Lesson 4 - Finite-State Automata Defined

Finite-state automata (sometimes called a finite-state machine) is a computation model that can be implemented with hardware or software and can be used to simulate sequential logic and some computer programs. Finite-state automata generate regular languages. It can be used to model problems in many fields including mathematics, artificial intelligence, games, and linguistics. Finite-state automata are the simplest computational models for computers with extremely limited amount of memory. From a mathematical perspective, finite-state automaton is a finite collection of states with transition rules that take you from one state to another that occur given an input.

A finite-state automaton consists of 5 tuple of objects:

$$(Q, \Sigma, \delta, q_0, F)$$

Specifically:

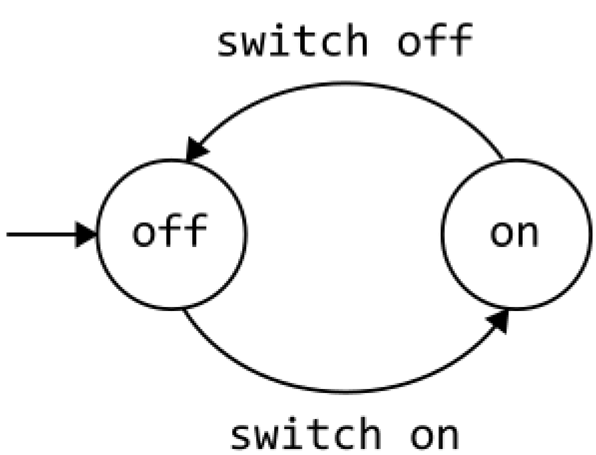
Symbol / Notation	Description
Q	Finite set called the **states**
Σ	Finite set called the **alphabet** or input
$f : Q \times \Sigma$	Transition function
$q_0 \in Q$	Initial / start state
$F \subseteq Q$	Final / accept states

Finite-state automata can be represented in two ways:

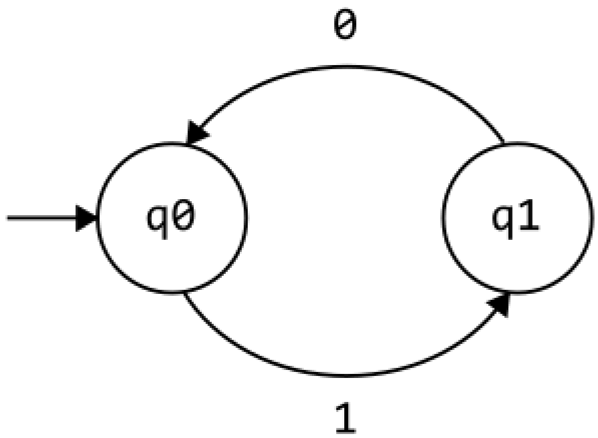
1. Transition diagram
2. Transition table

Transition diagram

Previously, we had a preview of an example of simple on-off switch model:



Let's try to simplify or represent it mathematically:



> States `off` and on became `q0` and `q1`, while `switch off` and `switch off` became `0` and `1`.

Then we can write the automaton above as:

$$Q = \{q_0, q_1\}$$

$$\Sigma = \{0, 1\}$$

$$q_0 = q_0$$

> The initial state `q0` happens to be the state `q0`.

$$F = \{\}$$

or

$$F = \emptyset$$

> We didn't defined any final state in this automaton.

For transitions, we can represent them as a set of ordered pairs:

$$\delta : Q \times \Sigma = \{(first), (second), ...\}$$

For example, the following are the two transitions represented as ordered pairs:

$$((currentstate, input), nextstate)$$

So, for the automaton of on-off switch:

$$((q_0, 1), q_1)$$

> q0 inputs 1, then went to q1

$$((q_1, 0), q_0)$$

> Similarly, q1 inputs 0, then went to q0

Finally, from this:

$$\delta : Q \times \Sigma = \{(first), (second), ...\}$$

To this:

$$\delta = \{((q_0, 1), q_1), ((q_1, 0), q_0)\}$$

We can also use this notation for transition:

$$T(q_0, 1) \rightarrow q_1$$

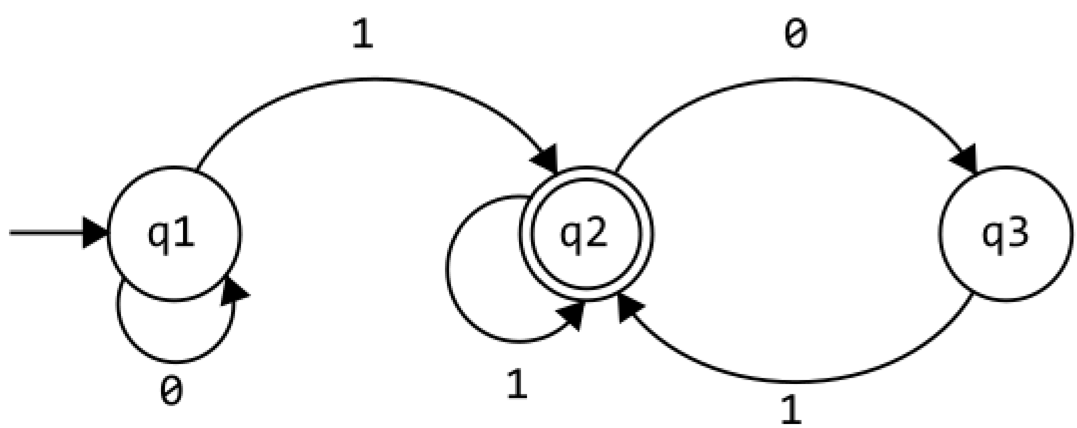
$$T(q_1, 0) \rightarrow q_0$$

So, when we say find T(q0, 1), the answer is q1.

Transition table

State	0	1
q_0	-	q_1
q_1	q_0	-

Second example:



$$Q = \{q_1, q_2, q_3\}$$

$$\Sigma = \{0, 1\}$$

$$q_0 = q_1$$

> This time, our start state is labeled as `q1`. So we say our `q0` or `initial state` is `q1`.

$$F = \{q_2\}$$

> Also noticed the state with two circle. Two circles in the diagram represents the `final` or `accepting` state. In this diagram, we only have 1 final state, but we can have 1 or more final state; that's why it's represented as set, because we can set of final states defined.

$$\delta = \{((q_1, 0), q_1), ((q_1, 1), q_2), ((q_2, 1), q_2), ((q_2, 0), q_3), ((q_3, 1), q_2)\}$$

OR:

$$T(q_1, 0) \rightarrow q_1$$

$$T(q_1, 1) \rightarrow q_2$$

$$T(q_2, 1) \rightarrow q_2$$

$$T(q_2, 0) \rightarrow q_3$$

$$T(q_3, 1) \rightarrow q_2$$

Transition table

State	0	1
q_1	q_1	q_2
q_2	q_3	q_2
q_3	-	q_2

Finite-state automata has two types, which will be discussed in detail in the next module:

1. Deterministic finite-state automata (DFA)
2. Non-deterministic finite-state automata (NDFA)

Summary

Symbol / Notation	Description
Q	Finite set called the **states**
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$F \subseteq Q$	Final / accept states