

Lesson 2 - Review on Relation and Graph

Sets

Set is collection of things.

For example, the items you wear: hat, shirt, pants, watches and many more.

Another example, is fingers. This includes index, middle, ring, pinky and thumb finger.

Notation

To define set, we use the name of the set. In the previous example, we use the word `items`. Then, we write = equal sign, followed by open curly braces, inside that we put the elements and close it with curly braces.

$$items = \{hat, shirt, pants, watches, \dots\}$$

We call the hat, shirt, pants, watches as an element or member of a set.

Notice the usage of ellipsis (the three dots) representing and many more. In this example, we have more items we can wear, so we just say there are many more items on set items. In this case, we want to say it is an infinite set.

Similarly, the set fingers is represented as:

$$fingers = \{index, middle, ring, pinky, thumb\}$$

But this time, we know fingers is only composed of 5. Thus, we can call this a finite set.

Not all usage of ellipsis means an infinite set. Let's take an alphabet set as an example. We know alphabet has 26 letters. Instead of writing all letters, we can write the letters in the middle as ellipsis:

$$alphabet = \{a, b, c, \dots, x, y, z\}$$

Thus, we used ellipsis even in a finite set.

Other notation

Let set A:

$$A = \{1, 2, 3, 4, 5\}$$

Based on the set above, we can say 1 is an element of A. Similarly, 2 is also an element of A. Using notation we can write:

$$1 \in A$$

$$2 \in A$$

Subset

A subset is set that exist on another set.

Let set A:

$$A = \{1, 2, 3, 4, 5\}$$

Let set X:

$$X = \{1, 2\}$$

We can say that X is a subset of A, since the elements in X are found (exist) in A.

What if our X is:

$$X = \{1, 2, 3, 4, 5, 6\}$$

This time, X is not a subset of A, since the element 6 does not exist in A.

However, reversing it, we found that A is a subset of X, since the elements 1 to 5 are found in X.

So, the notation for subset is (if we want to say X is a subset of A):

$$X \subseteq A$$

Rectangular coordinate system

We can draw rectangular coordinate system, as a graph with horizontal line (x-axis) and vertical line (y-axis).

Let's pick a point: 1 in x-axis and -2 in y-axis. We can represent this as:

$$(1, -2)$$

x-axis value is on first position and y-axis value in second position.

$$(x, y)$$

Rectangular coordinate system is also called Cartesian coordinate system.

Cartesian product of a sets

Cartesian product of a two set is the set of all ordered pairs (a, b) such that a belongs to A and b belongs to B.

For example: Let set Students and Subjects as:

$$Students = \{alice, bob, carl\}$$

$$Subjects = \{automata, intelsys\}$$

Then, their cartesian product of Students and Subjects is:

$$Students \times Subjects = \{(alice, automata), (alice, intelsys), (bob, automata), (bob, intelsys), (carl, automata), (carl, intelsys)\}$$

Imagine we are distributing the Students to Subjects or saying Students is taking Subjects. For example, Student alice is taking the Subject automata. In this case, the order matters or we call this as ordered pairs, since we can not say the Student automata is taking the subject alice.

Using numerical values as an example:

$$A = \{1, 2, 3\}$$

$$B = \{10, 20, 30\}$$

Then, their cartesian product is:

$$A \times B = \{(1, 10), (1, 20), (1, 30), (2, 10), (2, 20), (2, 30), (3, 10), (3, 20), (3, 30)\}$$

Relations

A relation is a set of ordered pairs that satisfy a relationship. Thus, we expect that a relation is written as:

$$R = \{(..., ...), (... , ...)\}$$

Let set A and B:

$$A = \{5, 2, 3\}$$

$$B = \{1, 3, 2\}$$

If we want to get the relation between A and B, then the relation is defined as:

$$R \subseteq A \times B$$

Where

$$A \times B$$

is the cartesian product between those two sets.

Relationship

But, what do we mean that satisfy a relationship? For example, we define a relationship A is greater than B:

$$A > B$$

First we get their cartesian product:

$$A \times B = \{(5, 1), (5, 3), (5, 2), (2, 1), (2, 3), (2, 2), (3, 1), (3, 3), (3, 2)\}$$

Again, we say that a relation is a subset of ordered pairs (the cartesian product) that satisfy a relationship ($A > B$).

So, what are the ordered pairs in the cartesian product that satisfy the relationship A is greater than B ?

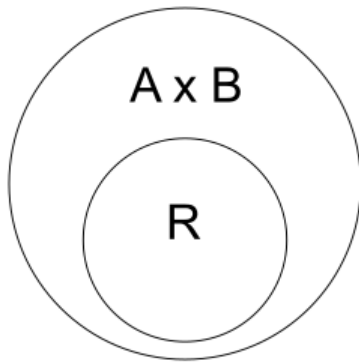
The following are the ordered pairs that satisfy the relationship, since the first value (5) in first pair (5, 1) is greater than second value (2).

$$(5, 1), (5, 3), (5, 2), (2, 1), (3, 1), (3, 2)$$

Thus, the relation of the set A and B above is simply written as:

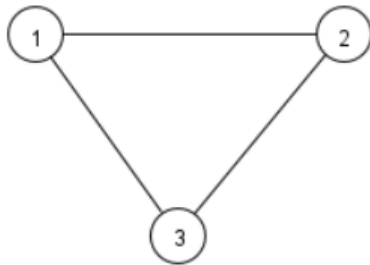
$$R = \{(5, 1), (5, 3), (5, 2), (2, 1), (3, 1), (3, 2)\}$$

But, what is the connection of the relation to automata? Simply, the relation is the set of edges. In this module, we will discuss what do we mean by edges and vertices in a graph. Also, all concepts discussed so far will be used all throughout this course.



Graph

Undirected graph



In a graph, we call the circles as vertex or node, and the lines as edges.

Thus, in the graph above, we can represent the vertices as a set:

$$V = \{1, 2, 3\}$$

And edge as a pair. Note that we didn't say it is an ordered pair since, in an undirected graph the order does not matter. So we can represent all edges as a set of pairs.

$$E = \{(1, 3), (1, 2), (2, 3)\}$$

Now, formally, we can represent a graph as:

$$G = (V, E)$$

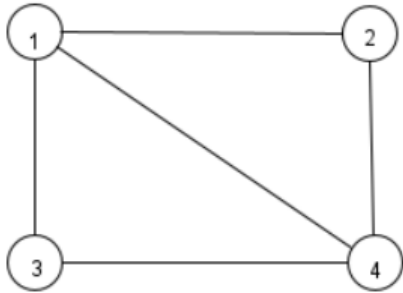
Where, V is set of vertices and E is set of edges. So, to represent the graph above:

$$G = (\{1, 2, 3\}, \{(1, 3), (1, 2), (2, 3)\})$$

We also have what call a degree. A degree of a node is the number of edges connected to that node. So, all vertices has a degree of 2 since each of them has 2 edges connected.

Node	Degree
1	2
2	2
3	2

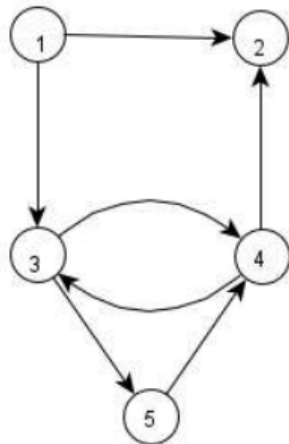
Another example of an undirected graph:



$$G = (\{1, 2, 3, 4\}, \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), \})$$

Node	Degree
1	3
2	2
3	2
4	3

In automata, we will mostly use what we call a directed graph or digraph for short. Digraph has now an arrow or direction on the edges. So when we represent an edge as pair, it should be now an ordered pair, the order matters now since the edge is specific to the direction. Usually, what is being pointed to is in the second value of an ordered pair.



$$G = (\{1, 2, 3, 4, 5\}, \{(1, 2), (1, 3), (3, 4), (4, 3), (3, 5), (5, 4), \})$$

In digraph we have indegree and outdegree. The indegree of a node is the number of edges pointing in to that node. While outdegree is the number of edges pointing away to that node.

Node	Indegree	Outdegree
1	0	2
2	2	0
3	2	2
4	2	2

Node	Indegree	Outdegree
5	1	1

Summary

Symbol/Notation	Description
$\{...\}$	Sets
\in	Element of
\subseteq	Subset of
$A \times B$	Cartesian product
R	Relations
G	Graph
$(..., ..)$	Pair