

Principal Stratification Proposal

Yujing Gao¹

¹Department of Statistics, North Carolina State University

1 Traditional continuous and binary outcome

Here we list the current principal stratification methods. The first two methods are for RCT data and the third method is both for RCT data and observational data.

- Bayesian method [Magnusson et al. \[2019\]](#): Their Assumption 1 is joint exchangeability, which only applies for the RCT data, not for the observational study.
- Principal Score method [Ding and Lu \[2017\]](#): Their Assumption 1 is randomization, which only applies for the RCT data, not for the observational study.
- Multiply Robust Estimator [Jiang et al. \[2022\]](#): Previously, [Ding and Lu \[2017\]](#) establish some preliminary results for estimating the PCEs in randomized experiments including an identification formula based on weighting and the corresponding estimators for each PCE with and without adjusting for covariates. Their model-assisted estimator adjusts for covariates but is neither doubly robust nor semiparametrically efficient. So even in randomized experiments, the theory for estimating the PCEs is incomplete. They discuss a broader class of treatment assignments in both randomized experiments and unconfounded observational studies, providing two additional identification formulas for each PCE and proposing more principled estimators based on the EIFs. Their new estimators outperform those in [Ding and Lu \[2017\]](#) and they recommend using them in data analyses.

[I still need to check the simulation result in Jiang et al. \[2022\].](#)

2 Time-to-event outcome

There are two ways to describe the time-to-event outcome. The first way is to use the survival function, another way is to use the hazard ratio, popularized by the Cox proportional hazards model. Although [Hernán \[2010\]](#) pointed that directly comparing the hazard between 2 treatment arms does not lead to a valid causal estimand and thus defining causal contrasts of hazards is generally controversial. We can still treat it as a possible way to deal with the time-to-event outcome.

Here we list the current principal stratification methods for the time-to-event outcome.

- Method 1 [Liu et al. \[2024\]](#): They use the ER assumption to deal with survival function.
- Method 2 [Cheng et al. \[2023\]](#): They use the principal ignorability assumption to deal with survival function.
- Method 3 - possible idea: [Wang et al. \[2023\]](#) proposed a method to estimate the causal hazard ratio. Maybe we can borrow their idea to deal with the hazard ratio, then combine it with the principal stratification. However, in their method, they use the instrumental variable to deal with the unmeasured confounders. Their frame work is quite different from the principal stratification method. [I need to dig more into this method.](#)

3 Continuous Post-Treatment Variables [Lu et al. \[2023\]](#)

[I just copied their abstract and key words.](#) Post-treatment variables often complicate causal inference. They appear in many scientific problems, including noncompliance, truncation by death, mediation, and surrogate endpoint evaluation. Principal stratification is a strategy to address these challenges by adjusting for the potential values of the post-treatment variables, defined as the principal strata. It allows for characterizing treatment effect heterogeneity across principal strata and unveiling the mechanism of the treatment's impact on the outcome related to post-treatment variables. However, the existing literature has primarily focused on binary post-treatment variables, leaving the case with continuous post-treatment variables largely unexplored. This gap persists due to the complexity of infinitely many principal strata, which present challenges to both the identification and estimation of causal effects. We fill this gap by providing nonparametric identification and semiparametric estimation theory for principal stratification with continuous post-treatment variables. We propose to use working models to approximate the underlying causal effect surfaces and derive the efficient influence functions of the corresponding model parameters. Based on the theory, we construct doubly robust estimators and implement them in an R package. Keywords: efficient influence function; principal ignorability; principal density; semiparametric efficiency bound.

4 Sensitivity Analysis for Principal Stratification [Nguyen et al. \[2023\]](#)

Maybe we can generalize this into the time-to-event outcome. [need to check this paper.](#)

5 Principal Stratification with confounded principal strata [Luo et al. \[2022\]](#)

Maybe we can generalize this into the time-to-event outcome. [need to check this paper.](#)

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B.1: Proof for Proposition 3