Introduction

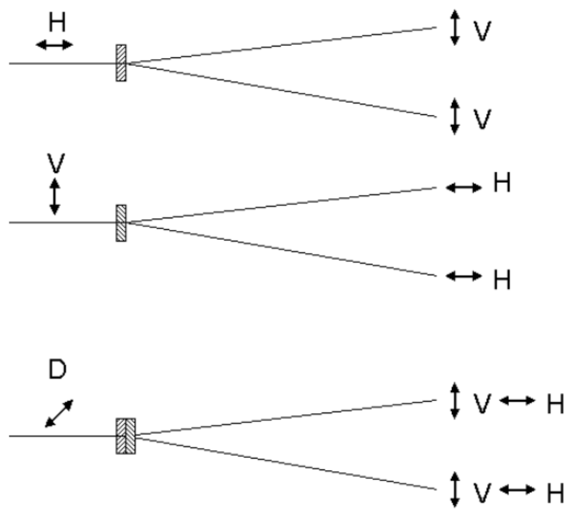
A characteristic feature of quantum mechanics is a phenomenon referred to as nonlocality. Two particles, not necessarily near each other, can instantaneously give information about the state the other is in upon knowledge of the state of the first. This non-intuitive notion has boggled the minds of physicists and laymen alike over the last century, and has required careful study to ensure no so-called hidden variables are overlooked that could otherwise explain the phenomenon. Bell's inequality is a calculation that shows when a certain sum of expectation values is greater than 2, there is sufficient evidence of entanglement. Our goal of this experiment is to reproduce results of a familiar double photon entanglement setup utilizing polarization and seek to attain more striking violations of Bell's inequality.

Theory

Where H and V denote photons polarized in the horizontal and vertical directions respectively, we can define a product state by example of two photons polarized in the vertical direction. In a product state, the wavefunction of the pair of polarized photons is the product of the wavefunctions of the individual particles. One such product state may be the following:

$$|\boldsymbol{\psi}_i> = |V>_1|V>_2.$$

If we seek to produce polarization-entangled states, the figure below depicts methods of doing so.



By shooting a photon into a crystal (details in procedure,) the first two situations are depictions of photons entangled by polarization in solely the horizontal or vertical direction, whereas the final case is a superposition of the first two. Such a case is described by the following wavefunction:

$$|\mathbf{\Phi}_{ent}\rangle = 1/\sqrt{2}\ [\ |H\rangle_1|H\rangle_2 + e^{i\delta}|V\rangle_1|V\rangle_2]$$

Where δ represents a phase between the two product states. Without much loss of generality, the following equations represent this entangled state for the cases δ=0 and δ=π:

$$|\mathbf{\Phi}^{+}\rangle = 1/\sqrt{2}\ [\ |H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2]\ (\boldsymbol{\delta=0})$$

$$= 1/\sqrt{2}\ [\ |D\rangle_1|D\rangle_2 + |A\rangle_1|A\rangle_2]$$

$$|\mathbf{\Phi}^{-}\rangle = 1/\sqrt{2}\ [\ |H\rangle_1|H\rangle_2 - |V\rangle_1|V\rangle_2]\ (\boldsymbol{\delta=\pi})$$

$$= 1/\sqrt{2}\ [\ |D\rangle_1|A\rangle_2 + |A\rangle_1|D\rangle_2]$$

Where |D> and |A> represent the exact diagonal and antidiagonal states:

$$|D\rangle = 1/\sqrt{2}\ [|H\rangle + |V\rangle]$$

$$|A\rangle = 1/\sqrt{2}\ [-|H\rangle + |V\rangle].$$

The following are corresponding equations assisting us in our measure of Bell's inequality. For our case, θ_1=0, θ_2=22.5, θ'_1=45.0, and θ'_2=67.5 (all measured in degrees.) N refers to the coincidence count of each measurement with various polarizer orientations (outlined in procedure.)

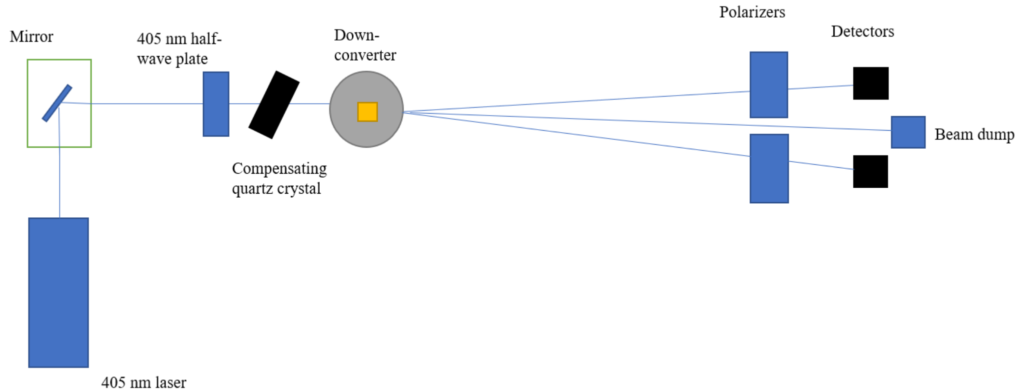
$$E(\theta_1,\theta_2) = \frac{N(\theta 1, \theta 2) + N(\theta 1 + \boldsymbol{\pi}, \theta 2 + \boldsymbol{\pi}) - N(\theta 1, \theta 2 + \boldsymbol{\pi}) - N(\theta 1 + \boldsymbol{\pi}, \theta 2)}{N(\theta 1, \theta 2) + N(\theta 1 + \boldsymbol{\pi}, \theta 2 + \boldsymbol{\pi}) + N(\theta 1, \theta 2 + \boldsymbol{\pi}) + N(\theta 1 + \boldsymbol{\pi}, \theta 2)}$$

$$S = E(\theta_1,\theta_2) - E(\theta_1,\theta'_2) + E(\theta'_1,\theta_2) + E(\theta'_1,\theta'_2)$$

$|S| \leq 2$ for classical states. If this is violated, we have evidence of entangled photons.

Equipment

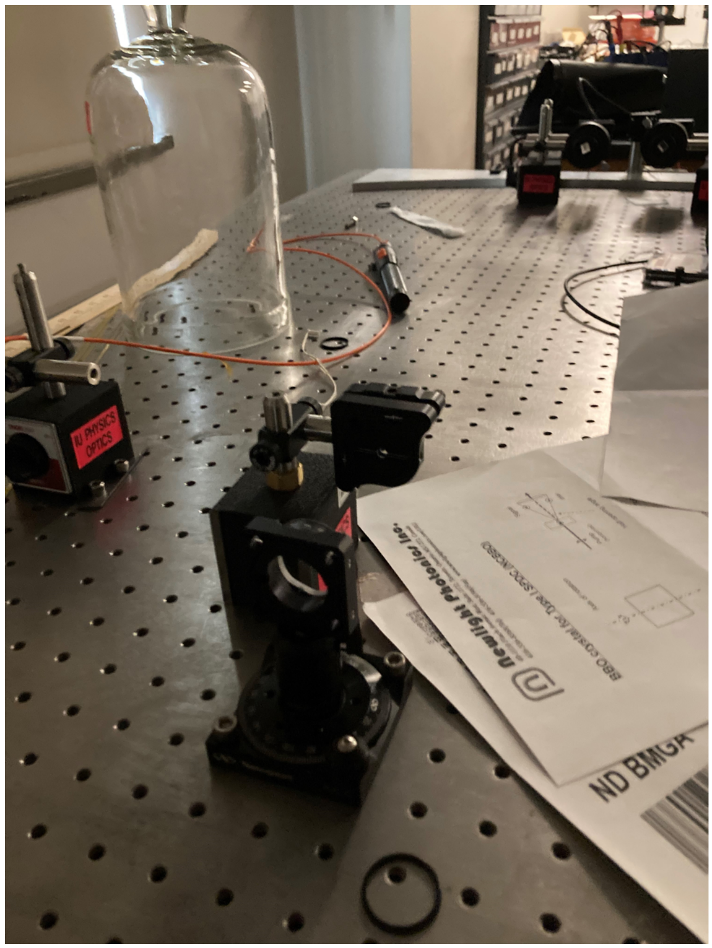
- Laser (150 mW), 405 nm
- Half Wave Plate (405 nm)
- Compensating Crystal: Quartz, 6-8mm thickness
- Down-Converter: Two type-I BBO crystals 0.5 m thick, with 29◦ phase matching rotated 90◦ relative to each other and mounted on a rotation plus tilting (mirror-like) mount. Axes of the crystals must be in horizontal/vertical planes.
- Linear Polarizer: ThorLabs (2x)
- Photon Detector (2x)
- Photometer/Coincidence Counter



Procedure

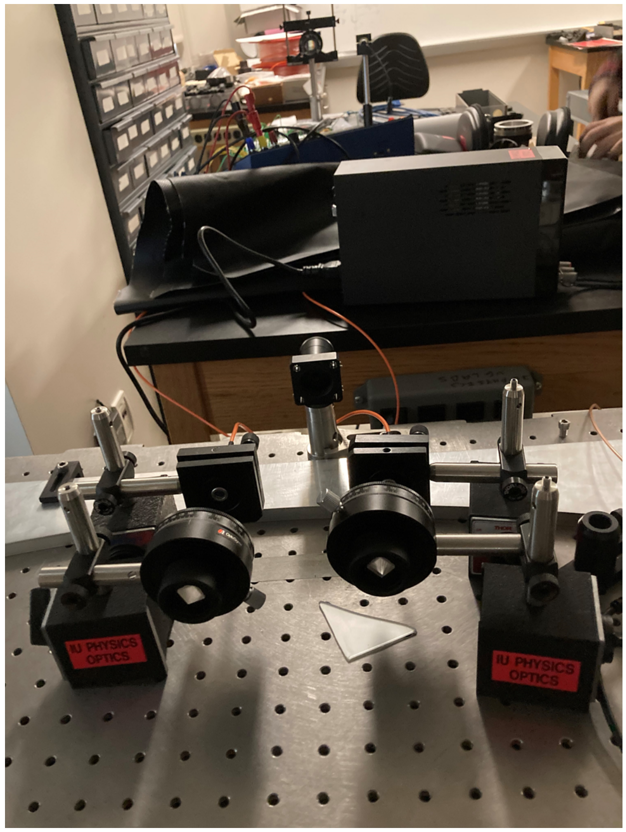
Our procedure began with learning to align optical equipment to extreme precision. To help us understand the delicate nature of alignment, we practiced aligning different interferometers. Starting with the Michelson interferometer, we learned to align an incident, a beam splitter, and two mirrors precisely enough to produce an interference pattern. Moving onto the Mach-Zender interferometer, the goal of producing an interference pattern became much harder to achieve with another beam splitter added into the setup. Experience with the Mach-Zender made us even more aware of how precise the geometry of the setup has to be in order to achieve a spatial overlap of the split beams while making sure they do not diverge too much. Once we worked enough on these two interferometers, we decided to practice alignment of the setup of this report.

We began practicing with a non-dangerous beam in place of the 405 nm one, as we had a lot of learning to do. Many factors had to be taken into account when trying to maximize the detected intensity and coincidences of the outgoing beams. We first left the downconverting crystal alone, and only worried about the alignment of the two detectors. A 405 nm beam perpendicularly incident on the beam splitter would produce two outgoing beams of 810 nm, each diverging at angle of 3 degrees from the normal on the other side of the crystal.



Down-converting crystal pictured above

Since these beams went radially outward from the crystal's position, it was important that our detectors were positioned somewhere on a circular arc (which depended on how far out they were and the 3 degree angles which the beams were moving along). The alignment on the circular arc was crucial to any detection at all.

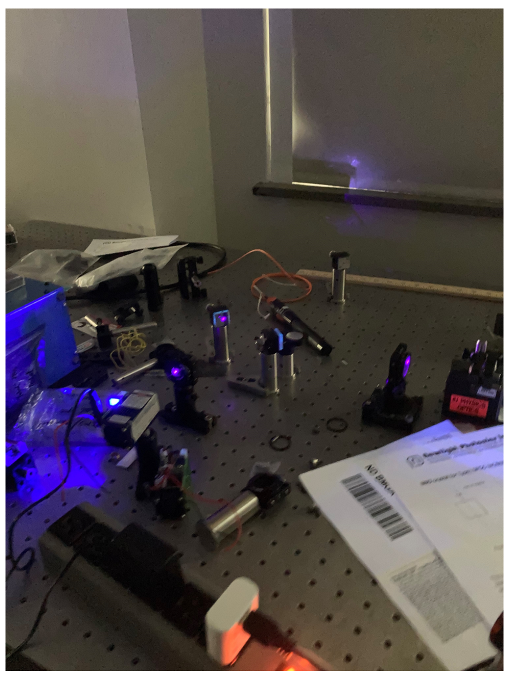


Detectors placed on circular arc, with polarizers in front (closer to the camera)

Once the detectors were made flush with the circular arc, we still had three degrees of freedom to worry about: the position of the detector about the arc, two directions of adjusting the angle which the detector faces: one vertical and one horizontal. Since we began with a non-dangerous laser, we aligned the detector's angles by placing a mirror in front of the detector's face, and made sure it reflected the beam right back to the down-converting crystal. Having the correct angles of the detectors' faces resulted in an immense increase in the detected intensities, and allowed us to start recording some coincidences as well. To start getting serious numbers for the detected coincidences, we had to start adjusting the positions of each detector along the circular arc. We learned that even if moving one detector along the circular arc made our coincidences grow, what was truly important was the collective positions of each detector. This meant we had to often go back and forth adjusting one detector or the other until we got serious increases in coincidence. As the order of magnitude of coincidences rose from 10^1 to 10^2 and so on, the adjustment of the detector position became much more subtle. Some final adjustments on the down-converter's angle brought our coincidence count up to 900-1,000 detected coincidences in a coin window of 10 nanoseconds and an overall dwell time of 2 seconds.

We then switched to our 405 nm laser and confirmed that our alignment worked for it as well. Also, introducing the polarizers in front of the detectors significantly cut down the detected intensity/coincidences, as expected since it only allows a certain polarization to pass through.

Inserting the half wave plate and quartz crystal before the down converter is necessary to ensure that we prepare the entangled state correctly. Our goal is to achieve a superposition state 1/sqrt(2)[|HH> + |VV>]for the two photons exiting the down converter. This state means that the output photons are either both horizontally polarized or both vertically polarized. The half wave plate's angle orientation is how we achieve this superposition. To check that the half wave plate is at the appropriate angle, we set polarizers both at 45 degrees (diagonal) in front of the detectors, check the coincidence count, and then repeat the process with the polarizers set at 315 degrees (anti-diagonal). The half-wave plate will be correctly oriented when the coincidence count for both polarizer orientations is as equal as possible. We also want to maximize this value which the coincidences are equal to, in a way such that when added together, they equal the total coincidence count when no polarizers were put in front of the detectors.This will mean that photons are equally likely to be detected as horizontal as they are vertical. The final issue then, is making sure that the phase of the superposition state is correctly prepared. This has to do with the fact we are measuring in a diagonal/anti-diagonal basis. For example: Even though 1/sqrt(2)[|HH> - |VV>]is an equal superposition of both horizontal and both vertical, when converting to the diagonal/anti-diagonal basis, the pair becomes a superposition of both orthogonal to each other (in two different ways). This will become an issue when taking data with the polarizers oriented at all sorts of angles and can lead to confusion in calculating averages. To remedy this, we must orient the quartz crystal correctly. Once the half-wave plate is correctly oriented from before, we can set the one detector's polarizer to diagonal and the other to anti-diagonal. If the quartz crystal is at the appropriate rotated angle, and our superposition state has the correct phase, we should expect zero coincidences detected with orthogonally oriented polarizers in front of the detectors.



Laser fires through half-wave plate, reflects off mirror, and goes through quartz crystal before entering down-converter.

Once we ensure all components are well aligned to produce the entangled state $|\boldsymbol{\Phi}^{+}\rangle = 1/\sqrt{2}\,[\ |H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2] = 1/\sqrt{2}\,[\ |D\rangle_1|D\rangle_2 + |A\rangle_1|A\rangle_2]$, we can begin to take data of the coincidences detected for numerous orientations of both polarizers.

Data/Analysis

θ_1 (degrees)	**θ_2 (degrees)**	**N_1 (counts)**	**N_2**	**N_3**
0	**22.5**	**1547.6**	**1264.6**	**1380.8**
45	**22.5**	**1441.2**	**1250.4**	**1272.8**
90	**22.5**	**236.6**	**204**	**203.8**
135	**22.5**	**409.8**	**374.8**	**370.2**
0	**67.5**	**231.4**	**230**	**228.8**
45	**67.5**	**1400.4**	**1345.2**	**1384**
90	**67.5**	**1536.2**	**1513**	**1535.6**
135	**67.5**	**336.2**	**305.8**	**314.8**
0	**112.5**	**246.6**	**260.4**	**264.6**
45	**112.5**	**343.8**	**326.4**	**324.8**
90	**112.5**	**1649.2**	**1660**	**1710**
135	**112.5**	**1555.6**	**1571.6**	**1568.6**
0	**157.5**	**1356.2**	**1371.4**	**1345.8**
45	**157.5**	**252.8**	**248.4**	**256.2**
90	**157.5**	**347.6**	**313.4**	**331.2**
135	**157.5**	**1522.4**	**1498.2**	**1508.6**

Following the equation for $E(\theta_1,\theta_2)$, we can set $\theta_1 = 0$, $\theta_2 = 22.5$ and calculate the first expectation value needed to find S using the values in the N_1(counts) column. If we set $\theta'_1 = 45$, $\theta'_2 = 67.5$, we can calculate the remaining three expectation values needed for finding the value of S from the same column. We then repeat this process for the two other N(counts) columns and average our three resulting S values. If our average S value is greater than 2, we have violated Bell's inequality. From our table:

S_1=2.666490

S_2=2.684604

S_3=2.688576

Average: S_{avg}=2.67989

Our S values indicate a significant violation of Bell's inequality, telling us that we do indeed have entangled photons and a minimal amount of mixed product states. We had a variety of random and systematic errors, though the quantitative measurement of these is rather difficult in practice. Qualitatively, random errors include excess light coming in from the windows and side room of the optics lab. Additionally, though there was a black curtain on the hallway door to avoid excess light as well as practice laser safety, there could have been extra light coming from that area as well. The down converter crystal is extremely sensitive and must be covered when not in use. However, when in use, it may wear down (though slowly.) Systematically, the coincidence number had an uncertainty on each measurement, evidenced by the fact that back-to-back measurements with no adjustments yielded slightly different results. Quantifying this was difficult, however, because the variation in values did not seem to follow any particular pattern. To remedy, we took 5 measure means for each $N(\theta_1,\theta_2)$, and averaged them together to record in the table. Finally, there is a nonzero uncertainty in the positioning of the polarization angles that is quantifiable. However, because Bell's inequality involved the coincidence counts- based on the particular polarizer orientations- it is difficult to find a way to use the polarizer uncertainties in a meaningful manner.

Conclusion

The delicacy of the setup was an invaluable learning experience for us as students, as it taught the importance of patience and carefulness in an experimental optics setting.
The reproducibility of the double photon entanglement setup seems to be quite high, as showing entanglement through a calculation of Bell's inequality ended up being the smallest task at hand. Much more surprising and significant is the concrete results we obtained, not too shy of the theoretical maximum value of 2.8! This experiment has shown us that nature is not realistic, objects can have two states at once, and most of all two objects' realities can be fundamentally intertwined with each other through quantum entanglement.