Quantitative Tools - Level 1 - Fall 2015
Louis de Charsonville

Lecture 10

Interest rates

Introduction & Definitions

An interest is the payment of a loan. There are two different kind of interest :

- **simple interest** when the interest are paid at the end of the contract. The interest are calculated *pro rata temporis* - in proportion of the loan's duration.
- **compound interest** when the interest are added to the principal so that the interest earns interest from then on

In this chapter, we will consider that a year has 360 days, divided in twelve months of 30 days each. We use the following notations : Let A_0 be the amount of money invested at the annual interest i, let A_n be the amount of money after n years.

1 Simple Interest

For simple interest, the interest are not added up to the principal.

1.1 Basic formulas - Principle

- After one year we have

$$A_1 = A_0 * (1 + i)$$

- After n years we have

$$A_n = A_0 * (1 + n * i)$$

- Let A_x be the amount of money invested for x months at the annual interest rate i. Then

$$A_x = (1 + \frac{x * i}{12})A_0$$

- Let A_d be the amount of money invested for d days at the annual interest rate i. Then

$$A_d = A_0 * (1 + \frac{d * i}{360})$$

1.2 Use of Basic formulas

Let A_0 be the amount of money invested for d days at the annual interest i. In this subsection, we will derive from the previous formula, formula for the interest rate, the length of the investment, for the present value of the investment, while knowing the others parameters of the problem. Indeed, the problem has four parameters : the amount of money invested, the annual interest rate, the length of the investment and amount of the interests at the end of the investment. Knowing three parameters, we are able to find the fourth.

- If we are looking for the present value of A_d, the money we'll get at the end of the d days, that is expressing A_0 as a function of A_d. We have

$$A_0 = \frac{A_d}{1 + \frac{d * i}{360}}$$

- If we look for the interest rate :

$$i = \frac{A_d - A_o}{A_0} * \frac{360}{d}$$

- If we look for the length d of the investment :

$$d = \frac{A_d - A_0}{A_0} * \frac{360}{i}$$

1.3 Interest checked off - effective rates of investment

When the interest are paid at the beginning of the investment, we used the term *Interest checked off*. Because of this payment, the money investment is not the same. Indeed, we *really* invest only the principal minus the interest checked off. As a consequence, the effective rate of investment has changed.
Let A_0 the capital invest at the beginning, at the annual interest i, during n days, with checked off interest rates.
The amount of money which is indeed invested is :

$$A_0 - \text{the interest paid} = A_0 * (1 - \frac{i * d}{360})$$

The interest rate have also changed, and applying the formula found above, we get a new effective rate of investment i':

$$i' = \frac{S_0 - S_0 * (1 - \frac{i * d}{360})}{S_0 * (1 - \frac{i * d}{360})}$$

$$i' = \frac{i}{1 - \frac{i * d}{360}}$$

1.4 Discount

To Be Continued

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