

CSPC 105

**Automata
Theory and
Formal
Languages**

L.P.Facun

Introduction

Review on Relations and Graphs

Sets: collection of things

name = { , , }

items : hat , pants , shirts , socks , and many more

fingers : pinky , ring , middle , index , thumb

items = { hat , pants , shirts , socks , ... } → infinite set
 ↑ ↑
 element ellipsis

fingers = { pinky , ring , middle , index , thumb } → finite set

alphabet = { a , b , c , ... , x , y , z } → finite set
 ↑ ↑
 26

Sets: collection of things

integers = $\{1, 2, 3, 4, \dots\}$ \rightarrow infinite set

1 is an element of integer
 \hookrightarrow

$1 \in \text{integer}$

$2 \in \text{integer}$

$0 \notin \text{integer}$

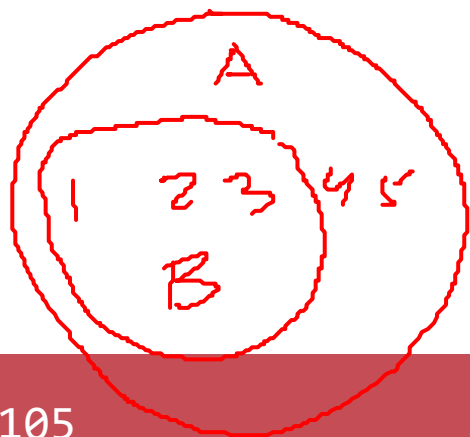
Sets: collection of things

Subsets = is a set that exist on another set

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{1, 2, 3\}$$

is B a subset of A? \checkmark



$$C = \{6, 7\}$$

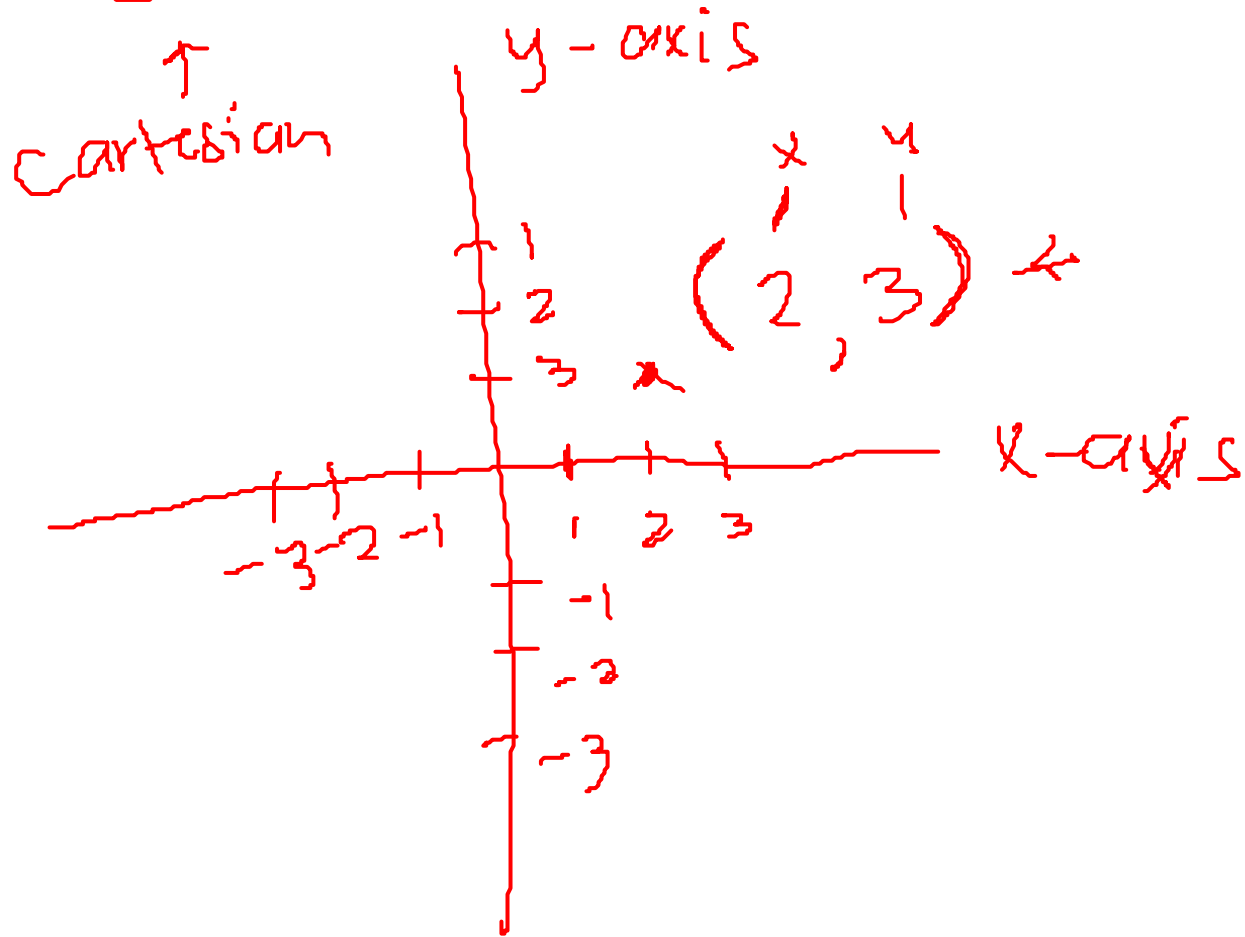
$$D = \{1, 2, 3, 4, 5, 6\}$$

is $C \subseteq A$? \times

is $D \subseteq A$? \times

is $A \subseteq D$ \checkmark

Rectangular coordinate system

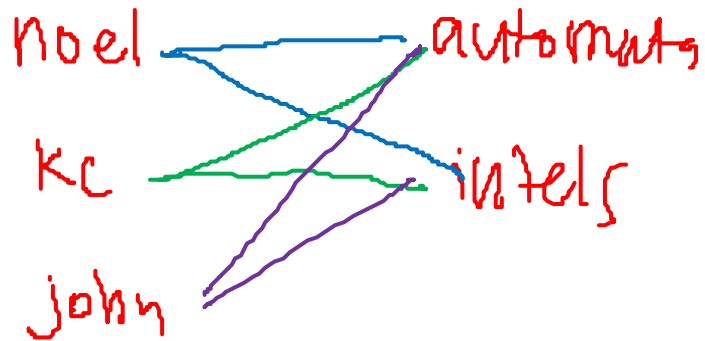


Cartesian product of sets

$\left\{ \begin{array}{l} \text{students} = \{ \text{noel}, \text{kc}, \text{john} \} \\ \text{subjects} = \{ \text{automata}, \text{intels} \} \end{array} \right.$

students

subjects



(automata, noel)

cartesian product



$\text{students} \times \text{subjects} = \{$
 $\underline{(\text{noel}, \text{automata})}, \underline{(\text{noel}, \text{intels})}$
 $[\text{kc}, \text{automata}], [\text{kc}, \text{intels}]$
 $(\text{john}, \text{automata}), (\text{john}, \text{intels})$
 $\}$

Cartesian product of sets

$$A = \{2, 4, 6\}$$

$$B = \{10, 20, 30\}$$

$$\underline{A \times B} = \{(2, 10), (2, 20), (2, 30), \\ (4, 10), (4, 20), (4, 30), \\ (6, 10), (6, 20), (6, 30)\}$$

$$\cancel{A} \times B = \{ \dots \}$$

Relation: set of ordered pairs that satisfy a relationship

$$\underline{R} = \{(\dots), (\dots), (\dots)\}$$

$$A = \{5, 2, 3\}$$

$$B = \{1, 3, 2\}$$

$$A \times B = \left\{ \begin{array}{l} \overset{a}{\underset{b}{(5,1)}}, \overset{a}{\underset{b}{(5,3)}}, \overset{a}{\underset{b}{(5,2)}}, \\ \overset{a}{\underset{b}{(2,1)}}, \overset{a}{\underset{b}{(2,3)}}, \overset{a}{\underset{b}{(2,2)}}, \\ \overset{a}{\underset{b}{(3,1)}}, \overset{a}{\underset{b}{(3,3)}}, \overset{a}{\underset{b}{(3,2)}} \end{array} \right\}$$

$R \subseteq A \times B$

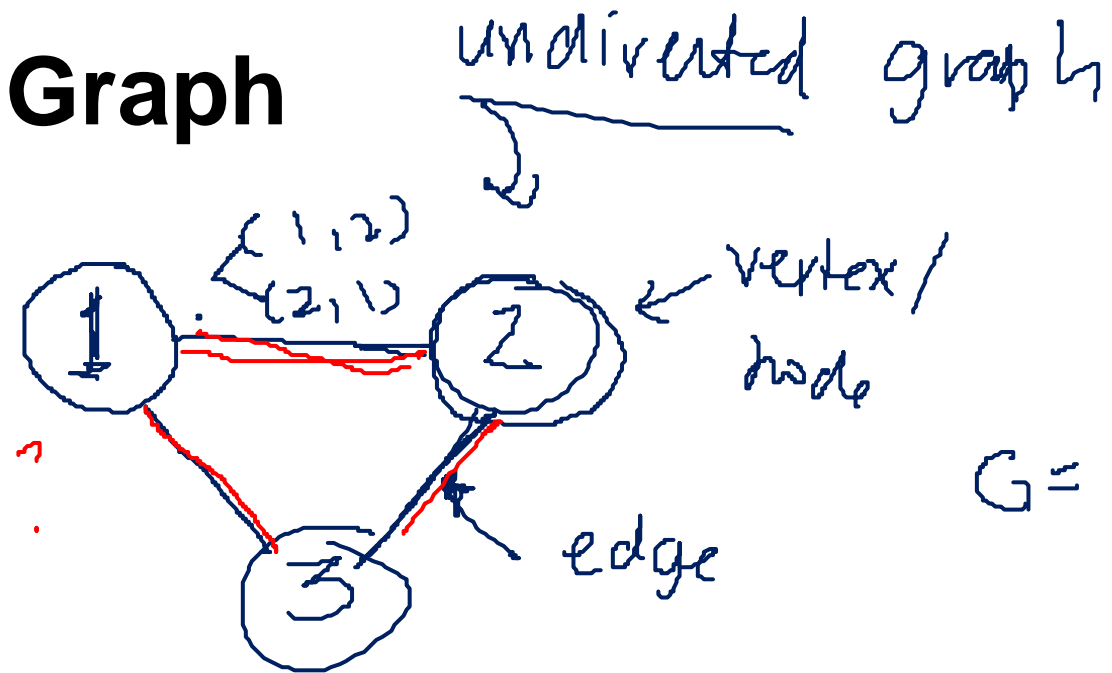
$$\cancel{A} > B$$

$$a < b \quad \checkmark$$

$$R = \left\{ \begin{array}{l} (5,1), (5,3), (5,2) \\ (2,1), (3,1), (3,2) \end{array} \right\}$$

$$R = \{ (2,3) \}$$

Graph



$$V = \{1, 2, 3\}$$

$$E = \{(1, 2), (1, 3), (2, 3)\}$$

$$G = (\{1, 2, 3\}, \{(1, 2), (1, 3), (2, 3)\})$$

$$G = (V, E)$$

↓ ↓

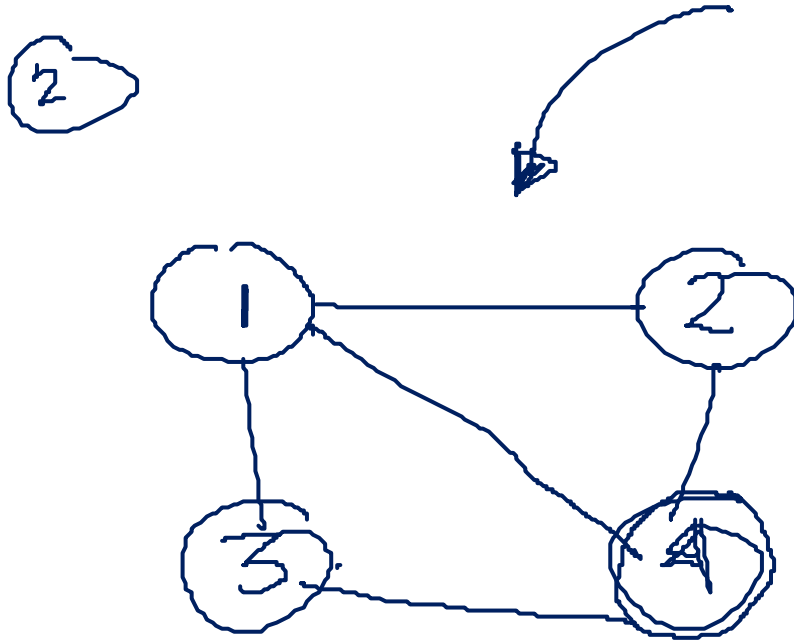
Set Set

of of

vertices edges

vertex	degree
1	2
2	2
3	2

Graph



$$G = (\{1, 2, 3, 4\},$$

$$\{ (1, 2), (1, 4), (1, 3), (2, 4), (3, 4) \})$$

vertex	degree
1	3
2	2
3	2
4	3

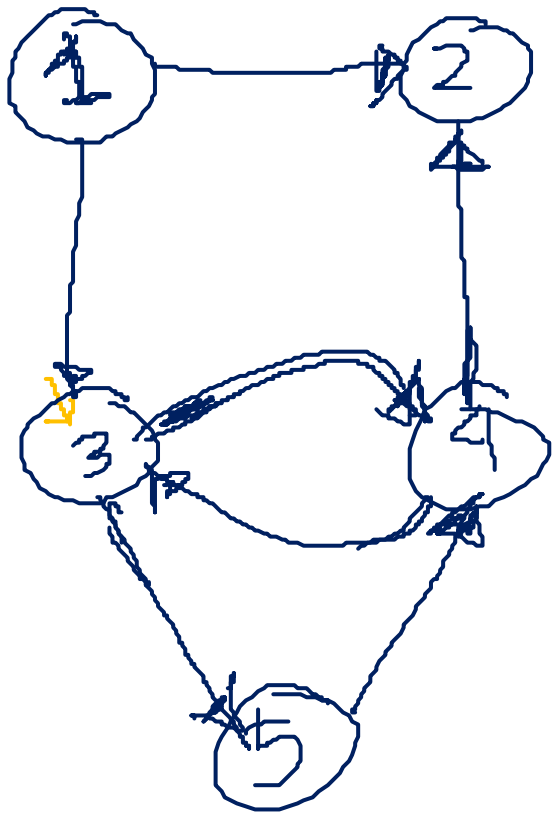
Graph

directed graph \rightarrow digraph,

$$G = (V, E)$$

$$G = (\{1, 2, 3, 4, 5\},$$

$$\{ (1, 3), (1, 2), (4, 2), (3, 5), (3, 4), (4, 3), (5, 4) \})$$



vertex	indegree	out degree
1	0	2
2	2	0
3	2	2
4	2	2
5	1	1

set
 $Q = \{on, off\}$
 $q_0 \in Q$

~~Activity!~~ Summarize

$\{ \dots \}$ - sets

(\dots) - pairs

\in - element of

\subseteq - subset

$A \times B$ - cartesian product

R - relation

G - graph