Comp Sci 538, Written Assignment 1

Problem 1:

digit = "0"| "1"| "2"| "3"| "4"| "5"| "6"| "7"| "8"| "9"
int = ["-"] digit { digit }
be = "tt"|"ff"
  | be "&&" be
  | be "||" be
  | ae "==" ae
  | ae ">" ae;
ae = int
  | ae "+" ae
  | ae "×" ae
  | "if" be "then" ae "else" ae;

Problem 2: (1)

When $v_i = int$, $\forall i \in \mathbb{Z}^+$, stop stepping.
If $ae_1 \to ae_1'$, then ae_1 (+ or ×) $ae_2 \to ae_1'$ (+ or ×) ae_2
If $ae_2 \to ae_2'$, then v_1 (+ or ×) $ae_2 \to v_1$ (+ or ×) ae_2'
If $v_1 = int_1, v_2 = int_2$, and int_1 (+ or ×) $int_2 = int$, then v_1 (+ or ×) $v_2 \to int$

When $b_i = tt$ or ff, $\forall i \in \mathbb{Z}^+$, stop stepping.
If $ae_1 \to ae_1'$, then ae_1 (== or >) $ae_2 \to ae_1'$ (== or >) ae_2
If $ae_2 \to ae_2'$, then v_1 (== or >) $ae_2 \to v_1$ (== or >) ae_2'
If $v_1 \to int_1$, $v_2 \to int_2$, and $int_1 = int_2$, then $v_1 == v_2 \to$ "tt". Otherwise, $v_1 == v_2 \to$ "ff"
If $v_1 \to int_1$, $v_2 \to int_2$, and $int_1 > int_2$, then $v_1 > v_2 \to$ "tt". Otherwise, $v_1 == v_2 \to$ "ff"

If $be_1 \to be_1'$, then be_1 (&& or ||) $be_2 \to be_1'$ (&& or ||) be_2
If $be_2 \to be_2'$, then b_1 (&& or ||) $be_2 \to b_1$ (&& or ||) be_2'
If b_1 = tt, b_2 = tt, then b_1 (&& or ||) $b_2 \to$ tt
If b_1 = tt, then b_2 = ff, then b_1 || $b_2 \to$ tt
If b_1 = tt, then b_2 = ff, then b_1 && $b_2 \to$ ff
If b_1 = ff, then b_2 = ff, then b_1 (|| or &&) $b_2 \to$ ff
lazy-if :
If $be_1 \to be_1'$, if be_1 then ae_1 else $ae_2 \to$ if be_1' then ae_1 else ae_2
If b_1 = tt, $ae_1 \to ae_1'$, if b_1 then ae_1 else $ae_2 \to$ if b_1 then ae_1' else ae_2
If b_1 = tt, $v_1 = int_1$, if b_1 then v_1 else $ae_2 \to int_1$
If b_1 = ff, $ae_2 \to ae_2'$, if b_1 then ae_1 else $ae_2 \to$if b_1 then ae_1 else ae_2'
If b_1 = ff, $v_2 \to int_2$, if b_1 then ae_1 else $v_2 \to int_2$

(2) *eager-if*
If $ae_1 \to ae_1'$, if be_1 then ae_1 else $ae_2 \to$ if be_1 then ae_1' else ae_2
If $ae_2 \to ae_2'$, if be_1 then v_1 else $ae_2 \to$ if be_1 then v_1 else ae_2'
If $be_1 \to be_1'$, if be_1 then v_1 else $v_2 \to$ if be_1' then v_1 else v_2
If b_1 =tt, $v_1 = int_1$, if b_1 then v_1 else $v_2 \to int_1$

If $b_1 =$ff, $v_2 = int_2$, if b_1 then v_1 else $v_2 \rightarrow int_2$

Problem 3:

(1) First of all, we replace f with the $\lambda x.\, x + 1$ and the whole lambda function becomes $(\lambda x.\, x + 1)$ 5, which evaluates to 6

(2) As in (1), we first replace f with the lambda function and get $(\lambda x.\ \lambda y.\ y - x)$ 5 3, then we replace x with 5 and get $(\lambda y.\ y - 5)$ 3, which evaluates to -2

(3) Plugging $(\lambda x.\ x\ x)$ into the expression of the first lambda function, we again get $(\lambda x.\ x\ x)(\lambda x.\ x\ x)$, which will go on infinitely.

Problem 4:

fib = fix f. λn. if $(n = 1||n = 2)$ then 1 else $f(n - 1) + f(n - 2)$

Reduction:

$$\begin{aligned}
fib\,3 &= \text{fix } f.\ \lambda n.\ \text{if } (n = 1||n = 2) \text{ then } 1 \text{ else } f(n-1) + f(n-2)\ 3 \\
&= \ \text{if } (3 = 1||3 = 2) \text{ then } 1 \text{ else } (\text{fix } f...)\ (3-1) + (\text{fix } f...)\ (3-2) \\
&= (\text{fix } f...)\ (3-1) + (\text{fix } f...)\ (3-2) \\
&= 1 + 1 \\
&= 2
\end{aligned}$$