Solving the Vehicle Routing Problem with Multiple Trips by Adaptive Memory Programming

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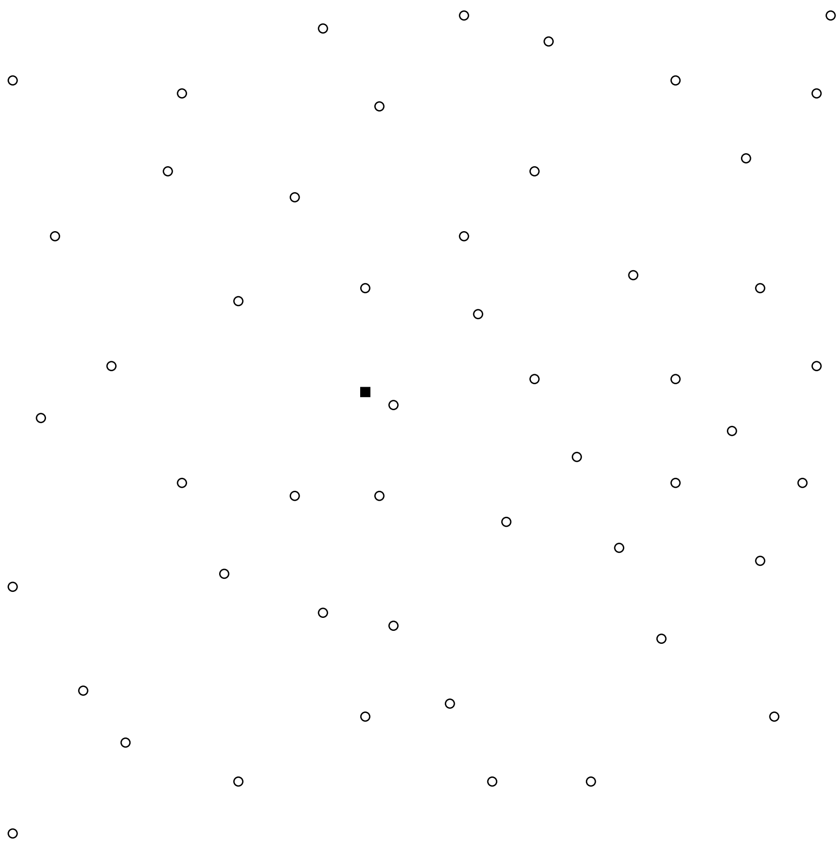
Motivation and problem description

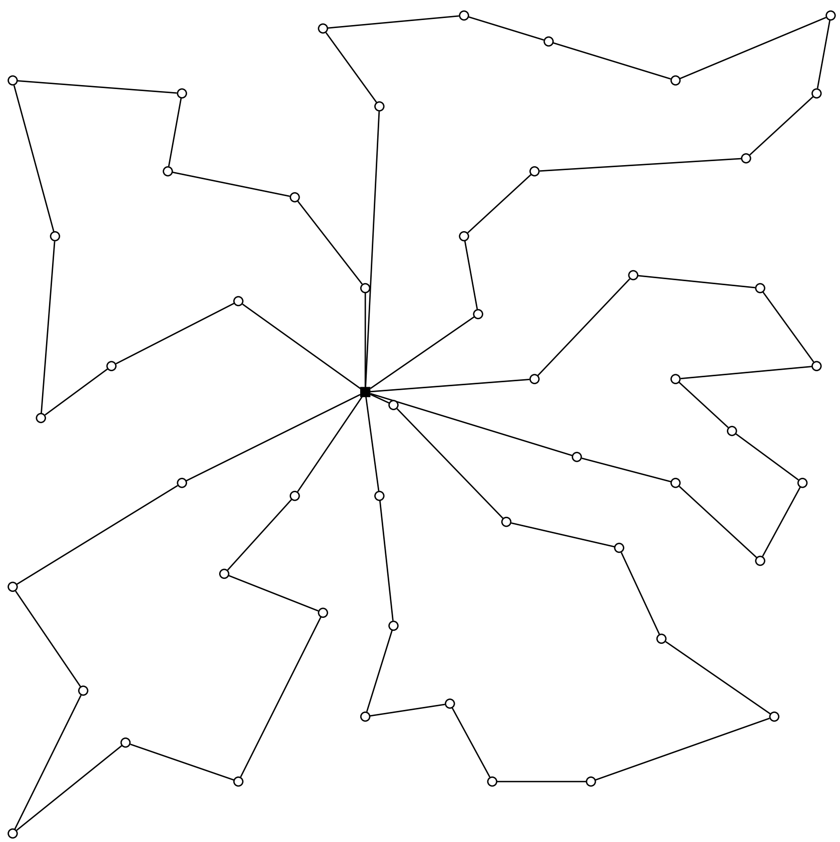
The classic Vehicle Routing Problem (VRP)

- Given:
  - A set of n customers and one depot.
  - Travelling costs c_{ij}.
  - Customer demands q_i and vehicle capacity Q.
- Find a set of routes such that:
  - Each route starts and ends at the depot.
  - Every customer belongs to exactly one route.
  - Vehicle capacity is not exceeded in any route.
  - The total cost is minimized.

A sample VRP instance

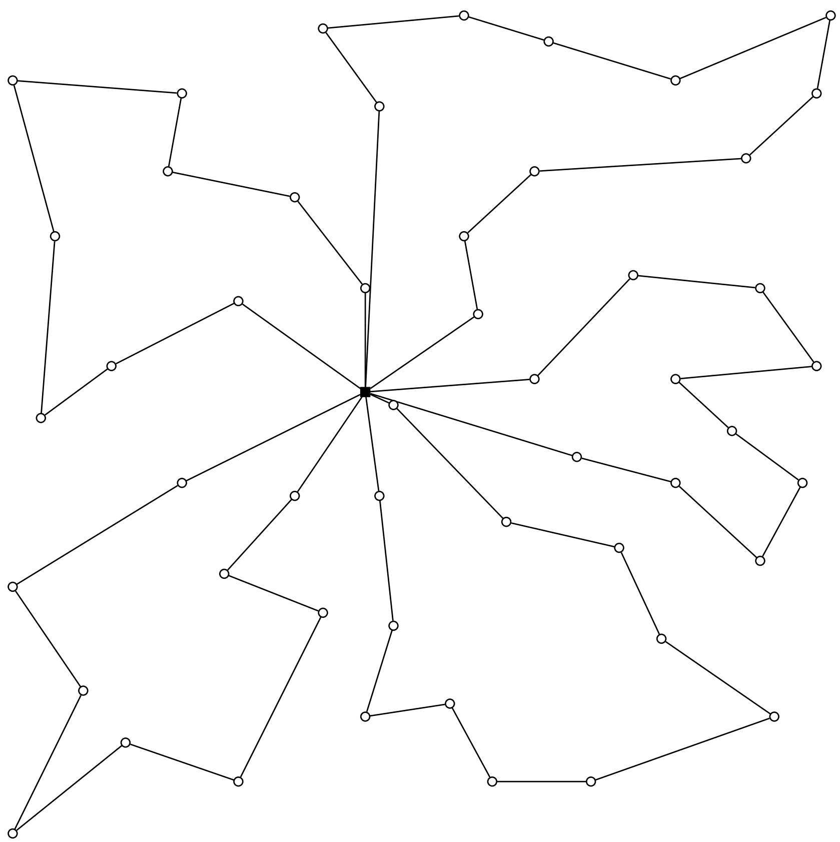


The optimal VRP solution



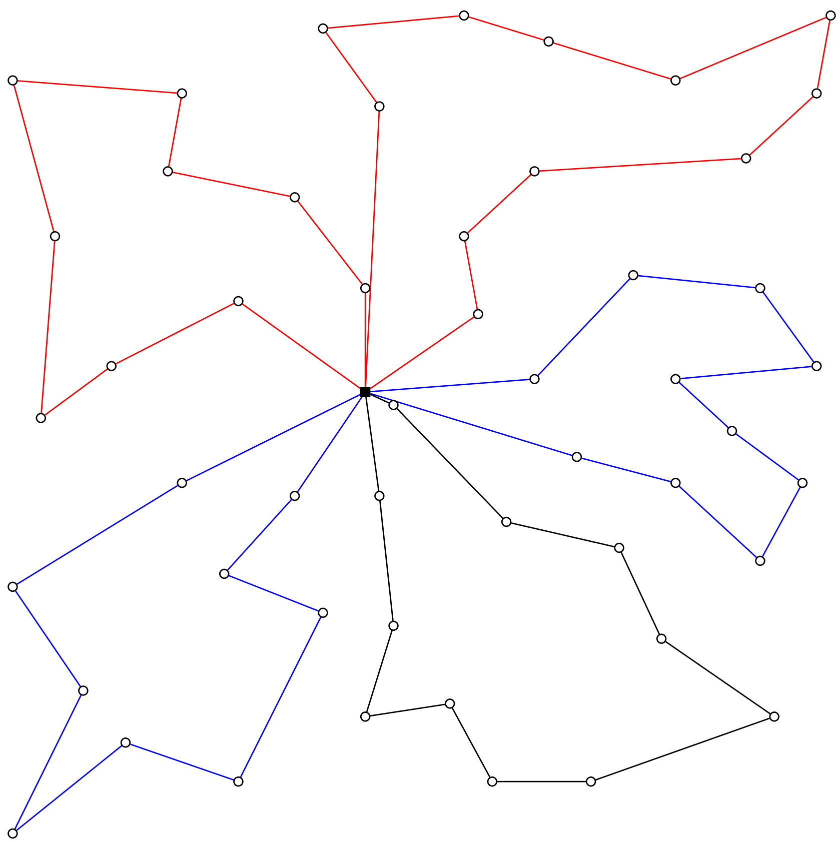
- Total cost 525 (5 routes).

The VRP solution in practice - finite fleet

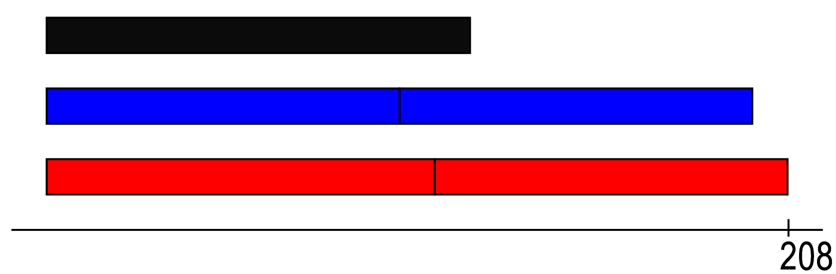


- Total cost 525 (5 routes).
- Problem: 3 vehicles available.
- Solution: ?

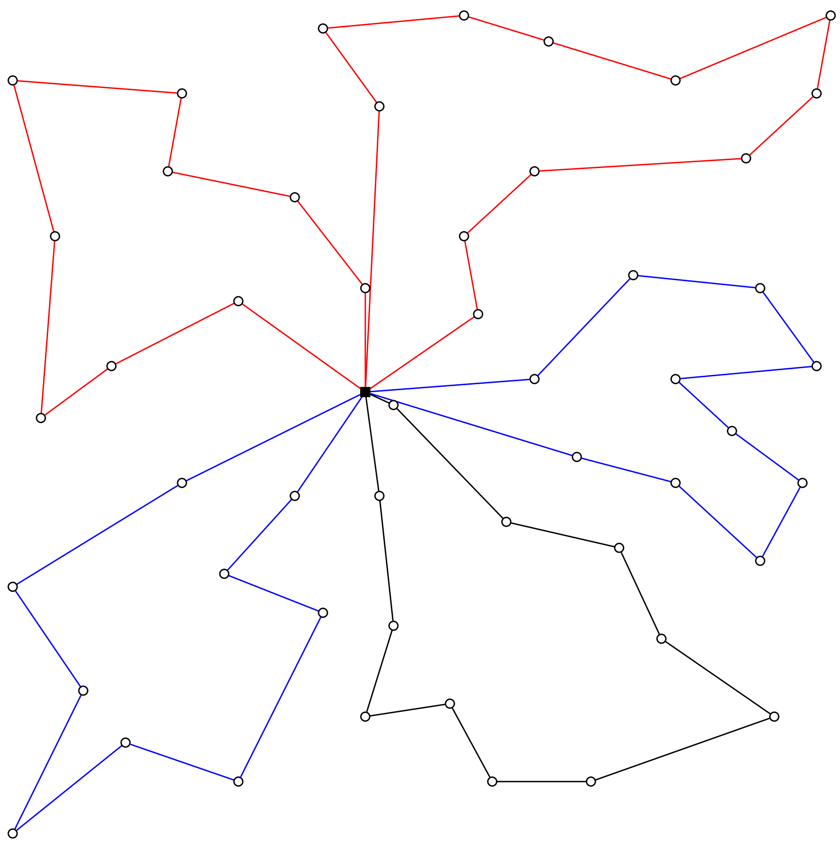
The VRP solution in practice - finite fleet



- Total cost 525 (5 routes).
- Problem: 3 vehicles available.
- Solution: Assign more than one route to the same vehicle.

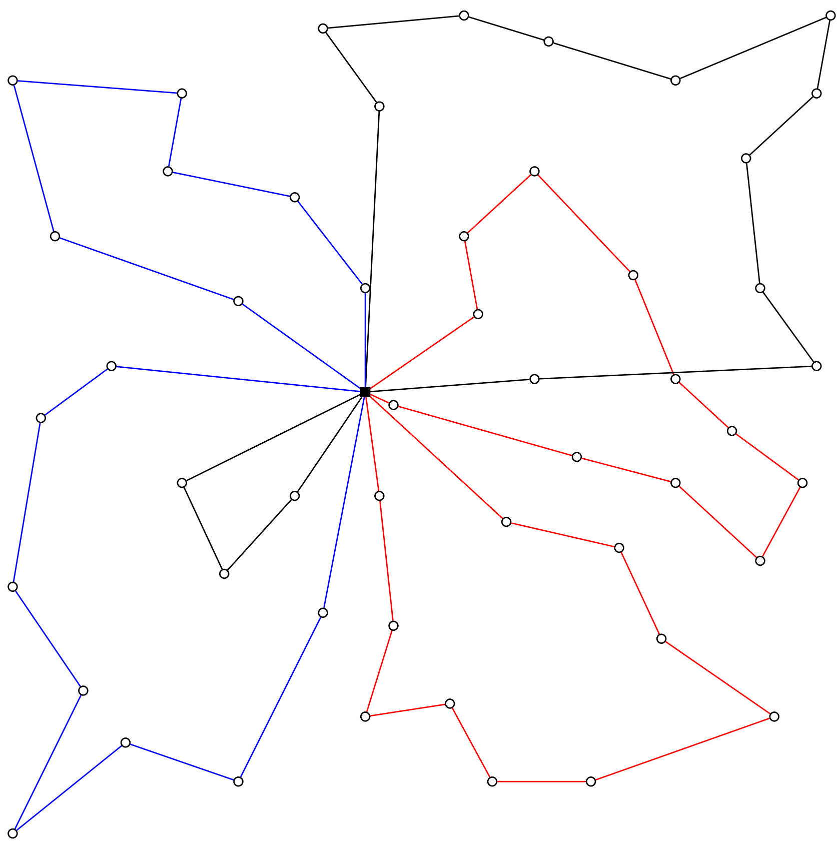


The VRP solution in practice - time horizon

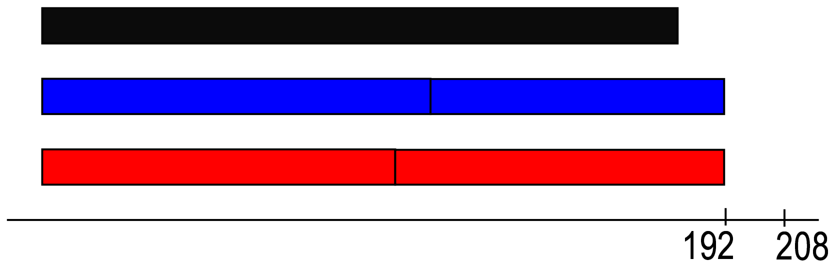


- Total cost 525 (5 routes).
- Problem: must finish the work before time 200.
- Solution: ?

The VRP solution in practice - time horizon



- Total cost 554 (6 routes).
- Problem: must finish the work before time 200.
- Solution: re-calculate!



VRP limitations

- The classic VRP model does not consider:
  - a finite number of vehicles,
  - a time horizon,
  - the assignment of routes to vehicles.
- It is not always possible to account for these constraints at a second stage.

The VRP with Multiple Trips (VRPMT)

- Besides the VRP constraints, the VRPMT considers:
  - A finite number of vehicles (m).
  - A time horizon (T).
- Solutions include an assignment of routes to vehicles.
- More than one route may be assigned to the same vehicle.
- The duration for each vehicle should not be greater than T.

Some remarks about the VRPMT

- The problem incorporates realistic constraints.
- Finding a *feasible* solution is NP-Hard (by reduction of the Bin Packing Problem).
- Little work reported in the OR literature.

Adaptive memory procedure

The Adaptive Memory Procedure (AMP)

- Proposed by Rochat and Taillard in 1995 for VRP and VRPTW.
- *Probabilistic intensification and diversification*.
- Enhancement of local search algorithms.
- Gives accurate solutions when used to solve other VRPs.

AMP - The algorithm

- A memory M with *components* of visited solutions is kept.
- A generic template:

1. **Initialize** M.
2. Repeat for AMP^{iter} iterations:
   (a) **Build** a solution s using M.
   (b) Apply **tabu search** to s.
   (c) **Update** M using the obtained solution.

AMP - The Memory

- The memory M contains individual routes.
- Routes belonging to better solutions are stored first.
- The size of the memory is bounded.

AMP - Initialization

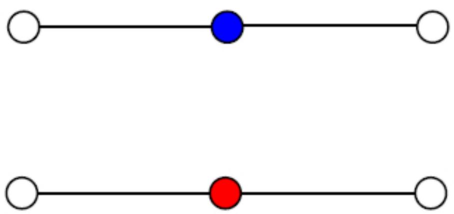
- Initially, the memory is empty.
- The following is repeated a fixed number of times:
  1. Build a solution using a fast randomized heuristic.
  2. Apply the **tabu search** to that solution.
  3. Add the routes of the new solution to M.

AMP - Solution building

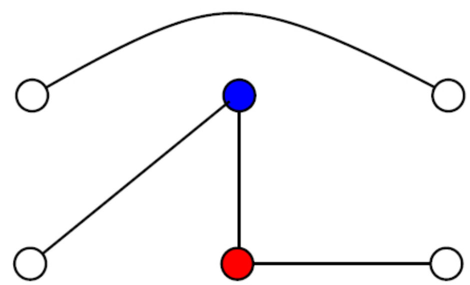
- Probabilistically select non-overlapping routes from M.
- Higher probability is assigned to the first routes.
- The selected routes are assigned to the vehicles, using a simple greedy heuristic.

Tabu Search - neighborhood definition

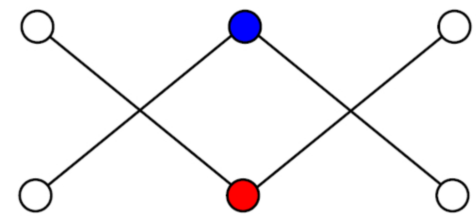
- For each solution s, a set of neighbors $N(s)$ is defined:



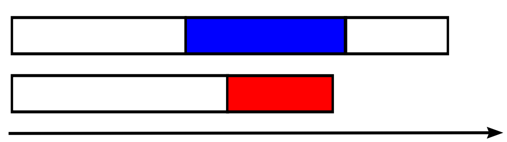
Original routes



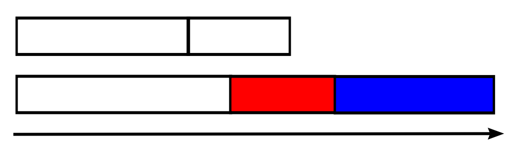
Customer insert



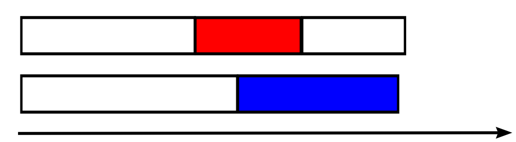
Customer swap



Original assignment



Route insert



Route swap

Tabu Search - exploration strategy

- Infeasible solutions may be visited and penalized in the objective function.

Repeat for TS^{iter} iterations:

1. Make the best non-tabu move.
2. Declare tabu the inverse of the move for δ iterations.

Experimental results

Benchmark problems

- The AMP was tested over a set of 104 benchmark problems.
- Highly constrained instances (finding a feasible solution is the main issue).
- Results were compared with:
  - Brandão and Mercer (BM),
  - Taillard, Laporte and Gendreau (TLG),
  - Petch and Salhi (PS).

Summarized results

	AMP	BM	TLG	PS
Feasible solutions	**95**	89	86	76
Average LTR (%)	**1.4**	5.2	6.5	4.9
Average Overtime	**3.9**	9.3	14.2	–

- Average GAP with respect to a VRP lower bound: 3.09%.
- The GAP standard deviation average was 0.76%.
- The AMP solved 6 previously unsolved instances.

Conclusions and future work

Conclusions

- We solved a hard and practical vehicle routing problem.
- The AMP was effective (once more).
- The algorithm seems to be flexible to handle more constraints.

Future work

- Add constraints to the VRPMT:
  - heterogeneous fleet,
  - multiple depots,
  - time windows,
  - control of depot congestion.

Questions