Quantitative Tools - Level 1 - Fall 2015
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Lecture 2

Percentages (continued) and applications

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1 Percentages and applications

1.1 Percentages (continued)

1.1.1 Relative difference

When doing comparisons between values, the relative difference is often used. For example, a economic journalist comparing US GDP with China's GDP would rather say that :

"US GDP is 81% higher than China GDP"

instead of

"The difference between US GDP and China's is 7'530 billions USD in 2013."

The 81% is the relative difference between China and US GDP with China GDP as the reference.

The **relative difference** between two values X and Y with X as a reference is

$$\frac{Y - X}{X_{reference}}$$

The reference chosen for calculating the relative difference is very important. For example, if Bob's income is $10,000$ and Alice's income is $20,000$. One can say that :

- Alice is earning two times more than Bob
- Bob is earning only 50% less than Alice

In the first sentence, the point of comparison is Bob's income, so :

$$\begin{aligned}\frac{W_{Alice} - W_{Bob}}{W_{Bob}} = \frac{20,000 - 10,000}{10,000} = 1 &\Leftrightarrow \text{Alice is earning 100\% more than Bob}\\ &\Leftrightarrow \text{Alice is earning two times more than Bob}\end{aligned}$$

In the second sentence, the point of comparison is Alice's income, so :

$$\begin{aligned}\frac{W_{Bob} - W_{Alice}}{W_{Alice}} &= \frac{10,000 - 20,000}{20,000}\\ &= -0,5\\ &\Leftrightarrow \text{Bob is earning 50\% less than Alice}\end{aligned}$$

1.1.2 The importance of the reference point in Percentages

A percentage is fraction with a numerator and denominator, so it's important to well ascertain both. In particular, the reference point is of first importance. We illustrate this point with two examples : the double-entry table and the "detergent ad puzzle".

Example 1.1. Double-entry table
Reading the table of Titanic sinking survivors and considering the 1^{st} Class passengers, one could main the following statement :

- 62% of 1^{st} Class passengers has survived the sinking.

- 29% of the survivors were 1^{st} Class passengers.

Table 1: Who survived from Titanic sinking ? (or why you'd always travel in 1st class)

		Survived	
		No	Yes
Class	1^{st} Class	122	203
	2^{nd} Class	167	118
	3^{rd} Class	528	178
	Crew	673	212

Example 1.2. The ad detergent puzzle
Advertising is often making the following statement :
"Enjoy 20% of extra detergent for the same price and benefit from a 20% discount".
Unfortunately, this statement is wrong. Imagine that the price of the detergent was \$5 for a $1kg$ pack of detergent, then the price per kg is \$5/kg. The special offer is a pack with 20% extra detergent from the same price, being \$5 for a $1.2kg$. So the price per kilo is $\frac{5}{1.2} = \$4.17/kg$.
The discount is

$$\frac{4.17-5}{5} = -17\% \text{ instead of a 20\% discount.}$$

1.1.3 Duration for a variable to double

Let X be a variable that grows at a constant rate each year i. We want to know in how many years X will have doubled. Let $X_{initial}$ be the first value of X. We want $X_{final} = 2 * X_{initial}$. Besides, after n years, we know that :

$$X_{final} = X_{initial} * (1+i)^n$$

So n is such that :

$$2 = (1+i)^n \rightarrow n = \frac{ln(2)}{ln(1+i)}$$

And the number of years n for a variable to be multiplied by c :

$$c = (1+i)^n \rightarrow n = \frac{ln(c)}{ln(1+i)}$$

Tip
For i close to zero (roughly speaking, if i is less than 10%), $ln(1+i) \approx i$. And $ln(2) \approx 0.7$. So a rule of thumb is

$$n = \frac{70}{i\%}$$

1.2 Applications : Nominal vs Real values

Percentages are widely used in economics, especially to transform nominal values into real values with the multiplying factor of prices or *deflator*.

- **Nominal value** : a nominal value (for example the GDP) is a value expressed in current currency values.
- **Real value** : a real value is a value expressed in constant currency values. It equals the nominal value adjusted from the general price level changes (or inflation). So, a real value at a date t is the nominal value at t divided by the general price level changes between a chosen reference date and date t.

The prices of things are, in average, increasing each year : it's called inflation. Due to inflation, comparing nominal values across long period of times does not make much sens : if Bob's income increased by 10%, but if in the sames times, prices increased by 10%, Bob purchase power has not changed. That is because, Bob nominal income has increased by 10% but Bob real income has not changed.
Formally, for a variable X :

$$\boxed{X^t_{real} = \frac{X^t_{nominal}}{multiplying\ factor\ of\ prices}}$$

And :

$$Real\ Growth\ Rate\ of\ X = \frac{Nominal\ Growth\ Rate\ of\ X}{multiplying\ factor\ of\ prices}$$

Example 1.3. Real GDP and real wages If Alaska GDP goes from \$40bn in year 1 to \$45 in year 2 while prices increase by 10%. What's Alaska real economic growth rate ?
The nominal economic growth rate of Alaska is

$$\frac{45-40}{40} = 12.5\%$$

The real economic growth rate is :

$$\frac{1.125}{1.1} \approx 1.02 \rightarrow 2\%$$

One could also calculate Alaska real GDP in year 1 dollars.

$$Real\ GDP = \frac{45}{1.1} \approx \$40.9bn$$

Between year 1 and year 2, an Alaska fisherman, Santiago, had a raise by 5%. Does Santiago real income has increased ?

$$Real\ Wage = \frac{1.05}{1.1} = 0.955 \rightarrow \text{ Santiago real income has decreased by 4.5\%}$$

Example 1.4. Inflation and exchange rate
The nominal exchange rate is the price of a currency in an other. The real exchange rate is the purchasing power of a currency relative to another. The difference between the two are due to different evolution and determinants of prices between countries.
The nominal exchange rate is usually expressed as the domestic price of a foreign currency. So if it costs a U.S. dollar holder \$2 to buy one euro, from a euroholder's perspective the nominal rate is €0.5 per dollar. The real exchange rate RER (from a euroholder's perspective) is :

$$RER = e * \frac{P^*}{P} \quad \text{where} \begin{cases} e \text{ is the nominal exchange rate €/\$} \\ P^* \text{ is the level of prices in the United States} \\ P \text{ is the level of prices in the Euro Zone} \end{cases}$$

In year 1, the nominal exchange rate between USA and ZE is €1=2\$. Inflation rate is on average 1% in the Euro Zone and 3% in the US. In year 2, what should be the nominal exchange rate so that the real exchange rate remains unchanged ?
If the real exchange rate has not changed, the nominal exchange rate will be (from a dollarholder's perspective) :

$$\text{nominal exchange rate} = \text{real exchange rate} * \frac{\text{multiplying factor of prices in the US}}{\text{multiplying factor of prices in the EZ}}$$

Indeed, imagine a good that costs €1 in year 1 in the Euro Zone. The price of exactly the same good in the United States is \$2 [1]. In year 2, due to inflation the price of this good is €1.01 in the EuroZone and \$2.06 in the US. The new nominal exchange rate is now :

$$\frac{2.06}{1.01} \approx 2.04$$

The intuition is that if prices are increasing at a faster pace in the US than in the Euro Area between year 1 and year 2, one can buy more goods in year 2 than in year 1 with the same amount of euro.

Warning : Percentages and relative changes make sense when :

- they are about great numbers : considering the first 10% of a class with 8 students does not make sense.
- the relative difference is bigger than the level of uncertainty. A lot of economic values, or statistic values (like results of polls for instance) are known with a degree of uncertainty. Thus, saying that GDP increased by 0.1% with a accuracy of 0.2% does not make sense.

[1] This is a strong assumption but there is no loss of generality for our example