AM 205 Final Project
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# Contents



# 1 Introduction to Beer

## 1.1 History

Most of us see beer as a lighthearted accompaniment to a night out with friends or to a hearty and satisfying meal; however, beer has long been the subject of serious scientific and historical study. These studies indicate that there may be a much more profound connection between beer and the development of civilization. There is archaeological evidence that humans were brewing with cereal grains as early as 9500 BC [1] and the genetic diversity of beer and bread yeasts indicate that these strains diverged from native yeast 10,000-12,000 years ago [2]. There are some well-developed theories that the desire for grain based luxury foods for feasts and other ceremonial activities was a major part of the shift from a hunter gatherer society to a agrarian society in both Europe and the near east [3, 4]. In short, it is quite reasonable to argue that beer was a major part of the agricultural revolution.

Not only has beer facilitated the development of civilization, but it is deeply intertwined in social interaction and culture. Not only is drinking near universally a social event, with social proscription of solitary drinking, but there is a cross-cultural similarity of drinking locations as bonding and egalitarian environments [5]. Beer has also largely influenced the economy throughout history being used as a currency in Sumeria [6] , wages in Egypt [7], and a commodity today. With the rise of the Greek city states and the Roman empire, wine usurped beer as the drink of choice around the mediterranean but the cultural heritage of beer survived in Gaul and in the regions controlled by the Germanic tribes [8, 9]. This manifests itself today as the strong culture centered around beer brewing in Germany, Belgium and northeast France.

Throughout most of history the taste and composition of beer varied by location with the availability of resources. The diversity in styles of beer we know today are a result of commercial brewing. While the need for commercial brewing was limited by the availability of beer at monasteries during the Middle

Ages, in early modern times commercial breweries began to emerge. This shift of brewing activity coincided with the famous 'Reinheitsgebot' (Bavarian purity laws) which limited the ingredients of beer to water, hops and barley (later amended to include yeast) [10]. This increase in commercial brewing also lead to wider distribution. With monastic breweries, innovation to improve taste was stifled by lack of competition. Commercial breweries began experimenting with flavors and as a result regional beer styles evolved. It is likely that regional styles were strongly influenced by the mineral content of the local water. For example, the water in Pilsen a city in the Czech Republic is particularly low in mineral content which favors the light color and gentle malt flavor of Pilsners. [11]. The focus on control of flavors continued and beer was dramatically impacted by the new technology available as a result of the industrial revolution. The first major development was made possible by the study of microbiology, which improved the understanding of yeast coupled, with the beginning of refrigeration and better process control. During this period lagering, a second, colder stage of fermentation, was discovered reducing off flavors and improving consistency. Today, lagers (e.g. Budweiser, Miller, Coors) are one the most widely consumed beer styles worldwide. The second major development was the development of canning and pasteurization which increased the shelf life of beer and permitted even wider distribution. It was during this time period that the macrobreweries of today were founded in the United States [10]. In the 19th century the production of beer slowed during prohibition and the world wars. It was during this period, with the scarcity of grain due to rationing that other adjuncts were used to supplement the supply of grain in a beer. The United States primarily turned to corn and rice while much of Europe used beets [10]. Recent years have shown a movement away from mass produced lager styles to smaller craft breweries with higher alcohol and more complex taste profiles. As the commercial offerings have grown, it has also spurred a devoted community of homebrewers to experiment with different grains, hops, and other additives to recreate favorite beers or create their own unique brew [11]. Today, beer is a 110 billion dollar a year industry offering consumers some incredible options. [12].

## 1.2 Brewing Process

While there are many variations, substitutions, and stylistic changes an individual brewer may make when crafting their brew, the basic process remains the same. It revolves around the four main ingredients of beer [11]:

**Malted Barley** Otherwise known as malt, this is the primary source of sugar and color for beer. Malt is partially germinated barley, where the natural enzymes present in the grain have converted the large starch chains to smaller carbohydrates. The germination process is halted by roasting the grain. Through variations on the roasting process it is possible to create a wide array of colors and flavors such as biscuit, caramel, or even chocolate.

**Hops** These are cone-like flowers that provide bitterness, flavor, aroma and some preservative qualities to beer. The variety of strains produce different types and levels of alpha and beta acids, the compounds brewers attempt to extract from this plant. By varying how the hops are used and choosing the type of hop brewers can create many different flavor combinations to balance the sweetness from the malt. Descriptions like citris, tropical fruit, pine, resin, herbal, spicy and many more come from choice of hop.

**Water** Beer is mostly water and because the taste of water changes wildly based on its mineral content it is important to control the composition of water used in the brewing process. Not only do you want the water to taste good, but water chemistry can dramatically affect the extraction of alpha acids from hops or the conversion of carbohydrates to fermentable sugars.

**Yeast -** Brewer's yeast (Saccharomyces cerevisiae) converts fermentable sugars to alcohol and $CO_2$. The many strains and varieties have different levels of alcohol tolerance and levels of attenuation (how much sugar is converted). Sometimes more exotic microorganisms (Brettanomyces, Lactobacillus, and/or Pediococcus) are used to provide "funky" flavors or sour the beer. The choice of yeast strain has a profound effect on the final flavor of the beer.

The process of converting these ingredients into a beer can be summed up into the following three steps.

### Mashing

Mashing is the process by which the simple carbohydrates from the malt become fermentable sugars which can be extracted in water. This process relies on the enzymes that are still present in the grain after roasting. There are two enzymes that are primarily responsible for this conversion process, alpha amylase and beta amylase. Alpha amylase works most effectively at temperatures between 154°F and 162°F where beta amylase works optimally in the 131°F to 150°F and so typically a compromise of 153°F is quoted as the optimal mashing temperature. Typically the mash water is heated to $\approx$ 160°F to account for the heat capacity of grain and the grain is allowed to steep for about an hour. This process allows for the bulk of the saccharides to be converted to simple fermentable sugars. The sugars are now rinsed from the grain in a process called lautering, the result is a sweet grain colored solution called wort.

### Boil

This wort is now brought to a boil, so that unwanted microorganisms are killed, and the alpha acids (AA) and beta acids (BA) from hops can be extracted. Typically, the boil lasts 60 minutes. Bittering hops are added with 60 minutes left in the boil, this allows the AA and BA from hops to be extracted and isomerized producing bitterness. Flavor hops are added with between 20-5 minutes left in the boil allowing AA and BA to be extracted but without enough time to isomerize. Finally, aroma hops are added to influence the smell of the beer at the end of the boil. The wort must now be cooled to 70°F to allow us to add the brewer's yeast without killing our yeast cells. If the wort is cooled quickly, small soluble proteins will precipitate and fall out of solution, this helps clarify the beer and improves long term stability.

### Fermentation

With the wort cooled down to room temperatures, the desired strain of yeast can be added. The beer is allowed to do a primary fermentation, where the bulk of the sugars are converted to alcohol in 10-14 days. Often a secondary fermentation is used to allow undesired flavors to mellow and to allow the finished beer to clarify. The beer can then be kegged and carbonated with added $C0_2$ or bottled with a small addition of corn sugar to naturally carbonate over 10 days.

## 2 Problem statement

For this project I have examined the problem that brewers face of cooling the wort to allow pitching of yeast. While the importance of heat transfer on an industrial scale is critical to many industries and has been studied in great detail [13], I have addressed this issue from the perspective of a home brewer where cost and practicality dominate engineering concerns.

I have modeled two common methods used by homebrewers to chill the boiling wort, an immersion wort chiller and a counter flow wort chiller. I have iteratively developed the model of the immersion chiller based on feedback provided by simple experimental measurements and experience. I will use this calculation to inform my design as I construct a counter flow wort chiller (though I have not documented this).

Figure 1: Immersion wort chiller made with copper tubing.

## 3 Immersion Wort Chiller

An immersion wort chiller is shown in Fig. 1. It is a coil of tubing, commonly 25ft long, that you attach to a garden hose. Cool tap water flows through the coil as the coil is immersed in the hot wort. A cross section of the wort chiller is shown in Fig. 2. Where showing the wall thickness (W), the inner diameter (R1), and the outer diameter (R2). As an initial assumption let us assume the coiling of the copper tube is insignificant, reducing the problem to a more manageable linear geometry.

For this problem I will be working with the dimensions R1 = $\frac{5}{16}$", R2 = $\frac{3}{8}$" and 25 ft of length.

## 3.1 Preliminary Work

I approach this problem by determining if I can separate the conductive heat transfer problem from the convective heat transfer. Conduction refers to heat transfer within a stationary material, while convection is the movement of heat through the movement of the medium.

Our first calculation is to establish the timescale on which a cross section of water would thermalize to the copper immersed in hot wort. To do this I solve the heat equation assuming the water is stationary, while the outside of the tube is held at a constant temperature. I can compare this time scale of thermalization to the timescale of water flowing through the copper pipe. If the timescales are vastly different it would allow us to separate the calculation of the individual processes. If thermalization occurs quickly compared to the waters motion, this would allow us to effectively solve for thermal equilibrium for every point along the pipe. If it occurs slowly, I can solve just the thermalization problem and the time axis would map to distance through the chiller. Based on a simple experimental measurement I typically run water through the cooler at a rate of 0.42 gallons per minute. While in practice I have the ability to modify this rate, I find it is a good compromise between water conservation and cooling time.

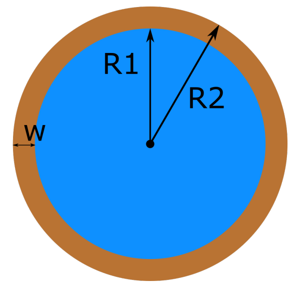



Figure 2: Cross section of immersion wort chiller. Materials are color coded, copper is copper and water is blue.

I begin by solving the heat equation in radial coordinates.

$$\frac{\partial T}{\partial t} = \frac{k}{c_p \rho}\left(\nabla^2 T\right) \tag{1}$$

where k is thermal conductivity, $\rho$ is the mass density, and $c_p$ is the specific heat capacity. For the physical materials, these values change as a function of temperature and Section 5 discusses how I determine these values. Our problem has cylindrical symmetry so the equation can be put into radial form

$$\frac{\partial T}{\partial t} = \frac{k}{c_p \rho}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{2}$$

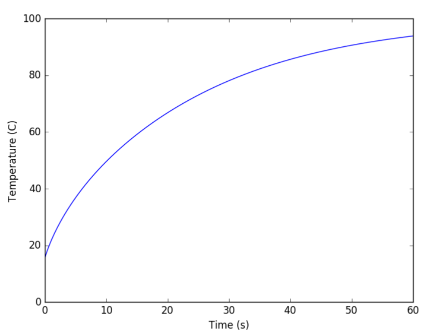



Figure 3: Average temperature as a function of time.

Motivated by class notes and experience on the homework I use a backward in time, centered in space (BTCS) finite element method as implicit methods generally have larger regions of stability [14].

$$\frac{T_j^{n+1} - T_j^n}{\Delta t} = \frac{k}{c_p \rho}\left(\frac{T_{j+1}^{n+1} - 2T_j^{n+1} + T_{j-1}^{n+1}}{\Delta r^2} + \frac{1}{r}\frac{T_{j+1}^{n+1} - T_{j-1}^{n+1}}{2\Delta r}\right) \tag{3}$$

I use initial conditions where the water is at 15.5°C and the copper has a linear thermal gradient from 15.5°C (60 °F) to 100°C. I use a Neumann boundary condition at r=0 and Dirichlet condition at r=R2:

$$\frac{\partial T}{\partial r}|_{r=0} = 0 \qquad\qquad T|_{r=R2} = 100°C$$

for the first boundary condition:

$$\frac{T_{j+1}^{n+1} - T_{j-1}^{n+1}}{2\Delta r} = 0 \tag{4}$$

$$T_{j+1}^{n+1} = T_{j-1}^{n+1} \tag{5}$$

for the second boundary condition I can simply enforce that T(R2,t) = 100 °C.

## 3.2 Stability Analysis

To insure that our solution is physical let us look at the stability of this discretization. Using our discretization in eq. 12 with the ansatz $T_j^n = \lambda(k)^n e^{ijk\Delta r}$ I obtain:

$$\lambda(k) - 1 = \frac{k\Delta t}{c_p\rho}\Big(\frac{\lambda(k) * e^{ik\Delta x} - 2\lambda(k) + \lambda(k)e^{-ik\Delta r}}{\Delta r^2} + \frac{1}{r}\frac{\lambda(k)e^{ik\Delta r} - \lambda(k)e^{-ik\Delta r}}{2\Delta r}\Big) \tag{6}$$

$$\lambda(k) = \frac{1}{1 - \frac{k\Delta t}{2c_p\rho\Delta r}\big(\frac{cos(k\Delta r)-4}{\Delta r} + \frac{sin(k\Delta r)}{2ir}\big)} \tag{7}$$

$$\lambda(k) = \frac{4c_p\rho\Delta r^2 ri}{\big(4c_p\rho\Delta r^2 r - k\Delta t2r(cos(k\Delta r) - 4)\big)i - \big(k\Delta t\Delta r(sin(k\Delta r)\big)} \tag{8}$$

For stability I require $|\lambda| \leq 1$

$$|\lambda(k)|^2 = \frac{(4c_p\rho\Delta r^2 r)^2}{\big(4c_p\rho\Delta r^2 r + k\Delta t2r(4 - cos(k\Delta r))\big)^2 + \big(k\Delta t\Delta r(sin(k\Delta r)\big)^2} \tag{9}$$

I see by looking at the left denominator term that $|\lambda|^2 \leq 1$ thus this system is unconditionally stable, a result of choosing an implicit method.

## 3.3 Preliminary Results and Modifications

Running this simulation with a grid of 200 points describing the radial coordinate and a time step of 10 ms I can extract the timescale of the thermalization. The average temperature of water as a function of time is plotted in Fig. 3. I can see that the time scale for thermalization is on the order of minutes, while the time the water actually spends in the immersion chiller is of order few seconds. In order to save computational time I can plot the results as a function of time step size. The results shown in Fig. 4 are in good agreement even with a coarse time step of 1 s. Similarly I can lower the precision of the grid on the radial coordinate, good agreement is found with a grid of 10 radial steps. For the bulk of this project I use 15 radial steps and 20 longitudinal steps.

## 3.4 Laminar Flow Model

### 3.4.1 Accounting for Convection

I now add a convection term to the expression. The Reynolds number for this system is $\approx 7400$. This indicates that the fluid motion is in the turbulent regime. I will start by adding a laminar convection term and proceed to 'imitate' turbulence.

I add a convection term to the original differential equation so that I now have:

$$\frac{\partial T}{\partial t} = \frac{k}{c_p\rho}\Big(\nabla^2 T\Big) - \vec{v}\cdot\nabla T \tag{10}$$

$$\frac{\partial T}{\partial t} = \frac{k}{c_p\rho}\Big(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\Big) - \vec{v}_z\frac{\partial T}{\partial z} \tag{11}$$

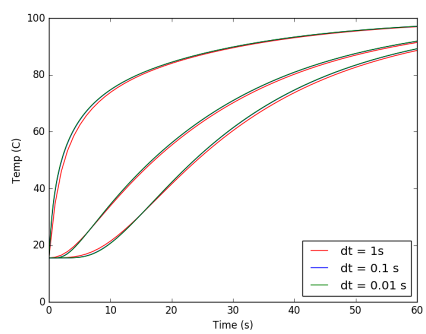



Figure 4: Temperature as a function of time for different time steps. Showing three different radial positions.

### 3.4.2 Stability

Again using a BTCS discretization. I obtain:

$$\frac{T_{i,j}^{n+1}-T_{i,j}^{n}}{\Delta t}=\frac{k}{c_p\rho}\Big(\frac{T_{i,j+1}^{n+1}-2T_{i,j}^{n+1}+T_{i,j-1}^{n+1}}{\Delta r^2}+\frac{1}{r}\frac{T_{i,j+1}^{n+1}-T_{i,j-1}^{n+1}}{2\Delta r}+\frac{T_{i+1,j}^{n+1}-2T_{i,j}^{n+1}+T_{i-1,j}^{n+1}}{\Delta z^2}\Big)-\vec{v}_z\frac{T_{i+1,j}^{n+1}-T_{i-1,j}^{n+1}}{2\Delta z} \tag{12}$$

Using $T_j^n=\lambda(f)^n e^{ijf\Delta r}e^{ijf\Delta z}$

$$\lambda(f)-1=\frac{k\Delta t}{c_p\rho}\Big(\frac{\lambda(f)Cos(f\Delta r)-4\lambda(f)}{2\Delta r^2}+\frac{1}{r}\frac{\lambda(f)Sin(f\Delta r)}{4i\Delta r}+\frac{\lambda(f)Cos(f\Delta z)-4\lambda(f)}{2\Delta z^2}\Big)-\vec{v}_z\frac{\lambda(f)\Delta tSin(f\Delta z)}{4i\Delta z} \tag{13}$$

$$|\lambda(f)|^2=\frac{|4r\Delta r^2\Delta z^2 r|}{|4r\Delta r^2\Delta z^2 r+2\frac{k\Delta t}{c_p\rho}r\Delta z^2(4-Cos(f\Delta r)+2\frac{k\Delta t}{c_p\rho}r\Delta r^2(4-Cos(f\Delta z)|^2+|f(\Delta t,\Delta r,\Delta z)|^2} \tag{14}$$

where f is a real function. This shows that this method is unconditionally stable. I have determined that this result is flawed. While I have not found any stability issues for the diffusion part of this expression, I have determined that my expression breaks down when $\vec{v}_z\Delta t > dz$.

### 3.4.3 Boundary Conditions

Critical to the success of this method is choosing boundary conditions that appropriately describe the system.

$$\frac{\partial T}{\partial r}|_{r=0}=0 \qquad T|_{r=R2}=T_{bath}(t)=100°C$$

Since the the heat capacity of the bath is so large I will assume that it is a constant (an assumption I will modify).

For the z boundary conditions I will assume

$$T|_{z=0}=15.5°C$$

I had difficulty with the boundary condition for the Z= end. Initially I used:

$$\frac{\partial T}{\partial z}|_{z=end}=0$$

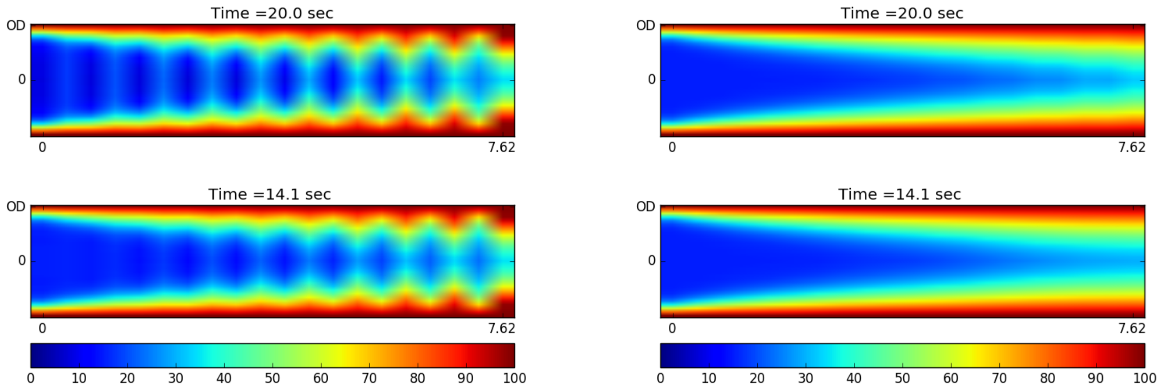
But I found that after "wave" of cold water reached the end I began to get reflections of the temperature off of the boundary. This was entirely due to forcing the boundary to obey the zero derivative condition. I also found that this boundary acted as a source/sink if I did not enforce this boundary condition.

As Fig. 5 shows, these interference effects largely disappear if after determining the next temperature map, I manually changed the temperature of the ghost nodes at Z=end to match the temperatures at Z = end-1. This can be thought of as removing heat from the system as the cooling water leaves the pipe.

Luckily, as these interference effects only occur after a time where water would have completed its journey through the pipe, they only occur after a steady state has been reached.

### 3.4.4 Results

The graphs in Fig. 5 used a constant velocity across the entire inner diameter of the tube. to obtain a more physical representation of the $\vec{v}$(r) I will use a laminar flow profile.




Simply Enforcing Boundary Condition

Temperature Matching

Figure 5: Demonstration of boundary effects. y-axis is pipe radius, x axis is pipe length. Flow goes left to right at a constant velocity (.54 m/s). Interference effects dominate the left simulation at long times. By setting the ghost node in the problem equal to the pipe temperature after the solving for the new temperatures, the interference effects largely disappear.

$$\vec{v}(r) = v_{peak}(1 - \frac{4r^2}{ID^2})$$

which produces an average velocity

$$v_{avg} = \frac{v_{peak}}{2}$$

Running this simulation I obtain a temperature distribution in the pipe that is similar to the distribution from a constant velocity (Fig. 5), but since the fluid in the center of the pipe is moving faster there is a larger fraction of cold water in the center of the pipe.

So far I have presented snapshots of the thermal profile. A better metric is cooling power or time to yeast pitching temperature. My simulation produces a steady state temperature map with a outside temperature as an input. I can also feedback upon the boundary conditions to get a temperature as a function of time.

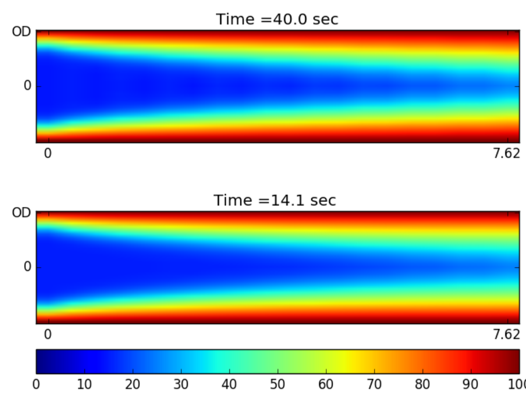



Figure 6: Temperature map of a pipe with laminar flow with $V_{avg} = 0.54m/s$

I can monitor the temperature of the water where it leaves the pipe. I can then converting this temperature to a heat and subtract it from the total heat contained in the wort. I can then set the outer radial source to be equal to this new temperature.

$$T^{n+1}_{wort} = T^{n}_{wort} - \frac{(T_{out} - T_{in})\vec{v}(r)(2r\Delta r)\Delta t Cp_{water}\rho_{water}}{V_{wort}\rho_{wort}Cp_{wort}} \tag{15}$$

In Fig. 6, I see the behavior of temperature as a function of time and a comparison of a stainless steel vs copper immersion chiller with laminar flow. Even though the thermal conductivity of stainless is lower than copper by about an order of magnitude, both of these metals still conduct heat significantly better than water. Thus the temperature profiles are nearly identical.

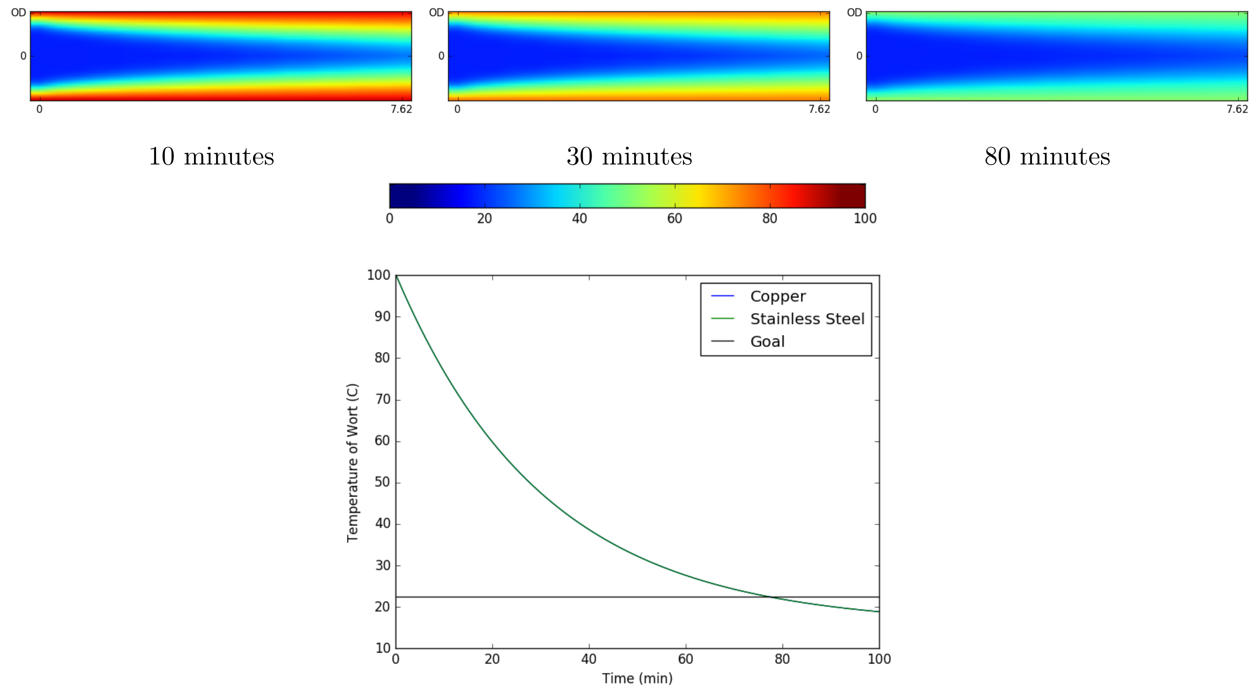



Figure 7: Temperature maps of a copper immersion chiller with laminar flow as at three times and a comparison of the Wort Temperature vs Time for copper and stainless steel tube walls. (the difference between copper and stainless steel is unresolvable)

#### 3.4.5 Fluid Mixing

Experimentally the Laminar flow model agrees fairly well with the experimental data (experimentally, a similar chiller will cool wort down about 30 minutes with .42 gallons/min flow). However in all of the temperature maps I see that there remains a large amount of cold water. This corresponds to potential cooling power that is not being utilized. To obtain better agreement I will look at a common passive enhancement of heat exchangers. An element like a coil or twist can be inserted into the pipe to somehow mix the chilling medium.I can model this by introducing a velocity expression that produces convective mixing.I need to ensure that the induced mixing respects the incompressibility of the medium so I are looking for an expression that maintains $\nabla \cdot \vec{v} = 0$.

I introduce the following expression

$$\vec{v}\begin{pmatrix} r \\ z \end{pmatrix} = scale\begin{pmatrix} \frac{aCos(az)Sin(br)}{r} \\ -\frac{bCos(br)Sin(az)}{r} \end{pmatrix} + \begin{pmatrix} 0 \\ v_m(1-\frac{r^2}{ID^2}) \end{pmatrix} \begin{matrix} \hat{r} \\ \hat{z} \end{matrix} \tag{16}$$

This velocity field satisfies the incompressibility condition $\nabla \cdot \vec{v} = 0$, the radial velocity goes to 0 as the fluid approaches the tube wall and the laminar term remains to ensure that there is overall flow through the tube. I chose the numbers a and b so that there are an integer numbers of vortices.

$$\frac{\partial T}{\partial t} = \frac{k}{c_p \rho}\Big(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z}\Big) - \vec{v}_r\frac{\partial T}{\partial r} - \vec{v}_z\frac{\partial T}{\partial z} \tag{17}$$

However, because I already know that my simulation breaks down when $\vec{v}_z\Delta t > dz$ and since these velocities are scaled by $\frac{1}{r}$ I will model a turbulent layer that is arbitrarily chosen to exclude $\frac{1}{4}$ of the pipes inner diameter. This prevents simulation breaking large velocities at small values of r. I can then look at how the temperature maps and cooling power change as a function of the number of vortices I induce along each axis. My initial guess was that as the number of vortices increased the cooling power would quickly approach the maximum value. Here I introduce plots that indicate temperature (color) as well as flow direction (arrows).

### 3.5 Results

Initially working with a turbulent flow on the order of 1% to 0.1% of the laminar flow I already get a dramatic change in the temperature maps inside the tube.

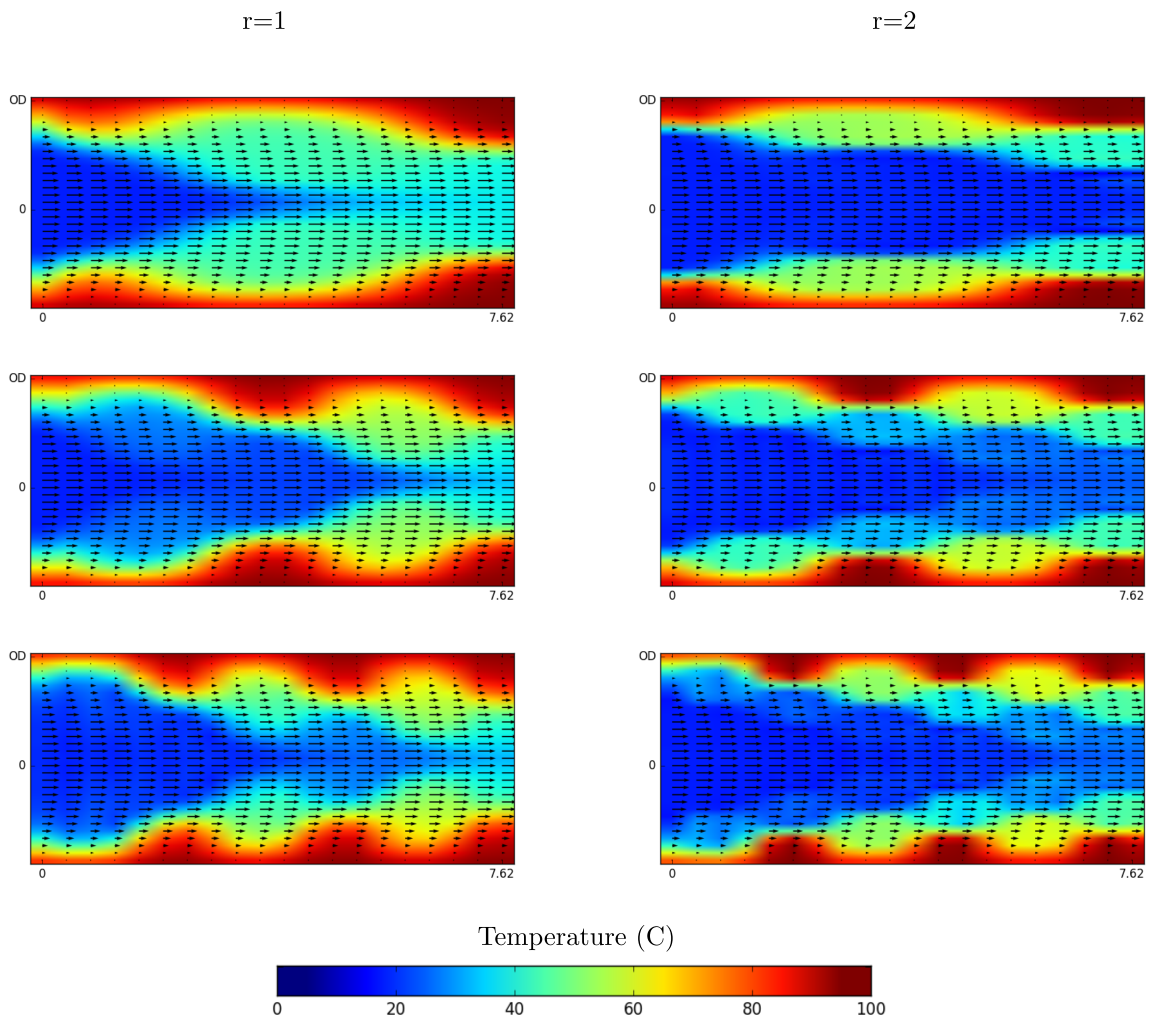



Figure 8: Temperature maps vs number of vortices. Columns show 1 and 2 radial vortices from left to right and rows show 1, 2 and 3 longitudinal vortices in descending order.

These types of flows offer similar cooling power to laminar flow approximately 1500 Watts, this value is ≈50% of maximum cooling rate given a flow of .42 gallons/min for cold water at 15.5 °C (60 °F) cooling water at boiling. However, if I increase scale of the turbulent mixing to be approximately equal to the laminar flow, I start to approach the maximum possible cooling rate. I also increased the number of grid points in both directions. This dramatically increased computational time but I only needed a few permutations of this simulation.

High Flow
4 Longitudinal vortices
2 Radial vortices

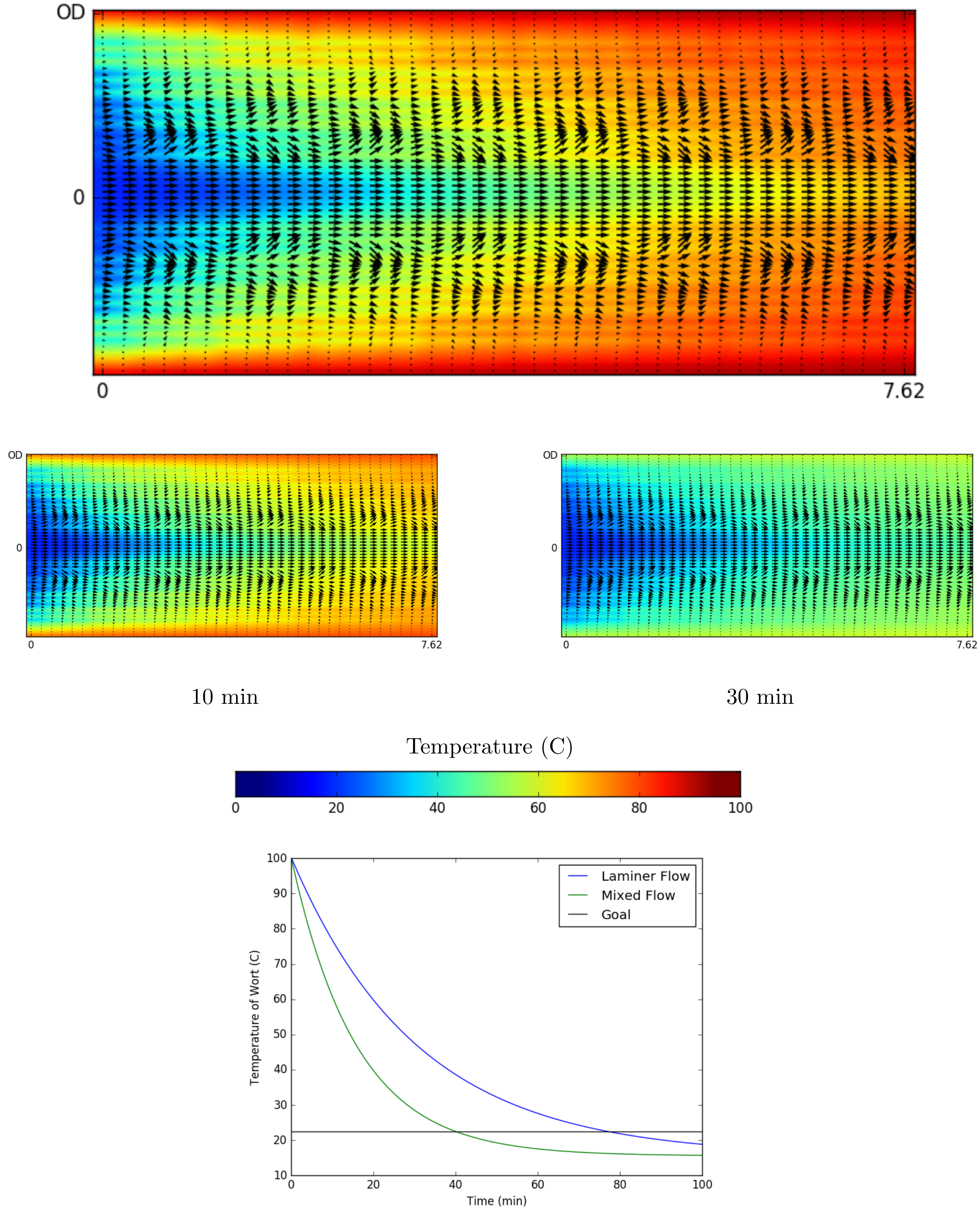



Figure 9: I see the increased performance of the turbulent chiller as a dramatic decrease in the time it takes to cool wort down to the goal temperature.

### 3.6 Error Estimation

I have strong suspicions that there are two primary sources of error. The first is that there are artifacts that result from my grid spacing. I would prefer to have run much of this code with a finer grid however I found that roughly my time goes as $\approx \frac{1}{\Delta t}$ and $\approx \frac{1}{\Delta r^2 \Delta z^2}$. These artifacts are especially apparent when looking at the turbulent flow model. Fig. 10 shows this error. The other source of error would be not taking in to account the change of physical parameters as a function of temperature. Both of these things were limited by calculation time. In order to analyze this problem I needed to run this code many times so this was a strong limitation. I am specifically avoiding listing the computational errors because I believe that those errors are small compared to other uncertainties in this project.

Another type of error that I have not accounted for is heat loss through the walls of the kettle. For both types f chiller the hot wort is sitting in a large pot and steam is escaping. While these effects would be most prominent at the initial higher temperatures I do not have a good estimate of how they influential they are in the cooling process. now that I have reasonable confidence in the chiller model, it would be reasonable to add an additional heat loss term to the wort reservoir that is proportional to the temperature difference of the wort to the room and the surface area of the wort.

Coarse Grid (15 (r), 20 (z)) Fine Grid (30 (r), 40 (z))

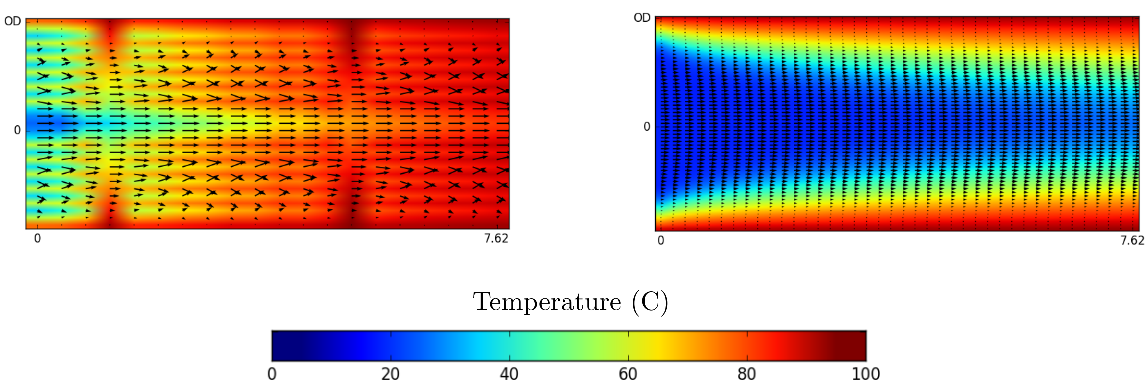



Figure 10: Temperatures maps with 1 longitudinal and 500 radial vortices. Mesh size in this case matters dramatically as the individual vortices cannot be resolved.

While much of this information is at least qualitatively intuitive, the result that surprised me was that the stainless steel and the copper performed equally well. Intuitively, I would have expected the order of magnitude difference in their thermal conductivities to matter greatly; however it appears that the thermal conductivity of water is the limiting parameter in this problem. Perhaps the difference will become visible with a more finely gridded tube wall.

# 4 Counter Flow Wort Chiller

I will use what I have learned during the study of the immersion chiller to predict how effective the counter flow chiller will be. For this application I will be using a constant flow model for the wort as inserting something to mix the wort will make it nearly impossible to sanitize the chiller. However, I will assume that the mixing inside of the counter flow chiller comes from turbulent liquid flow. The cross section is shown in Fig. 11. Again I will use the dimensions R1 = $\frac{5}{16}$", R2 = $\frac{3}{8}$", R3 = $\frac{1}{2}$" and 25 ft of length. The outer coating is a rubber hose. I have used similar boundary conditions as the immersion chiller. Sources will be temperature maintained. Sinks will be monitored for heat removal and will duplicate the adjacent longitudinal site. The inner radius will maintain $\frac{\partial T}{\partial r} = 0$ for symmetry. The outer diameter will be perfectly insulated by the rubber hose for $\frac{\partial T}{\partial r} = 0$.

Again I was surprised at the similar performance of stainless steel and copper. In my mind this really settles the debate I have seen on many homebrewing forums that weight longevity and ease of cleaning of

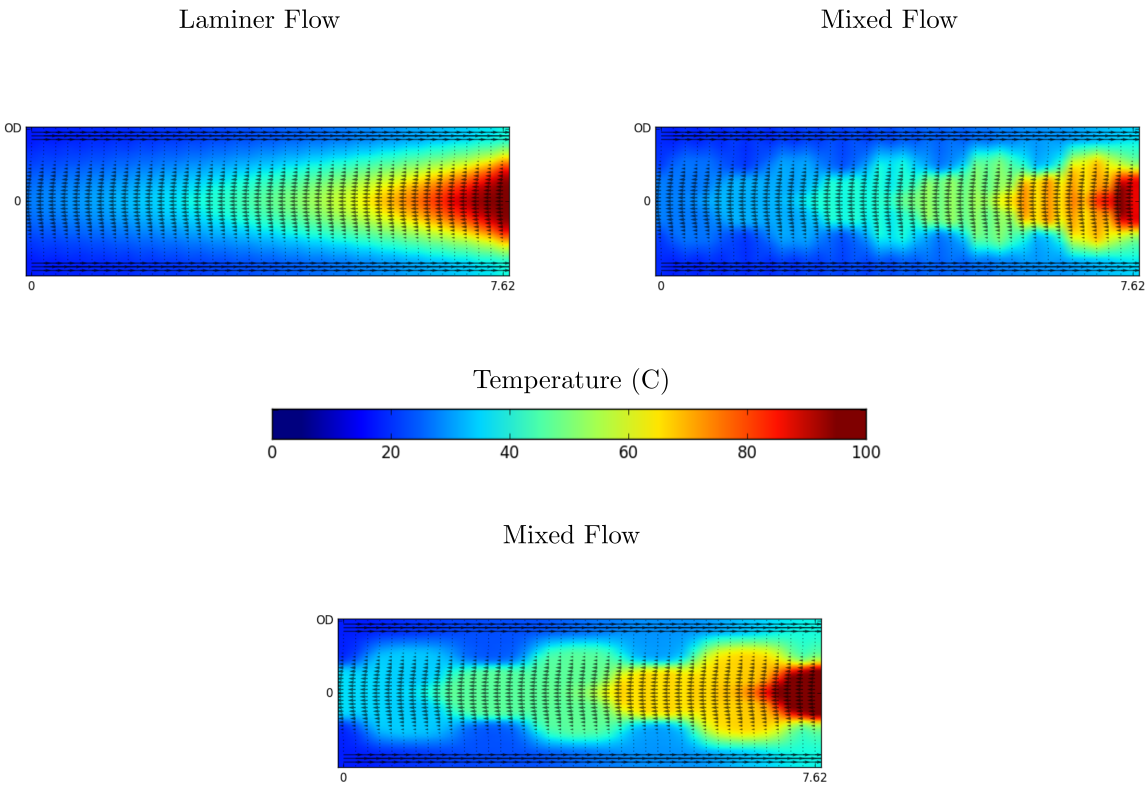



Figure 12: Temperatures maps for a counter flow chiller with variable numbers of longitudinal vortices.

stainless steel with the increased thermal conductivity of copper. The counter flow chiller does a good job of removing the heat from the wort in about 2 minutes, the time it takes to move through the chiller. I see the opposite trend in behavior as I increase turbulence in the counter flow chiller vs the immersion chiller. I interpret this as the counter flow chiller works by maintaining a thermal gradient, if the wort is mixed, the thermal gradient is not as consistent and the chiller loses efficiency. While this method should take about the same amount of time for a 5 gallon batch, it it much easier to implement larger batch sizes and the cooling happens rapidly preventing unwanted bacteria from entering newly finished beer. This technique is also scalable; multiple chillers can operate in parallel without losing efficiency.

The main disadvantage of the counter flow chiller is that it only uses a small fraction of the cooling potential of the chilling water. This tells me that perhaps the best method to employ when brewing beer is to use my newly constructed counter flow chiller in series with my immersion chiller. This way I can use my chilling water much more efficiently. Another disadvantage is that with an immersion chiller the surfaces that are in contact with the wort are accessible for cleaning and sanitation. This provides a small benefit to peace of mind if a brewer is especially worried about infections in their wort.

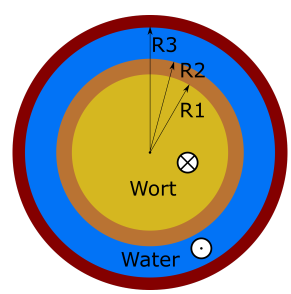



Figure 11: Cross section of counter flow wort chiller.

# 5 Physical Constants

Many of the physical constants that appear in this work change as a function of temperature some by non-negligible amounts. For instance the thermal conductivity of water changes by almost 20%. To obtain more physically accurate results I wanted to use temperature dependent parameters. However for the bulk of this work, it was computationally untenable to use variable parameters. Solving an equation Mx=b many time is reasonable with a large matrix M, however with variable temperatures you must reconstruct the matrix M each iteration dramatically increasing wall time. I have included a script that returns the parameters of the materials of interest as a function of temperature. While it would be trivial to implement the code, in developing this project I did not have the time to wait for the calculations to run. The parameter data was acquired from the literature and the script uses a cubic spline to interpolate it at relevant temperatures.

Listed below are physical constants for 50°C [15, 16, 17, 18] that have been used throughout this project :

| constant | water | wort | copper | stainless | description |
|---|---|---|---|---|---|
| k | 0.6424 | 0.6810 | 400 | 16.5 | thermal conductivity (W/m-K) |
| $\rho$ | 988.1 | 1047.4 | 8940 | 7750 | density (kg/m$^3$) |
| $c_p$ | 4182 | 4182 | 391.2 | 520 | specific heat (J/kg-K) |
| $\mu$ | $5.4710^{-3}$ | $9.2910^{-4}$ | - | - | dynamic viscosity(Ns/m$^2$) |
| $\nu$ | $5.5410^{-7}$ | $8.8710^{-7}$ | - | - | kinematic viscosity(m$^2/s$) |
| flow(ImC) | .42 | - | - | - | gallons/min |
| v (ImC) | .54 | - | - | - | m/s in a 5/16 ID tube |
| flow(CFC) | 1.68 | 0.084 | - | - | gallons/min |
| v (CFC) | 1.191 | 0.108 | - | - | m/s in a 5/16 ID tube |

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