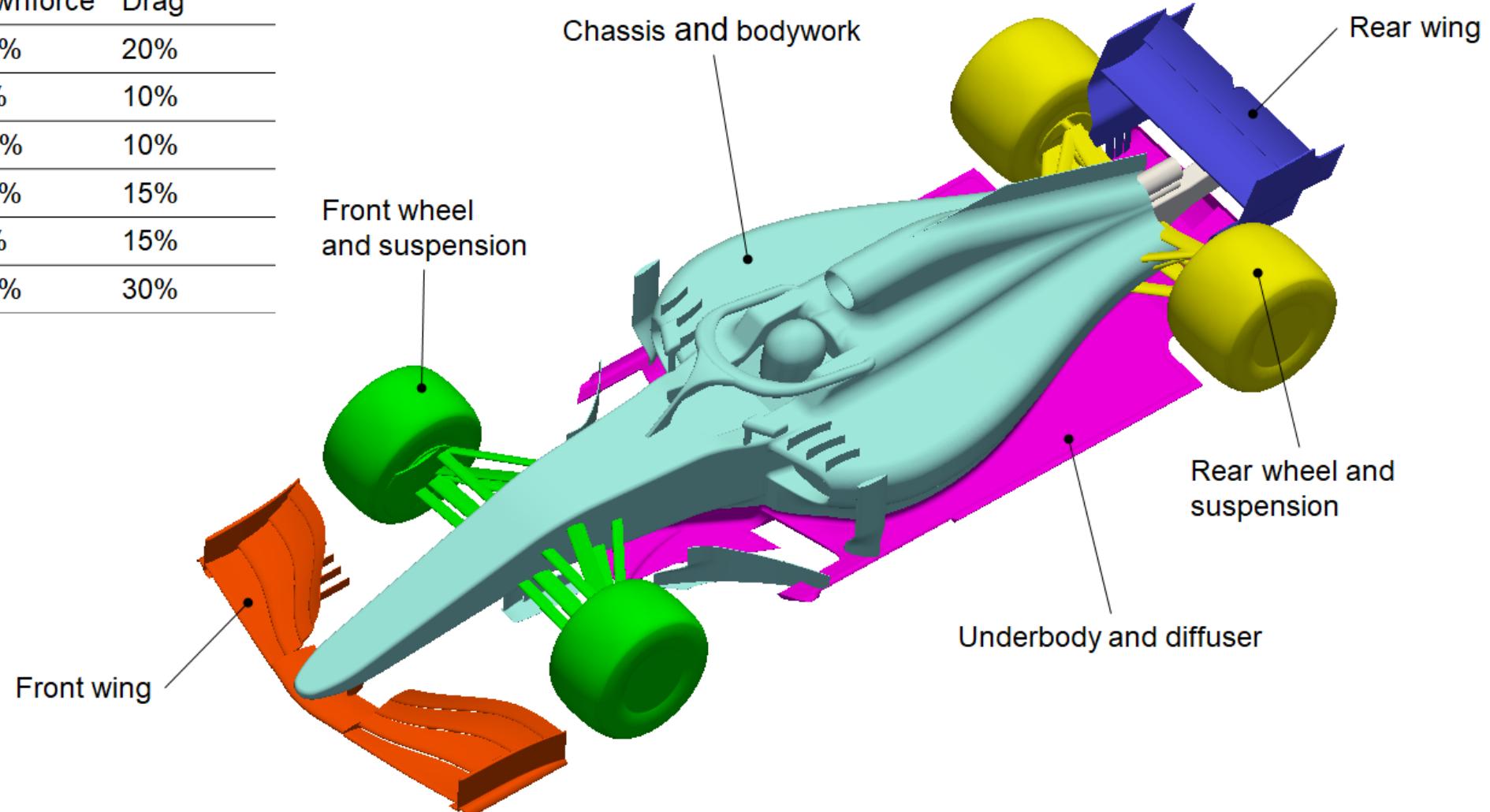
(a)



(b)

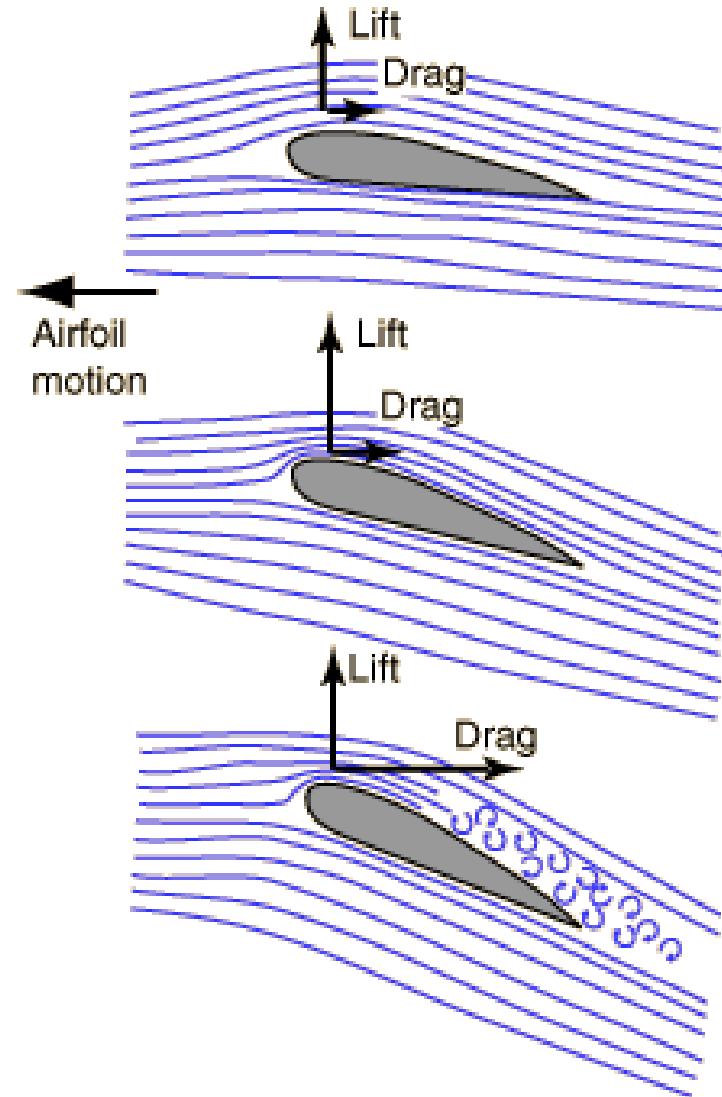
# Modstand og downforce på racerbiler – Formel 1

Downforce	Drag
25%	20%
-2%	10%
-10%	10%
65%	15%
-3%	15%
25%	30%



<https://www.f1technical.net/features/21667>

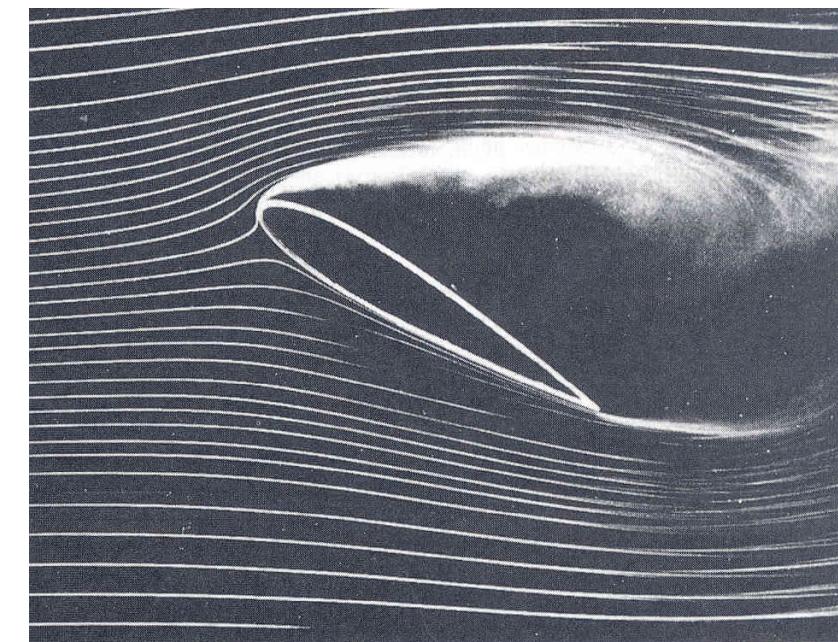
# Lift på vinger



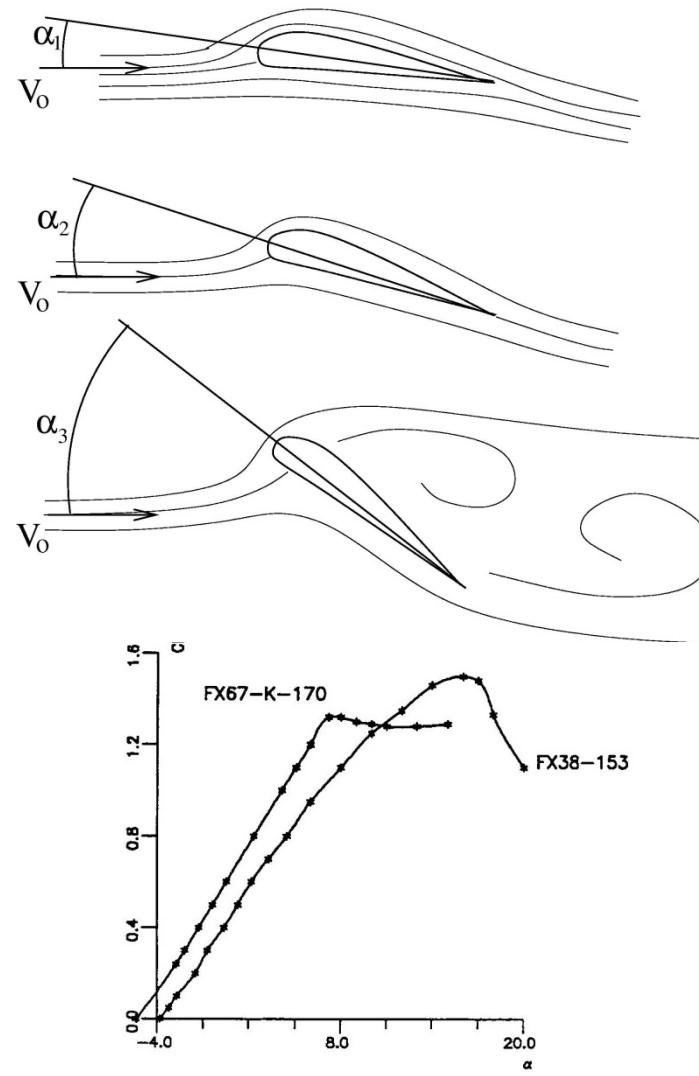
Vinger: L/D>10

Løft – Lift force:

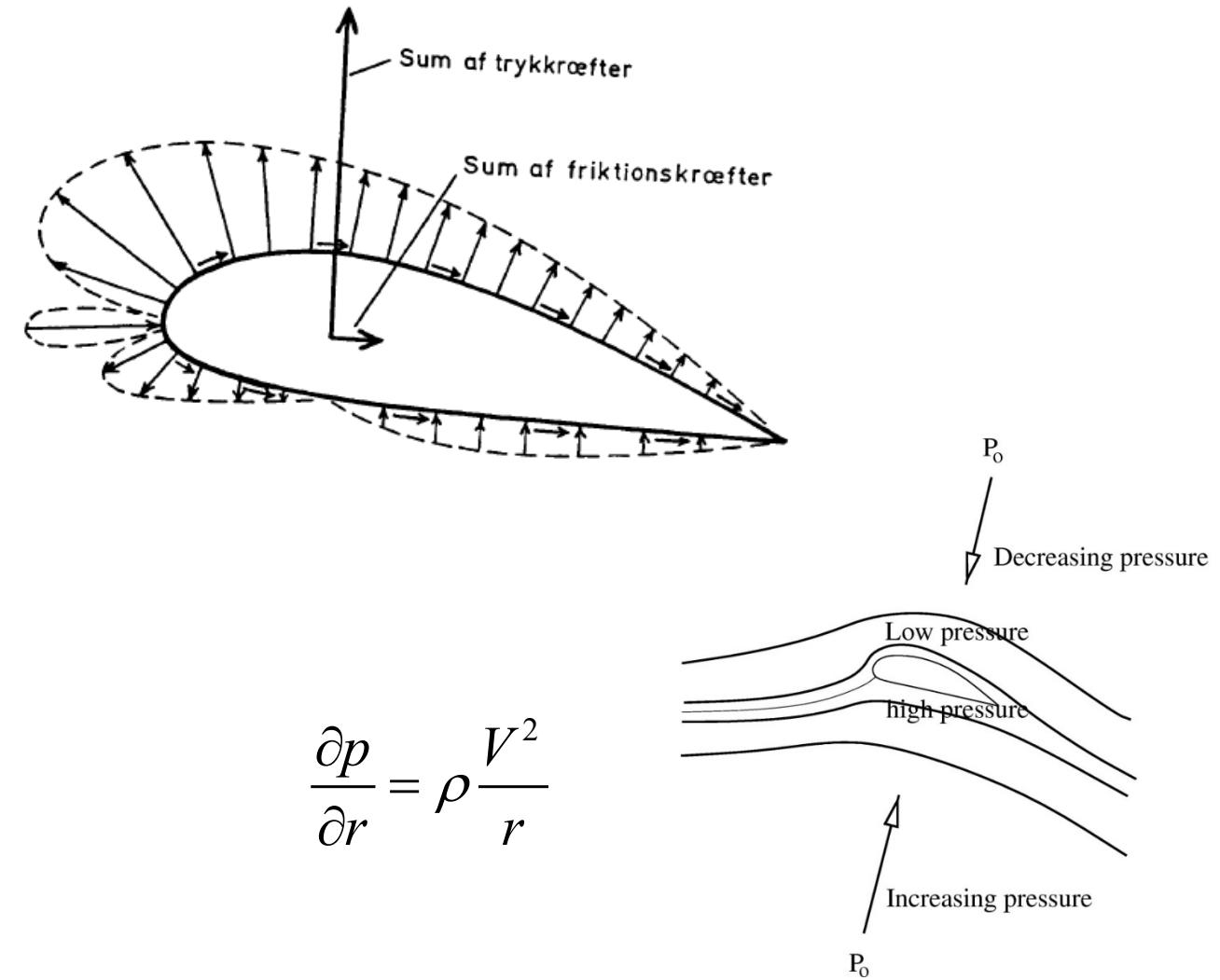
$$L = C_L \cdot \frac{1}{2} \rho V^2 A \quad \Leftrightarrow \quad C_L = \frac{L}{\frac{1}{2} \rho V^2 A}$$



# Tryk og flow omkring vinger

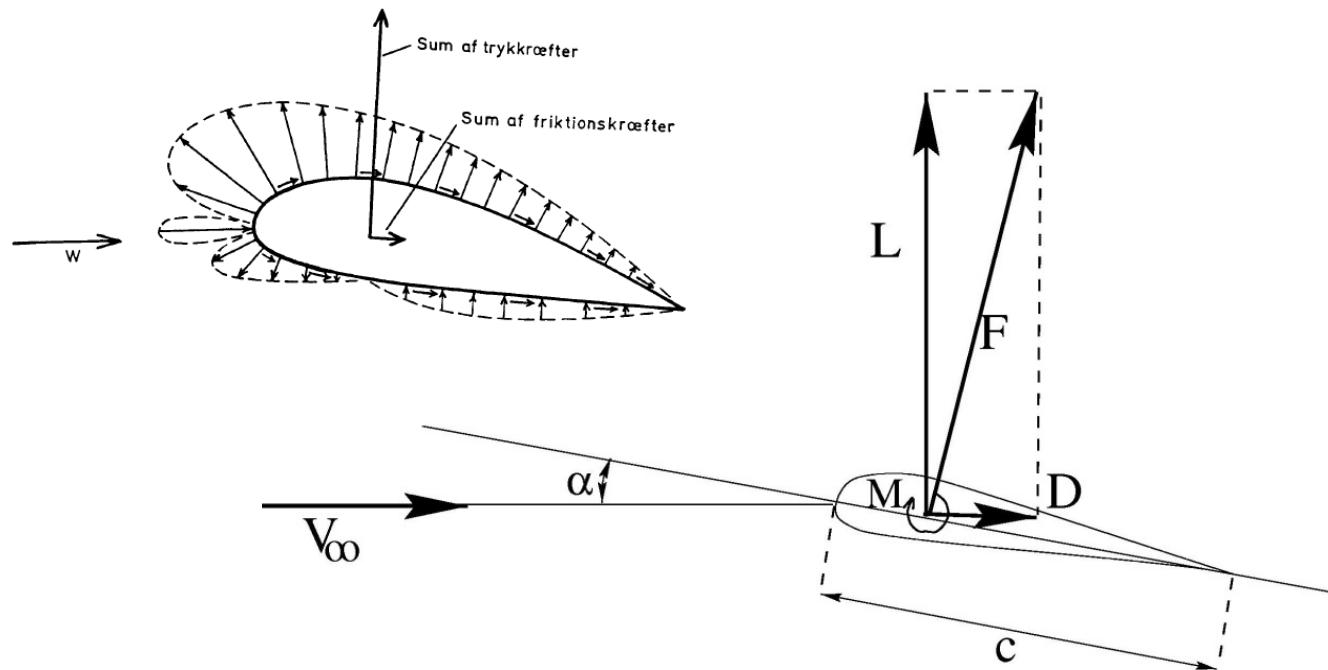


W →



$$\frac{\partial p}{\partial r} = \rho \frac{V^2}{r}$$

# Aerodynamiske koefficienter



$$Re = \frac{\rho V c}{\mu} \quad \text{Mach} = \frac{V}{c_a}$$

$$\rho \sim 1.2 \text{ kg/m}^3 \quad \mu \sim 1.8 \times 10^{-5} \text{ Pa}\cdot\text{s} \quad c_a \sim 340 \text{ m/s}$$

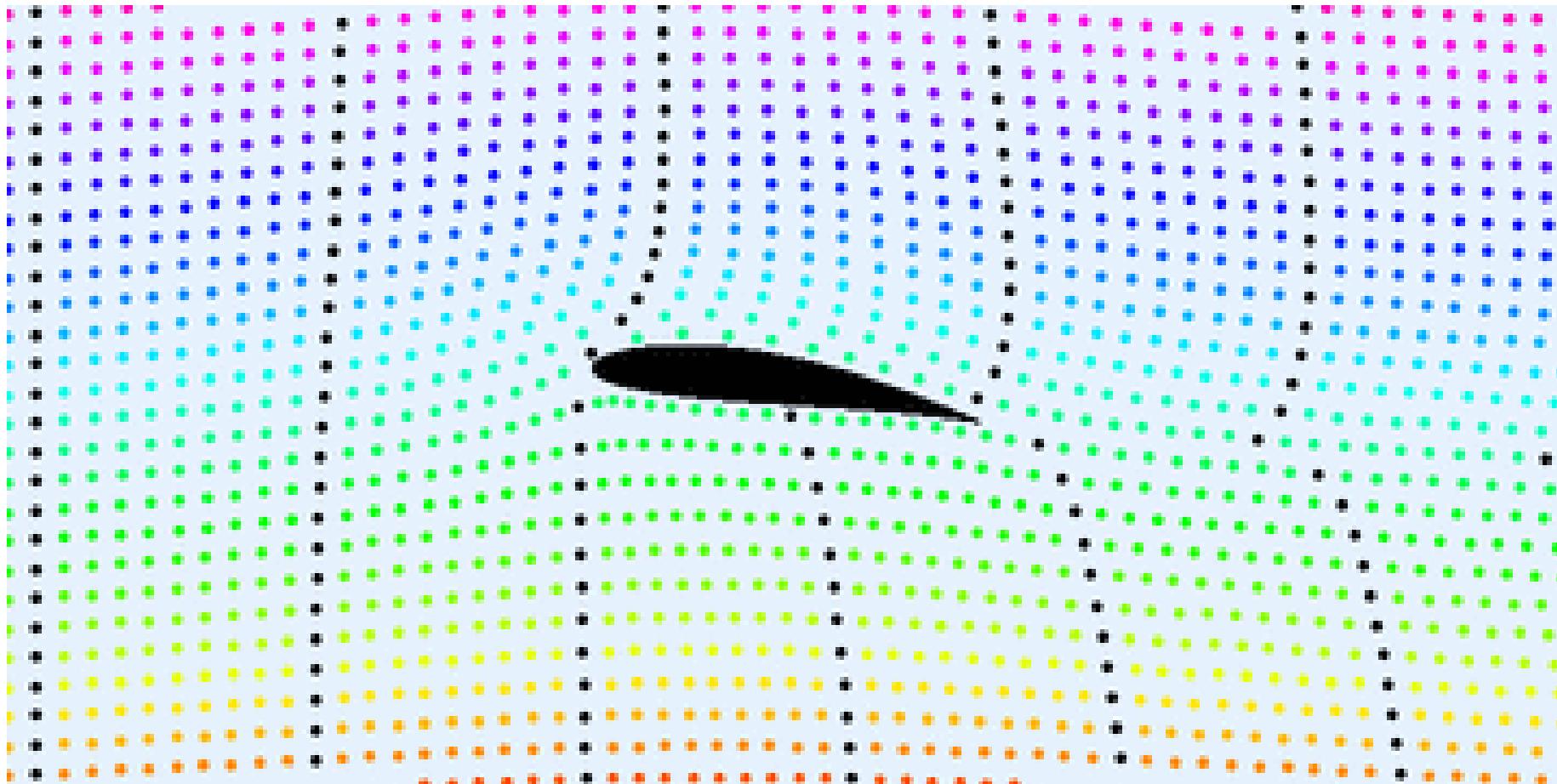
$$C_l(\alpha, Re, M, "shape") = \frac{l}{\frac{1}{2} \rho V^2 c}$$

$$C_d(\alpha, Re, M, "shape") = \frac{d}{\frac{1}{2} \rho V^2 c}$$

$$C_m(\alpha, Re, M, "shape") = \frac{m}{\frac{1}{2} \rho V^2 c^2}$$

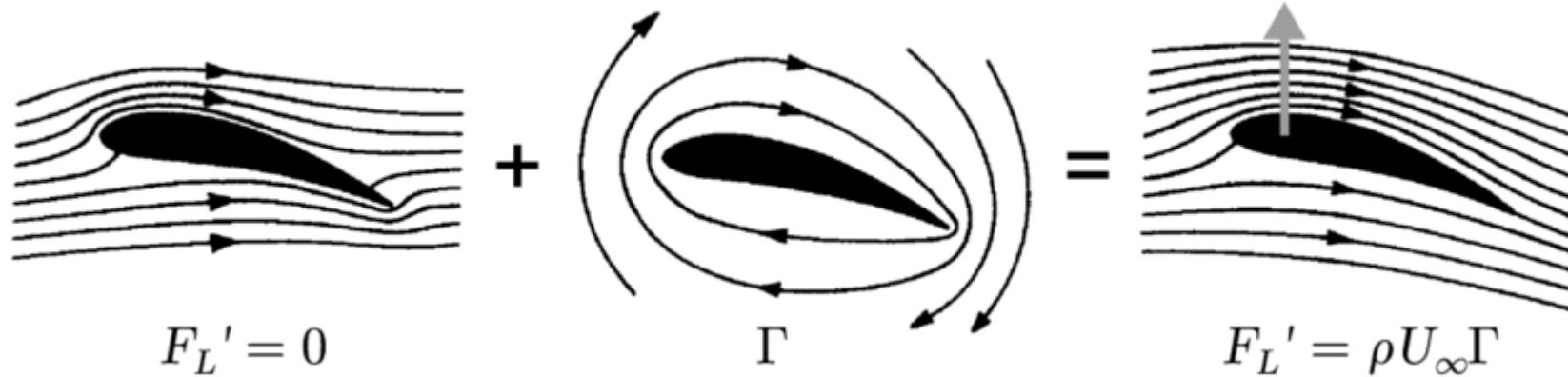
Dimensioner:  $l$  og  $d$  [N/m]  $m$  [Nm/m]

# Flow omkring en 2D vinge



[https://en.wikipedia.org/wiki/Lift\\_\(force\)#/media/File:Karman\\_trefftz.gif](https://en.wikipedia.org/wiki/Lift_(force)#/media/File:Karman_trefftz.gif)

# Lift fra circulation – Kutta-Joukowski



$$F_L' = 0$$

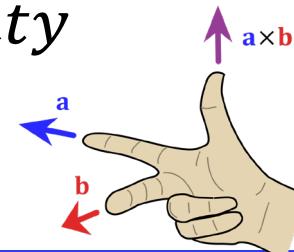
$$\Gamma$$

$$F_L' = \rho U_\infty \Gamma$$

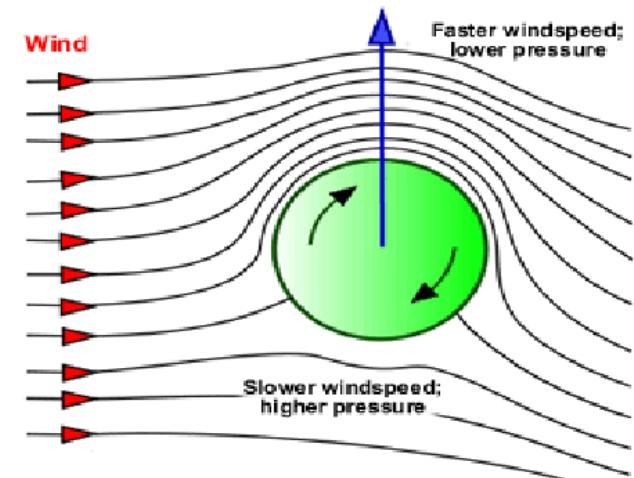
Kutta - Joukowski

$$\text{Lift} \quad \tilde{\text{L}} = \rho \cdot \tilde{\Gamma} \times \tilde{V}$$

*Circulation*      *Cylinder Velocity*



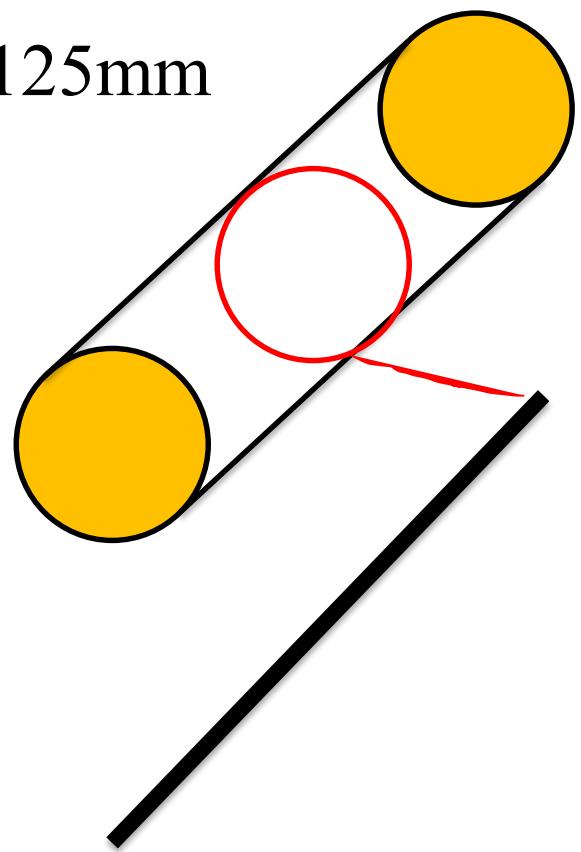
Lift from a cylinder



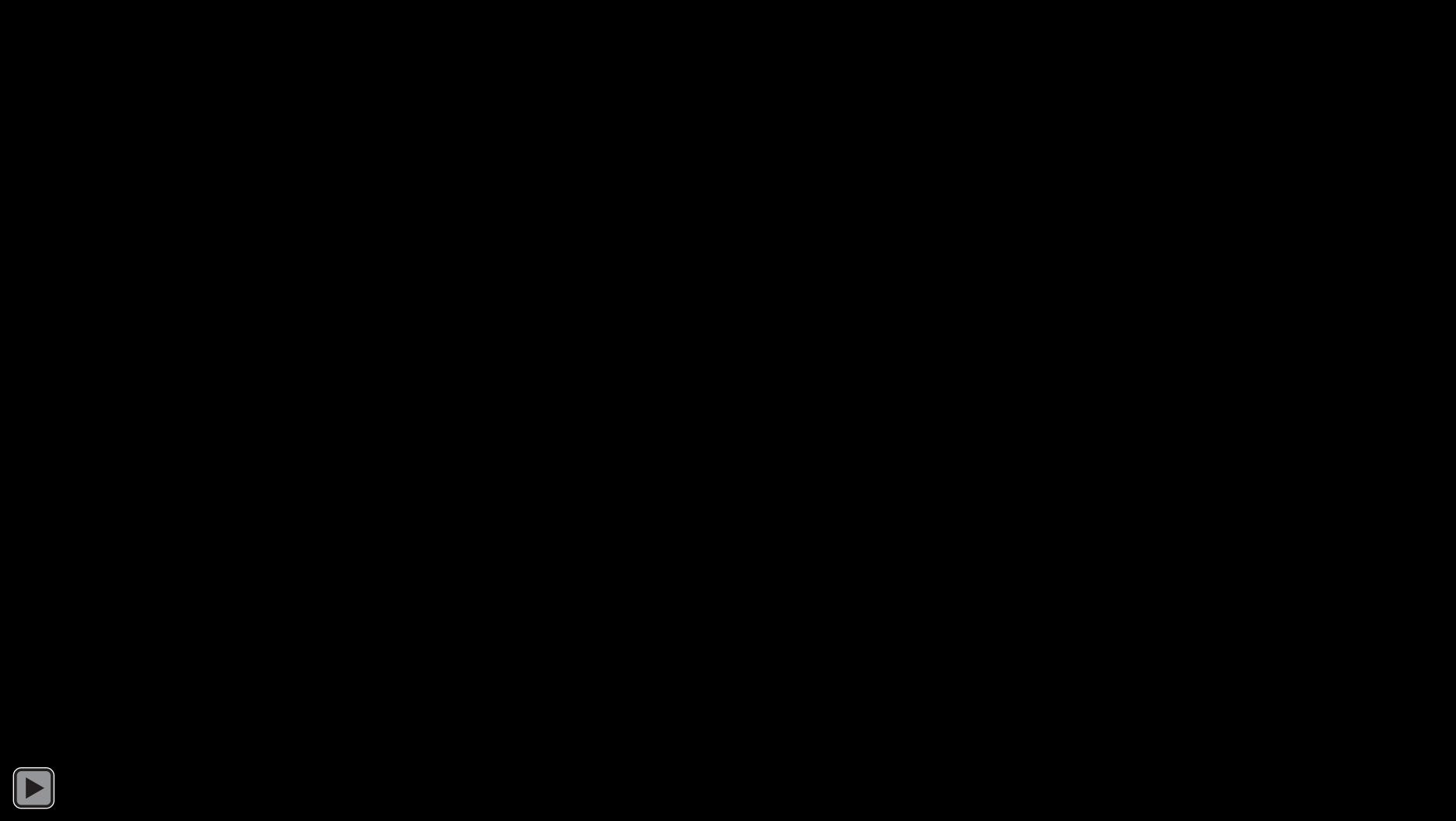
# Lift fra en cylinder – en lille test



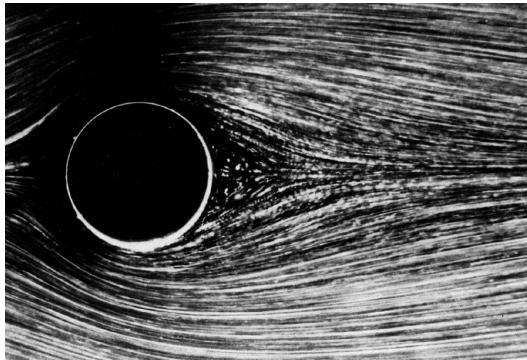
$D=105\text{mm}$   
 $L=1125\text{mm}$



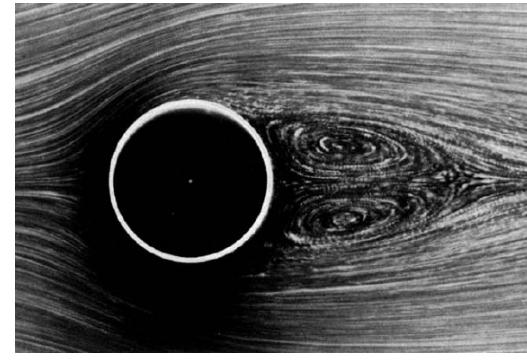
# Roterende cylinder – Magnus kraft



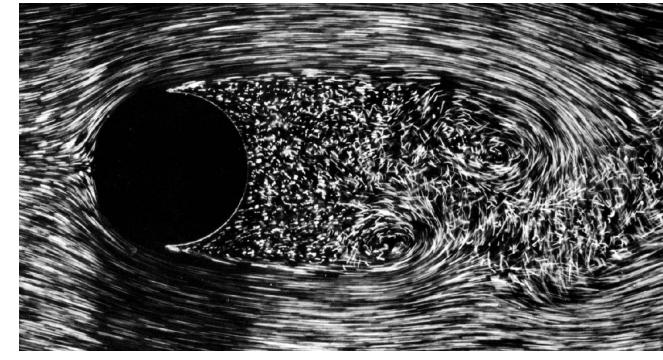
# Reynolds tal – strømning omkring cylinder



**Re=9.6**

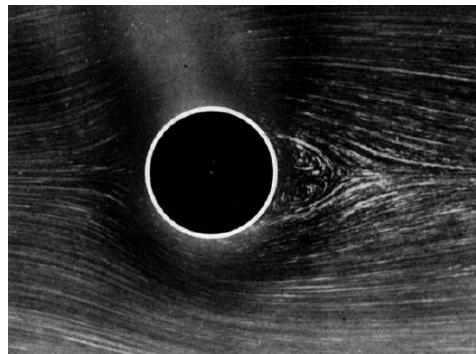


**Re=26**

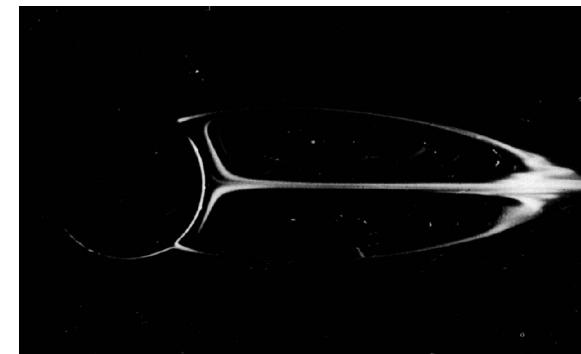


**Re=2.000**

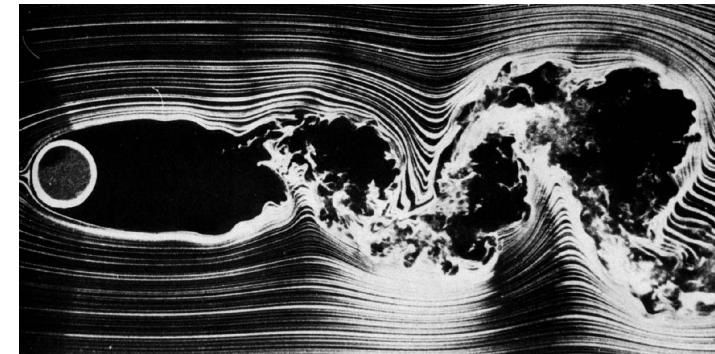
$$\text{Re} = \frac{\rho V c}{\mu}$$



**Re=13.1**



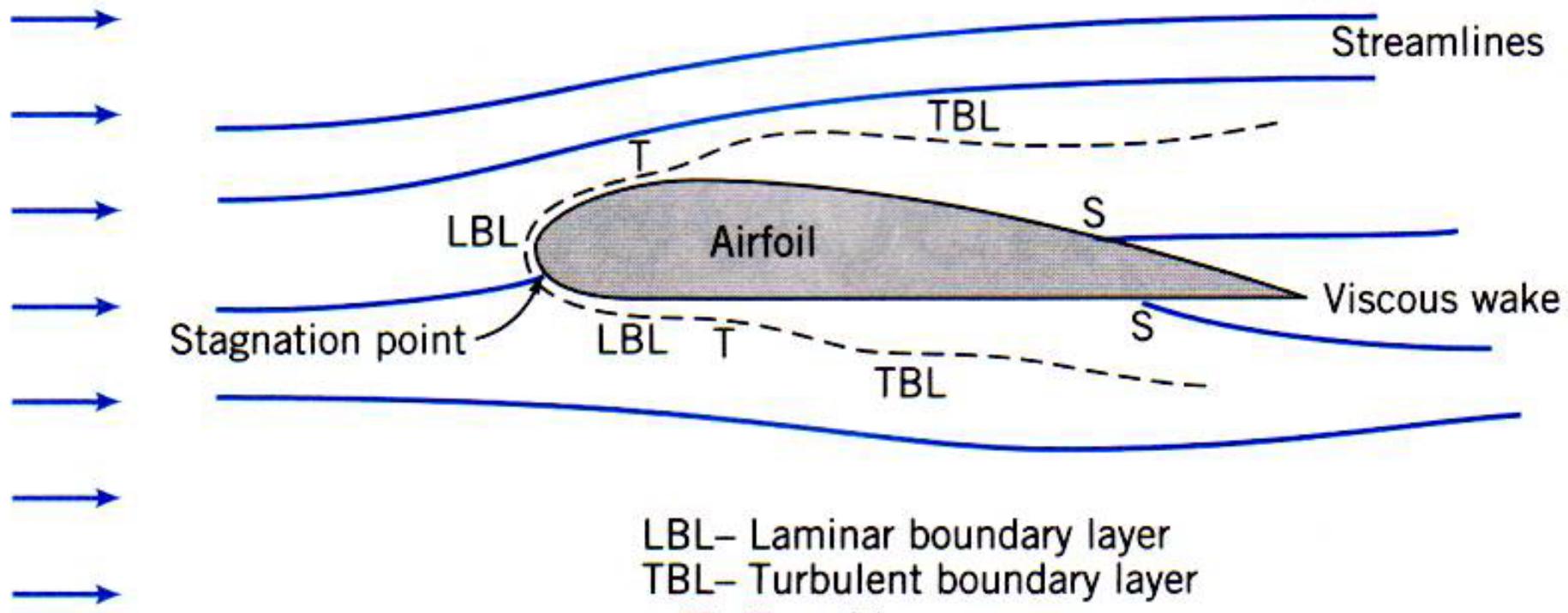
**Re=41**



**Re=10.000**

# Grænselag omkring en vinge

$U_\infty$ -Uniform velocity field upstream

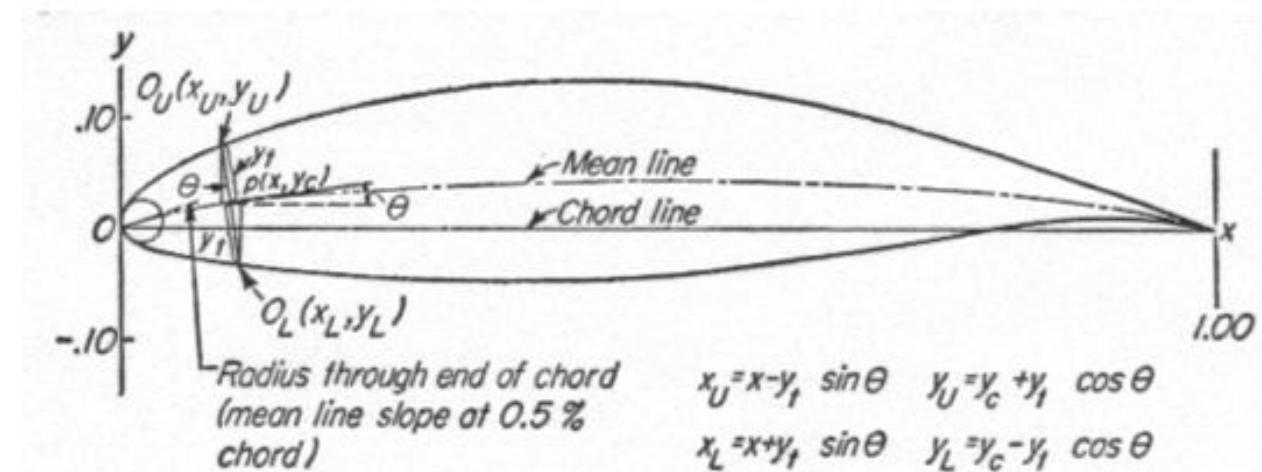
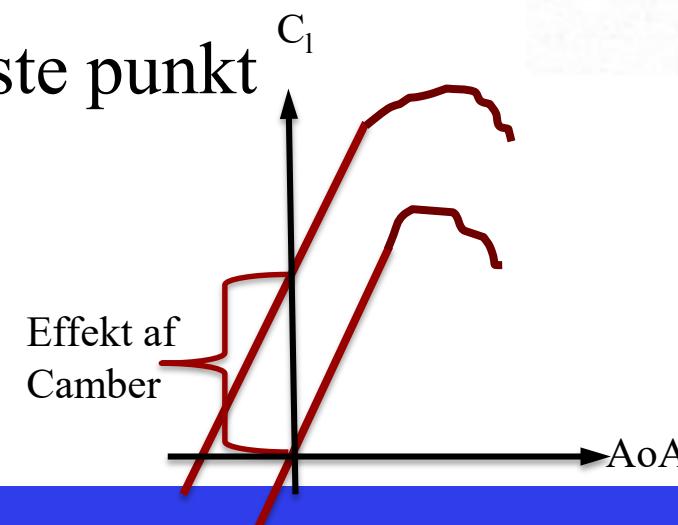


→ LBL- Laminar boundary layer  
→ TBL- Turbulent boundary layer  
→ T- Transition  
→ S- Separation point

# Vinge geometri

## Karakterisering af vingeprofiler

- Profil familie (mange tusinde, NACA, DU, FFA, RAF,...)
  - Høj/lav Reynolds tal
  - Kompressibel / Inkompressibel
  - Laminar
  - ...
- Tykkelse,  $t/c$  [%]
- x-Position af tykkeste punkt
- Camber



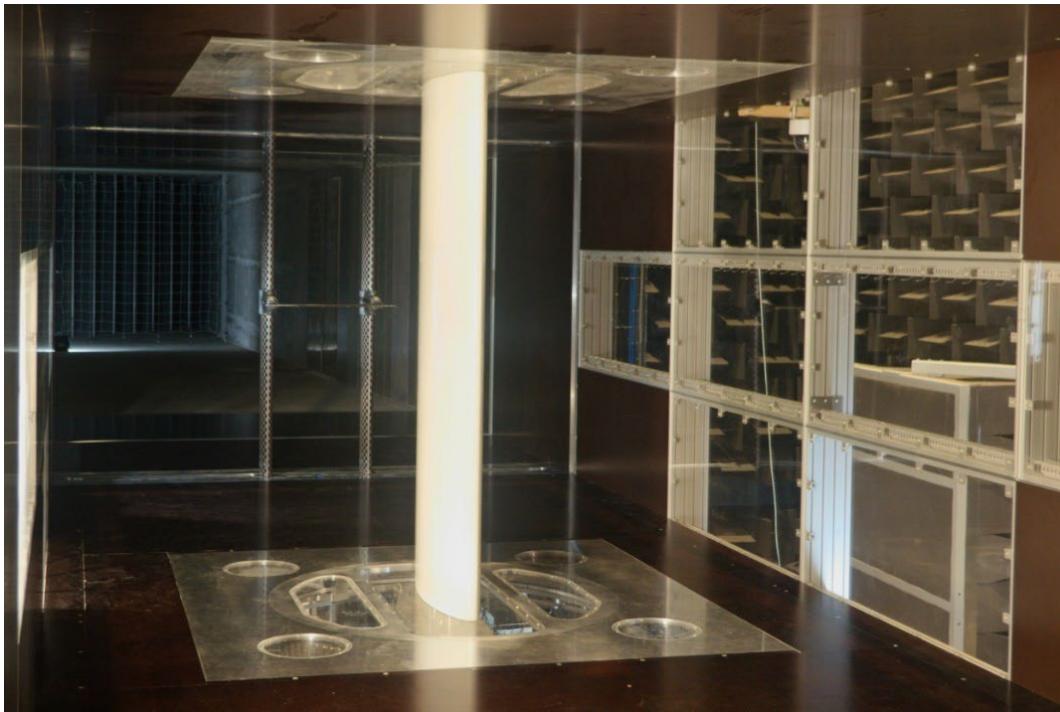
NACA 63-418

Familie – 6-series      x-max t/c      camber      tykkelse

NACA 6      30%       $C_l=0.4$  at 0 aoa      18%

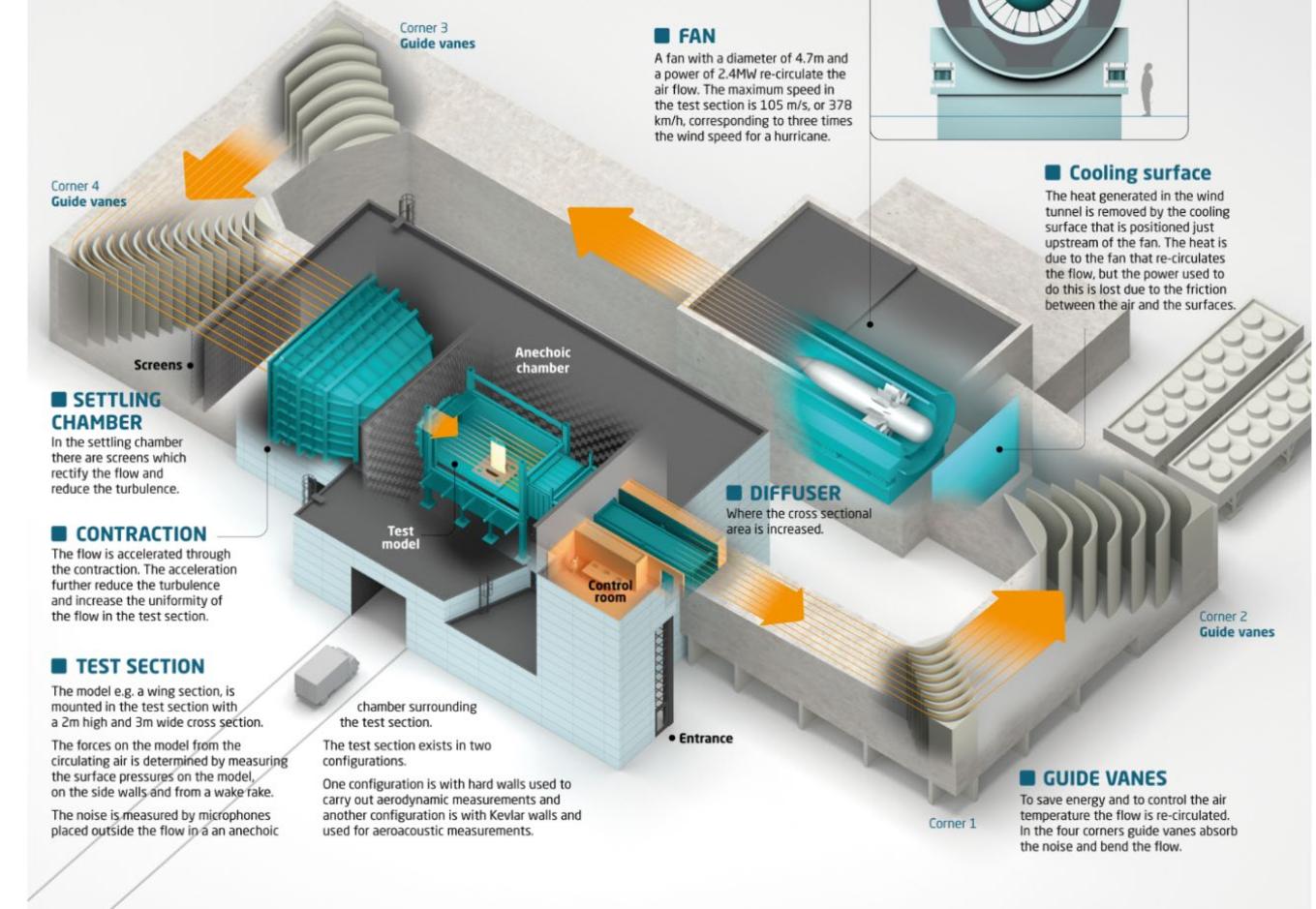
# Vindtunnel måling af 2D vinger

Test sektion - [www.plct.dk](http://www.plct.dk)

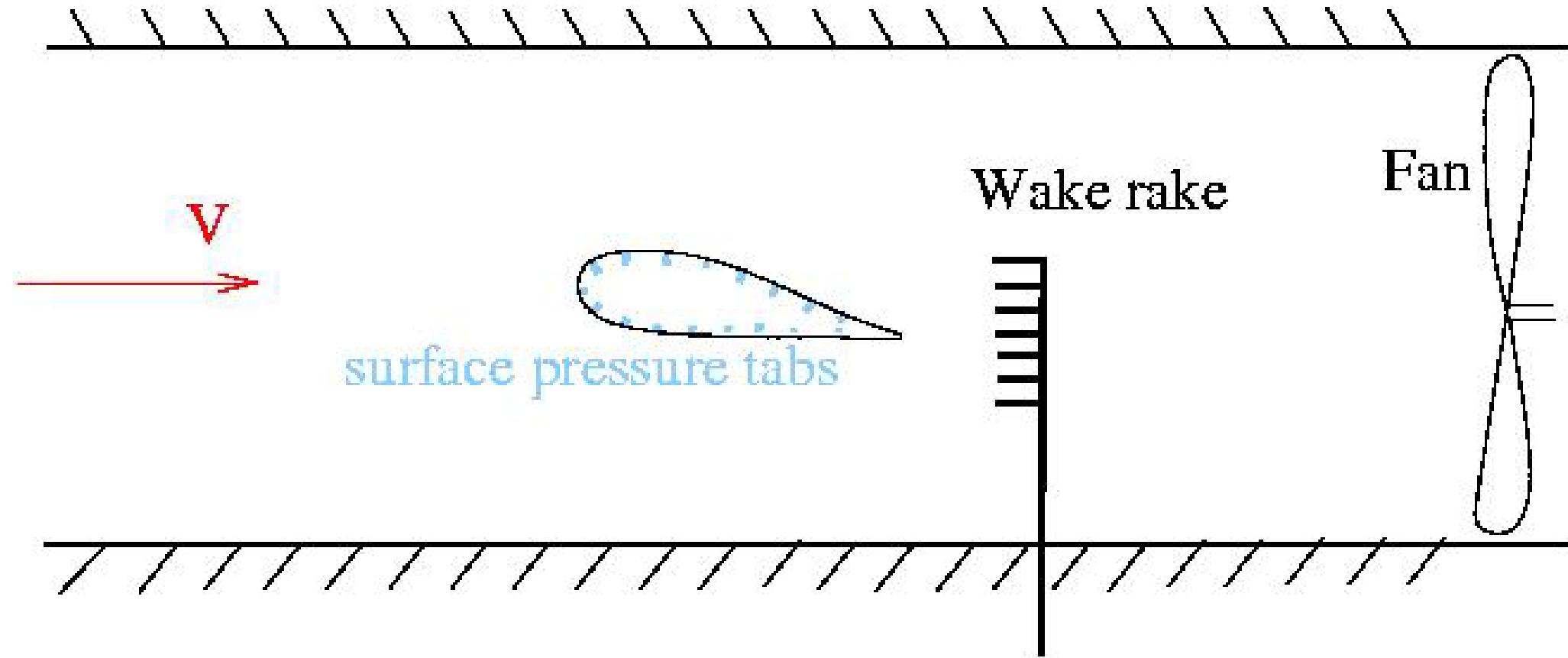


## Poul La Cour Tunnel

From September 2018, DTU Risø Campus will accommodate a new wind tunnel; the Poul La Cour Tunnel. The wind tunnel will be among the biggest university owned tunnels in the world, and the combination of test possibilities makes the wind tunnel one of a kind, not just in Denmark, but globally. This is due to the size of the wind tunnel and high flow speed as well as the possibility to combine measurements of aerodynamics and noise.

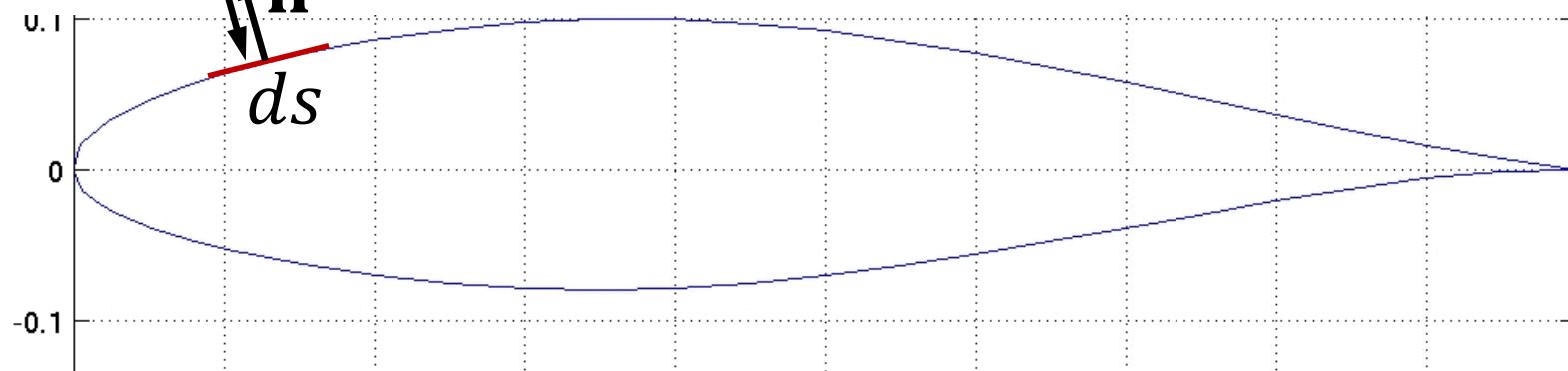


# Måling på 2D vinge



# Kræfter fra trykfordeling - Integration af tryk

$$d\mathbf{F} = -p \mathbf{n} ds$$



$$\mathbf{F} = \oint d\mathbf{F} = - \oint p \mathbf{n} ds$$

# Integration af tryk langs en vilkårlig lukket overflade

$$\Delta s_i^2 = \Delta x_i^2 + \Delta y_i^2$$

- 2D kontur defineret som  $s=s(x,y)$   $\Delta x_i = x_{i+1} - x_i$   $\Delta y_i = y_{i+1} - y_i$
- De samlede tryk kræfter findes ved integration

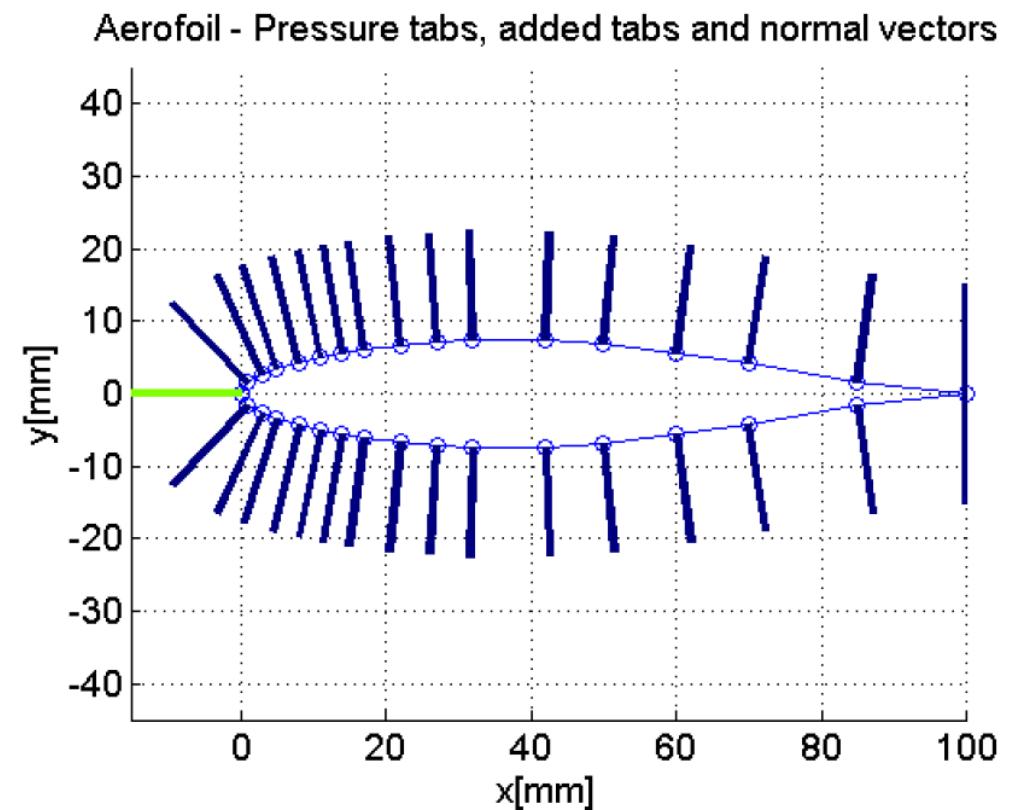
$$\mathbf{F}_p = \begin{Bmatrix} F_{p_x} \\ F_{p_y} \end{Bmatrix} = - \int p \mathbf{n} ds$$

- Normal vektor

$$\mathbf{n} = \begin{Bmatrix} n_x \\ n_y \end{Bmatrix} = \begin{Bmatrix} -dy/ds \\ dx/ds \end{Bmatrix} = \frac{d\hat{s}}{|ds|}$$

$$\mathbf{n} = \begin{Bmatrix} n_{x_i} \\ n_{y_i} \end{Bmatrix} = \begin{Bmatrix} -\Delta y_i / \Delta s_i \\ \Delta x_i / \Delta s_i \end{Bmatrix} \rightarrow \begin{Bmatrix} -\Delta y_i \\ \Delta x_i \end{Bmatrix} = \begin{Bmatrix} \Delta s_i \cdot n_{x_i} \\ \Delta s_i \cdot n_{y_i} \end{Bmatrix}$$

- Diskretiseret til  $N-1$  segmenter med  $N$  punkter



# Integration af tryk langs en vilkårlig lukket overflade

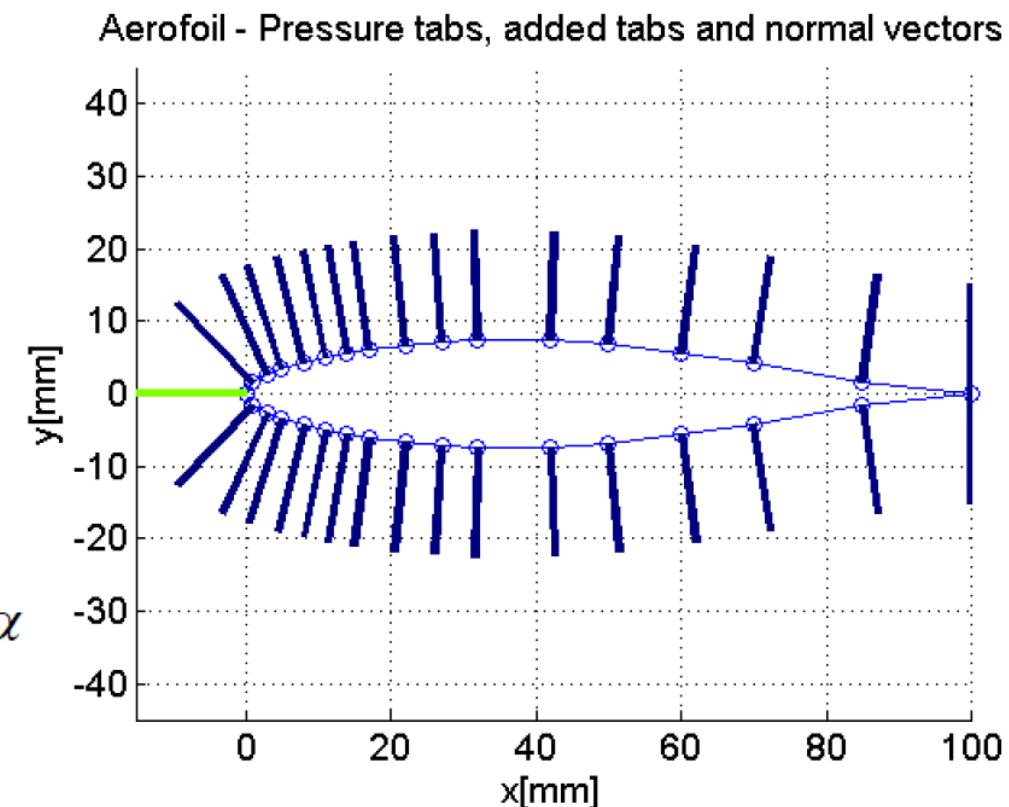
Den numeriske / diskrete version af integralet er (der antages linear variation af n og p...)

$$\begin{Bmatrix} F_{p_x} \\ F_{p_y} \end{Bmatrix} = \begin{aligned} & \sum_{i=1}^{n-1} \left[ \frac{1}{2} (p_i + p_{i+1}) \Delta y_i \right] \\ & - \sum_{i=1}^{n-1} \left[ \frac{1}{2} (p_i + p_{i+1}) \Delta x_i \right] \end{aligned}$$

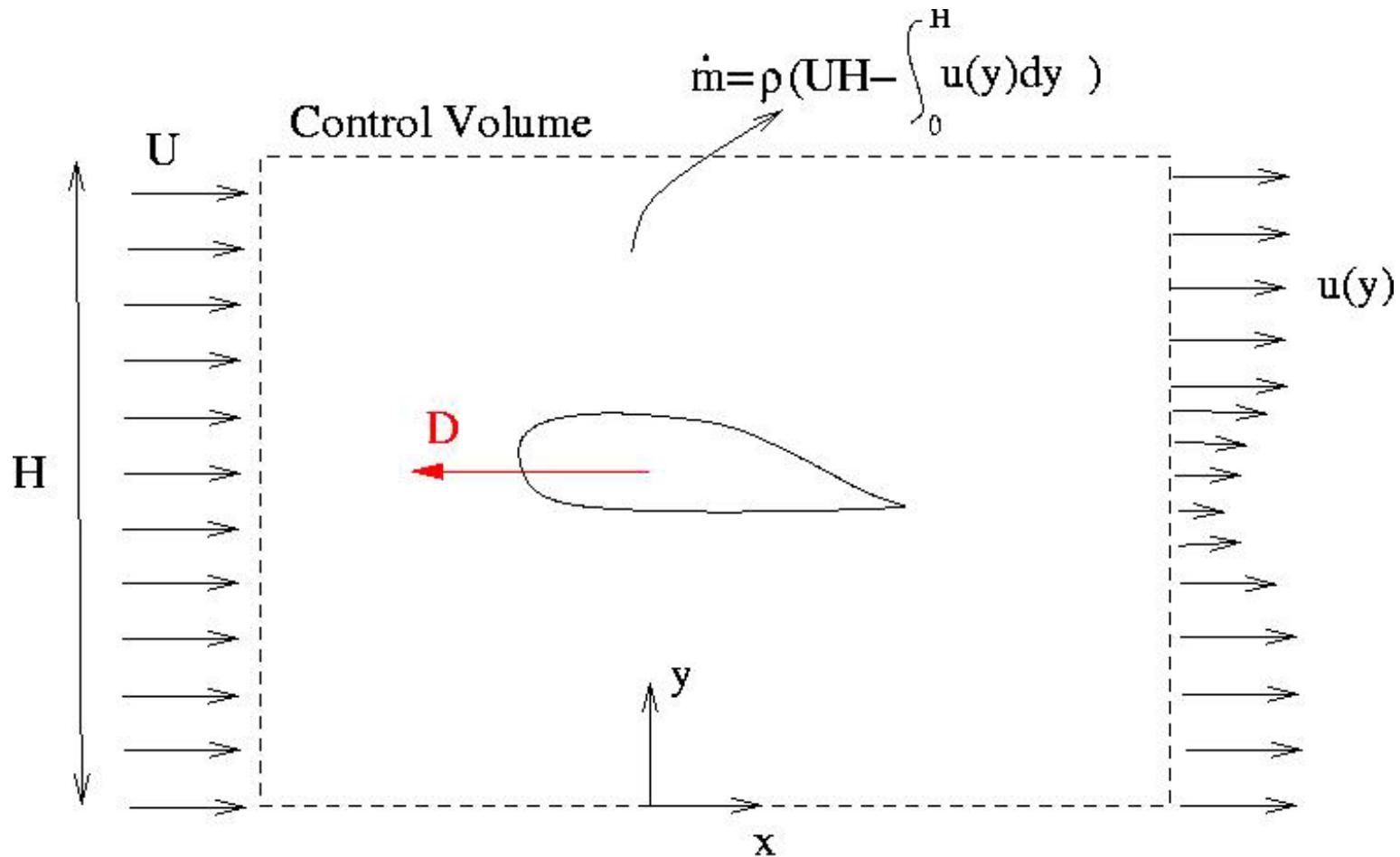
Her er x og y profile koordinater. Lift and Drag fra tryk findes ved projektion af kræfterne

$$L = F_{p_y} \cos \alpha - F_{p_x} \sin \alpha \quad , \quad D = F_{p_x} \cos \alpha + F_{p_y} \sin \alpha$$

hvor  $\alpha$  er "angle of attack" AoA af vingens koordinate system i vingen, i forhold til vind tunnelens koordinate system



# Måling af drag



$$-D = \int_0^H \rho u^2(y) dy + \dot{m}U - \rho U^2 H$$

$$D = \int_0^H \rho u(y)(U - u(y)) dy$$

$$C_d = \frac{D}{\frac{1}{2} \rho U^2 c} = \frac{2}{c} \int_0^H \frac{u(y)}{U} \left(1 - \frac{u(y)}{U}\right) dy$$

# Måling af drag fra slipstrøm - wake fra 2D vinge

- En kontrol volumen analyse af momentum ligningen i x-retningen kan bruges til at bestemme drag som:

$$\theta = \int \rho u(V - u) dy$$

- Med lidt omskrivning kan drag koefficienten skrives som

$$C_D = \frac{\theta}{\frac{1}{2} \rho V^2 c} = 2 \int \frac{u}{V} \left(1 - \frac{u}{V}\right) d\left(\frac{y}{c}\right)$$

- Der anvendes numerisk trapez integration skrevet som

$$C_D \cong \sum_{i=1}^{n-1} \left[ \frac{u_i}{V} \cdot \left(1 - \frac{u_i}{V}\right) + \frac{u_{i+1}}{V} \cdot \left(1 - \frac{u_{i+1}}{V}\right) \right] \cdot \frac{\Delta y_i}{c}$$

- ...men vi mangler hastigheden  $u(y)$  i slipstrømmen... Bernoulli?

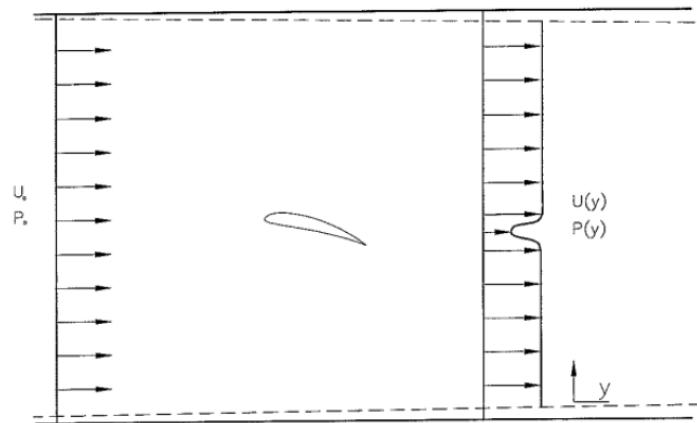


Figure 1 Flow about 2D objects in a wind tunnel (aerofoil, cylinder, etc.)

# Måling af drag fra slipstrøm - wake fra 2D vinge

Bernoulli ligning anvendes - vi måler tryk og skal konvertere til hastighed

$$p_i = p_o + \frac{1}{2} \rho u_i^2 \quad \Rightarrow \quad u_i = \sqrt{\frac{p_i - p_o}{\frac{1}{2} \rho}}$$

Trykket  $p_o$  er det statiske reference tryk på kontrolfladen, det bliver fratrukket alle tryk på vingen, når slanger forbinder til trykskanner.

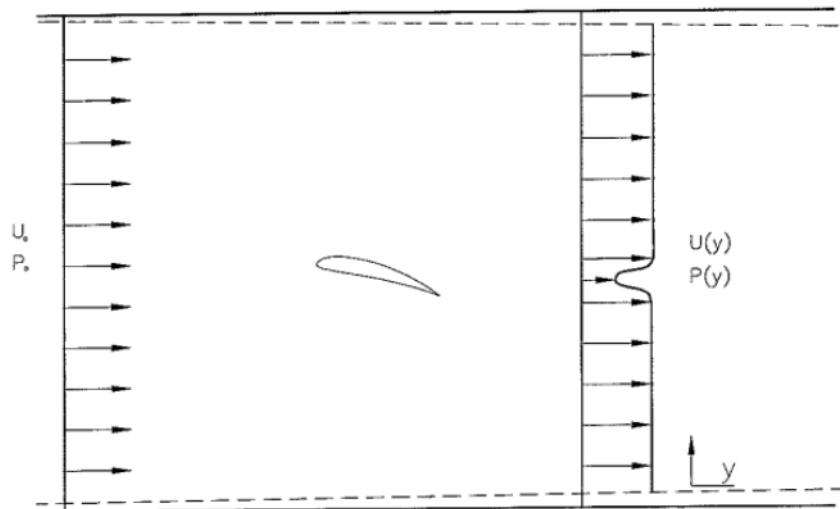
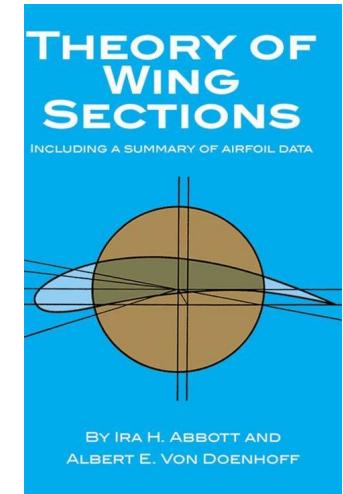
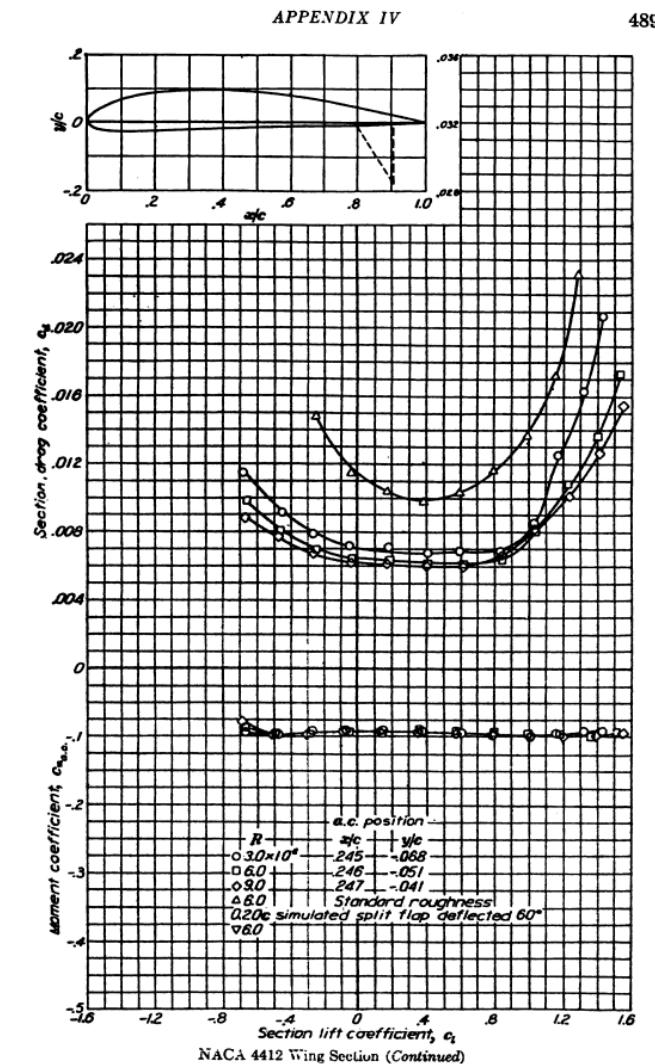
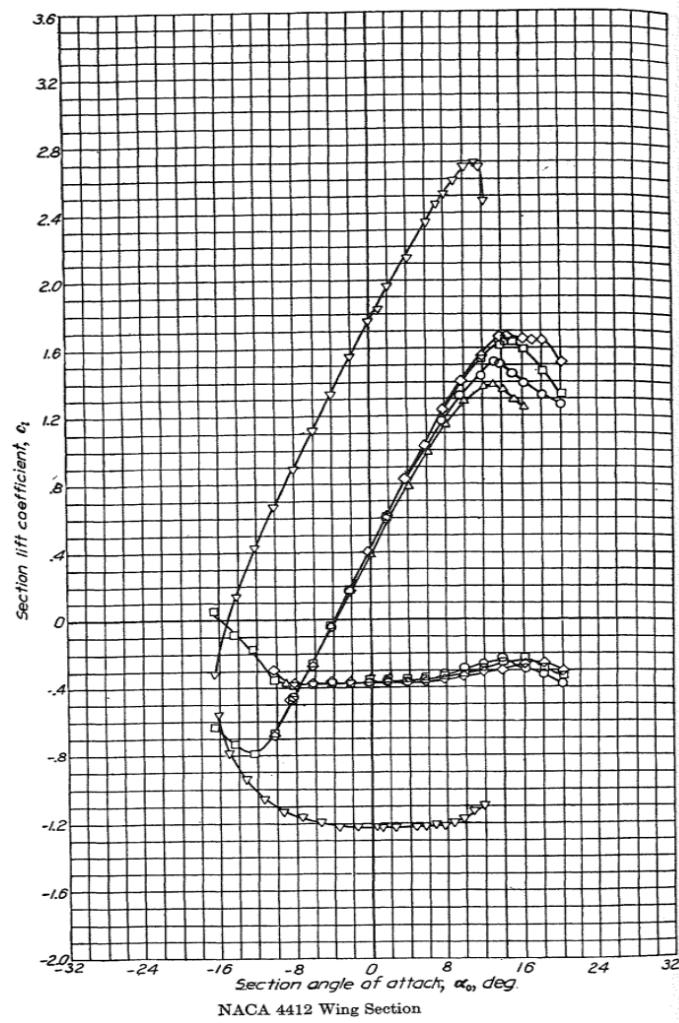


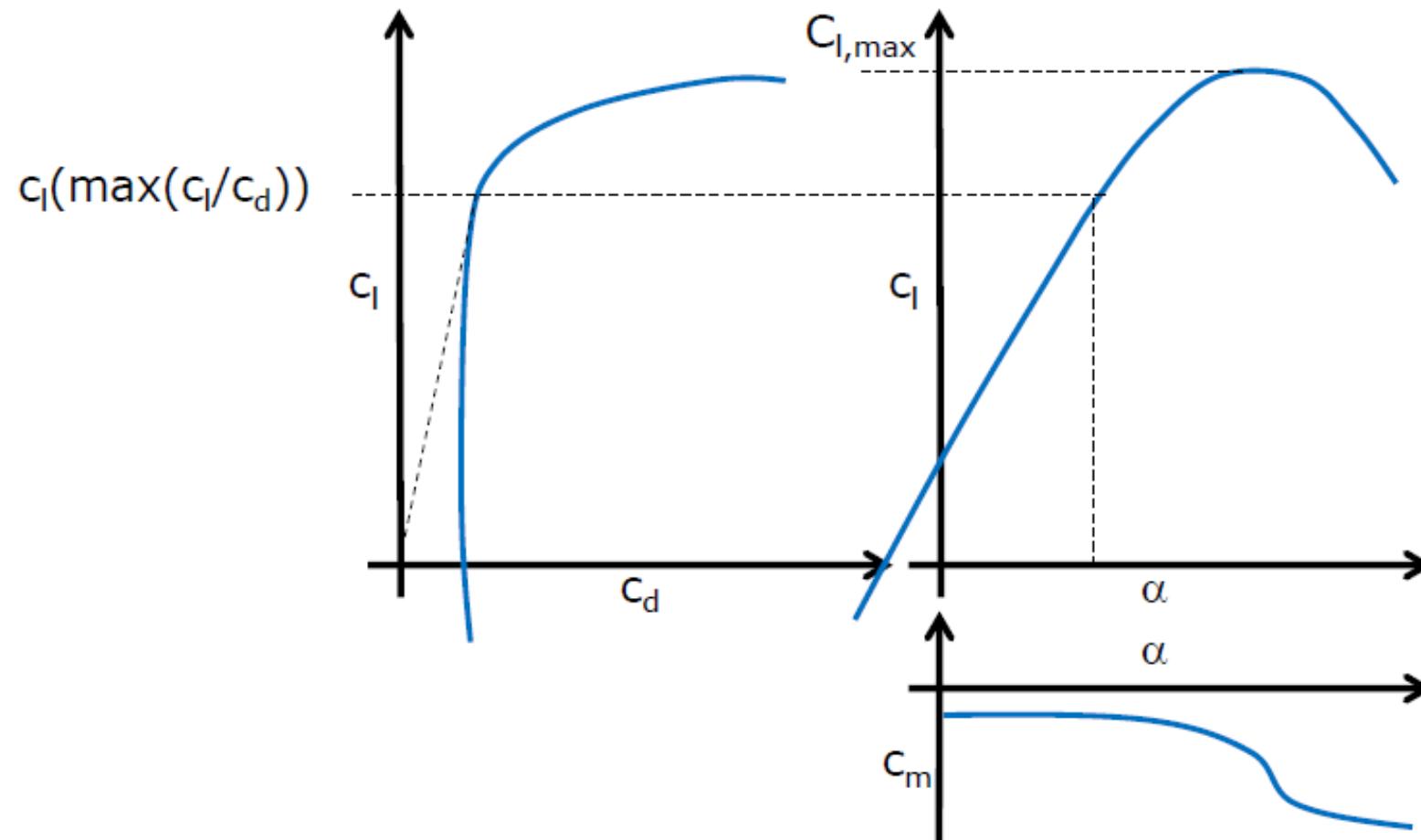
Figure 1 Flow about 2D objects in a wind tunnel (aerofoil, cylinder, etc.)

# Katalog med 2D vinge data - NACA

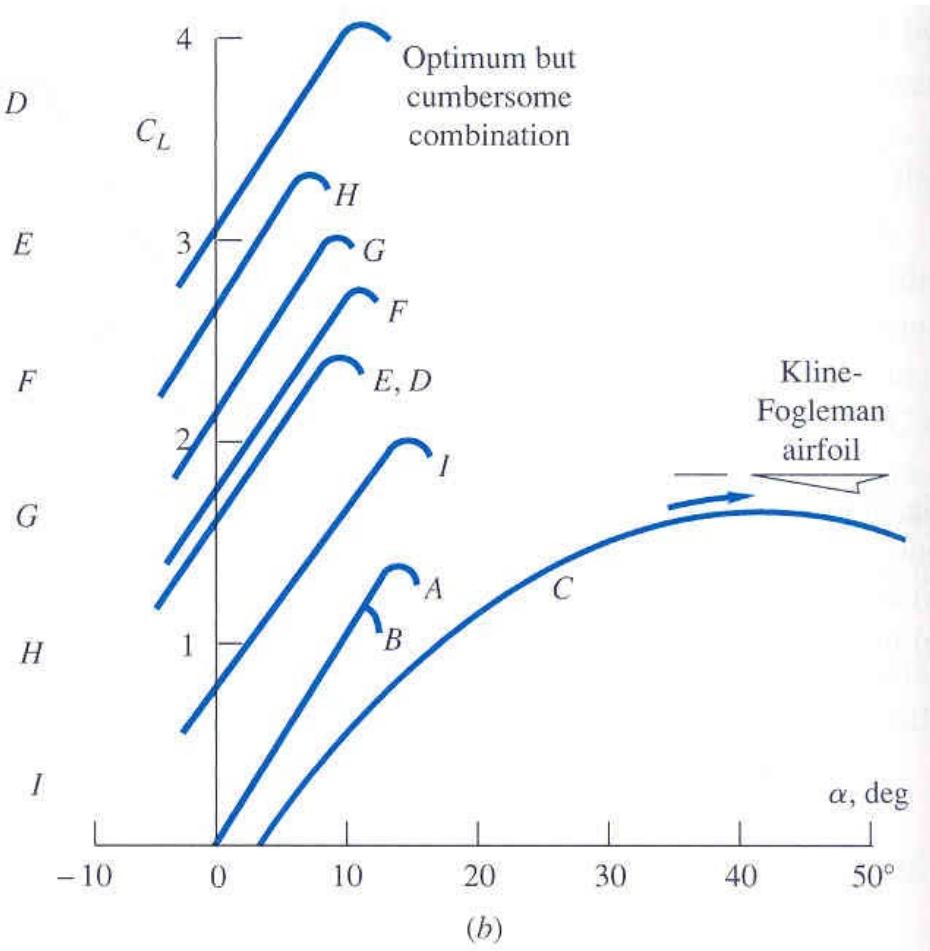
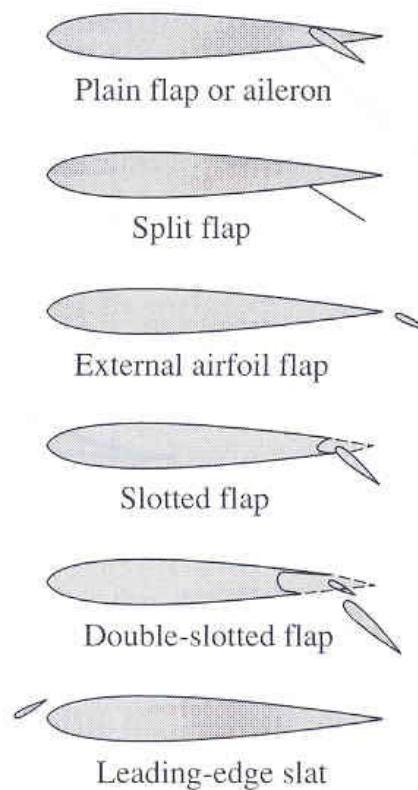


NACA report 824

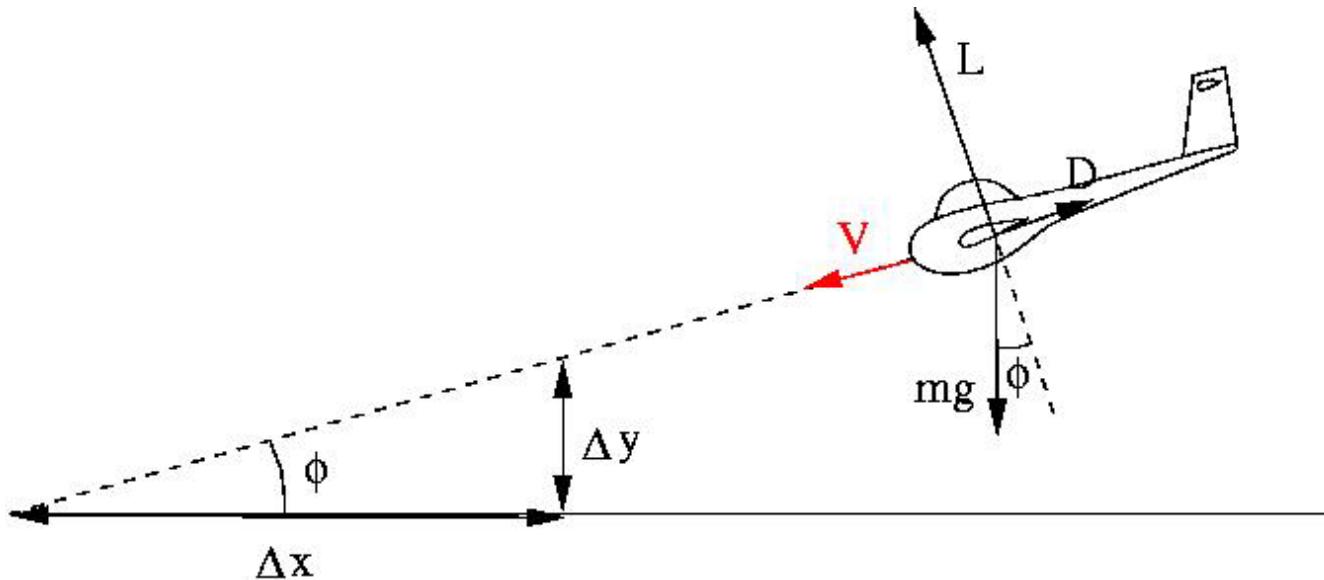
# 2D vingepolar – valg af design $C_l$



# Ændring af 2D vinge – high lift devises



# Glide tal – lift /drag forhold/ratio



$$L = \frac{1}{2} \rho V^2 A C_l = mg \cos \phi$$

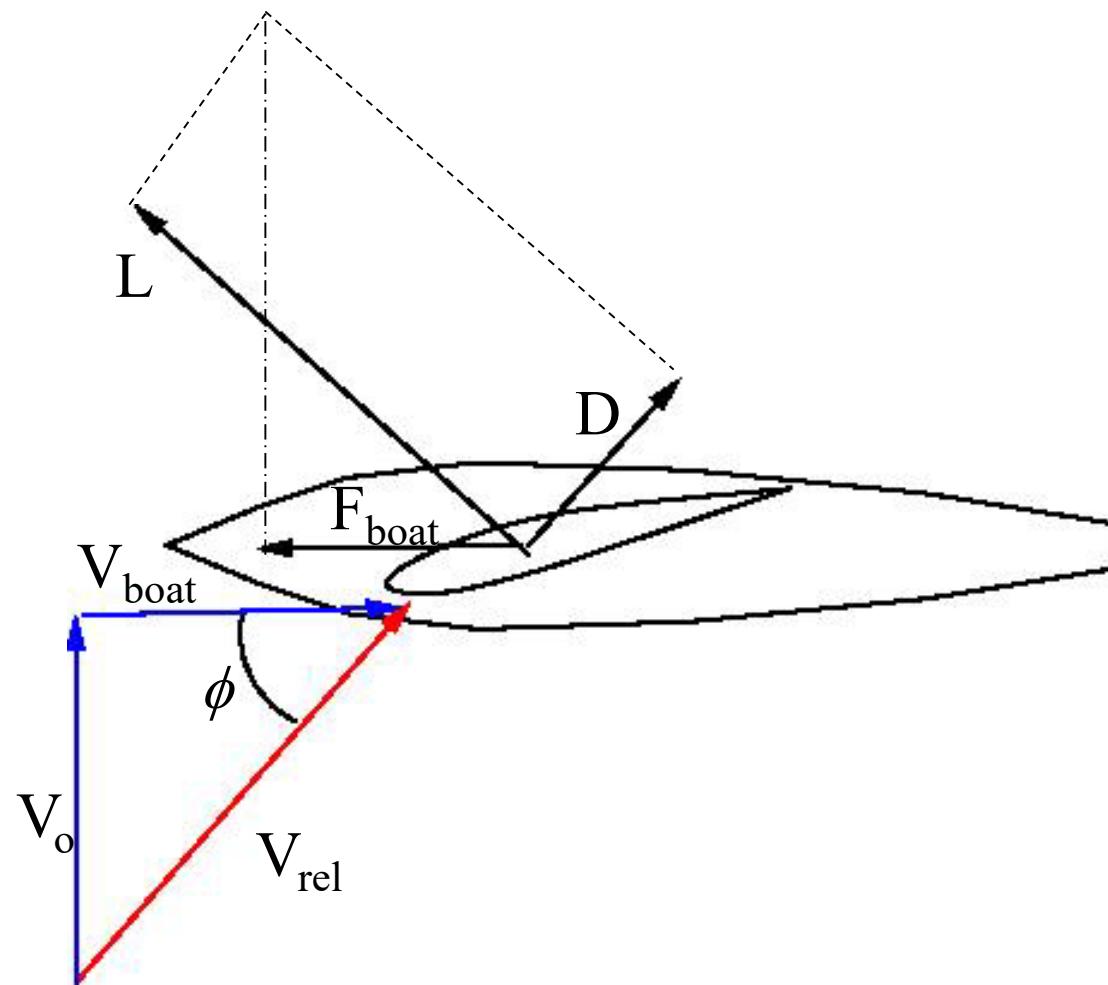
$$D = \frac{1}{2} \rho V^2 A C_d = mg \sin \phi$$

$$\tan \phi = \Delta y / \Delta x$$

$$\Rightarrow \frac{\Delta x}{\Delta y} = \frac{C_l}{C_d}$$

2D vinger, high Re:  $\Delta x / \Delta y \approx 100-150$   
Svævefly :  $\Delta x / \Delta y \approx 40-60$

# Fremdrift af en sejlbåd i sidevind

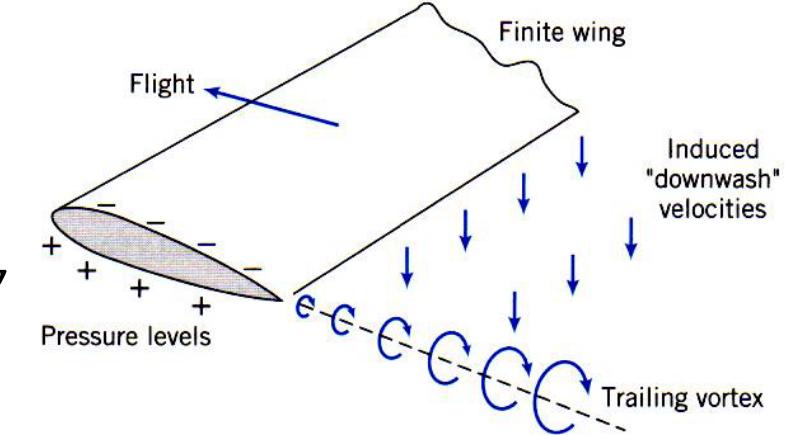


$$F_{boat} = L \sin \phi - D \cos \phi$$

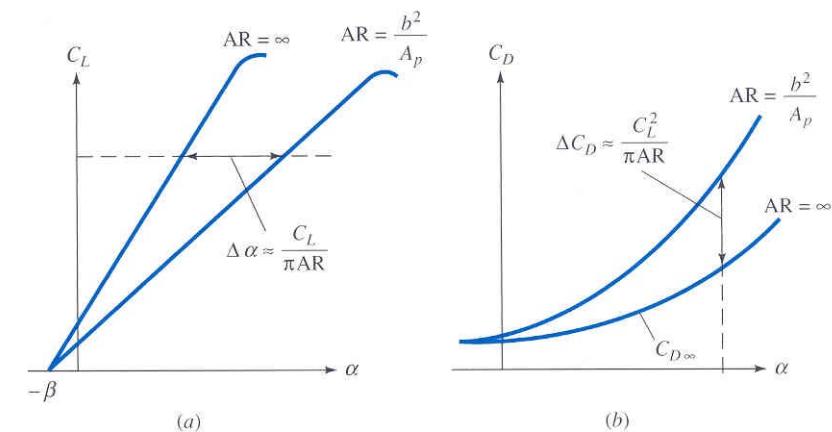
# 3D vinge – Tip hvirvel



$$\text{Lift} = \rho \cdot \text{Circulation} \times \text{Velocity}$$



3D vinger:  
Lavere hældning  
på lift kurve



# Kompressible strømning – shock – nær M~1



# Referencer – opslags værker

**Abbott & Doenhoff**

: Theory of Wing Sections

**Riegel**

: Airfoil Sections

**Hoerner**

: Fluid Dynamic Drag

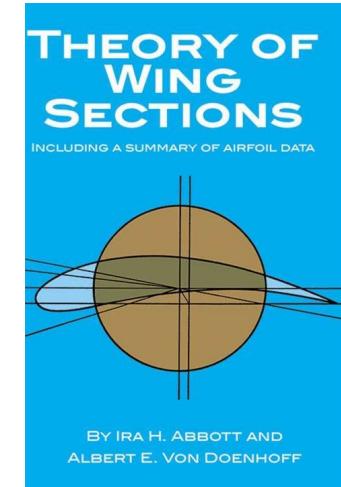
**Eppler**

: Fluid Dynamic Lift

**Selig**

: Airfoil Design and Data

: Low Reynolds no. airfoil

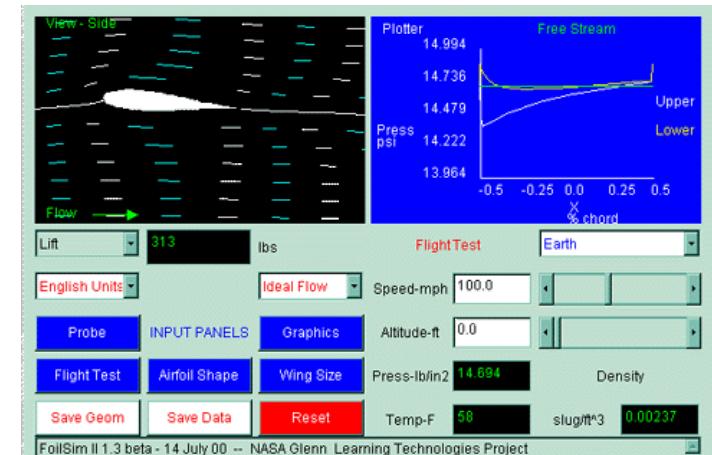


**Xfoil**

: Mark Drela, MIT

**NASA: FoilSim**

<https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/foilsimstudent/>



# Energi omsætning – lift eller drag? Hvad er bedst?



# Energi omsætning – lift eller drag? Hvad er bedst?

Is båd – kun drag

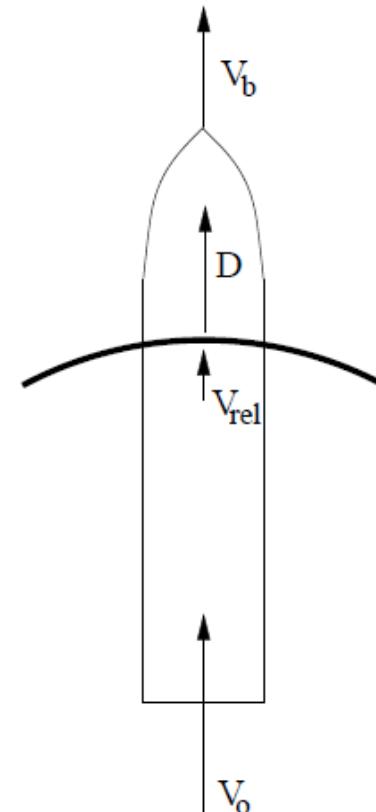
Vis at effekten P er (antag  $C_d=2$ )

$$D = \frac{1}{2} \rho V_{rel}^2 A C_d$$

$$P = \frac{1}{2} \rho V_b (V_o - V_b)^2 A C_d$$

$$C_p = \frac{V_b}{V_o} \left(1 - \frac{V_b}{V_o}\right)^2 C_d$$

$$C_{p,\max} = ?$$



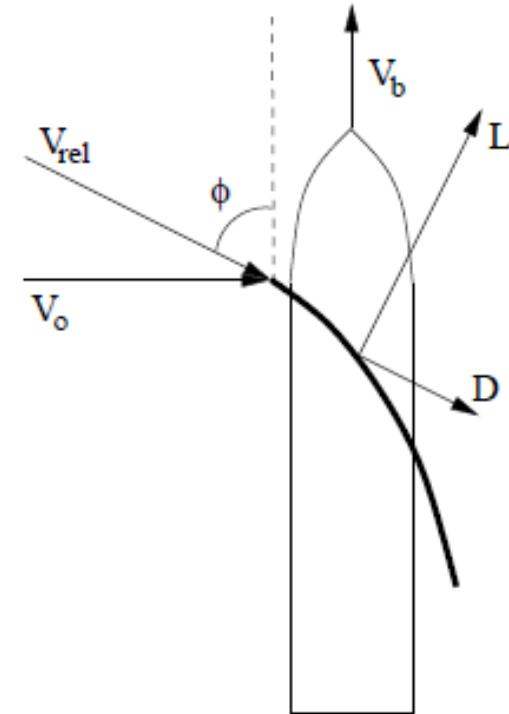
# Energi omsætning – lift eller drag? Hvad er bedst?

Is båd – lift fra sejl

Vis at (antag  $C_l=1$ ,  $C_d=0.1$ )

$$P = V_b (L \sin \phi - D \cos \phi)$$

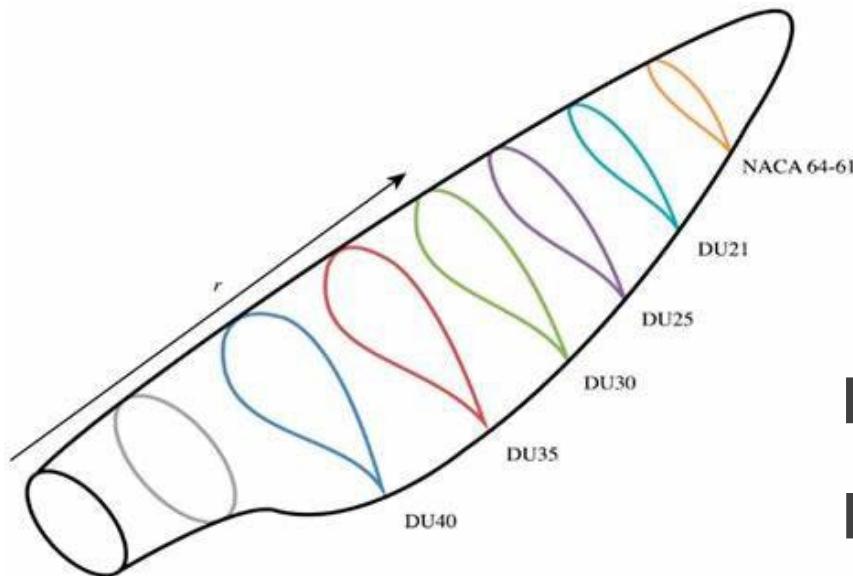
$$C_p = \frac{V_b}{V_o} \sqrt{1 + (V_b / V_o)^2} (C_l - C_d \frac{V_b}{V_o})$$



$$\text{Tip: } \sin \phi = V_o / V_{rel} \quad \cos \phi = V_b / V_{rel} \quad V_{rel}^2 = V_o^2 + V_b^2$$

# Vindmølle aerodynamik

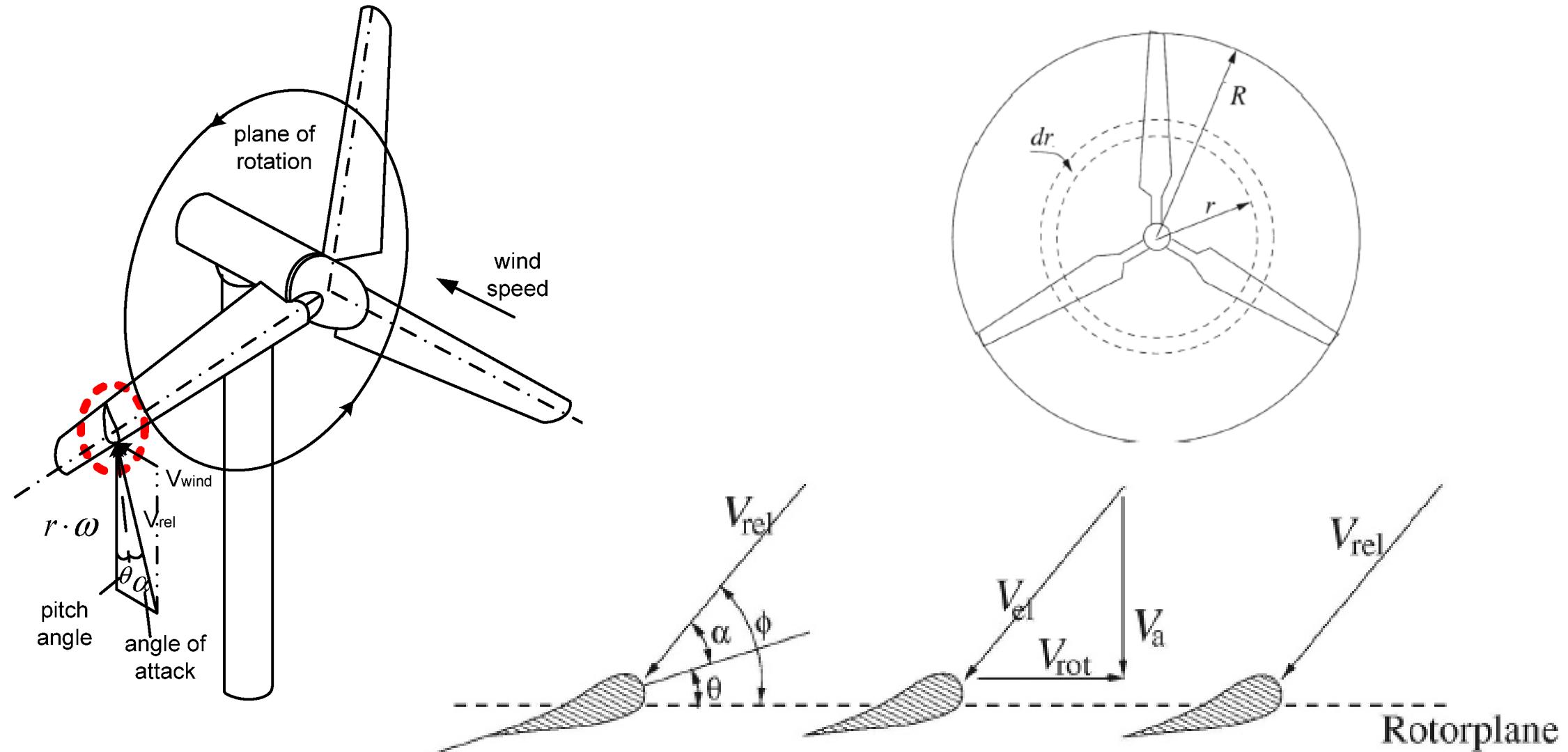
En vindmøllevinge – en 3D vinge  
lavet af 2D vinge profiler



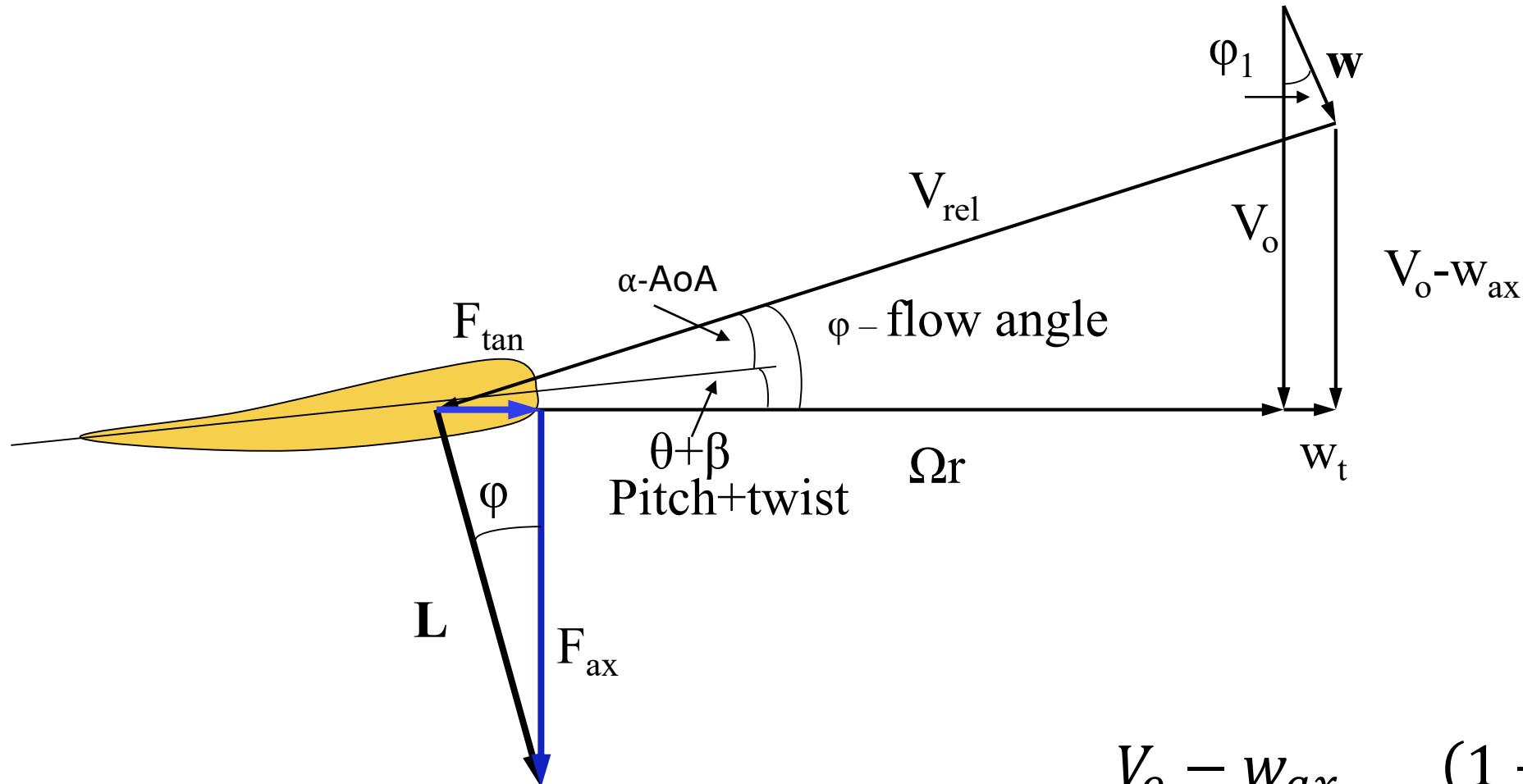
Hvad driver en vindmølle:  
Lift eller drag kræfter?



# Hastigheds trekant



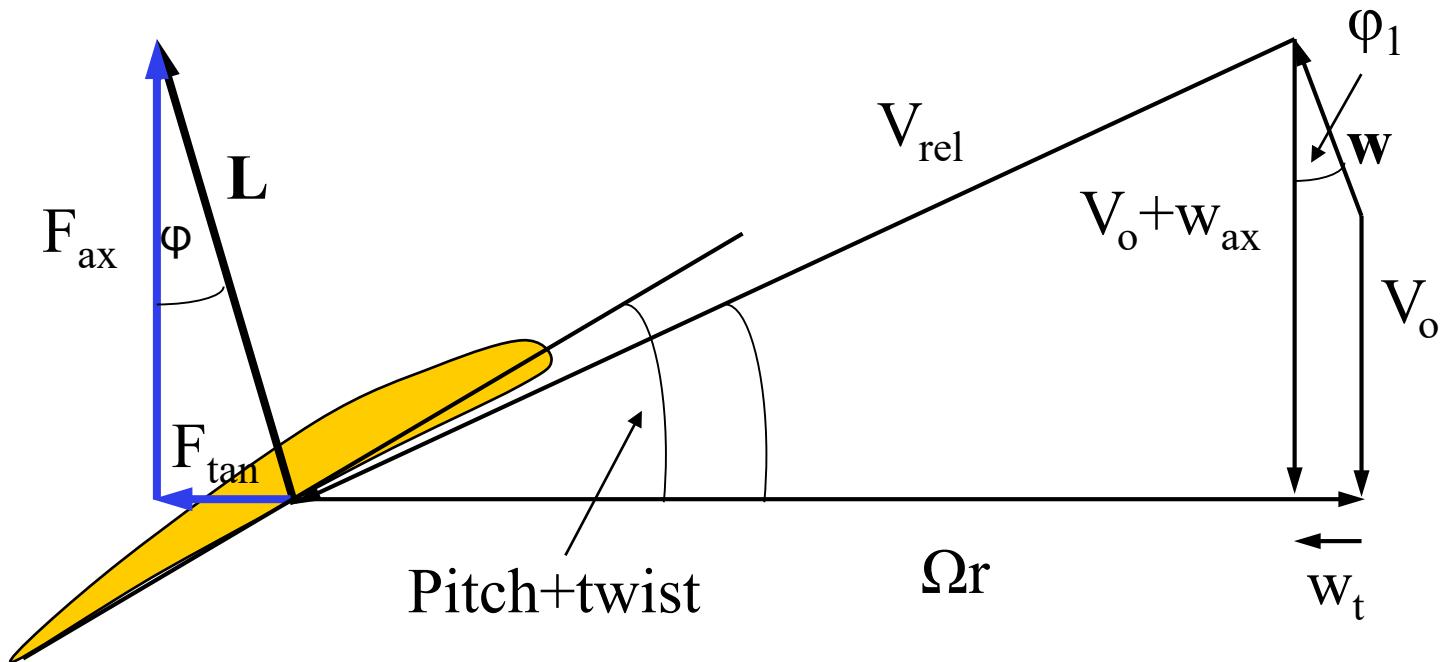
# Hastigheds trekant for en vindmølle



$$\tan \varphi = \frac{V_o - w_{ax}}{\Omega r + w_t} = \frac{(1 - a)V_o}{(1 + a')\Omega r}$$

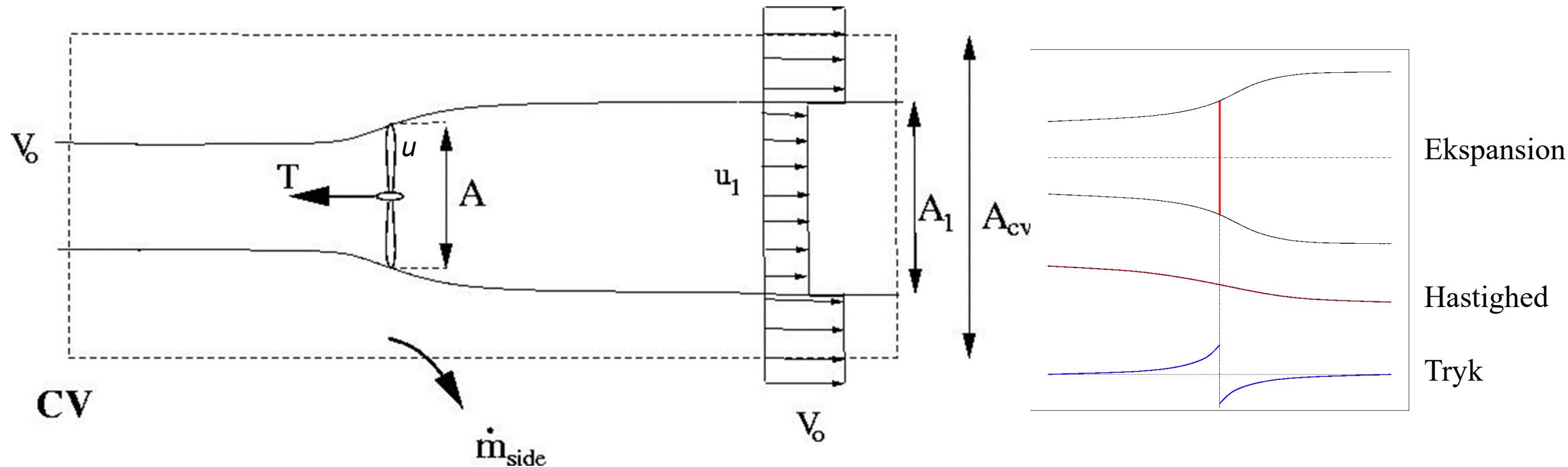
# Hastigheds trekant for en propeller

$$(\mathbf{V} - \mathbf{e}_\theta \Omega r) \cdot \mathbf{F} = 0$$



$$\tan \varphi = \frac{V_o + w_{ax}}{\Omega r - w_t} = \frac{(1 + a)V_o}{(1 - a')\Omega r}$$

# Flow igennem en vindmølle



$$\dot{m} = \rho u \pi R^2 = \rho u_1 A_1$$

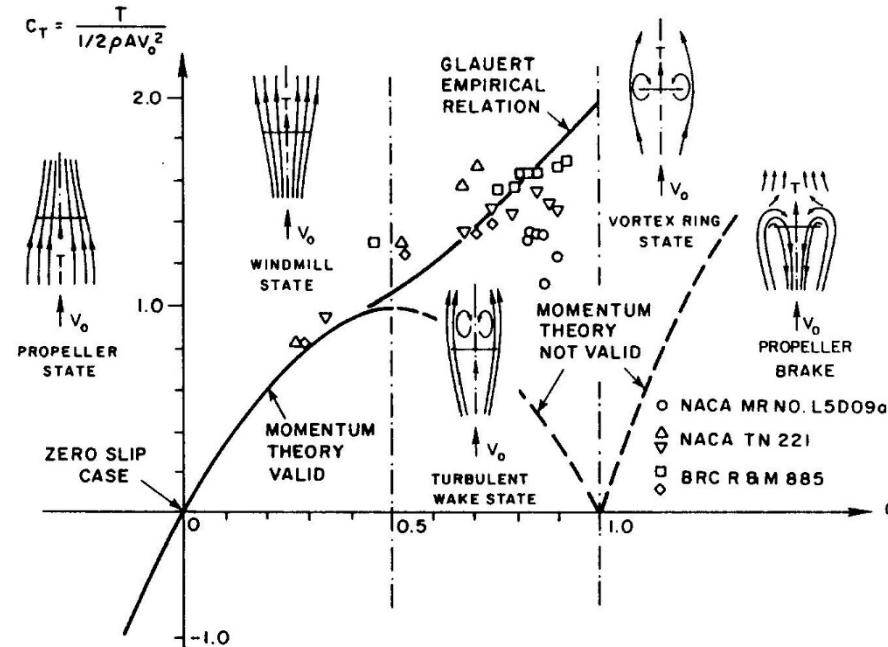
$$T = \dot{m}(V_o - u_1), \quad P = \frac{1}{2} \dot{m}(V_o^2 - u_1^2)$$

$$P = \frac{1}{2}(V_o + u_1)T = uT \quad \Rightarrow \quad u = \frac{1}{2}(V_o + u_1) = V_o(1-a) \quad b = 2a$$

$$C_T = \frac{T}{\frac{1}{2} \rho V^2 \pi R^2} = 4a(1-a)$$

$$C_P = \frac{P}{\frac{1}{2} \rho V^3 \pi R^2} = 4a(1-a)^2$$

# Vindmølle slipstrøm - wake



Propeller :  $C_T = 3$

Turbine :  $C_T = -0.8$

Turbulent – Vortex

ring state :  $C_T = -1.3$

# Modelforsøg – ligedannethed?

## Eksperiment med model og fuld skala prototype

### 1. Geometrisk ligedannethed

- Model og prototype har samme form
- Lineære dimensioner på model og prototype er ens med en konstant faktor til forskel

### 2. Kinematisk ligedannethed

- Strømlinjer og hastigheder omkring model og prototype er ens med en konstant faktor til forskel

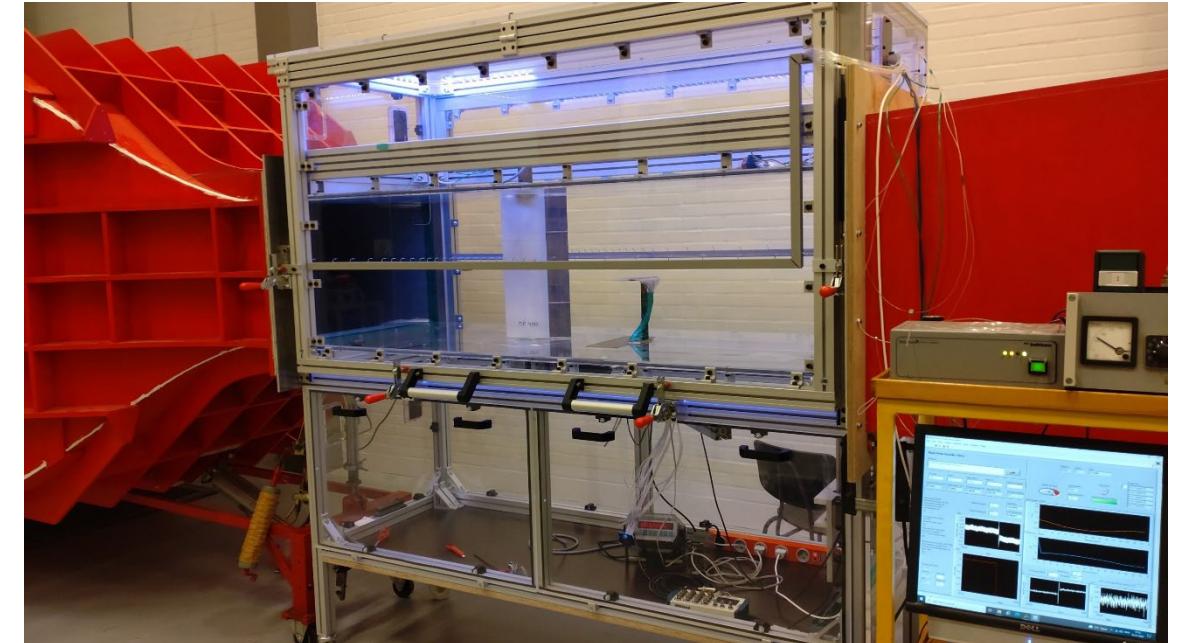
### 3. Dynamisk ligedannethed

- Kræfter på model og prototype er ens med en konstant faktor til forskel

# Vind tunnel forsøg - setup

## Red Wind Tunnel, RWT

- Lukket test sektion
- NACA64015 airfoil
- $c=0.15\text{m}$
- $\text{Span}=0.5\text{m}$
- Testsektions tværsnit  $0.75*0.5 \text{ m}^2$
- Max hastighed  $V=40\text{m/s}$
- Vinge model med tryk huller i ovefladen forbundet med slanger
- Måling af slipstrøm
- Kraft gage – lift måling



# Vind tunnel forsøg

## Måle opgave

- Densitet
- Måling af kræfter på vinge
- NACA64015 airfoil – trykmåling
- Slipstrøm



Trykdata, SI units: Pa, N, m/s, kg/m<sup>3</sup>, s

Tryk foil	wake	AoA	Lift	Flap	Pdyn	Vtun	rho	t[s]
• 1-32 33-64		65	66	67	68	69	70	71