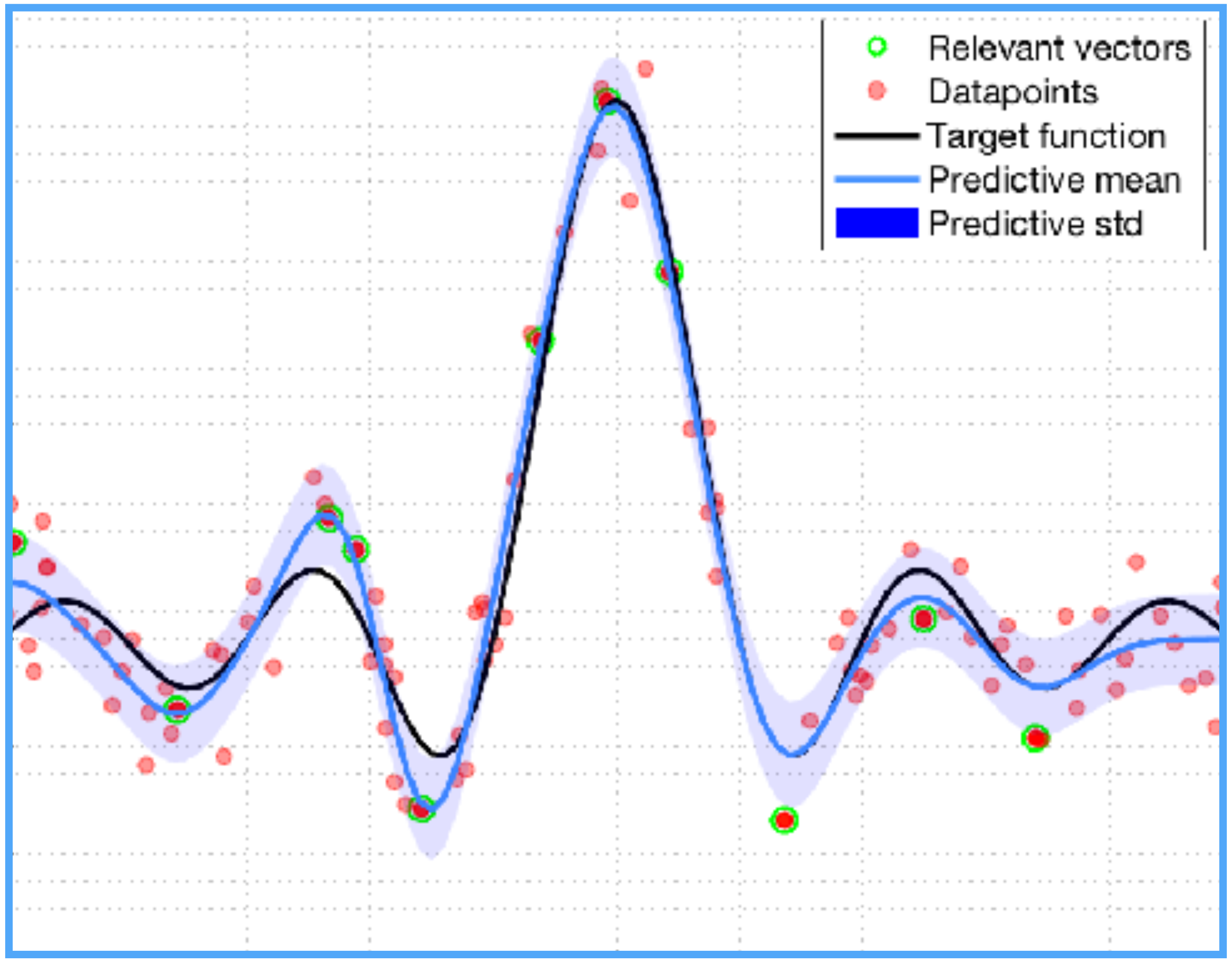
Support Vector Regression vs. Relevance Vector Regression

a sparsity / performance study



L.Faury G.Gallois-Montbrun H.Hendrikx

26/05/2017

Outline

- **Theoretical** reminders on both methods
- Datasets presentation & test runs
- Introduction to a **sparse-regression** metric, experimental justification
- Sparse-regression metric based **cross-validation**
- Comparison outcomes

Regression

Learn $f : \mathbb{R}^d \to \mathbb{R}$ thanks to a dataset $\{X, t\} \in (\mathbb{R}^d)^n \times \mathbb{R}^n$

Assuming a Gaussian **conditional p.d.f** around a linear transformation of features :

$$p(t \,|\, x, w) = \mathcal{N}(t \,| w^T \phi(x), \beta^{-1})$$

the maximum-likelihood estimator (MLE) writes :

$$\begin{aligned} \hat{w} &= \text{argmax}_w \, p(t \,|\, X, w) \\ &= \text{argmin}_w \, \frac{1}{2} \sum_{i=1}^{n} \|w^t \phi(x) - t\|^2 \end{aligned}$$

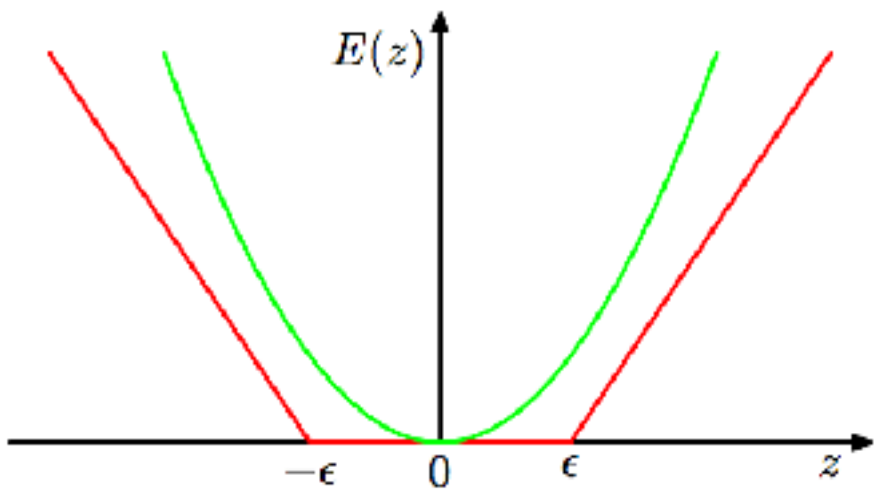
Support Vector Regression

- Introduce the ε-insensitive loss-function.

  Equivalent to the QP program :

$$min_w \frac{C}{n} \sum_n (\xi_n + \hat{\xi}_n) + \frac{1}{2}\|w\|^2$$

$$\text{s.t} \quad \begin{cases} \xi, \hat{\xi} \geq 0 \\ w^T \phi(x_n) + \xi_n + \varepsilon \geq t_n \\ w^T \phi(x_n) - \hat{\xi}_n - \varepsilon \leq t_n \end{cases}$$



Source : Bishop, Pattern Recognition and Machine Learning (2006)

- Inactive constraints leads to a sparse model. Only points outside the ε-tube are used for predictions :

$$y(x) = \sum_{n \in \mathcal{S}} (a_n - \hat{a}_n) k(x, x_n) \longrightarrow \text{Posterior } \textbf{decision}$$

Relevance Vector Regression

- Provide the predictor with a Gaussian prior : $w \sim \prod_i \mathcal{N}(w_i \,|\, 0, \alpha_i^{-1})$

$$y(x) = \sum_n w_n k(x, x_n)$$

- Use **type-2 likelihood** *(evidence approximation)* to determine :

$$(\alpha^*, \beta^*) = \text{argmax}_{\alpha,\beta} \left[p(t \,|\, \alpha, \beta) = \int_w p(t \,|\, w, \beta) p(w \,|\, \alpha) \right]$$

- Automatic Relevance Detection : drives some α_i to $+\infty$ (sparse model)

- Compute posterior and **predictive distribution**