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Model Predictive Control
Exercice #1 - Group 2

We consider the linear discrete-time LTI defined by :

$$\begin{aligned} x_{i+1} &= Ax_i + Bu_i \\ y_i &= Cx_i \end{aligned} \tag{1}$$

with :

$$A = \begin{bmatrix} \frac{4}{3} & -\frac{2}{3} \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} -\frac{2}{3} & 1 \end{bmatrix} \tag{2}$$

We can already state that the **system is controlable and observable** since the Kalman matrix and the observability matrix both are full rank :

$$\mathcal{C} = \begin{bmatrix} 1 & \frac{4}{3} \\ 0 & 1 \end{bmatrix}, \quad \mathcal{O} = \begin{bmatrix} -2/3 & 1 \\ 1/9 & 4/3 \end{bmatrix} \tag{3}$$

Figure 1 shows the trajectory of the uncontrolled system (i.e $u = 0$). We see that the system is stable which is confirmed by the analysis of matrix A. The norm of λ_1 and λ_2, its eigenvalues, is inferior to 1:

$$\lambda_1 = 0.6667 + 0.4714i \text{ and } \lambda_2 = 0.6667 - 0.4714i \tag{4}$$

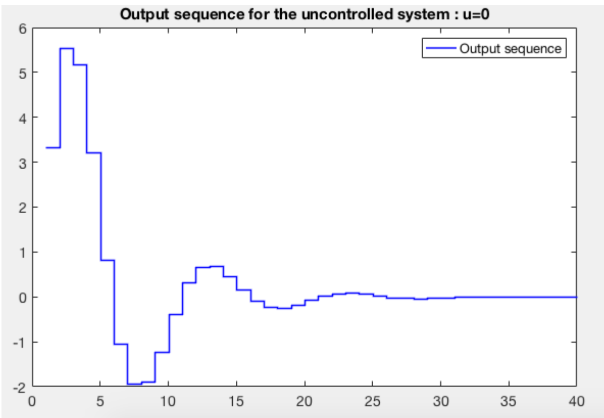

Figure 1: Uncontrolled system output sequence

We consider the LQR optimization problem. Given an horizon N, it writes :

$$u^* = \min_u \left\{ V(x,u) = \sum_{i=0}^{N-1} \left[x_i^T Q x_i + u_i^T R u_i \right] + x_N^T P_f x_N \right\} \tag{5}$$

with $Q = C'C + 0.001\mathbb{I}_2$, $P_f = Q$ and $R = 0.001$.

1 Exercice 1

We implemented the discrete-time Riccati recursion to compute the linear feedback law that solves the LQR optimization problem. The update rules are given by, assuming a horizon N :

$$\begin{aligned} &H_N = P_f \\ i = N-1 \dots 0 : \quad &\begin{cases} K_i = -(R + B^T H_{i+1} B) - B^T H_{i+1} A \\ H_i = Q + K_i^T R K_i + (A + BK_i)^T H_{i+1} (A + BK_i) \end{cases} \end{aligned} \tag{6}$$

2 Exercice 2

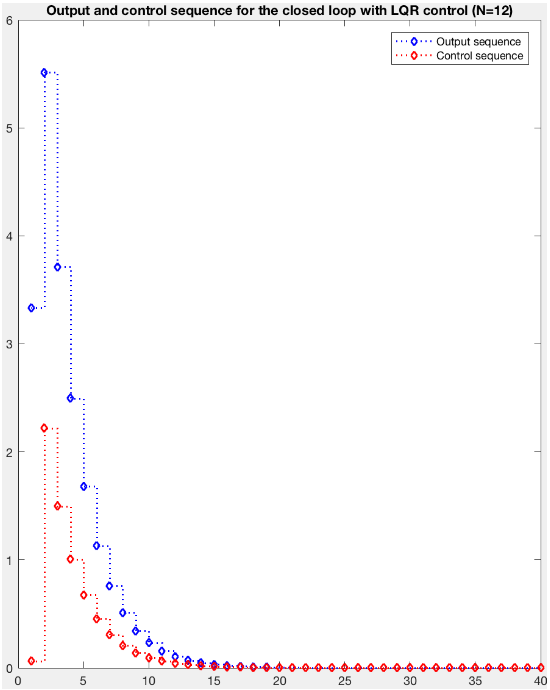

Figure 2: Output and control sequences for N=12

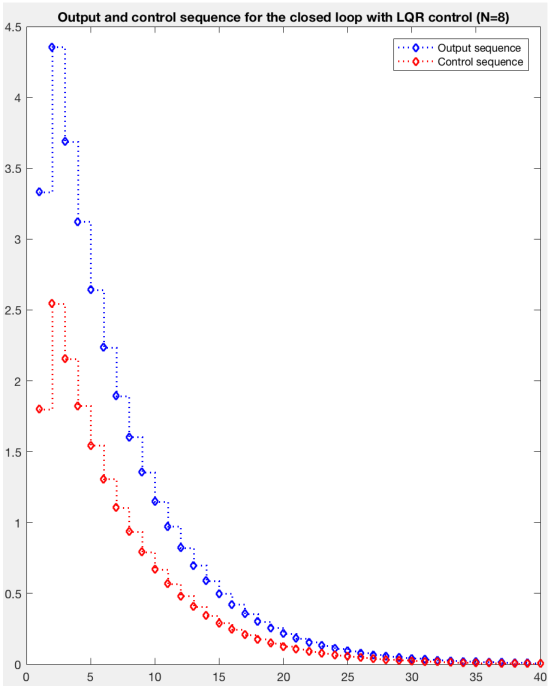

Figure 3: Output and control sequences for N=8

We then implemented the LQR control in a receding horizon fashion, starting at state $x_0 = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$. It is then easy to see that **the minimum horizon length that stabilizes the system is** $N^* = 7$. We give a few examples of trajectories on figure (2) and (3).

We show the result we obtain when plotting predicted trajectories for different values of N on figure (4) and (5). On these figures, dashed lines represent open loop trajectories on anticipated by the receding horizon controller while blues line represent real closed-loop trajectories. By increasing N, we see that our open-loop controller computes smoother and more realistic trajectories, as it is able to account for more futur dynamics. We indeed see that the greater the horizon is, the more the open-loop and closed-loop curve fit with one another, as future, possible unstable dynamic is accounted for.

Also, *as illustrated in the course*, there are no result proving that if the horizon N^* stabilizes the system, than for any horizon $N > N^*$, the system will be stabilized.

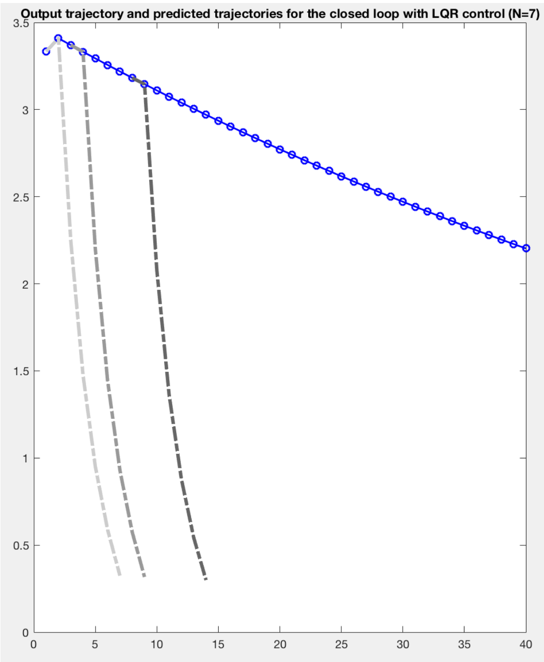

Figure 4: Predicted and actual output trajectorie for N=7

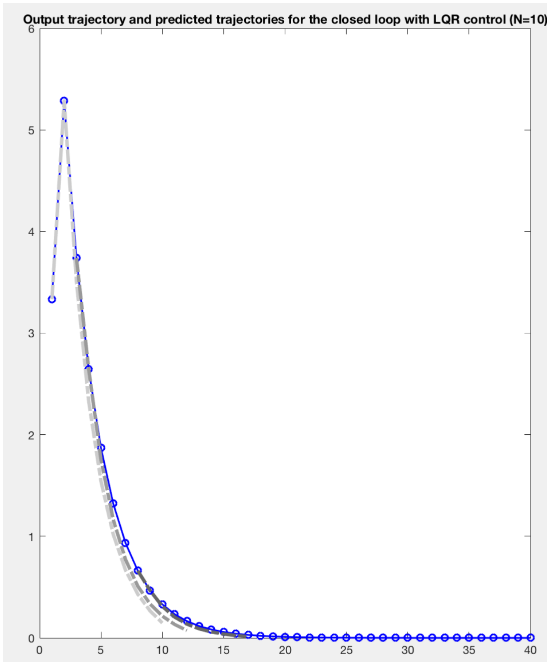

Figure 5: Predicted and actual output trajectorie for N=10