Statistical Reasoning

Week 4

Sciences Po - Louis de Charsonville

Spring 2018

Week 4 : Distributions

Research Paper

Distributions and graphs

Measures of central tendency

- Mean
- Median
- Mode

Measures of Variability

- Range(s)
- Standard deviation

Normal distribution

Research Paper

Research Paper

Timeline

Research Proposal	**Today**
1^{st} draft	**6 March**
2^{nd} draft	**10 April**
Final draft	**24 April**

Submission's Rules

- A **word** document (following template on the Google Drive).
- A **do-file** showing *all* commands in Stata with comments in green.

The Word document

- Provisional paper title
- Introduction stating and accounting the research question
- Brief theory section describing your hypotheses
  - Describe how you think the independent variable you chose are supposed to influence the dependent variable (better if you have a few references).
- Brief description of the dataset
  - Objectives of the survey, date, data collecting methods, sampling, etc.
- Description of the dependent and independent variables as they exist unmodified in the original data
  - Names, codes, values, what they measure, missing values
- Description of all variable renamings, recodings, how missing values have been managed ;
- Univariate statistics on all variables with 1/2 sentence(s) describing their distribution.

Exporting results from Stata into Word

- Tables : select `copy table`, paste in Excel, edit, paste in Word.
  - Add footnotes.
- Graphs : save in `.tif` format, insert as a picture in Word
- *More details :* section 13.4 in the **Stata Guide**.

Distributions and graphs

Distributions

- A distribution is a collection of data, or scores, on a variable.
- Scores are usually arranged in order from smallest to largest.

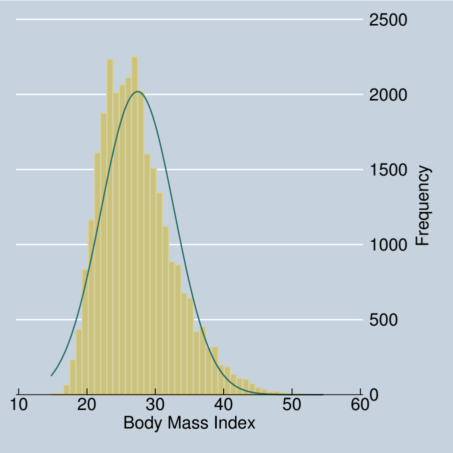

Figure – Distribution of BMI

Example - Gun control in the US

Would you like to see gun laws in the US made more strict, less strict, or remain as they are ?

- the same or less strict : 52%
- more strict : 46%
- no opinion : 2%

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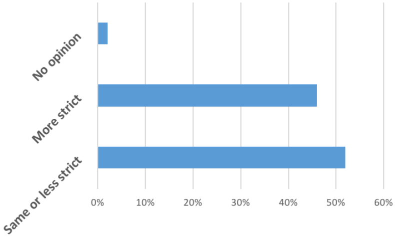

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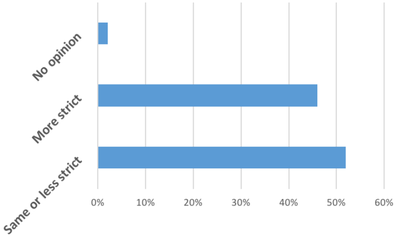

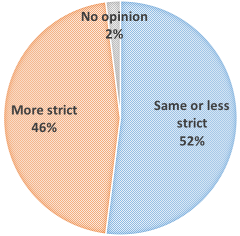

Distribution of Qualitative Variables

Frequency distributions

- Provide the number of observations in each category and/or the corresponding percentage
  - Be careful : percentages should sum to 100%
  - How have you dealt with *missing values* ?
- **Cumulative frequencies or percentages :**provide the number/percentage of observations below or equal to a given value or category (only with *ordinal data*)
- **Stata** : these statistics are obtained with `tab` or `fre`, and can be visualized using **bar graphs** and **histograms**

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  - Example : earnings in `nhis9711`
- Compute measures of central tendency and variability.
- Plot the probability density function (*kernel density*)

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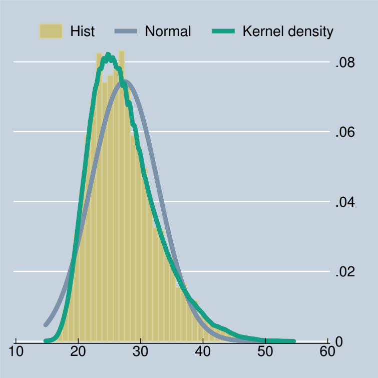

Measures of central tendency

Measures of central tendency

- Collection of scores of a variable : **distribution**
- How spread out the scores are ?
- What is the most common score ?
- etc.

One set of distribution characteristics that research are interested in is central tendency :

- mean
- median
- mode

Stata

- use `sum` or `tabstat`
- *primarily appropriate for quantitative variables*

Mean

- Arithmetic average of a distribution of scores :

$$\bar{x} = \sum_{i=1}^{N} \omega_i x_i \,, \text{ with } \omega_i \text{ weight of obs } i$$

- Most commonly used
- Denoted μ for the *population mean* and $\bar{x}$ for the *sample mean*

Weaknesses

- Sensitive to extreme values (*outliers*)
- A distribution may have very few scores near the mean

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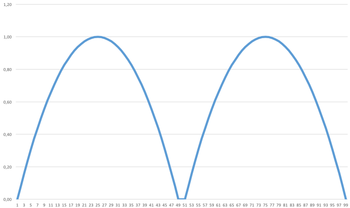

$$X_1 = \{2,3,5,6\}$$

$$X_2 = \{0,3,5,8\}$$

X_1, X_2 have the same mean.

Median

- The score in the distribution that marks the 50th percentile
- 50% of the scores in the distribution fall above the median and 50% fall below it.
- Not sensitive to outliers.
- Comparing the mean and the median gives an idea whether the distribution is skewed or not.

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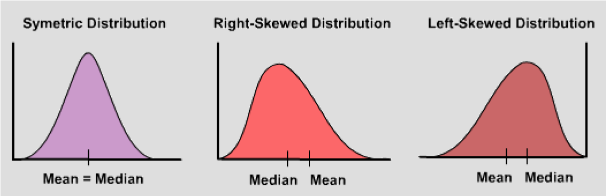
Skewed or not ?

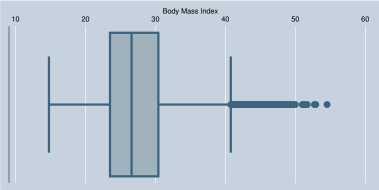


Beyond the median

- **Quartiles** : divide the sample into 4 equal parts
- **Deciles** : divide the sample into 10 equal parts
- **Percentiles** : divide the sample into 100 equals parts.

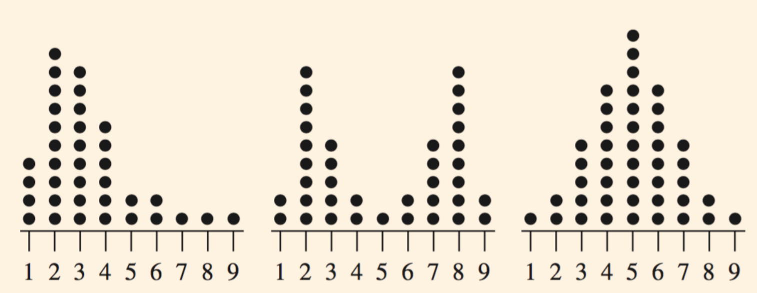
Stata

- use `summarize` (with options `details`)
- or draw a boxplot with `graph hbox`



Mode

- The **most frequent value** in the sample
- A series of values can be unimodal (one mode), bimodal(two modes) or multimodal (several modes).
- Not used a lot.



Example

$$X = \{86, 90, 95, 100, 100, 110, 110, 115, 120\}$$

- Mean ?
- Median ?
- Unimodal ? Bimodal ?

Measures of Variability

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Old saying

"If your head is in the freezer and your feet are in the oven, on average you're comfortable."

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- Measures of central tendency do not inform us on the dispersion of scores in the distribution

Measures of dispersions

- Range
- Variance
- Standard deviation (most informative and widely used)

Range

- Range = difference between the largest score and the smallest score.

$$Range = Max - Min$$

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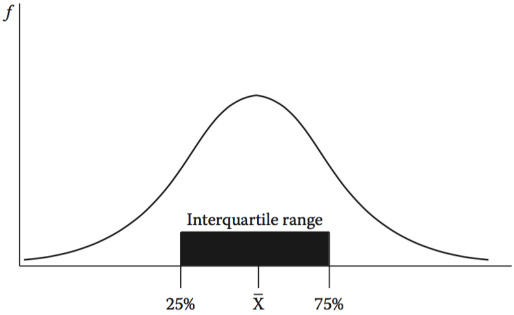
- Another common measure : **Interquartile range (IQR)** :

$$IQR = Q_3 - Q_1$$



Standard deviation

- **Deviation** : refers to the distance between an individual score and in the average score
- **Standard** : means *average*
- Standard deviation is the average distance between individual observation and the mean of the distribution.

Formula

Population :

$$\sigma = \frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N}$$

Estimate based on a *sample :*

$$\sigma = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

Shape

The shape of the distribution refers to how the observations are distributed around the mean

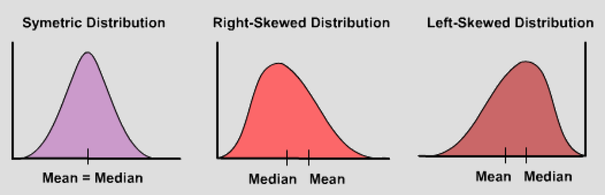
- symmetrically distributed ?
- Are the widely spread around the mean ? (Outliers ?)

Describing the shape :

- Skewness (asymmetry)
- Kurtosis (flatness)

Skewness

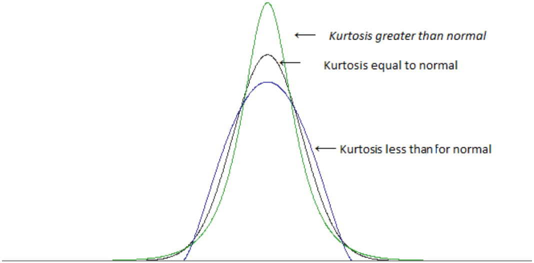
- **Right-skewed** (*positive skew*) : outliers pull the mean upwards (a few very high values), graphically the mean is pulled to the right, the right-hand tail is longer. Most observations are clustered at the lower end.
- **Left-skewed** (*negative skew*) : outliers pull the mean downwards (a few very low values), graphically the mean is pulled to the left, the left-hand tail is longer. Most observations are clustered at the higher end.



Kurtosis

The shape of a distribution of scores in terms of its flatness or peakedness (compared to the normal distribution)

- A normal distribution has a kurtosis of 3.
- **Leptokurtic** : a higher peak and thinner tails (than the normal curve, $kurtosis > 3$)
- **Platykurtic** : a lower peak and thicker tails (than the normal curve, $kurtosis < 3$)



Normal distribution

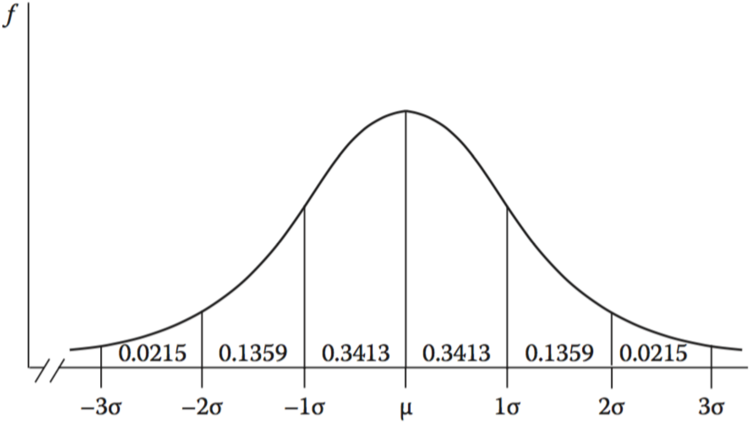
Normal distribution $\mathcal{N}(\mu, \sigma^2)$

- **Normal distribution** : extremely important to statistics
- often referred as the **bell curve**

Properties

- **symmetric** and **unimodal**
- **mean = median = mode**
- $\mathcal{N}(0,1)$ **standard normal distribution**

Normal curve



Normality assessment

Visual assessment

- Distributions : `hist, normal, kdensity, gr (h)box`
- Diagnostics : `symplot, qnorm, g(ladder)`

Formal assessment

- Use `su x, d` to assess the symmetry ($skewness \sim 0$) and flatness ($kurtosis \sim 3$) of a variable.
- Use `tabstat x y, s(skew kurt) c(s)` to compare a variable with its transformation (often to log-units)

PRACTICE