Business cycles and labor market search (David Andolfato)

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1 The model

The model nest a labor-market-search framework into an standard RBC model. He assume that there are households distributed uniformly on the unit interval and that they have preferences for consumption and leisure. In the model, households face the standard consumption-saving problem, but face altogether different opportunities for exchanging labor services. Individuals either have a job opportunity or not, and job opportunities come and go at random, depending to some extent on individual search effort, the availability of jobs, and plain luck. Firms also face a standard wealth maximization problem, except that, because finding new workers takes time and effort, firms view their existing workforce as a capital asset. Given constant returns in the production technology, it may be assumed without loss that each firm comprises a single job; in what follows, the terms firm and job will be used interchangeably.

1.1 The search process

In order to produce output, each job requires a worker. Let n_t, denote the number of jobs that are matched with a worker at the beginning of period t. Job-worker pairs are assumed to separate at the exogenous rate $0 < \sigma < 1$. Replenishing this stock takes time and consumes resources.

A firm interested in filling an available job must undertake recruiting and screening activities, which are necessary for finding a suitable employee. Let v_t, denote the number of jobs vacancies during period t, each of which incurs a flow cost equal to $\kappa > 0$, measured in units of physical output. In the version of the model studied here, workers are assumed to search passively. Letting e denote search effort per worker seeking employment, aggregate search effort by workers is given by $(1 - n_t)e$. The rate at which new job matches form is governed by an aggregate matching technology, $M(v, (1 - n)e)$, so that employment evolves according to the following dynamic equation:

$$n_{t+1} = (1 - \sigma)n_t + M(v_t, (1 - n_t)e) \tag{1}$$

The matching technology is assumed to be a non-decreasing, concave function of aggregate search and recruiting effort and is assumed to display constant returns to scale.

1.2 The Social Welfare Problem

The representative household has preferences represented by a utility function of the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(c_t) + \phi(t)H(1 - x_t)\right] \tag{2}$$

where c_t denotes consumption, x_t, denotes the fraction of time spent in nonleisure activities and $0 < \beta < 1$ is a discount factor. The functions U and H are increasing and concave. The value of the parameter $\phi(t) > 0$ depends on a household's employment status: $\phi(t)$ is equal to ϕ_1 if the household is employed and is equal to ϕ_2 if the household is unemployed.

Output is produced according to a standard neoclassical production technology, $y_t = F(k,\, n_t l_t;\, z_t)$, where k_t is the aggregate capital input; l_t is average hours worked by those employed; and z_t is a parameter reflecting the current state of technology, which evolves stochastically according to the transition function $G(z', z) = Pr[z_{t+1} \leq z' \mid z_t = z]$. The capital stock depreciates at rate $0 < \delta < 1$, so that the economy-wide resource constraint is then given by:

$$c_t + k_{t+1} + \kappa v_t = y_t + (1-\delta)k_t \tag{3}$$

$s \equiv (k,\ n\ , z)$ denote the vector of the state variables of the economic system, E_G denote the expectation operator associated with the transition function G, $W(s_0)$ denote the maximum value of (2), given an arbitrary initial condition (s_0), which is obtained by solving the welfare problem stated above. The primed variables denote 'next period values'.

1.3 Equations of the model to log-linearize

$$U_c(c) = \beta E_G \left[F_k(k', n'l';\ z') + (1-\delta)\right] U_c(c') \tag{4}$$

$$\phi_1 H_l(1-l) = F_l(k,\ nl;\ z)U_c(c) \tag{5}$$

$$\kappa v U_c(c) = \mu\alpha M(v,\ (1-n)e) \tag{6}$$

$$\mu = \beta E_G \left\{\phi_1 H(1-l') - \phi_2 H(1-e) + U_c(c')F_{n'}(k', n'l';\ z')l' + \mu'\left[1-\sigma-(1-\alpha)p(v', n')\right]\right\} \tag{7}$$

$$c + k' + \kappa v = F(k,\ nl;\ z) + (1-\delta)k \tag{8}$$

$$n' = (1-\sigma)n + M(v,\ (1-n)e) \tag{9}$$

$$w = (1-\alpha)F_2(k,\ nl;\ z) + \alpha\left[\frac{[\phi_2 H(1-e) - \phi_1 H(1-l)] + p(v,\ n)(1-\alpha)\mu}{U_c(c)}\right]\frac{1}{l} \tag{10}$$

The functional forms for U, H, F, M and G are as follows:

- $U(c) = log(c)$
- $H(1-x) = \frac{(1-x)^{(1-\eta)}}{(1-\eta)}$
- $F(k,\ nl;\ z) = exp(z)\zeta k^{\theta}(nl)^{(1-\theta)}$
- $M(v,\ (1-n)e) = min\left\{v,\ 1-n,\ \chi v^{\alpha}((1-n)e)^{(1-\alpha)}\right\}$
- $p(v,\ n) = \frac{\chi v^{\alpha}((1-n)e)^{(1-\alpha)}}{1-n}$

Additionally, the productivity shock is assumed to be governed by the following stochastic process:

- $z' = \rho z + \tilde{\varepsilon}$

where $0 < \rho < 1$ and $\tilde{\varepsilon}$ is an independently and identically distributed random variable. In particular, assume that $\tilde{\varepsilon} \in \{-\varepsilon,\ \varepsilon\}$,$\varepsilon > 0$, and $prob(\varepsilon) = prob(-\varepsilon) = \frac{1}{2}$.
Then equations (4) to (10) take the form:

- (4') $\frac{1}{c} = \beta E_G\left[\theta exp(z')\zeta(\frac{n'l'}{k'})^{(1-\theta)} + (1-\delta)\right]\frac{1}{c'}$
- (5') $\phi_1\,(1-l)^{-\eta} = (1-\theta)exp(z)\zeta k^{\theta}(nl)^{-\theta}\frac{1}{c}$
- (6') $\kappa v\frac{1}{c} = \mu\alpha\chi v^{\alpha}((1-n)e)^{(1-\alpha)}$
- (7') $\mu = \beta E_G\left\{\phi_1\frac{(1-l')^{(1-\eta)}}{(1-\eta)} - \phi_2\frac{(1-e)^{(1-\eta)}}{(1-\eta)} + \frac{1}{c'}(1-\theta)exp(z)\zeta k'^{\theta}(n'l')^{-\theta}l' + \mu'\left[1-\sigma-(1-\alpha)\frac{\chi v'^{\alpha}((1-n')e)^{(1-\alpha)}}{1-n'}\right]\right\}$
- (8') $c + k' + \kappa v = exp(z)\zeta k^{\theta}(nl)^{(1-\theta)} + (1-\delta)k$
- (9') $n' = (1-\sigma)n + \chi v^{\alpha}((1-n)e)^{(1-\alpha)}$
- (10') $w = (1-\alpha)(1-\theta)exp(z)\zeta k^{\theta}(nl)^{-\theta} + \alpha\left[\left(\phi_2\frac{(1-e)^{(1-\eta)}}{(1-\eta)} - \phi_1\frac{(1-l)^{(1-\eta)}}{(1-\eta)}\right) + (1-\alpha)\frac{\chi v^{\alpha}((1-n)e)^{(1-\alpha)}}{1-n}\mu\right]\frac{c}{l}$

1.4 Equations at the steady state

- (4") $1=\beta\left[\theta\zeta(\frac{n*l*}{k*})^{(1-\theta)}+(1-\delta)\right]$
- (5") $\phi_1\,(1-l*)^{-\eta}=(1-\theta)\zeta(k*)^{\theta}(n*l*)^{-\theta}\frac{1}{c*}$
- (6") $\frac{\kappa v*}{c*}=\mu*\alpha\chi(v*)^{\alpha}((1-n*)e)^{(1-\alpha)}$
- (7") $\mu*=\beta\left\{\phi_1\frac{(1-l*)^{(1-\eta)}}{(1-\eta)}-\phi_2\frac{(1-e)^{(1-\eta)}}{(1-\eta)}+(1-\theta)\zeta(k*)^{\theta}(n*l*)^{-\theta}\frac{l*}{c*}+\mu*\left[1-\sigma-(1-\alpha)\frac{\chi(v*)^{\alpha}((1-n*)e)^{(1-\alpha)}}{1-n*}\right]\right\}$
- (8") $c*+\delta k*+\kappa v*=\zeta(k*)^{\theta}(n*l*)^{(1-\theta)}$
- (9") $\sigma n*=\chi(v*)^{\alpha}((1-n*)e)^{(1-\alpha)}$
- (10") $w*n*l*=(1-\theta)-\frac{\kappa v*}{\sigma}\left[\left(\frac{1-(1-\sigma)\beta}{\beta}\right)\right]$

1.5 Log-linearization around the steady state

we denote $\frac{\partial X_t}{X*}=\widetilde{X}_t$, and X the value of X at the steady state while X_t is the value of X at time t.

1.5.1 Equation (4)'

From (4') and (10") we have :

$$\frac{1}{c_t}=\beta E_t\left[\theta exp(z_{t+1})\zeta\left(\frac{n_{t+1}l_{t+1}}{k_{t+1}}\right)^{(1-\theta)}+(1-\delta)\right]\frac{1}{c_{t+1}}$$

$$-\frac{\tilde{c}_t}{c}=\beta E_t\left[\theta\zeta(\frac{nl}{k})^{(1-\theta)}\widetilde{z_{t+1}}+(1-\theta)\theta\zeta(\frac{nl}{k})^{-\theta}\frac{ln}{k}\frac{\partial n_{t+1}}{n}+(1-\theta)\theta\zeta(\frac{nl}{k})^{-\theta}\frac{ln}{k}\frac{\partial l_{t+1}}{l}+(1-\theta)\theta\zeta(\frac{nl}{k})^{-\theta}\left(nl\frac{-\partial k_{t+1}}{k^2}\right)\right]\frac{1}{c}$$

$$-\widetilde{c_t}=\beta E_t\left\{\zeta*\theta*\left(\frac{nl}{k}\right)^{(1-\theta)}\left((1-\theta)(\widetilde{l_{t+1}}+\widetilde{n_{t+1}}-\widetilde{k_{t+1}})+\widetilde{z_{t+1}}\right)+1-\delta\right\}-\underbrace{\beta\left(\zeta*\theta*\left(\frac{nl}{k}\right)^{(1-\theta)}+1-\delta\right)}_{=1}E_t\widetilde{c_{t+1}}$$

$$\widetilde{c_t}=E_t\widetilde{c_{t+1}}-\beta E_t\left\{\zeta*\theta*\left(\frac{nl}{k}\right)^{(1-\theta)}\left((1-\theta)(\widetilde{l_{t+1}}+\widetilde{n_{t+1}}-\widetilde{k_{t+1}})+\widetilde{z_{t+1}}\right)+1-\delta\right\}$$

1.5.2 Equation (5)'

$$\phi_1\,(1-l)^{-\eta}=(1-\theta)exp(z)\zeta k^{\theta}(nl)^{-\theta}\frac{1}{c}$$

$$-\eta\phi_1\,(1-l)^{-\eta-1}\,(\frac{-\partial l_t l}{l})=(1-\theta)\zeta k^{\theta}(nl)^{-\theta}\frac{1}{c}\widetilde{z}_t+\theta(1-\theta)\zeta k^{\theta-1}(nl)^{-\theta}\frac{1}{c}\partial k_t-\theta(1-\theta)\zeta k^{\theta}(nl)^{-\theta}\frac{1}{c}\,(\partial n_t n+\partial l_t n)$$
$$-(1-\theta)\zeta k^{\theta}(nl)^{-\theta}\frac{\partial c_t}{c^2}$$

$$\eta\phi_1\,(1-l)^{-\eta-1}\,\widetilde{l}_t l=(1-\theta)\zeta k^{\theta}(nl)^{-\theta}\frac{1}{c}\left[\widetilde{z}_t+\theta\left(\widetilde{k}_t-\widetilde{n}_t-\widetilde{l}_t\right)-\widetilde{c}_t\right]$$
$$\frac{l\eta}{1-l}\widetilde{l}_t=\left[\widetilde{c}_t-\widetilde{z}_t+\theta\left(\widetilde{n}_t+\widetilde{l}_t-\widetilde{k}_t\right)\right]$$

1.5.3 Equation (6)'

$$\kappa v\frac{1}{c} = \mu\alpha\chi v^{\alpha}((1-n)e)^{(1-\alpha)}$$
$$\kappa v\frac{1}{c}\left(\widetilde{v}_t-\widetilde{c}_t\right) = \alpha\mu\alpha\chi v^{\alpha}((1-n)e)^{-\alpha}(\widetilde{\mu}_t+\widetilde{v}_t)+(1-\alpha)\mu\alpha\chi v^{\alpha}((1-n)e)^{-\alpha}(-\frac{\partial n_t ne}{n})$$
$$\kappa v\frac{1}{c}\left(\widetilde{v}_t-\widetilde{c}_t\right) = \alpha\mu\alpha\chi v^{\alpha}((1-n)e)^{1-\alpha}\left[\widetilde{\mu}_t+\alpha\widetilde{v}_t-(1-\alpha)\frac{ne}{(1-n)e}\widetilde{n}_t\right]$$
$$(1-\alpha)\widetilde{v}_t-\widetilde{c}_t = \widetilde{\mu}_t-(1-\alpha)\frac{n}{(1-n)}\widetilde{n}_t$$

1.5.4 Equation (7)'

$$\mu = \beta E_G\left\{\phi_1\frac{(1-l')^{(1-\eta)}}{(1-\eta)}-\phi_2\frac{(1-e)^{(1-\eta)}}{(1-\eta)}+\frac{1}{c'}(1-\theta)exp(z')\zeta k'^{\theta}(n'l')^{-\theta}l'+\mu'\left[1-\sigma-(1-\alpha)\frac{\chi v'^{\alpha}((1-n')e)^{(1-\alpha)}}{1-n'}\right]\right\}$$

$$\begin{aligned}\widetilde{\mu}_t\mu = \beta\Bigg\{&-(1-\eta)\phi_1\frac{(1-l)^{(-\eta)}}{(1-\eta)}\widetilde{l}_{t+1}l+\frac{1}{c}(1-\theta)\zeta k^{\theta}(nl)^{-\theta}lE_t\left[\widetilde{z}_{t+1}+\theta\left(\widetilde{k}_{t+1}-\widetilde{n}_{t+1}-\widetilde{l}_{t+1}\right)-\widetilde{c}_{t+1}\right]\\ &+E_t\widetilde{\mu}_{t+1}\left[1-\sigma-(1-\alpha)\frac{\chi v\alpha((1-n)e)^{(1-\alpha)}}{1-n}\right]-\mu(1-\alpha)\frac{\chi v\alpha((1-n)e)^{(1-\alpha)}}{1-n}\left(\alpha E_t\widetilde{v}_{t+1}\right)\\ &-\mu(-\alpha)(1-\alpha)\chi v^{\alpha}e^{1-\alpha}(-\widetilde{n_{t+1}}n)*(1-n)^{-\alpha-1}\Bigg\}\end{aligned}$$

$$\begin{aligned}\widetilde{\mu}_t\mu = \beta\Bigg\{&\phi_1\frac{l}{(1-l)^{eta}}E_t\left(z\tilde{_{t+1}}+\theta(k\tilde{_{t+1}}-n\tilde{_{t+1}}-l\tilde{_{t+1}})-c\tilde{_{t+1}}\right)+\left(1-\sigma-(1-\alpha)\frac{\sigma n}{1-n}\right)E_t\mu\tilde{_{t+1}}\\ &-\frac{\sigma n\alpha(1-\alpha)}{1-n}E_t\Bigg(v\tilde{_{t+1}}+\frac{n\tilde{_{t+1}}n}{1-n})\}\end{aligned}$$

1.5.5 Equation (8)'

$$c+k'+\kappa v = exp(z)\zeta k^{\theta}(nl)^{(1-\theta)}+(1-\delta)k$$
$$c\widetilde{c}_t+k\widetilde{k}_{t+1}+\kappa v\widetilde{v}_t = \zeta k^{\theta}(nl)^{(1-\theta)}\left(\widetilde{z}_t+\theta\widetilde{k}_t+(1-\theta)(\widetilde{n}_t+\widetilde{l}_t)\right)+(1-\delta)k\widetilde{k}_t$$
$$c\widetilde{c}_t+k\widetilde{k}_{t+1}+\kappa v\widetilde{v}_t = \zeta k^{\theta}(nl)^{(1-\theta)}\left(\widetilde{z}_t+\theta\widetilde{k}_t+(1-\theta)(\widetilde{n}_t+\widetilde{l}_t)\right)+(1-\delta)k\widetilde{k}_t$$

1.5.6 Equation (9)'

$$n' = (1-\sigma)n+\chi v^{\alpha}((1-n)e)^{(1-\alpha)}$$
$$n\widetilde{n}_{t+1} = (1-\sigma)n\widetilde{n}_t+\sigma n\left[\alpha\widetilde{v}_t-(1-\alpha)\frac{n}{(1-n)}\widetilde{n}_t\right]$$
$$\widetilde{n}_{t+1} = (1-\sigma)\widetilde{n}_t+\sigma\left[\alpha\widetilde{v}_t-(1-\alpha)\frac{n}{(1-n)}\widetilde{n}_t\right]$$
$$\widetilde{n}_{t+1} = \sigma\alpha\widetilde{v}_t+\frac{(1-n)-\sigma(1-\alpha n)}{(1-n)}\widetilde{n}_t$$

1.5.7 Equation (10)'

$$w = (1-\alpha)(1-\theta)exp(z)\zeta k^{\theta}(nl)^{-\theta} + \alpha \left[\left(\phi_2 \frac{(1-e)^{(1-\eta)}}{(1-\eta)} - \phi_1 \frac{(1-l)^{(1-\eta)}}{(1-\eta)} \right) + (1-\alpha) \frac{\chi v^{\alpha}((1-n)e)^{(1-\alpha)}}{1-n} \mu \right] \frac{c}{l}$$

$$\begin{aligned} w\widetilde{w}_t = &(1-\alpha)(1-\theta)\zeta k^{\theta}(nl)^{-\theta} \left[\widetilde{z}_t + \theta \left(\widetilde{k}_t - \widetilde{n}_t - \widetilde{l}_t \right) \right] \\ &+ \alpha \left[\phi_1 (1-l)^{-\eta} \widetilde{l}_t l + (1-\alpha) \frac{\chi v^{\alpha}((1-n)e)^{(1-\alpha)}}{1-n} \mu \left[\mu \widetilde{\mu}_t + \alpha \widetilde{v}_t \right] \right] \frac{c}{l} \\ &+ \alpha \left[\left(\phi_2 \frac{(1-e)^{(1-\eta)}}{(1-\eta)} - \phi_1 \frac{(1-l)^{(1-\eta)}}{(1-\eta)} \right) + (1-\alpha) \frac{\chi v^{\alpha}((1-n)e)^{(1-\alpha)}}{1-n} \mu \right] \frac{c}{l} \left(\widetilde{c}_t - \widetilde{l}_t \right) \end{aligned}$$

$$\begin{aligned} w\widetilde{w}_t = &(1-\alpha)\phi_1 (1-l)^{-\eta} c \left[\widetilde{z}_t + \theta \left(\widetilde{k}_t - \widetilde{n}_t - \widetilde{l}_t \right) \right] \\ &+ \alpha \left[\phi_1 (1-l)^{-\eta} \widetilde{l}_t l + \frac{(1-\alpha)\sigma n}{(1-n)} \mu \left[\widetilde{\mu}_t + \alpha \widetilde{v}_t \right] \right] \frac{c}{l} \\ &+ \alpha \left[\mu \left((1-\sigma) - \frac{1}{\beta} \right) + \phi_1 (1-l)^{-\eta} \right] \frac{c}{l} \left(\widetilde{c}_t - \widetilde{l}_t \right) \end{aligned}$$

$$\begin{aligned} w\widetilde{w}_t = &(1-\alpha)\phi_1 (1-l)^{-\eta} c \left[\widetilde{z}_t + \theta \left(\widetilde{k}_t - \widetilde{n}_t - \widetilde{l}_t \right) \right] \\ &+ \alpha \left[\phi_1 (1-l)^{-\eta} \widetilde{l}_t l + \frac{(1-\alpha)\sigma n}{(1-n)} \mu \left[\widetilde{\mu}_t + \alpha \widetilde{v}_t \right] \right] \frac{c}{l} \\ &+ \alpha \left[\mu \left((1-\sigma) - \frac{1}{\beta} \right) + \phi_1 (1-l)^{-\eta} \right] \frac{c}{l} \left(\widetilde{c}_t - \widetilde{l}_t \right) \end{aligned}$$

Annex

Wage equation at the steady state

In this section, we prove that the wage equation at the steady state equals (10").

$$w* = (1-\alpha)(1-\theta)\zeta(k*)^{\theta}(n*l*)^{-\theta} + \alpha\left[\left(\phi_2\frac{(1-e)^{(1-\eta)}}{(1-\eta)} - \phi_1\frac{(1-l*)^{(1-\eta)}}{(1-\eta)}\right) - (1-\alpha)\frac{\chi(v*)^{\alpha}((1-n*)e)^{(1-\alpha)}}{1-n}\mu*\right]\frac{c*}{l*}$$

$$w* = (1-\alpha)(1-\theta)\zeta(k*)^{\theta}(n*l*)^{-\theta} + \alpha\left[\left(\phi_2\frac{(1-e)^{(1-\eta)}}{(1-\eta)} - \phi_1\frac{(1-l*)^{(1-\eta)}}{(1-\eta)}\right) - (1-\alpha)\frac{\chi(v*)^{\alpha}((1-n*)e)^{(1-\alpha)}}{1-n*}\mu*\right]\frac{c*}{l*}$$

From (7") we have

$$\begin{aligned}\frac{\mu*}{\beta} - \mu*(1-\sigma) = \phi_1\frac{(1-l*)^{(1-\eta)}}{(1-\eta)} - \phi_2\frac{(1-e)^{(1-\eta)}}{(1-\eta)}\\ + \frac{1}{c*}(1-\theta)\zeta(k*)^{\theta}(n*l*)^{-\theta}l* + \mu*\left[-(1-\alpha)\frac{\chi(v*)^{\alpha}((1-n*)e)^{(1-\alpha)}}{1-n*}\right]\end{aligned}$$

$$\begin{aligned}\mu*(1-\sigma) - \frac{\mu*}{\beta} + \frac{1}{c*}(1-\theta)\zeta(k*)^{\theta}(n*l*)^{-\theta}l* = \phi_2\frac{(1-e)^{(1-\eta)}}{(1-\eta)} - \phi_1\frac{(1-l*)^{(1-\eta)}}{(1-\eta)}\\ - \mu*\left[(1-\alpha)\frac{\chi(v*)^{\alpha}((1-n*)e)^{(1-\alpha)}}{1-n*}\right]\end{aligned}$$

$$\begin{aligned}\mu*\left((1-\sigma) - \frac{1}{\beta}\right) + \frac{1}{c*}(1-\theta)\zeta(k*)^{\theta}(n*l*)^{-\theta}l* = \phi_2\frac{(1-e)^{(1-\eta)}}{(1-\eta)} - \phi_1\frac{(1-l*)^{(1-\eta)}}{(1-\eta)}\\ - \mu*\left[(1-\alpha)\frac{\chi(v*)^{\alpha}((1-n*)e)^{(1-\alpha)}}{1-n*}\right]\end{aligned}$$

Replacing in the previous eq. we have

$$\begin{aligned}w* &= (1-\alpha)(1-\theta)\zeta(k*)^{\theta}(n*l*)^{-\theta} + \alpha\left[\mu*\left((1-\sigma) - \frac{1}{\beta}\right) + \frac{1}{c*}(1-\theta)\zeta(k*)^{\theta}(n*l*)^{-\theta}l*\right]\frac{c*}{l*}\\ w* &= (1-\alpha)(1-\theta)\zeta(k*)^{\theta}(n*l*)^{-\theta} + \alpha(1-\theta)\zeta(k*)^{\theta}(n*l*)^{-\theta} + \alpha\left[\mu*\left((1-\sigma) - \frac{1}{\beta}\right)\right]\frac{c*}{l*}\\ w* &= (1-\theta)\zeta(k*)^{\theta}(n*l*)^{-\theta} + \alpha\left[\mu*\left(\frac{(1-\sigma)\beta - 1}{\beta}\right)\right]\frac{c*}{l*}\\ w*n*l* &= (1-\theta)\zeta(k*)^{\theta}(n*l*)^{1-\theta} + \alpha\left[\mu*\left(\frac{(1-\sigma)\beta - 1}{\beta}\right)\right]c*n*\end{aligned}$$

From :

$$\frac{\kappa v*}{c*} = \mu*\alpha\chi(v*)^{\alpha}((1-n*)e)^{(1-\alpha)}$$

We have :

$$\begin{aligned}w*n*l* &= (1-\theta)\zeta(k*)^{\theta}(n*l*)^{1-\theta} + \frac{\kappa v*}{c*\left[\chi(v*)^{\alpha}((1-n*)e)^{(1-\alpha)}\right]}\left[\left(\frac{(1-\sigma)\beta - 1}{\beta}\right)\right]c*n*\\ w*n*l* &= (1-\theta)\zeta(k*)^{\theta}(n*l*)^{1-\theta} + \frac{\kappa v*}{[\sigma n*]}\left[\left(\frac{(1-\sigma)\beta - 1}{\beta}\right)\right]n*\\ w*n*l* &= (1-\theta)\zeta(k*)^{\theta}(n*l*)^{1-\theta} - \frac{\kappa v*}{\sigma}\left[\left(\frac{1-(1-\sigma)\beta}{\beta}\right)\right]\end{aligned}$$

With the technology parameter ζ chosen to normalize the steady-state level of output to unity, labor share can be computed as :

$$w * n * l* = (1 - \theta) - \frac{\kappa v*}{\sigma} \left[\left(\frac{1 - (1 - \sigma)\,\beta}{\beta} \right) \right]$$