Lesson 2 - Review on Relation and Graph

Sets

Set is `collection` of things.

For example, the items you wear: `hat`, `shirt`, `pants`, `watches` and many more.

Another example, is `fingers`. This includes `index`, `middle`, `ring`, `pinky` and `thumb` finger.

Notation

To define set, we use the name of the set. In the previous example, we use the word `items`. Then, we write = equal sign, followed by open curly braces, inside that we put the elements and close it with curly braces.

$$items = \{hat, shirt, pants, watches, ...\}$$

We call the `hat`, `shirt`, `pants`, `watches` as an `element` or `member` of a set.

Notice the usage of `ellipsis` (the three dots) representing `and many more`. In this example, we have more `items` we can wear, so we just say there are many more items on set items. In this case, we want to say it is an `infinite` set.

Similarly, the set `fingers` is represented as:

$$fingers = \{index, middle, ring, pinky, thumb\}$$

But this time, we know fingers is only composed of 5. Thus, we can call this a `finite` set.

Not all usage of ellipsis means an `infinite` set. Let's take an `alphabet` set as an example. We know alphabet has 26 letters. Instead of writing all letters, we can write the letters in the middle as `ellipsis`:

$$alphabet = \{a, b, c, ..., x, y, z\}$$

Thus, we used `ellipsis` even in a `finite` set.

Other notation

Let set `A`:

$$A = \{1, 2, 3, 4, 5\}$$

Based on the set above, we can say `1` is an element of `A`. Similary, 2 is also an element of `A`. Using notation we can write:

$$1 \in A$$

$$2 \in A$$

Subset

A `subset` is set that exist on another `set`.

Let set `A`:

$$A = \{1, 2, 3, 4, 5\}$$

Let set `X`:

$$X = \{1, 2\}$$

We can say that `X` is a subset of `A`, since the `elements` in `X` are found (exist) in `A`.

What if our X is:

$$X = \{1, 2, 3, 4, 5, 6\}$$

This time, `X` is `not` a subset of `A`, since the `element` 6 does not exist in `A`.

However, reversing it, we found that `A` is a subset of `X`, since the `elements` 1 to 5 are found in `X`.

So, the notation for subset is (if we want to say `X` is a subset of `A`):

$$X \subseteq A$$

Rectangular coordinate system

We can draw rectangular coordinate system, as a graph with horizontal line (`x-axis`) and vertical line (`y-axis`).

Let's pick a point: 1 in `x-axis` and -2 in `y-axis`. We can represent this as:

$$(1, -2)$$

`x-axis` value is on first position and `y-axis` value in second position.

$$(x, y)$$

Rectangular coordinate system is also called `Cartesian coordinate system`.

Cartesian product of a sets

Cartesian product of a two set is the set of all ordered pairs `(a, b)` such that `a` belongs to `A` and `b` belongs to `B`.

For example: Let set `Students` and `Subjects` as:

$$Students = \{alice, bob, carl\}$$

$$Subjects = \{automata, intelsys\}$$

Then, their `cartesian product` of `Students` and `Subjects` is:

$$Students \times Subjects = \{(alice, automata), (alice, intelsys), (bob, automata), (bob, intelsys), (carl, automata), (carl, intelsys)\}$$

> Imagine we are distributing the `Students` to `Subjects` or saying `Students` is taking `Subjects`. For example, `Student alice` is taking the Subject `automata`. In this case, the `order` matters or we call this as `ordered pairs`, since we can not say the `Student automata` is taking the `subject alice`.

Using numerical values as an example:

$$A = \{1, 2, 3\}$$

$$B = \{10, 20, 30\}$$

Then, their cartesian product is:

$$A \times B = \{(1, 10), (1, 20), (1, 30), (2, 10), (2, 20), (2, 30), (3, 10), (3, 20), (3, 30)\}$$

Relations

A relation is a set of ordered pairs that satisfy a `relationship`. Thus, we expect that a relation is written as:

$$R = \{(..., ...), (..., ...)\}$$

Let set `A` and `B`:

$$A = \{5, 2, 3\}$$

$$B = \{1, 3, 2\}$$

If we want to get the relation between `A` and `B`, then the relation is defined as:

$$R \subseteq A \times B$$

Where

$$A \times B$$

is the cartesian product between those two sets.

Relationship

But, what do we mean `that satisfy a relationship`? For example, we define a relationship `A` is greater than `B`:

$$A > B$$

First we get their cartesian product:

$$A \times B = \{(5,1),(5,3),(5,2),(2,1),(2,3),(2,2),(3,1),(3,3),(3,2)\}$$

Again, we say that a relation is a subset of ordered pairs (the cartesian product) that satisfy a relationship (A > B).

So, what are the ordered pairs in the cartesian product that satisfy the relationship `A is greater than B`?

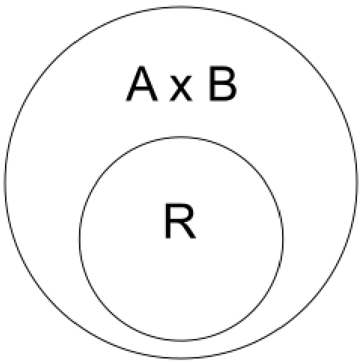
The following are the ordered pairs that satisfy the relationship, since the `first` value (5) in `first` pair (5, 1) is greater than `second` value (2).

$$(5,1),(5,3),(5,2),(2,1),(3,1),(3,2)$$

Thus, the relation of the set `A` and `B` above is simply written as:

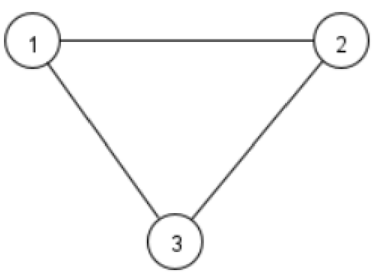
$$R = \{(5,1),(5,3),(5,2),(2,1),(3,1),(3,2)\}$$

> But, what is the connection of the relation to automata? Simply, the relation is the `set of edges`. In this module, we will discuss what do we mean by `edges` and `vertices` in a graph. Also, all concepts discussed so far will be used all throughout this course.



Graph

Undirected graph



In a graph, we call the circles as `vertex` or `node`, and the lines as `edges`.

Thus, in the graph above, we can represent the `vertices` as a set:

$$V = \{1,2,3\}$$

And `edge` as a `pair`. Note that we didn't say it is an `ordered` pair since, in a `undirected` graph the order does not matter. So we can represent all `edges` as a set of pairs.

$$E = \{(1,3),(1,2),(2,3)\}$$

Now, formally, we can represent a graph as:

$$G = (V, E)$$

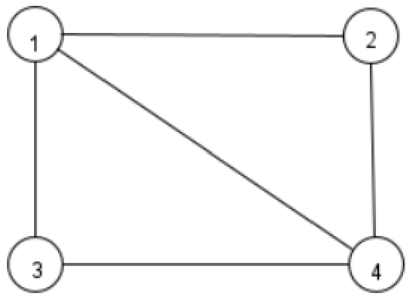
Where, `V` is set of `vertices` and `E` is set of `edges`. So, to represent the graph above:

$$G = (\{1,2,3\},\{(1,3),(1,2),(2,3)\})$$

We also have what cell a `degree`. A `degree` of a node is the number of edges connected to that node. So, all `vertices` has a a degree of `2` since each of them has `2` edges connected.

Node	Degree
1	2
2	2
3	2

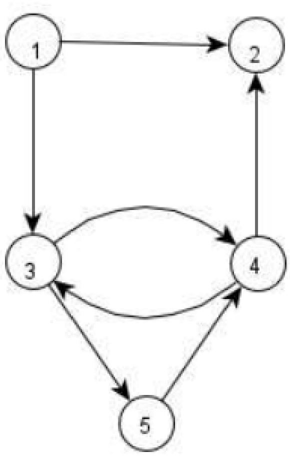
Another example of an undirected graph:



$$G = (\{1,2,3,4\}, \{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4),\})$$

Node	Degree
1	3
2	2
3	2
4	3

In automata, we will mostly used what we call a `directed` graph or `digraph` for short. `Digraph` has now an arrow or `direction` on the `edges`. So when we represent an `edge` as pair, it should be now an `ordered` pair, the order matters now since the edge is specific to the direction. Usually, what is being pointed is in the second value of an `ordered` pair.



$$G = (\{1,2,3,4,5\}, \{(1,2),(1,3),(3,4),(4,3),(3,5),(5,4),\})$$

In `digraph` we have `indegree` and `outdegree`. The `indegree` of a node is the number of edges `pointing in` to that node. While `outdgree` is the number of edges `pointing away` to that node.

Node	Indegree	Outdegree
1	0	2
2	2	0
3	2	2
4	2	2

Node	Indegree	Outdegree
5	1	1

Summary

Symbol/Notation	Description
$\{\ldots\}$	Sets
$\in$	Element of
$\subseteq$	Subset of
$A \times B$	Cartesian product
R	Relations
G	Graph
$(.., ..)$	Pair