Translating a Polygon Proportionally to Rotation

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1 Derivation

To ensure that the perfect road for a polygon is displayed when the polygon is rotated and translated (in the x direction), the amount translated for an increment is related to how much was rotated. This is displayed through multiple figures, the first shown in Figure (1).

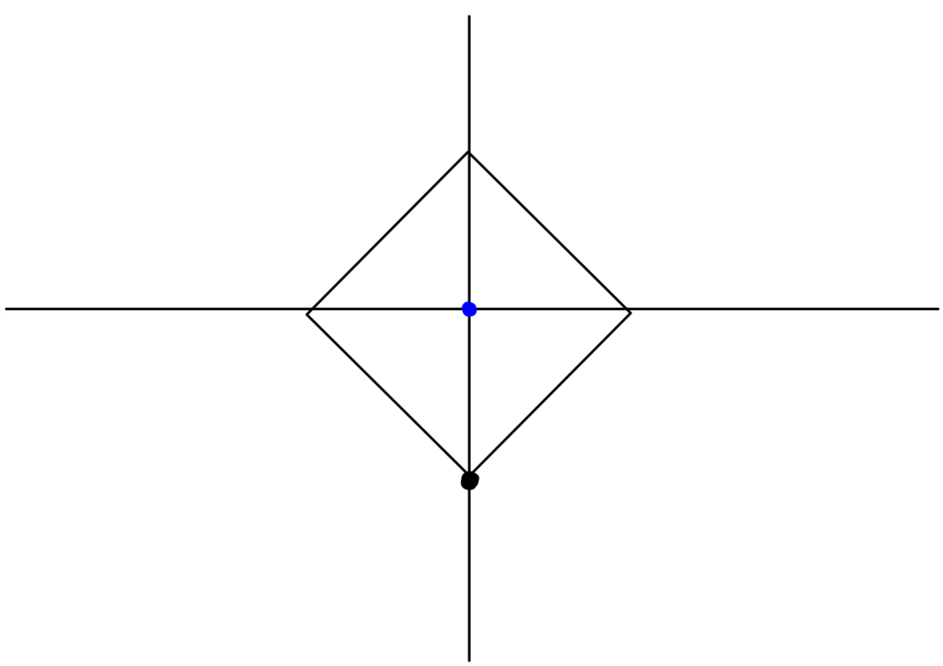

Figure 1: The black point indicates the contact point, since it is on a vertical line with the blue axle point (centre of the square).

A square in Figure (1) is rotated an increment $d\theta$ in Figure(2). This creates a new contact point.
To relate the angle increment rotated $d\theta$ with the translational increment the square makes dx, the squares are superimposed onto each other to show that the arc the contact point travels during rotation is related to dx. This is shown in Figure (3).

The equation in Figure (3) results in an arc translation for large $d\theta$, making increments in $d\theta$ as small as possible results in translations that are approximately straight lines which yields a good approximation

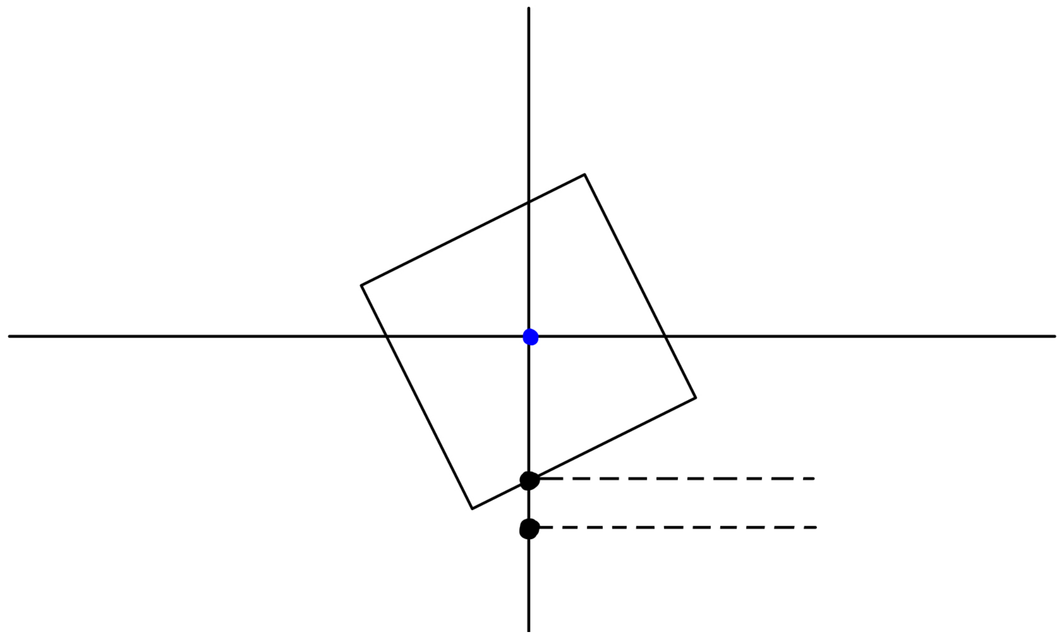

Figure 2: The square is rotated an angle $d\theta$ from Figure (1). The black points indicate the contact points, the one which is on the edge of the square is the one after rotation. This is the contact point since it is on a vertical line with the blue axle point (centre of the square). The two dashed horizontal lines, show that the contact point has risen in the y axis, and the new point where where the road is drawn is at this new height.

to the perfect road for a square. This same logic can be applied to any shape, even non-polygons.

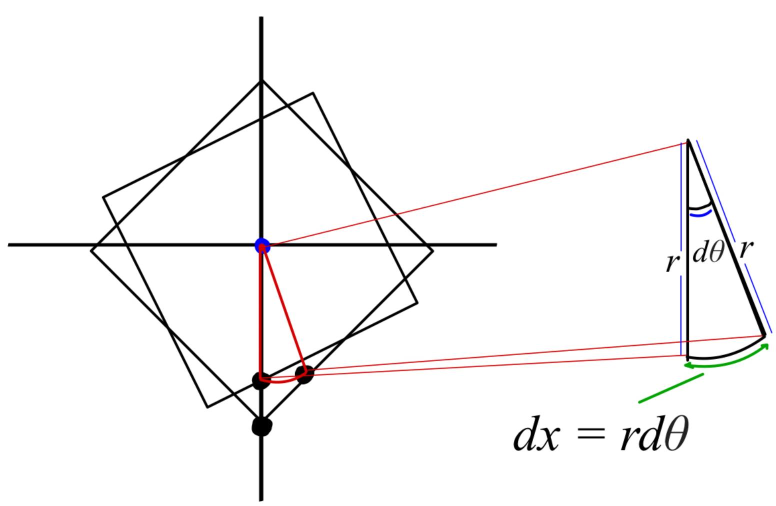

Figure 3: Figure (1) and Figure (2) superimposed on each other. Showing the arc that the contact point travels during rotation. The equation then displays the translational and rotational dependence on one another.